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S. Bosi et N.-S. Pham

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Stefano Bosi
EPEE (University of Evry)

Ngoc-Sang Pham[†]
LEM (University of Lille 3) and EPEE (University of Evry)

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Abstract

We study the interplay between taxation, bubble formation and economic growth. A rational bubble may be beneficial when growth is fuelled by public investment (or R&D externalities) and the government levies taxes on bubble returns to finance this investment. Our main result challenges the conventional view about the negative effect of bubbles in endogenous growth (Grossman and Yanagawa, 1993).

Keywords: taxation on financial revenue, public R&D, endogenous growth.
JEL Classifications: E44, H23, O30.

1 Introduction

A pure bubble arises when the equilibrium price of an asset bringing no dividends is strictly positive.[1] In the mid of Eighties, Tirole (1985) found out that a pure bubble may emerge in OLG economies under capital overaccumulation. Proposition 2 of his influential paper pointed out that the asymptotically bubbly equilibrium is efficient while any asymptotically bubbleless equilibrium is not.[2] Grossman and Yanagawa (1993) extended Tirole (1985) with externalities from physical capital, a well-known engine of endogenous growth, while showing that the existence of a bubble may delay this growth and worsen the welfare of any generation.

[†]Corresponding author. Email: pns.pham@gmail.com. Address: University of Lille 3, UFR MIME; 3, Rue du Barreau, 59650 Villeneuve-d'Ascq, France.
[1]The reader is referred to Miao (2014) for an introduction to bubbles in infinite-horizon models and to Brunnermeier and Oehmke (2013) for a survey on bubbles in OLG models with asymmetric information or heterogeneous beliefs.
[2]An allocation is efficient if it is not possible to improve the welfare of all generations and strictly improve for at least one of them.

Our paper reconsiders these results in an OLG model with fiscal policy. We study the impact of taxes on bubble dynamics and endogenous growth. Differently from Grossman and Yanagawa (1993), the growth fuel is the government spending in R&D (in the spirit of Barro (1990)). R&D investments are financed through the taxes not only on labor and capital income but also on returns on the bubble asset. The novelty of our paper rests on this additional fiscal instrument and its consequences. A comparison between these different taxes is also of interest.

First, we find that there is room for bubbles if and only if the aftertax interest rate in the bubbleless equilibrium is lower than the population growth rate. Therefore, bubbles appear if the tax rate on capital income is sufficiently high while they are ruled out if the tax rates on labor income or on the bubble return are sufficiently large. Moreover, with some specifications, we provide a full characterization of equilibrium dynamics, that is a global analysis of capital and bubble dynamics. The size of the bubble is explicitly computed.

Second, we figure out the cases where bubbles may harm or promote economic growth. If bubbles do not exist and, *de facto*, the government is prevented from using bubble taxes, while it is allowed to play only with low tax rates on capital and labor income, R&D activities turn out to be underfunded with detrimental effects on economic growth. Conversely, positive bubbles may ensure additional fiscal revenues and R&D expenditures sufficient to trigger a beneficial self-sustained growth. This result challenges the conventional view supported by Grossman and Yanagawa (1993) about a negative effect of bubbles in endogenous growth. However, under a higher tax rate on capital income, bubbles dampen the economic growth: thus, we recover the main conclusion by Grossman and Yanagawa (1993).

Our paper contributes to the literature on the positive effects of bubbles. Among others, Farhi and Tirole (2012), Martin and Ventura (2012) consider OLG models and point out that, under financial market imperfections, bubbles may be beneficial through the reallocation of funds from less to more productive investments in the private sector. Hirano and Yanagawa (2015) study an infinite-horizon model and show that the effects of asset bubbles depend on financial market conditions: if the pledgeability level is relatively low (high), bubbles enhance (decrease) the economic growth rate. Hirano et al. (2015) develop Hirano and Yanagawa (2015) to take in account the connection between bailout policies and bubbles.

2   Framework

Consider a two-period OLG model of rational bubbles in the spirit of Tirole (1985) and Weil (1987).

A representative firm maximizes the profit under a complete capital depreciation: $F_t(K_t, L_t) - R_t K_t - w_t L_t$, where K_t and L_t denote the aggregate capital and the labor forces, while R_t and w_t represent the return on capital and the wage rate. For simplicity, the production function is Cobb-Douglas:

$F_t(K_t, L_t) \equiv A_t K_t^{\alpha} L_t^{1-\alpha}$. Profit maximization yields

$$R_t = \alpha A_t k_t^{\alpha-1} \text{ and } w_t = (1-\alpha) A_t k_t^{\alpha-1} \tag{1}$$

where $k_t \equiv K_t/L_t$ denotes the capital intensity.

At period t, N_t individuals are born. Each consumer-worker lives two periods. When young, she supplies one unit of labor, earns a labor income taxed at a constant rate τ, consumes c_t and saves through capital s_t and a long-lived asset a_t. When old, she consumes d_{t+1}, that is the gross returns on capital and financial asset (which brings no dividend). These returns are taxed at the constant rates τ_k and τ_b. The price of consumption good is normalized to one while q_t denotes the price of asset in consumption units at time t. Preferences are rationalized by a separable intertemporal utility function $\ln c_t + \beta \ln d_{t+1}$, where β represents the discount rate. The agent faces two budget constraints (one per period):

$$c_t + s_t + q_t a_t \leq (1-\tau) w_t$$
$$d_{t+1} \leq (1-\tau_k) R_{t+1} s_t + (1-\tau_b) q_{t+1} a_t$$

to maximizes her utility with respect to s_t, a_t, c_t and d_{t+1}.

Solving the program, we find the sharing between consumption and savings

$$c_t = \frac{1}{1+\beta} (1-\tau) w_t \tag{2}$$

$$s_t + q_t a_t = \frac{\beta}{1+\beta} (1-\tau) w_t \tag{3}$$

jointly with the (equilibrium) no-arbitrage condition

$$(1-\tau_k) R_{t+1} q_t = (1-\tau_b) q_{t+1} \tag{4}$$

and the budget constraints, now binding.

The government levies taxes on labor income and gross returns on capital and the asset to finance public investment good:

$$G_t = \tau w_t + \tau_k \frac{R_t s_{t-1}}{n} + \tau_b \frac{q_t a_{t-1}}{n} \tag{5}$$

where G_t is the public investment good as a pure productive externality and $n \equiv N_{t+1}/N_t$ denotes the population growth rate, supposed to be constant.

We focus on a simple model of public investment (R&D, for instance) in the spirit of Barro (1990): $A_t = \theta G_t^{1-\alpha}$ for any t. Thus, G_t affects the TFP, the product and the revenues from labor, capital and financial speculation. These revenues are supposed to affect in turn, within the same period, the tax receipt and the public spending G_t at the end. This functional specification promotes endogenous growth dynamics.

Definition 1. *1. An equilibrium is a positive sequence*

$$(q_t, R_t, w_t, c_t, d_{t+1}, a_t, s_t, K_{t+1}, L_t, G_t)_{t \geq 0}$$

satisfying (1), (2), (3), (4), (5), the market clearing conditions:

$$asset : N_{t+1}a_{t+1} = N_t a_t$$
$$physical \ capital : K_{t+1} = N_t s_t$$
$$labor : L_t = N_t$$
$$consumption \ good : s_t + c_t + d_t/n + G_t = f_t(k_t),$$

and budget constraints are binding for any $t \geq 0$.

2. *If $q_0 > 0$, the equilibrium is said to be bubbly, otherwise it is said to be bubbleless.*[3]

Equilibria in the asset and capital markets write $na_{t+1} = a_t$ and $s_t = nk_{t+1}$. The asset volume shrinks exponentially: $a_t = a_0 n^{-t}$. Let $b_t \equiv q_t a_t$ denote the value of financial asset. Therefore, the equilibrium system writes:

$$nk_{t+1} + b_t = \sigma A_t k_t^\alpha \tag{6}$$

$$b_{t+1} = \frac{1 - \tau_k}{1 - \tau_b} \frac{\alpha A_{t+1} k_{t+1}^{\alpha-1}}{n} b_t \tag{7}$$

$$A_t = \theta G_t^{1-\alpha}, \quad G_t = \tau w_t + \tau_k R_t k_t + \tau_b b_t \tag{8}$$

where σ is the propensity to save in the bubbleless equilibrium (i.e., when $b_t = 0$):

$$\sigma \equiv \frac{s_t}{f_t(k_t)} = (1 - \tau)(1 - \alpha)\frac{\beta}{1 + \beta}$$

We see that a positive sequence $(q_t, R_t, w_t, c_t, d_{t+1}, a_t, s_t, K_{t+1}, L_t, G_t)_{t \geq 0}$ is driven by (1), (6), (7) and (8). In short, $(k_{t+1}, b_t)_{t \geq 0}$ will denote an equilibrium sequence.

3 Equilibrium analysis

Our model bridges two theories: rational bubbles (à la Tirole (1985)) and endogenous growth (à la Barro (1990)). The main proposition rests on the balanced growth rates with and without bubbles. More precisely, we introduce the growth factors corresponding to the cases without bubble and with maximal bubble:

$$\rho_0 \equiv \sigma\frac{\theta}{n}\left(\theta\left[\alpha\tau_k + (1-\alpha)\tau\right]\right)^{\frac{1-\alpha}{\alpha}}$$

$$\rho_1 \equiv \frac{\sigma}{\gamma}\frac{\theta}{n}\left(\theta\left[\alpha\tau_k + (1-\alpha)\tau + \sigma\tau_b\frac{\gamma-1}{\gamma}\right]\right)^{\frac{1-\alpha}{\alpha}}$$

where

$$\gamma \equiv \frac{\sigma}{\alpha}\frac{1 - \tau_b}{1 - \tau_k}$$

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[3]We notice that $q_0 > 0$ iff $q_t > 0$ for any t.

captures how bubbly the equilibrium is, and

$$\bar{b}_0 \equiv \frac{\gamma - 1}{\gamma} \frac{\beta}{1 + \beta} (1 - \tau) w_0$$

is the maximal bubble in the sense of Tirole (1985).

All is clarified by the following result which provides a complete characterization of all equilibria.

Proposition 1 (global dynamics). *1. If $\gamma \leq 1$, there are no bubble (i.e., $b_t = 0$ for any t). The equilibrium is unique and given by $k_t = \rho_0^t k_0$ for any $t \geq 0$.*

2. If $\gamma > 1$, any equilibrium must satisfy $b_0 \leq \bar{b}_0$. And there is equilibrium indeterminacy: any sequence $(k_{t+1}, b_t)_{t \geq 0}$ driven by (1), (6), (7) and (8) with $b_0 \in [0, \bar{b}_0]$ is an equilibrium. Moreover,

> *(a) If $b_0 = 0$, then the equilibrium is bubbleless and still determined by $k_t = \rho_0^t k_0$ for any $t \geq 0$.*
>
> *(b) If $b_0 \in (0, \bar{b}_0)$, then the equilibrium is bubbly but bubble is asymptotically negligible, in the sense that $\lim_{t \to \infty} (b_t/k_{t+1}) = 0$. Moreover, $\lim_{t \to \infty} (k_{t+1}/k_t) = \rho_0$.*
>
> *(c) If $b_0 = \bar{b}_0$, then the equilibrium is bubbly. Moreover, the endogenous BGP (Balanced Growth Path) is given by $b_t = (\gamma - 1)nk_{t+1}$ with $k_t = \rho_1^t k_0$ for any $t \geq 0$.*

Proof. See Appendix 4. □

Definition 2. *We call $\bar{b}_0$ the size of bubble.*[4]

3.1   Fiscal policy and the existence of bubbles

According to Proposition 1, a bubble exists if and only if $\gamma > 1$. We see that both γ and $\bar{b}_0$ are increasing in τ_k but decreasing in τ and τ_b. So, bubbles are more likely to appear when tax rates on asset bubble and labor income are low and/or the tax rate on capital income is high. The intuition is straightforward: when the capital income tax increases, consumers invest less in capital and more in the bubble (portfolio substitution effect). By contrast, when the bubble income tax τ_b is high enough, agents no longer invest in the asset bubble and, then, bubbles are ruled out.

We also observe that capital taxation promotes instability (in the sense of equilibrium multiplicity), while taxation on financial assets and labor income promotes stability (in the sense of equilibrium uniqueness).

It is easy to see that $\gamma > 1$ is equivalent to

$$\gamma = \frac{1 - \tau_b}{1 - \tau_k} \frac{n}{f'(k^*)} > 1 \tag{9}$$

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[4]Tirole (1985) calls this threshold the *maximal feasible bubble.*

Therefore when $\tau_b = \tau_k = 0$, we recover part (b) of Proposition 1 in Tirole (1985). Tirole (1985) works with general production and utility functions, and proves the existence of a threshold $\bar{b}_0$ without computing it. Our specific production and utility functions allow us to provide the explicit expression for $\bar{b}_0$ in terms of the fundamental parameters and, hence, to compute the effects of these parameters on the bubble. To the best of our knowledge, this outcome is also new in the theoretical literature.

We finally observe that, under a tax distortion ($\tau_b \neq \tau_k$), the overaccumulation inequality $n > f'(k^*)$ is no longer sufficient to ensure the existence of a bubbly equilibrium. A similar result holds in Kunieka (2011) with a significant difference: we provide a global analysis (of transition dynamics) instead of a steady state analysis.

3.2 Fiscal policy, bubbles and growth

Focus on the second case of Proposition 1 ($\gamma > 1$) and study the relationship between ρ_0 and ρ_1 (the growth factors when bubbles are respectively asymptotically negligible and non-negligible).

Noticing that γ does not depend on θ, we find an immediate consequence.

Corollary 1. *The growth rates ρ_0 and ρ_1 are increasing in the government's efficiency θ.*

ρ_0 depend on the fiscal pair (τ, τ_k) and ρ_1 on the triplet (τ, τ_b, τ_k). The functions $\rho_0(\tau, \tau_k)$ and $\rho_1(\tau, \tau_b, \tau_k)$ are continuous in (τ, τ_b, τ_k). Moreover, $\rho_0(0,0) < \rho_1(0,0,\tau_b)$ with $\tau_b > 0$. This leads to a comparative proposition.

Proposition 2. *If*
$$(1 - \tau_b) \frac{1 - \alpha}{\alpha} \frac{\beta}{1 + \beta} > 1$$

there exist $\bar{\tau} > 0$, $\bar{\tau}_k > 0$ and $\underline{\tau}_b > 0$ such that, for any triplet (τ, τ_k, τ_b) satisfying $\tau \in (0, \bar{\tau})$, $\tau_k \in (0, \bar{\tau}_k)$ and

$$\underline{\tau}_b < \tau_b < \bar{\tau}_b \equiv 1 - \frac{\alpha}{1 - \alpha} \frac{1 + \beta}{\beta} \tag{10}$$

we have $\gamma > 1$ and $\rho_1 > \rho_0$: the growth rate with asymptotically non-negligible bubble exceeds the rate with asymptotically negligible bubble.

Condition $\tau_b < \bar{\tau}_b$ ensures $\gamma > 1$ when τ_k and τ are low enough, which implies in turn that a bubbly equilibrium exists.

Proposition 2 deserves some economic intuitions. Since the R&D process is financed by taxes, the growth rate depends on the fiscal policy. When the labor and capital income taxes are low, the key instrument becomes the tax rate τ_b on the asset bubble. In this case, when this tax rate is sufficiently high ($\tau_b > \underline{\tau}_b$), the growth factor ρ_1 (with asymptotically non-negligible bubble) turns out to be higher than the growth factor ρ_0 (with asymptotically negligible bubble).

Our findings suggest that the existence of a bubble (such as a housing bubble) may be beneficial to economic growth. This point of view challenges the one in Grossman and Yanagawa (1993) where it is shown that an asset bubble absorbs the savings of a market economy experiencing underaccumulation (because of positive productive externalities), and, in the end, makes the situation worse.

Consider eventually the case where the government applies a higher tax rate on capital income. It is easy to check that $\rho_0 > \rho_1$ if and only if

$$\gamma^{\frac{\alpha}{1-\alpha}} - 1 > \frac{\gamma - 1}{\gamma} \frac{\sigma \tau_b}{\alpha \tau_k + (1 - \alpha) \tau} \tag{11}$$

Ceteris paribus, γ tends to infinity when τ_k tends to one. In the limit, condition (11) is satisfied, or, equivalently, $\rho_0 > \rho_1$.

Proposition 3. *Fix all the parameters except τ_k. Then, $\rho_0 > \rho_1$ when τ_k exceeds a threshold.*

Under the condition in Proposition 3, the growth rate with asymptotically negligible bubble is higher than the one with asymptotically non-negligible bubble. In this case, we recover the conventional result by Grossman and Yanagawa (1993).

4   Appendix: Proof of Proposition 1

We have

$$G_t = \tau w_t + \tau_k R_t k_t + \tau_b b_t = \tau (1 - \alpha) \theta G_t^{1-\alpha} k_t^{\alpha} + \tau_k \alpha \theta G_t^{1-\alpha} k_t^{\alpha} + \tau_b b_t$$

$$= [\alpha \tau_k + (1 - \alpha) \tau] \theta G_t^{1-\alpha} k_t^{\alpha} + \tau_b \frac{1 - \tau_k}{1 - \tau_b} \frac{\alpha \theta G_t^{1-\alpha} k_t^{\alpha-1}}{n} b_{t-1}$$

and, then,

$$\left(\frac{G_t}{k_t}\right)^{\alpha} = \theta [\alpha \tau_k + (1 - \alpha) \tau] + \tau_b \frac{1 - \tau_k}{1 - \tau_b} \frac{\alpha \theta}{n} \frac{b_{t-1}}{k_t}$$

Consider two cases.

Case 1: $\gamma \leq 1$. There is no bubble. Indeed, a bubble exists iff $b_t > 0$ for any t. In this case, by combining (6) and (8), we get

$$x_{t+1} = \gamma x_t - 1 \tag{12}$$

where $x_t \equiv n k_{t+1}/b_t$. Since $\gamma \leq 1$, x_t becomes negative soon or later: this leads to a contradiction.

The equilibrium system becomes

$$nk_{t+1} = \sigma A_t k_t^{\alpha}$$

$$A_t = \theta G_t^{1-\alpha} = \theta \left([\alpha \tau_k + (1 - \alpha) \tau] \theta\right)^{\frac{1-\alpha}{\alpha}} k_t^{1-\alpha}$$

which implies $k_{t+1} = \rho_0 k_t$.

Case 2: $\gamma > 1$. Consider only the case where a bubble exists, i.e., $b_t > 0$ for any t. We have $x_{t+1} = \gamma x_t - 1$ and, hence,

$$x_t = \gamma^t x_0 - \frac{1 - \gamma^t}{1 - \gamma} = \frac{[(\gamma - 1) x_0 - 1] \gamma^t + 1}{\gamma - 1} \tag{13}$$

A positive solution (x_t) exists iff $x_0 \geq 1/(\gamma - 1)$ or, equivalently, iff

$$b_0 \leq (\gamma - 1) n k_1 = (\gamma - 1) \left[\frac{\beta}{1 + \beta} (1 - \tau) w_0 - b_0 \right]$$

Solving this inequality for b_0, we find $b_0 \leq \bar{b}_0$. Conversely, it is easy to see that when $b_0 \leq \bar{b}_0$, the sequence $(b_t, k_{t+1})_{t \geq 0}$, determined by (1), (6), (7) and (8), is positive. Hence, it is an equilibrium.

Focus on two sub-cases.

Case 2.b: $b_0 \in (0, \bar{b}_0)$. The equilibrium is bubbly. According to (12) and $\gamma > 1$, we have $\lim_{t \to \infty} (k_{t+1}/b_t) = \infty$ and, thus, $\lim_{t \to \infty} (G_t/k_t)^\alpha = \theta [\alpha \tau_k + (1 - \alpha) \tau]$. Using (6) and $A_t = \theta G_t^{1-\alpha}$, we obtain

$$n \frac{k_{t+1}}{k_t} \left(1 + \frac{b_t}{n k_{t+1}} \right) = \sigma \theta \left(\frac{G_t}{k_t} \right)^{1-\alpha}$$

Therefore, $\lim_{t \to \infty} (k_{t+1}/k_t) = \rho_0$.

Case 2.c: $b_0 = \bar{b}_0$. The equilibrium is bubbly. According to (12), we find $n k_{t+1}/b_t = 1/(\gamma - 1)$ for any $t \geq 0$. Therefore,

$$\left(\frac{G_t}{k_t} \right)^\alpha = \theta \left[\alpha \tau_k + (1 - \alpha) \tau + \sigma \tau_b \frac{\gamma - 1}{\gamma} \right] \tag{14}$$

By combining (6) and $A_t = \theta G_t^{1-\alpha}$, we have

$$\frac{k_{t+1}}{k_t} = \frac{\sigma}{\gamma} \frac{\theta}{n} \left(\frac{G_t}{k_t} \right)^{1-\alpha}.$$

and, then by using (14), we obtain that $(k_{t+1}/k_t) = \rho_1$ for any $t \geq 0$.

References

Barro, J. B., 1990. *Government spending in a simple model of endogenous growth*. The Journal of Political Economy, vol. 98, no 5, pt 2.

Brunnermeier, M.K. and Oehmke, M., 2009. *Bubbles, financial crises, and systemic risk*, in: Constantinides, G. M., Harris, M., and Stulz, R.M. (Eds.), Handbook of the Economics of Finance, vol 2. Elsevier.

Farhi E. and Tirole J., 2012. *Bubbly liquidity*, Review of Economic Studies, vol. 79, 678-706.

Grossman G. and Yanagawa N., 1993. *Asset bubbles and endogenous growth.* Journal of Monetary Economics, vol. 31, 3-19.

Hirano T., Inaba M., and Yanagawa N., 2015. *Asset bubbles and bailouts.* Journal of Monetary Economics, vol. 76 Supplement, S71-S89.

Hirano T., and Yanagawa N., 2015. *Asset Bubbles, Endogenous Growth, and Financial Frictions.* Link: $http://dx.doi.org/10.2139/ssrn.2432788$.

Kunieka S., 2011. *Speculative Bubbles and Tax Policy*, Hitotsubashi University working paper series.

Martin A. and Ventura J., 2012. *Economic growth with bubbles*, American Economic Review, vol. 102, 3033 - 3058.

Miao J., 2014. *Introduction to economic theory of bubbles.* Journal of Mathematical Economics, vol. 53, 130-136.

Tirole J., 1985. *Asset bubbles and overlapping generations.* Econometrica, vol. 53, 1499-1528.

Weil F., 1987. *Confidence and the real value of money in an overlapping generations economy.* Quarterly Journal of Economics, vol. 102, 1-22.