Life-Cycle of Goods and Intergenerational Externalities

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Abstract

This paper analyses recycling optimal policy in a general equilibrium model with intergenerational environmental externalities. We consider a generalized life-cycle framework: households live two periods (young and old) and goods live two periods (virgin and recycled). First, we show that the life-cycle assumption for agents and goods requires a definition of a green modified golden rule suitable for intergenerational externalities and recycling problems. Indeed, as recycling decreases waste pollution in the future, it acts as an incentive to increase consumption and waste in the present. Secondly, we show that the recycling sector is a tool for sustainable development. Indeed, if the recycling technology is efficient enough, recycling activities offset the environmental externalities. In this case, the optimal level of the Pigovian tax rate on pollution is zero.

Keywords: Environmental externalities, Overlapping generations model, Recycling, Tax, Waste.

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1 Introduction

Waste recycling activity corresponds to an industrial reality. Recycling is a growing industry for a number of raw materials. Some countries achieve record numbers in the recycling of glass (96% in Switzerland, 92% in Sweden, 88% in Germany and in Belgium). Moreover, the world production of recycled aluminium is greater than the production of virgin aluminium. In addition to industrial concerns, recycling is a tool for a sustainable development. Hence, governments make use of many aids in order to develop the recycling sectors: free sorting by the households, European laws on eco-packaging and eco-taxation on packaging.

The objective of this article is to analyze the trade-off between industrial concerns and environmental benefits in order to determine whether the development of recycling sectors could be a solution to internalize intergenerational environmental externalities.

Economic literature analyses recycling from three theoretical points of view. First, recycling is integrated into the analysis of industrial organization; following the Alcoa case, studies have analyzed the erosion of monopoly power by competitive firms of recycled products (Gaskins, 1974, Swan 1980, Suslow 1986, Grant 1999). These works examine the industrial concentration problem and do not deal with the environmental concerns: the development of a competitive fringe of recyclers reduces the monopoly power of Alcoa. Hence, these studies propose public aid for the competitive fringe.

Secondly, natural resource economics studies point out the usefulness of recycling for economies facing a decrease in the availability of resources; recycling postpones the extraction of mining resources and reduces the intensity of forestry exploitation (Mäler, 1974, Dasgupta and Heal, 1979). Again, recycling generates social benefits and has to be subsidized.

Thirdly, environmental economics analyses recycling as an instrument to reduce environmental externalities. Lusky (1976) considers recycling via the problem of domestic waste. Fullerton and Wu (1998) present a general equilibrium model in which firms produce a good using raw and recycled materials, while choosing their level of packaging and recycling. These general or partial equilibrium models consider recycling as a means for reducing environmental externalities arising from the disposal of waste. They all suggest economic instruments such as subsidizing for its social benefits of recycling.
Therefore, according to these studies, government should help the development of recycling activities. But, the input of the recycling sector is waste for which the price determination might be non-competitive.¹ For many kinds of waste, there is a market failure problem, the waste price is non competitive but fixed by institutional conventions. Consequently, on the one hand, private markets do not generate exactly the optimal amount of waste; on the other hand, private markets fail to recycle an optimal amount of material (Porter, 2002). These characteristics are not considered by these studies.

This article takes into account three characteristics of the recycling sector in a general equilibrium framework. We consider (i) recycling as an instrument for reducing an environmental externality; (ii) inputs are waste coming from the production of the preceding period; (iii) waste prices could be competitive or not.

The framework is an overlapping generations model. Indeed, environmental decisions have an impact on the welfare of both current and future generations, since environmental quality is a public good that different generations share. These intergenerational issues on environmental externalities have already been widely studied in the economic literature but the overlapping generations approach, suggested by Solow (1974), (1986), began only to be used for analyzing environmental policies concerning polluting goods by John and Pecchenino (1994), (1997), John et alii (1995), Howarth and Norgaard (1995). The main result of these studies is that environmental policy implies such a welfare loss for the older generations experiencing the fiscal reform that its implementation cannot be desired as the generation which would decide it would also bear the heaviest burden. Moreover, considering egoistic generations increases the intertemporal externalities, consequently the social planner’s objectives are harder to achieve in a decentralized economy, requiring further economic instruments.

In the present paper, we argue that the development of recovery and recycling sectors could be one of the instruments of internalization of the intergenerational externalities. This paper considers both the recycling sector facing waste market failures and intergenerational issues of environmental policies: for these reasons it differs from the previous literature.

We suppose that production of virgin goods emits pollution. One way to internalize this externality would be to tax virgin good producers. The use of such a Pigovian tax would
lead firms to produce a smaller quantity. Overall production (and consumption) decrease, in order to maintain the quality of the environment. We show that the government can correct market failure by inducing recycled good producers to produce a larger quantity. In this case, the appropriate response is a subsidy. That instrument (subsidizing recycled goods, rather than taxing virgin goods) leads to a larger overall quantity of production (virgin and recycled). Thanks to recycling, overall production (and consumption) do not decrease, while keeping the quality of the environment unchanged.

In particular, we will show that the government must subsidize the recycling sector if it does not demand the whole of the waste available. Inversely, one could imagine a recycling tax if costs linked to the recycling activity or the price for waste were low and did not reflect the availability of the resource to be recovered.

Actually, public incentives to sort waste have reduced the marginal cost of waste recovering (for example, European Directive of 2002 concerning electrical and electronic waste products). However, public incentives are not always sufficient, depending on different countries and different kinds of waste, to increase the level of recycling. Our model suggests that there is a need to increase public assistance for recycling such waste (in order to diminish the cost of recovery and recycling), whereas these subsidies are unnecessary in the sectors where recovery and recycling are profitable.

The paper is organized as follows. Section 2 presents the model and characterizes the competitive equilibrium. Section 3 describes the social optimum. Section 4 presents the optimal level of the tax rates to decentralize the optimum. The last section concludes.

2 The Model

This section presents the behavior of households, the production of virgin goods and recycled goods, the evolution of environmental quality, government budget constraint and finally the competitive equilibrium. The framework is an overlapping generations model, which allows a simultaneous taking into account of the life-cycles of agents and goods.
2.1 Households

Our economy relies on standard assumptions of the overlapping generations framework. When young \((y)\), the representative agent born in period \(t\) supplies one unit of labour, earns a wage \(w_t\) and a profit \(\pi_t^{R,y}\). Income is used for consumption expenditures \(c_t^y\) and saving \(s_t\). When old \((o)\), the agent consumes \((c_{t+1}^o)\) his savings and the interest (the real interest rate is \(r_{t+1}\)). \(T^i_t\) \((i = y, o)\) is a lump-sum tax. The intertemporal discount factor is \(\beta \in [0, 1]\). Let \(\theta \in [0, 1]\) be the weight of the environmental quality in the welfare function. The quality of the environment is an externality for agents. The intertemporal utility function of an agent born at \(t\) is:

\[
U(c_t^y, Q_t, c_{t+1}^o, Q_{t+1}) = u(c_t^y) + \theta z(Q_t) + \beta [u(c_{t+1}^o) + \theta z(Q_{t+1})]
\]

Assume that utility functions \(u\) and \(z\) exhibit the usual properties: they are increasing in their argument, strictly concave, homothetic and satisfy the Inada conditions. For each period, the budget constraints are:

\[
\begin{align*}
&w_t - T_t^y + \pi_t^R = c_t^y + s_t \\
&c_{t+1}^o = (1 + r_{t+1}) s_t + T_{t+1}^o
\end{align*}
\]

The agent born at \(t\) maximizes his lifetime utility subject to the budget constraints. The first order conditions determine the optimal consumption path of the household:

\[
u_{c^y}' - \beta (1 + r_{t+1}) u_{c^o}' = 0 \tag{1}
\]

2.2 Virgin Production

The virgin production sector consists of one competitive representative firm, characterized by a production function \(f(.)\) which has constant returns to scale and satisfies the Inada conditions. Production per capita \((y^V)\) is a function of capital per capita \((k)\):

\[
y_t^V = f(k_t) \quad \text{with} \quad f'(.) > 0; \quad f''(.) \leq 0
\]
Assuming full depreciation and taking the output price as *numeraire*, the maximization problem of the representative firm is:

$$Max_{k_t} \pi_t = (1 - \tau_t) f(k_t) - w_t - (1 + r_t) k_t$$

where $\tau$ is a tax on virgin production. Since markets are competitive, capital and labor earn their marginal products:

$$
\begin{align*}
(1 - \tau_t) f'(k_t) &= 1 + r_t \\
(1 - \tau_t) (f(k_t) - k_t f'(k_t)) &= w_t
\end{align*}
$$

(2)

### 2.3 Recycling Sector

Let us assume that recycled goods and virgin goods are perfect substitutes. A recycling sector competes directly with the virgin production sector. Its output is $y^R$. The representative firm includes recovery and recycling activities. Following Swan (1980) and Martin (1982), we suppose that the production factor is the recovered waste and that the returns are decreasing (a characteristic of the technology of recovery). The program of the recovery - recycling firm is:

$$
\max_{R_t} \pi_t^R = g(R_t) - \left( p_t^j + \gamma \right) R_t \\
\text{s.t. } R_t \leq \phi y_{t-1}^V
$$

(3)

where $R_t$ is the quantity of waste purchased. The recycling function $g(.)$ is strictly concave and satisfies the Inada conditions. The constraint shows that the virgin good lasts for one period before it is recycled. Consequently, the available waste in $t$ depend on the amount of the virgin goods produced during the preceding period $t - 1$: it is supposed equal to $\phi y_{t-1}^V$. As $\phi \in [0, 1]$, a constant proportion $(1 - \phi)$ of the virgin good is lost each period at the waste recovery stage (depreciation cost, Martin, 1982).

$p_t^j$ is the waste net price. Assume that the waste are collected then supplied to the recovery-recycling sector by the government *via* the municipal solid waste services\(^3\) at the price $p_t^j$. This price can either be set by the competitive conditions of the waste market ($j = M$), or fixed by the government ($j = F$) as an environmental policy tool.\(^4\) $\gamma \in [0, 1]$ represents costs linked to the recycling activity: a share is lost in the secondary production (shrinkage cost, Martin,
The factor demand is:

\[ R_t^* = \min \left[ \phi y_{t-1}^V ; g_{t-1} \left( p_t^j + \gamma \right) \right] \quad \forall j = M, F \tag{4} \]

In case of competitive price \( j = M \), the waste market is balanced and the quantity of waste recovered and recycled is equal to the supply of waste. The first order condition gives (optimal demand is quoted superscript \( d \), the effective demand is quoted superscript \( * \)):

\[ R_t^d = g_{t-1}^{-1} (p_t^M + \gamma) \tag{5} \]

When \( p_t^j \) is fixed \( j = F \), the resource availability constraint requires to study two cases \( (F^a) \) and \( (F^b) \) (see Table 1).

In the case where the availability constraint is not binding (non constrained case, \( F^a \)), we have:

\[ R_t^d < \phi y_{t-1}^V \]

The waste supply is in excess; the surplus is destroyed and disappears definitely from the economy. The demand is:

\[ R_t^d = g_{t-1}^{-1} (p_t^F + \gamma) = R_t^d < \phi y_{t-1}^V \]

which is constant as \( p_t^F \) is fixed. The recovery-recycling sector does not wish to recover all the available waste stock.\(^7\) If it did so, it would find itself in the position where the marginal cost was higher than the price and it therefore decreases the profit. This case corresponds to that of plastic, for example.

In the case where the availability constraint is binding (constrained case \( F^b \)), we have:

\[ R_t^d \geq \phi y_{t-1}^V \]
The sector cannot satisfy its production factor demand and we then obtain:

\[ R_t^d \geq R_t^* = \phi y_{t-1}^V \]

The whole of the waste available to be recovered is recycled. The recovery-recycling sector demands all of the available waste. Either this corresponds exactly to the situation in which it equalizes its marginal cost and the price, or it demands a superior amount to that available but it is then faced with the waste availability constraint. This case corresponds to, for example, metals, glass and some types of paper and cardboard.

In the case of a non-competitive price \( j = F \) the fact that the availability constraint is either binding or not is endogenous. We show that the availability constraint is binding if:

\[ \phi y_{t-1}^V < g' \left( p_t^F + \gamma \right) \]  

(6)

This result can be explained as follows. The recovery-recycling sector demands all of the available waste stock if: (i) the quantity which can be recovered \( \phi \) is small; (ii) the quantity of the virgin goods produced in the preceding period \( y_{t-1}^V \) is small; (iii) the cost of recycling \( \gamma \) and the price \( p_t^F \) are low. Condition (6) determines the threshold value of the capital intensity for which the sector moves from the constrained case to the non constrained one:

\[ k_{t-1} < f^{-1} \left( g' \frac{p_t^F + \gamma}{\phi} \right) \text{ (}= k^c) \]  

(7)

Thus, if the capital stock in \( t-1 \) is beneath the threshold (noted \( k^c \)), the supply of the recoverable waste in \( t \) will be too small, and the recycling firm will face a constraint in its factor demand.

The profits \( \pi^{R,j} \ (\forall j = M, F) \) are paid to the young households through lump-sum transfers. In the constrained case \( (F^b) \), since the recovery-recycling sector does not equalize the real cost of the factor to its marginal productivity, the profit can be zero, or even negative, which results in non production by this sector. The constraint of non negativity of profit in the constrained
Price j

<table>
<thead>
<tr>
<th>Resource Constraint</th>
<th>M: Competitive Price</th>
<th>F: Fixed Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>case $F^a : k_{t-1} \geq k^c$</td>
<td>case $F^b : k_{t-1} &lt; k^c$</td>
<td></td>
</tr>
</tbody>
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Marginal Productivity

- $g'(.) = p^M + \gamma$
- $g'(.) = p^F + \gamma$
- $g'(.) > p^F + \gamma$

Input Optimal Demand

- $R^d_t = R^*_t = \phi f (k_{t-1})$
- $R^d_t = R^*_t \leq \phi f (k_{t-1})$
- $R^d_t > R^*_t = \phi f (k_{t-1})$

Table 1: Waste Market Cases

case is given by:

$$\frac{g(\phi f (k_{t-1}))}{\phi f (k_{t-1})} > p^F_t + \gamma$$

This condition imposes an upper bound to $\gamma$ so that the recycling activity can be maintained. Under certain conditions applying to $g(.)$ (concave with positive values), we show that the condition determined by the relation (7) is a sufficient condition for the non negativity of profit ($\pi^{R,F} \geq 0$). In other terms, the demand for recoverable goods is constrained when it is beneath a threshold (defined by eq. 7); this threshold corresponds to a sufficiently weak cost thereby ensuring the non negativity of profits.

2.4 Dynamics of the Environmental Quality

Assume that the production of virgin goods is responsible of pollution emissions. The law of evolution of the quality of the environment is described by the following law of motion:

$$Q_{t+1} = \hat{Q} + [1 - h] Q_t - b V_t$$

(8)

$\hat{Q} > 0$ is a constant and represents the long-term environmental quality when the environment is undisturbed by human activity ($b = 0$). $h \in [0, 1]$ is the constant rate of the autonomous evolution of environmental quality. $b > 0$ is the rate of degradation of the environment due to the virgin production sector. The degradation of the environment is linked to the sector of production of virgin goods (emissions into the atmosphere, discharges into the water table, etc.), whereas the production of recycled goods does not affect the quality of the environment. Our assumption then considers $b$ as a net rate of polluting emissions between the production of virgin and recycled goods.
2.5 Government

Assume that government spending is financed by taxes. The government budget constraint is such that its transfers must equal, at each period $t$, its tax revenues. The equilibrium of the government’s budget is:

$$T_t^\tau = \tau_t y_t^V + p_t^j R_t + T_t^y \quad \forall j = (M, F)$$

This budget takes into account lump-sum transfers, the tax on virgin production and the waste price ($p_t^j > 0$) paid by the recovery - recycling sector.

2.6 Competitive Equilibrium

The equilibrium condition of the capital market is $s_t = k_{t+1}$. The equilibrium condition of the goods market is $y_t^V + y_t^R = c_t^y + c_t^o + k_{t+1} + \gamma R_t$. The goods market equilibrium takes into account the part of the resource lost during the recycling activity (shrinkage cost $\gamma$).

The general equilibrium is defined by a sequence of decisions, of prices and taxes

$$\left\{ y_t^V, y_t^R, k_t, R_t, c_t^y, c_t^o, s_t, w_t, r_t, T_t^y, \tau_t, p_t^j \right\}_{t=1}^\infty$$

such that, at each date $t = 1, 2...$: (i) agents maximize their utility under their budget constraints; (ii) firms $V$ and $R$ maximize their profit; (iii) markets are in equilibrium; (iv) the environmental quality index evolves according to its law; (v) $\{k_0, Q_0, R_0\}$ are given.

The steady-state equilibrium is such that $k_{t+1} = k_t = k^*$. We assume the existence, uniqueness and stability of the steady-state equilibrium\textsuperscript{10}. The steady state equilibrium is defined by
the following system $\forall j = (M, F)$:

$$
\begin{align*}
(1 - \tau) f'_k &= 1 + r^* \\
(1 - \tau) (f(k^*) - k^*f'_k) &= w^* \\
w^* + \pi R_* - Ty &= c^y + s^* \\
c^o^* &= (1 + r^*) s^* + T^o \\
k^* &= s^* \\
\frac{u'_w}{u'_o} &= \beta (1 + r^*) \\
\tau y V^* + Ty + p^j R^* &= T^o \\
R^* &\leq \phi f (k^*) \\
g' &\geq p^j + \gamma \\
\pi R_* &= g (R^*) - (p^j + \gamma) R^* \\
Q^* &= \frac{\bar{Q} - b f (k^*)}{n}
\end{align*}
$$

In the case of price rigidity ($j = F$), two steady states exist according to the value of $k^*$ relative to $k^c = f^{-1} \left[ \frac{g^{-1}(p^F + \gamma)}{\phi} \right]$. If $k^* < k^c$ (case $F^b$) then $R^* = \phi f (k^*)$. In case $F^a$ ($k^* \geq k^c$), then $R^* = g^{-1} (p^F + \gamma) \leq \phi f (k^*)$.

3 The Optimal Allocation

The objective of the social planner is to maximize a discounted sum of the life-cycle utility of all current and future generations. $\delta \in ]0, 1[ \text{ represents the social discount factor.}$ $k_0, Q_0$ and $R_0$ are given. The program of the social planner is:

$$
\max_{\{c^y, c^o, k, Q, R\}} \sum_{t=1}^{+\infty} \delta^{t-1} \left( u (c^y_t) + \theta z (Q_t) + \beta \left[ u (c^o_{t+1}) + \theta z (Q_{t+1}) \right] \right)
$$
Let \( \lambda_{k,t} \) denotes the Lagrangean multiplier associated to the resources constraint of the period \( t \); and let \( \lambda_{Q,t} \) denotes the Lagrangean multiplier associated to the constraint of evolution of \( Q_t \).

We only give the steady-state solutions. The first order conditions give the two shadow prices:

\[
\lambda_Q = \frac{(1 - h) \left( \frac{1 - \frac{\beta}{\delta}}{\frac{1}{\delta} - 1 + h} \right) \theta z'(\cdot)}{1} > 0
\]

(10)

\[
\lambda_k = \frac{\frac{bd(\delta + \beta) f'(. \theta z'(\cdot))}{1 - \delta(1 - h)} - \frac{1}{\delta}}{f'(\cdot)(1 + \delta (g'(\cdot) - \gamma) \phi) - \frac{1}{\delta}} > 0
\]

(11)

\( \lambda_Q \) is the shadow price associated to the environmental quality. It measures the marginal social benefit of a shift of the environmental quality. Thus, \((1 - h) \theta z'(\cdot)\) represents the long-term consequences of the environmental quality on the two coexisting generations, 1 and \( \frac{\beta}{\delta} \) are the corresponding weights. The denominator \( \frac{1}{\delta} - 1 + h \) represents the discount rate relevant to the long-term environmental policy analysis as defined by Marini and Scaramozzino (1995); this discount rate is equal to the sum of the pure social discount rate \( \frac{1}{\delta} - 1 \) and of the natural rate of evolution of the environmental quality \( h \).

The shadow price of capital \( \lambda_k \) is defined by eq. (11) and captures the effect of recycling on environmental quality and, therefore, on social welfare. This price could be positive or negative as the denominator \( 1 + \delta (g'(\cdot) - \gamma) \phi \) depends on the sign of \( g'(\cdot) - \gamma \) which measures the net social benefit of recycling: when the planner allows one additional unit of resource to the recycling sector, it allows a greater production but with the social cost \( \gamma \).

Straightforward substitution of conditions (10) and (11) in the first order conditions leads to the optimal intergenerational trade-off rule defined by:

\[
u_{e} = \frac{\beta}{\delta} \nu'_{e}
\]

(12)
This rule gives the optimal relation between $c^y$ and $c^o$. Using \((12), (10)\) and \((11)\), one determines the optimal expression of consumptions $^c y$ and $^c o$ as a function of the optimal capital, defined by the green modified golden rule:

$$ f' \left( \hat{k} \right) = \frac{1}{\delta} \frac{1}{1 + \delta \left( g' \left( . \right) - \gamma \right) \phi - \frac{b(\delta + \beta) \theta z' \left( . \right)}{\omega' \left( 1 - \delta(1 - h) \right)}} \quad (13) $$

Let $\hat{k}$ denotes the optimal capital stock and recall that the modified golden rule determines the optimal capital stock $\hat{k}_{mgr}$ in a standard Overlapping generations model such that $f'_{k_{mgr}} \left( . \right) = \frac{1}{\delta}$. Notice first that if there are no environmental externalities ($b = 0$) and no recycling of waste ($\phi = 0$), we obtain $f' \left( \hat{k} \right) = f' \left( \hat{k}_{mgr} \right)$.

We then have $f' \left( \hat{k} \right) = f' \left( \hat{k}_{mgr} \right) / \left( 1 + \delta \left( g' \left( . \right) - \gamma \right) \phi - \frac{b(\delta + \beta) \theta z' \left( . \right)}{\omega' \left( 1 - \delta(1 - h) \right)} \right)$. As the denominator could be smaller or greater than one, depending on the sign of $g' - \gamma$ among other terms, the optimal capital stock could be greater or smaller than the one defined by the modified golden rule. Hence, when $(g' \left( . \right) - \gamma) \phi$ increases, $\hat{k}$ grows higher than $\hat{k}_{mgr}$. If the net social benefit of recycling $g' \left( . \right) - \gamma$ increases or if the recovery rate $\phi$ increases, then $f' \left( \hat{k} \right)$ decreases inducing a raise of the optimal capital stock: the planner will fix a higher capital stock. Indeed, when the recycling is raised (ie. higher marginal productivity $g' \left( . \right)$ or higher recovery rate $\phi$), then less virgin goods are being produced, reducing the emission of pollution. Hence, the optimal capital stock should increase. Moreover, the increase in the amount of recovered waste $\hat{R}$ reduces the marginal productivity of recycling $(g' \left( . \right))$, therefore $\hat{k}$ also decreases as well as the production of virgin goods: there is a substitution effect between the recycled goods and the virgin goods.

We also show that the optimum capital stock is a decreasing function of the ratio $\theta z' \left( . \right)/u'_v$: as the environmental sensitivity gets higher, the social planner has to decrease the pollution externality. Finally, we have $\hat{k} = k \left( g' \left( . \right) - \gamma, \phi, z' \left( . \right), u'_v \right)$. The steady state solution to the planner’s program is summarized by the following system
which determines \( \{ \hat{c}^y, \hat{c}^o, \hat{k}, \hat{R}, \hat{Q} \} \):

\[
\frac{u'_c}{u'_{co}} = \frac{\beta}{\delta}
\]

\[
f'(\hat{k}) = \frac{1}{\delta} \left[ \frac{1}{(1+\delta(g'(\cdot)-\gamma)\phi)-\delta(1-h)\beta u''(\cdot)} \right]
\]

\[
\hat{R} = \phi f(\hat{k})
\]

\[
g(\hat{R}) + f(\hat{k}) = \hat{c}^o + \hat{c}^y + \hat{k} + \gamma \hat{R}
\]

\[
\hat{Q} = \frac{\hat{Q} - bf(\hat{k})}{h}
\]

### 4 Optimal Tax Policy

Let us focus on the economy characterized by a waste market failure such that waste price is fixed by the government at a non competitive level. The government has to determine the tax rates \((\hat{T}^y, \hat{\tau})\), and the waste price \(\hat{p}^F\) such that the competitive equilibrium (system 9) corresponds to the long term optimum (system 14). The government faces three economic inefficiencies\(^{11}\): polluting virgin production; non competitive waste market implying that the waste competitive production does not correspond to the optimal amount of waste; dynamic inefficiency which induces the over or under accumulation of the capital stock. The optimal policy therefore consists of three economic tools \((\hat{T}^y, \hat{p}^F\) and \(\hat{T}^\circ\)): the Pigovian tax \(\hat{\tau}\) internalizes the flow of pollution emissions arising from the virgin production; \(\hat{p}^F\) incites the recycling sector to recover exactly the amount of available waste, as we consider the case of waste missing market; \(\hat{T}^y\) fixes the capital intensity at the level defined by the green modified golden rule (eq. 13). A fourth instrument \((\hat{T}^\circ)\) balances the government budget.

By comparison of system (9) and system (14), we obtain the following optimal value for the Pigovian tax:

\[
\hat{\tau} = \frac{b (\delta + \beta) \theta z'(\cdot)}{u'_{cs} (1 - \delta (1 - h))} - \delta \left( g'(\cdot) - \gamma \right) \phi \geq 0
\]

This tax corrects the environmental externality: it therefore increases with the ratio between the marginal environmental damage \(b (\delta + \beta) \theta z'(\cdot))\) and the marginal utility of consumption \(u'_{cs}\).
On the other hand, the more efficient the recovery-recycling sector is, the more the environmental externality decreases to the extent of disappearing. Hence a higher marginal productivity \( (g') \) of the recycling sector, with a strong rate of recovery \( (\phi) \), can make the pollution externalities vanish. The government needs one instrument less (ie. \( \tau = 0 \)). Consequently, the development of recovery and recycling sectors could be one of the instruments of internalization of the intergenerational externalities.

As there is no competitive waste market \( (j = F) \), the optimal waste price \( (\hat{p}^F) \) is defined by:

\[
\hat{p}^F = g' \left( \phi f \left( \hat{k} \right) \right) - \gamma \vDash 0
\]

This price (decided by the government) could be positive (as a tax) or negative (subsidy), according to the level of the recovery cost. The recycling subsidy gives an incentive to recycle the whole waste (case \( F^\alpha \)). Indeed, the steady state solution to the planner’s problem requires \( \hat{R} = \phi g^V \). The optimal policy must ensure that the whole of the available waste stock is recycled. The instrument allows the recycling sector to maximize its profit when recovering and recycling exactly the quantity of available waste. The case of subsidies corresponds to the French policy of Eco-Packaging.

Conversely (case \( F^\beta \)), this price is positive if the recycling costs \( (\gamma) \) are sufficiently low or the marginal productivity of recycling is high. The recycling sector, in this case, demands more waste than available: the government must increase the waste price in order to compensate for the low cost, and correct the consequences of the non competitive waste market. Moreover, two additional reasons justify the increase of the waste price. First, if the recycling sector is a heavy industry characterized by massive growth, it needs more and more waste to produce recycled goods. If the waste supply is too low, this disequilibrium requires that the virgin sector produce more goods in order to supply more waste. This situation creates more pollution and more intergenerational environmental externalities. Hence, the government has to increase the waste price \( \hat{p}^F \) in order to reduce the development of the recycling sectors. Secondly, the more profit the recycling sector makes \( (g' \text{ high enough vis-à-vis } \gamma) \), the higher the revenue households receive, which increases their total consumption. This is a revenue-effect that creates more demand and more pollution for future generations.
Finally, the capital market equilibrium fixes the lump-sum tax on the younger generations. This relation is such that:

$$\dot{k} = s \left( w(\dot{k}) - \dot{T}^y + \dot{T}^R + \frac{\dot{T}^o}{1 + r}, r(\dot{k}) \right) \quad \forall j = (M, F)$$

This condition determines the level of the tax rate that corrects the dynamic inefficiency of the equilibrium and allow the decentralized capital stock to reach the one defined by the green modified golden rule.

5 Conclusion

This article sets forth three characteristics of the recovery-recycling sector in order to analyze intertemporal trade-offs in the use of environmental resources. We show that if the recycling activity is efficient enough (high marginal productivity and/or high rate of recovery, for example), it reduces environmental externalities which leads to a decrease in the Pigovian tax rate. We also show that the recovery-recycling sector must be subsidized when there is no competitive market for waste and when the recovery-recycling costs are too high. On the other hand, the government has to increase the waste price in order to reduce the development of the recycling sectors if costs are too low. Moreover, as recycling decreases pollution in the future, it acts as an incentive to increase consumption and waste in the present.

Public intervention in this sector ceases when a competitive price exists for waste: this requires a new legal definition of waste so that the legal notion of property rights can be applied.

Notes

1. This characteristic is a consequence of the legal definition of waste. According to the law, “Any substance or object the holder discards, intends to discard or is required to discard” is waste under the Waste Framework Directive (European Directive 2006/12/EC), which repeals the European Directive 75/442/EC as amended. Waste are therefore of no interest to economic agents, which give to the waste a negative value.
2. France adopted the European Directive of 2002 concerning the disposal of electrical and electronic waste. Since November 2006, producers of these goods contribute to one of the four State-registered eco-organisations (Eco-Systèmes, ERP, Ecologic, and Recyclum); actually the consumer have to pay a contribution through the payment of a tax upon purchasing the goods concerned. This eco-contribution was designed to fill the gap between the re-sale price of the secondary materials and the marginal cost of their recovery.

3. We use this assumption only for simplicity in order to avoid the collecting sector problem. It has no consequence on the results. It appears that the government collects costless the waste material and resells it to producers. This seems unrealistic (recovery is not free of charge), but \( p_j^* \) is a “net” price. This is a simplifying assumption.

4. These two ways for determining the price of waste correspond to industrial realities in many countries. In the competitive case, the waste prices fluctuate in the same way as those of raw materials. The fixed price case corresponds to the french “buyback guarantee” system of Eco-Emballages before 2002.

5. This cost corresponds to an iceberg cost: a part of the goods disappears during transportation. This cost has consequences on the goods market equilibrium.

6. In the competitive case, there is equality between the two terms of eq. \[ 4 \] :
   \[
   \phi y_{t-1}^V = g^{t-1} \left( p_i^j + \gamma \right).
   \]

7. This case is characterised by a social inefficiency as a free resource is destroyed.

8. This model does not incorporate the externality from disposal of waste that is not recycled. But there is “waste” that is not recycled, namely the scrap that does not get re-used in production, equal to \( y_{t-1}^V - (1 - \gamma) R_t \). If the availability constraint is binding, \( i.e. \) the whole of the waste to be recovered is recycled (that is the objective of the public policy), then the “waste” is equal to \( (1 - \phi (1 - \gamma)) y_{t-1}^V \). Set \( b = (1 - \phi (1 - \gamma)) \). Then the quality of the environment incorporates the externality from disposal of waste that is not recycled.

9. In the case of steel, the production of one ton of primary steel emits four times as much \( CO_2 \) into the atmosphere as the production of one ton of recycled steel. However, for some materials,
there can be a recycling threshold beyond which recycling pollutes more (*ie.* \(b < 0\)). This particular case could result in a change in the sign of the optimal value of the tax instruments, so as to reduce recycling. We do not examine that particular case, but it would not be a problem.

10. To ensure the existence, the uniqueness and the stability of the *laissez-faire* steady-state equilibrium, one must verify that the rate of intertemporal substitution between young and old consumption is not too weak (normal goods).

11. In case of competitive waste market, the economy faces only two inefficiencies since the waste price is competitive. The results concerning the other taxes \(\hat{T}_Y^p\) and \(\hat{\tau}\) remain the same.

12. If \(p^j\) is a competitive price \((j = M)\), using the first order condition of the recycling sector, we have (eq. 5): \(g'(.) = \hat{p}^M + \gamma\) and \(\hat{R}^d = \hat{R}^* = \phi \hat{y}^V\).

**References**


