Government Spending
in a Monetary Model of Endogenous Growth: a Note

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Government Spending in a Monetary Model of Endogenous Growth: A Note*

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Abstract

Few endogenous growth models are able to encompass unbalanced transitional dynamics. In Barro (1990) public spending is a productive externality and growth is only regular. The second best tax rate equals the public spending return.

We provide a monetary version of Barro (1990), where short-run fluctuations are due to money and long-run effects to technology. Barro rule is found to be surprisingly robust within transition.

Keywords: cash-in-advance, endogenous growth, indeterminacy.

JEL Classification: D90; E32; E50.

1 Introduction

Business cycle models focus usually on short-run fluctuations, i.e. aim at accounting for fluctuations in prices and quantities occurring at relative high frequencies, e.g. a quarter or a year. On the other hand, new growth theories, in the shed of Romer (1986, 1990) and Rebelo (1991) early contributions,

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rationalize long-run trends without caring too much about the short-run. Very few attempts in literature have been devoted to reconcile these two approaches, namely to treat transition, and associated patterns like cycles, within models originally set to mimic long-run phenomena.\footnote{Among the few exceptions the Jones and Manuelli (1990) one-sector model with an ad hoc production function to account for transition, the Lucas (1988) two-sector economy with human capital accumulation, the two-sector version of Grossman and Helpman (1991) with taste for variety, the Barro and Sala-i-Martín (1995) two-country model with catch-up, the papers by Benhabib and Rustichini (1994) and Cazzavillan (1996) focusing on equilibrium indeterminacy.}

The aim of this paper is to partly fill this theoretical gap, and try to account for short-run phenomena, like sunspot fluctuations, in models originally conceived to display sustained growth but neglecting the study of transitional dynamics and the possibility of attractors other than balanced growth paths. For this purpose, we exploit the role of money and financial market in short-run dynamics and, at the same time, the persistent impact of technology on long run growth. In this respect we set up a simple monetary model of endogenous growth where demand for nominal balances is motivated by the introduction of a cash-in-advance constraint on consumption and production is affected by public good externalities, public spending being financed through a flat income tax, as originally framed in Barro (1990).

The positive externality represents a wedge between private returns (decreasing and thus compatible with a profit maximizing behavior) and social returns (which are increasing). The straightforward Barro’s conclusion is that the tax rate has to be set equal to the public good “productivity”\footnote{The public good elasticity in production.} to internalize the externality. However, Barro (1990) is a pure stationary economy displaying neither transition nor fluctuations.

How the policy maker has to manage transition is a question of interest, but Barro, since he focuses only on long-run, does not provide any answer. Our monetary version displays richer dynamics, namely irregular growth and persistent cycles. We are able to check the robustness of the rule under transition and we obtain a surprising result: Even in presence of fluctuations the best policy still remains the Barro’s one prescribing the equality between tax pressure and public spending return. Moreover, although in our model unbalanced equilibria are always multiple, yet the best rule is invariant to this multiplicity. Such a finding comes from an explicit computation of transition paths and relevant welfare.
Balanced growth is Pareto-dominated by a continuum of unbalanced equilibria: The economy may jump indeed on higher growth rates and thus compensate the underaccumulation characterizing Barro’s regime. To complete the paper, we compute the best transition through mild approximations.

2 The Model

Barro (1990) is reset in discrete time to highlight the transaction timing. An infinite-lived representative agent maximizes the intertemporal utility function

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $\beta \in (0, 1)$ is the discount factor, $c$ consumption, and $u$ the per-period consumption utility. The utility function is assumed to be increasing and strictly concave, and usual boundary conditions do hold. The consumer computes how much to consume and to invest in nominal balances $M_{t+1} - M_t$ and in physical equipment $k_{t+1} - k_t$, whose depreciation rate is assumed for simplicity to be zero. Capital and consumption are produced within the same technology and share the same price $p$. The inflation factor and real balances are denoted respectively $\pi_{t+1} = p_{t+1}/p_t$ and $m_t = M_t/p_t$. The consumer faces in each period $t$ the usual budget constraint

$$\pi_{t+1} m_{t+1} - m_t + k_{t+1} - k_t + c_t \leq (1 - \rho) (r_t k_t + w_t) + \tau_t$$

and the additional cash-in-advance constraint

$$c_t \leq m_t,$$

where $r$ stands for the real return on capital and $w$ for the real wage. Labor supply is assumed to be inelastic and normalized to one. $\rho$ is the income flat tax rate, while $\tau$ denotes the real monetary transfers “helicoptered” to individuals. In words, the left-hand side of the budget constraint details the portfolio demand, while the right-hand side represents the disposable income.

A constant returns to scale production function is specified as in Barro (1990): $F(k_t, l_t) = A g_t^{1-\alpha} k_t^{\alpha} l_t^{1-\alpha}$, where $l_t$ is the labor demand and $\alpha$ is the

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Stockman (1981) is the first model of intertemporal utility maximization under a cash-in-advance constraint à la Clower (1967).
capital share on total income. \( g_t \) is the public good, a positive externality in production of elasticity \( 1 - \alpha \). The intensive production function is given by \( f (h_t) \equiv Ag_t^{1-\alpha}h_t^\alpha \), where \( h_t \equiv k_t/l_t \) denotes the capital intensity. As usual firm equilibrium requires \( r_t = f'(h_t) \) and \( w_t = f(h_t) - f'(h_t)h_t \). At equilibrium labor demand equals the labor supply and we thus obtain \( h_t = k_t \).

Since the tax is the only way to finance the public spending, the budget equilibrium requires \( g_t = \rho (r_t k_t + w_t) = (\rho A)^{1/\alpha} k_t \). Therefore the production turns out to be linear in capital: \( f (k_t) = A^{1/\alpha} \rho^{1/\alpha-1} k_t \), and the interest rate to be constant: \( r_t = \alpha A^{1/\alpha} \rho^{1/\alpha-1} \equiv r \).

Monetary policy is simply reduced to a constant monetary growth factor \( \mu \equiv M_{t+1}/M_t \).

### 3 Dynamic System

Deriving the demand functions and letting the market to clear, we obtain the system:

\[
\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{i_t}{i_{t+1}/R_{t+1}}, \quad (4)
\]

\[
k_{t+1} - k_t + c_t = (1 - \rho) f(k_t), \quad (5)
\]

\[
\lim_{t \to \infty} (\lambda_t + \nu_t)k_t = 0, \quad (6)
\]

where \( R_t \equiv 1 + (1 - \rho) r_t \) and \( i_t \equiv \pi_t R_t \) are respectively the real and the nominal interest factor, while \( \lambda \) and \( \nu \) are the Lagrangian multipliers associated to constraints (2) and (3).

Equation (4) is an Euler condition augmented to take in account the role of inflation on the return on investment. The left-hand side is the marginal rate of substitution between the present and the future good. The right-hand side is the price ratio, the cost of the present good \( i_t \) over the discounted price of the future good \( i_{t+1}/R_{t+1} \).

By Walras’ law the equilibrium budget constraint (2) is the sum of money market equilibrium \( \pi_t = \pi_{t+1} m_{t+1} - m_t \) and good market equilibrium (5). Finally, (6) is the transversality condition.

As usual the positive elasticity of intertemporal substitution \( \sigma \) is assumed to be constant: \( u(c_t) \equiv c_t^{-1/\sigma} / (1 - 1/\sigma) \). System (4-5) then becomes

\[
x_{t+1} = \left(xx_t^{-\sigma}\right)^{1/(1-\sigma)}, \quad (7)
\]
\[ y_{t+1} = \frac{x_t y_t}{a - y_t} \]  

(8)

where \( x_t \equiv c_{t+1}/c_t \) and \( y_t \equiv c_t/k_t \). \( x \equiv (\beta R)^\sigma \) is the balanced growth factor and \( a \equiv 1 + (R - 1)/\alpha \) is a constant. Capital dynamics are specified by \( \gamma_t \equiv k_{t+1}/k_t = a - y_t \). The steady state is given by \( (x, y) = (x, a - x) \) and the stationary growth is balanced: \( \bar{c}_0 = (1 - \rho) f(k_0) - (x - 1) k_0 \). \( \mu \) does not affect \( x_t \). In words money is superneutral even during transition, but, unlike in Barro (1990) transition now matters thanks to money.

We require three restrictions around the steady state.

1. \( a > x \) (positivity of \( c_t/k_t \)),
2. \( R > x \) (transversality condition),
3. \( i > 1 \) (binding cash-in-advance).

The transversality condition is always satisfied for \( \sigma < 1 \) (income effects) and, since \( a > R \), it implies the positivity of \( c_t/k_t \).

4 Indeterminacy and Cycles

\( M_0 \) and \( k_0 \) are given initial conditions. At the beginning of period \( t \), the consumption levels \( c_t \) and \( c_{t+1} \) are non-predetermined, while \( k_t \) is inherited from the past. Therefore \( y_t \equiv c_t/k_t \) is non-predetermined as well and, even if we freely peg \( y_t \) and so \( c_t \), the growth factor \( x_t = c_{t+1}/c_t \) remains non-predetermined, because \( c_{t+1} \) is independently non-predetermined with respect to \( c_t \). The usual condition for that equilibrium multiplicity one calls local indeterminacy, is a dimension of stable manifold strictly greater than the number of predetermined variables, now zero.

The eigenvalues of system (7)-(8) are \( \lambda_1 = \sigma / (\sigma - 1) \) and \( \lambda_2 = a/x > 1 \). Therefore the steady state is a saddle (and there is indeterminacy) if and only if \( \sigma < 1/2 \) (large income effects).

For \( \sigma < 1/2 \), the short-run consumption path fluctuates around a long-run trend (the transition function (7) is negative-sloped) and eventually converges.

For \( \sigma = 1/2 \), dynamics are captured by the symmetric hyperbola \( x_t x_{t+1} = x^2 \) and a continuum of two-period cycles arises in the \( (x_t, x_{t+1}) \)-plane, one for each non-predetermined starting point \( x_0 \).

For \( \sigma > 1/2 \), the unique rational expectations equilibrium is the Barro’s balanced growth.
Since local indeterminacy depends only on $\sigma$, there is room for neither monetary nor fiscal policy, to refine the multiplicity. In order to understand the mechanism leading to indeterminacy, let us consider again the dynamic equation (7). Suppose now that the economy is initially at the steady growth and let us try to construct an alternative equilibrium in which agents in period $t$ anticipate an increase in next period consumption factor of growth, $x_{t+1}$. Suppose also that $\sigma < 1/2$. Under this conjecture, it is easy to verify that the re-establishment of (7) requires a very large contraction of consumption growth factor in period $t$, $x_t$. This, of course, will give raise to converging, although oscillatorily, forward dynamics.

5  Dealing with Transition

Let $\sigma < 1/2$. Solving the difference equation (7), we obtain the transition path:

$$c_t = c_0x^t(x_0/x)^{(1-\sigma)[1-(\frac{x}{x_0})^t]},$$

where $c_0 = (1 - \rho)f(k_0) - (\gamma_0 - 1)k_0$.

The stable manifold is one-dimensional and, given $x_0$, a unique $y_0$ satisfies the saddle path, the rational expectations equilibrium. We compute the tangent line to the downward-sloped saddle path in the steady state:

$$y_0 = m - nx_0,$$

where

$$m \equiv (a - x) \left[ 1 + \frac{(1 - \sigma)x}{\sigma x + (1 - \sigma)a} \right] > 0,$$

$$n \equiv \frac{(a - x)(1 - \sigma)}{\sigma x + (1 - \sigma)a} > 0,$$

and we approximate the equilibrium initial growth factor for capital $\gamma_0 \equiv a - m + nx_0$.

5.1 Welfare Analysis

Consumers have the same tastes and endowments, and the representative agent’s utility function (1) becomes a social welfare function. To evaluate
the welfare, we define two approximated consumption paths.

\[ \tilde{c}_t = \tilde{c}_0 x^t (\frac{x_0}{x})^{(1-\sigma)[1-(\frac{\sigma}{\sigma-1})]} \]

\[ \hat{c}_t = \hat{c}_0 x^t (\frac{x_0}{x})^{1-\sigma} \]

\[ \tilde{c}_0 = (1 - \rho) f (k_0) - (\tilde{\gamma}_0 - 1) k_0 \approx c_0. \]

Transition (10) takes in account the linearized saddle path (9), while the less fine approximation (11) is obtained from (10) by suppressing the fluctuation mechanism \[ [\sigma / (\sigma - 1)]^t \]. Welfare maximization is not substantially affected by the higher order effects neglected by these approximations.

The welfare function (1) is first evaluated along the fictitious transitions (10) and (11) with \( x_0 \neq x \), and then along the balanced growth path with \( x_0 = x \).

\[ \tilde{W} = -\sigma [1-\alpha] \sum_{t=0}^{\infty} \frac{1}{t!} \frac{\ln (x_0/x)}{1 - \sigma} \frac{(1-\sigma)^2}{\sigma} \]

\[ \hat{W} = -\sigma [1-\alpha] \frac{1}{1 - x/R} \]

\[ \tilde{W} = -\sigma [1-\alpha] \frac{1}{1 - x/R} \]

where

\[ d = \frac{x + (1-\sigma)(a-x_0)}{x + (1-\sigma)(a-x)} \left( \frac{x_0}{x} \right)^{1-\sigma} \]

5.2 Robustness of Barro Rule

In Barro (1990) the government maximizes the welfare of stationary growth (14) and finds a simple rule: \( \rho^* = 1 - \alpha \). In words, the tax rate is set to equal the return on public spending \( 1 - \alpha \) in terms of elasticity. The higher the productivity of public good, the stronger the fiscal pressure to finance its provision.

Since real economies experience endless transitions, we have to answer the question about policy robustness. Is this thumb rule wrong and possibly dangerous in the face of irregular growth and multiple equilibria? The surprising result we obtain is that transition doesn’t matter and the best
rule doesn’t change. The policy maker has not to care about business cycles, multiple equilibria and sunspots. The rule is invariant to animal spirits.

To prove the point, we provide an analytical argument jointly with a numerical simulation. More precisely we maximize first analytically the proxy (13) and then numerically the finer proxy (12).

Substituting \( x = [\beta (\alpha a + 1 - \alpha)]^\sigma \) in (13), \( \tilde{W} \) becomes a reduced function of \( a \). The fiscal pressure \( \rho \) affects the welfare value \( \tilde{W} (a) \) only through \( a \). We set the following bounds to the initial condition \( x_0 \).

\[
\begin{align*}
x_0 & \equiv x - x \frac{x + (1 - \sigma) (a - x)}{x + (1 - \sigma) (b - x)}, \\
\bar{x}_0 & \equiv x + \frac{x + (1 - \sigma) (a - x)}{1 - \sigma},
\end{align*}
\]

(15) (16)

where \( b \equiv R / (\alpha \sigma) > a \). Clearly \( x_0 < x < \bar{x}_0 \). Straightforward but tedious computations show that \( x_0 \in (x_0, \bar{x}_0) \) implies \( \tilde{W}' (a) > 0 \). The solution for \( \rho \) to maximizes \( a \) (and therefore \( \tilde{W} \)) is interestingly \( 1 - \alpha \).²

One may question whether the result depends on the rough approximation we make, by suppressing the term \(- [\sigma / (\sigma - 1)]^\sigma \). The doubt is drop by refining the proxy and performing a numerical check. Using now (12) and setting \( A = k_0 = 1, \alpha = \sigma = 1/3, \beta = 0.99 \) (on line with quarterly data), we find that also \( \tilde{W} (a) \) is strictly increasing. The rule \( \rho^* = 2/3 \) is still optimal, whatever the choice for \( x_0 \) around \( x \).

### 5.3 Imperfections Balance

Are the multiple equilibria Pareto-ranked and is the regular growth Pareto-dominated? In other terms how is the best path shaped with respect to the balanced one? Could the credit market imperfection counterbalance the externality in production and paradoxically improve the economy?

Indeterminacy of initial conditions, entails equilibrium multiplicity. To find the best path, simply we need to compute the best starting point \( x_0^* \). As above we consider the simpler but rougher proxy (13) to provide a rather elegant analytical solution and the relevant interpretation. Maximizing \( \tilde{W} \)

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²In Futagami, Morita and Shibata (1993) the transitional equilibrium is unique and \( \rho^* \neq 1 - \alpha \).
with respect to $x_0$, we obtain that the approximated optimal accumulation $\tilde{x}_0^*$ lies within the bounds (15) and (16) and strictly dominates the steady state $x$, whatever is the policy rule $\rho$.

$$x_0 < x < \tilde{x}_0^* = \frac{x + (1 - \sigma) a}{2 - \sigma} < x_0.$$  

In words there are achievable and better paths with higher growth factors around $\tilde{x}_0^*$ than the stationary one. A higher investment is possible under and thanks to indeterminacy and, compensating the underaccumulation of Barro’s regime, enhances the social welfare. Since indeterminacy is due to a cash-in-advance, while the underaccumulation to a public good externality, we could say that one imperfection counterbalances the other. Whether the best growth is implemented by agents’ coordination is clearly a matter of chance and we don’t introduce a game-theoretic mechanism to refine the equilibrium multiplicity.

Do we have to care about the loss of information in (13)? Luckily, our analytical finding is confirmed by a numerical maximization of the finer proxy (12) with the parameters set as above and, for simplicity, $\rho = 2/3$. Figure 1 illustrates the conclusion.
6 Conclusion

Barro (1990) focuses exclusively on lung-run growth. Equilibrium is unique but inefficient because of public good externality in production. The policy maker has at disposal a second best taxation rule to face capital underaccumulation: Optimal fiscal pressure has to be equal to public spending return.

Our model is a monetary extension of Barro (1990) characterized by irregular growth and equilibrium multiplicity. The balanced growth path is Pareto-dominated by a manifold of transitions with higher accumulation rates. We compute the optimal transition and we show that the Barro’s optimal policy does surprisingly still hold whatever transition the economy experiences.

7 References


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