Optimal Privatization Design and Financial Markets

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January 13, 2003

Abstract

This paper provides a simple general equilibrium analysis of privatization, exploring its real effects. They derive from the expansion of risk-sharing opportunities (within an incomplete markets setting) that are created by the addition of a market in the public project property rights. The principal conclusion is that an optimal combination of voucher and share issue privatization can implement the first-best.

JEL classification: G1; H4; L33

Keywords: Privatization; Public good provision; Risk diversification; Financial market development

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*This research is part of the project “Privatisation and Financial Market Development”, funded by the European Commission (contract n. HPSE-CT-1990-00007). We thank two anonymous referees, Ingrid Rossellini and participants at the 18èmes Journées de Micro-économie Appliquée, the T2M 2001 Conference, the 16th Annual Congress of the European Economic Association and the 50ème Congrès de l’Association Française de Sciences Économiques.

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“Because different assets have different distributions of returns, privatization is a way of allocating risks across members of the economy.” Maskin [2000]

1 Introduction

Most of the theoretical research on privatization analyzes the microeconomic efficiency of privatization. However, massive privatizations, i.e. large property transfers, may have a sensitive macroeconomic impact. Among others, previous approaches taking into account macroeconomic aspects focused either on the necessary critical mass of privatization (Roland and Verdier [1994]), on insider privatizations (Blanchard and Aghion [1996]), on the impact of privatization on aggregate absorption, public finance and the balance of payments (Verdier and Winograd [1998]), or on the impact on aggregate labor productivity of having government, rather than private industry, produce investment goods (Schmitz [2001]).

The aim of this paper is to explore why and how privatization may have an effect on risk-sharing, in order to state what many researchers in the field had suspected: that even if privatization had no direct implications in terms of productive efficiency and industrial organization, it would be not neutral, because of indirect general equilibrium consideration (in this case, risk-sharing).

Some recent works have suggested this idea of privatization as a way of allocating risks across members of the economy (Maskin [2000], Perotti and van Oijen [2001]). Our approach is also related to the financial and macroeconomic literature on incomplete asset markets and risk-sharing, as well as to the literature on asset trade under uncertainty (Acemoglu and Zilibotti [1997], Martin and Rey [2000], Pagano [1993] or Saint-Paul [1992]).

The simplest way to approach this issue is to keep in mind the famous proverb saying that nobody should keep all his eggs in one basket: whilst the investor views privatizations as a new “basket”, the government accordingly views revenues from privatization as new “eggs”.

Intuitively, one basic financial mechanism is a diversification effect (the “basket story”): if the returns on the new stocks, issued thanks to privatization, are imperfectly correlated with preexisting ones, individuals (who do not like risk, by assumption) will be interested in this new diversification opportunity. *Ceteris paribus*, they will decrease their demand for all preexisting private assets (including their own) to increase their demand for privatized assets. If thanks to privatization revenue, the government buys a diversified portfolio (as we will see, with no efficient taxation, such a policy allows to ensure future public spending), the diversification effect will be supplemented by a demand effect (the “eggs story”): government demand for a diversified portfolio mechanically increases the price of private assets and thereby their supply.

In some cases these financial mechanisms may have real consequences, in the sense that privatization may have an effect on variables such as consumption levels or welfare measures. Suppose, for instance, that the future provision of public good is not initially perfectly ensured, that the size of the public sector is initially too high compared to the desired amount of public
good, and that there is no tax system at all.

In this context, there is always an *optimal privatization mix*, optimal because that it allows the economy to reach the most favorable state in terms of welfare. What is the composition of this optimal privatization mix? First, some distribution of free shares of the privatized assets allows for the adjustment of the public sector to the desired amount of public good. Second, some share issue privatization (SIP), whose revenues are invested in a diversified portfolio, allows future public good provision to be secured. In other words, this part of the optimal policy consists in the reallocation of the government’s portfolio: without efficient taxation, the government should not keep all its “eggs” in one “basket”.

These basic principles and intuitions are conceptually, empirically and historically relevant.

First, the situation in which the government cannot finance the public good via taxation seems interesting. In our model, there is no tax system at all. This is of course an extreme assumption, but it helps to understand what happens in economics where the tax system is known to exhibit some inefficiencies. Indeed, there are numerous examples of countries where raising domestic revenues from consenting citizens to provide public goods is a challenge. As Kahn, Silva, and Ziliak [2001] write, effective tax collection is a prerequisite for a sound economy, but many countries have been plagued by the inability to collect sufficient tax revenues to finance government expenditures. They report that the Brazilian economy prior to the reform of the collection system (1989) was characterized by widespread tax evasion. More generally, according to Andreoni, Erard, and Feinstein [1998], the problem of tax compliance is as old as taxation itself, and available evidence suggests that non-compliance is particularly acute in many countries¹, and clearly, significantly reduces government revenues. In this respect Gauthier and Gersovitz [1997] write that many observers view erosion of the tax base in developing countries as a major feature of the tax systems in these states. Studies from different developing countries, cited by Fjeldstad and Semboja [2001], indicate that it is not uncommon for half or more of the potential tax revenues to remain uncollected. Moreover, the expanding underground economic activities may also affect implementation and outcomes of economic policies. For instance, Ashlund, Boone, and Johnson [2001] write that in parts of East-Central Europe and most of the former Soviet Union, high levels of underground activity keep tax revenue so low that the government cannot afford to provide public goods.² More generally, a large unofficial economy means that few pay taxes, and in many countries, the estimates of the share of the unofficial economy (as % of GDP) given by Friedman, Johnson, Kaufmann, and Zoido-Lobaton [2000] are dramatic: Nigeria 76.0%, Thailand 71.0, Egypt 68.0, Bolivia 65.6, Philippines 50.0, Tunisia 45.0, Sri Lanka, 40.0, Malaysia 39.0, Morocco 39.0, Korea 38.0. Tax non-compliance and/or a

¹For case studies on developing or transition countries, see, among others, Aitken [2001] on Russia, Fjeldstad and Semboja [2001] on Tanzania and Gauthier and Gersovitz [1997] on Cameroon. Furthermore, an article by Besley, Preston, and Ridge [1997], based on the UK’s experience, reminds that even in an economy with a relatively well developed detection and legal system, one cannot take tax compliance for granted.

²Roland [2001] also refers to the strong increase in the size of the hidden economy in former Soviet republics, and the resistance of large Russian enterprises to tax collection.
large unofficial economy in developing or transition countries are basic examples of inefficiencies affecting their fiscal system. More generally, and even in developed countries, the well-known existence of tax distortions is another justification for our tax-constrained environment.

Secondly, it is useful to examine privatization by a voucher system as a mechanism for decreasing the expected size of the public sector initially fixed at some non-optimal level is useful: from the stylized facts presented in Verdier and Winograd [1998] one learns that the common way to implement the transfer of assets from the public sector to the private sector was in several eastern economies free distribution of public assets to private individuals. This divestment method has been extensively used in transition economies, and has brought about fundamental changes in the ownership of business assets. Roland [2001] evokes, for instance, the free distribution of assets in Russia. Mackenzie [1998] writes also that the comparative thinness of financial markets in some countries and the great size of some privatization programs have led some countries to adopt privatization by voucher. In the simplest voucher program one can imagine, participants receive free vouchers that make them owners of a given fraction of an enterprise or group of enterprises. Obviously, in the case of voucher privatizations, private agents do not always receive free shares of the privatized assets. More generally, in a voucher program, eligible citizens can utilize vouchers, distributed free or at a nominal cost, to bid for shares of state-owned enterprises and of other assets that are being privatized.3 But the analysis of our extreme case (distribution of free shares) helps to understand what happens in intermediary situations.

So, the relevant case seems to be the one where voucher privatization is positive. In contrast, we do not examine “voucher nationalization” of the private returns to production in order to finance production of the public good. Indeed, historically, nationalizations (possibly viewed as negative voucher privatizations) are all linked with very specific political events.

Thirdly, the simultaneity of voucher privatization and SIP is not unrealistic. The stylized facts presented in Verdier and Winograd [1998] among others confirm that both types of privatization have been implemented in some countries, for instance in Poland, Hungary, Slovakia and Romania. Besides, it is not conceptually irrelevant, or irrational to provide lump sum distributions of shares in the government projects to the public and at the same time to use SIP to raise revenue from the same population.

Fourthly, the investment of privatization revenues in a diversified portfolio of private assets is also realistic. For instance in France, receipts from the saving banks privatization and from the “waves privatization” (sale of licenses for UMTS mobile phone) are directed to retirement reserve funds. The debate is still unresolved, but the government recognizes that the need of better returns diverts these funds to the stock market. The US and Canada are also equipped with such retirement reserve funds, which are partially invested in the stock market.

Now, in order to discuss the results, we need to fully explain the model setup and the ineffi-

3See Brada [1996] for details; see Filer and Hanousek [2001] concerning Czech voucher privatization, for example.
ciencies that may arise in equilibrium. Formally, in order to take into account the risk-sharing opportunities created by the privatization processes, we introduce state-owned property rights and public good production in a simple two-period, general equilibrium model with incomplete capital markets and risk-sharing opportunities. In our model, we consider identical consumers and the government, both with high-risk holdings. The returns on the government project are used to provide a public good. Consumers trade a variety of securities in order to limit risk, and in equilibrium they choose to hold a completely diversified portfolio. Payoffs are mutually exclusive so that in equilibrium only one risky asset pays a return. The existence of government complicates things in a number of ways:

- the government also holds a risky asset so that without privatization consumers cannot fully diversify and therefore still face risk in terms of their private consumption;
- similarly, the households face risk in terms of public good consumption since the public good is financed entirely by the return on the government’s project;
- the expected return on the government’s project is unrelated to consumer demand for the public good so that even if the government could completely diversify its investments and returns, the level of the public good provided may not be optimal.

A partial privatization of government returns by a voucher system in which the shares are simply distributed lump sum to households affords partial diversification in consumption for households, but does not diversify consumption of the public good and also reduces consumption of the public good when the public good is provided.

SIP, in which the government also trades on financial markets affords diversification over both private and public good consumption.

With ex-post lump sum redistribution/taxation, the public sector can freely choose the level of public good and redistribute between the government and consumers. Therefore, the government can reach the first-best, and privatization has no real effects in terms of consumption and/or welfare. In other words, if the government is armed with non-distorting ex-post, state-conditioned taxes, there really is not much of a privatization story to tell. That is precisely why we will focus upon the tax-constrained environment in this paper.

The principal result is that a combination of voucher (free distribution) and share issue (public sale) privatization can implement the first-best allocation in an economy where the government has no access to lump sum taxation as fiscal tools. The two privatization instruments allow the planner to address the two failures in the no-government equilibrium: incomplete diversification of consumption across states and incomplete diversification between private and public goods within states. This result demonstrates that implementation using ex-ante lump sum redistribution via voucher property right transfers, combined with appropriate diversification-enhancing share trading by the government is equivalent to share trading with ex-post lump sum redistribution by taxation.
Our results can be easily extended in order to take into account a lower efficiency of the government as a shareholder. This is of interest because somehow or other, the usual literature on privatization is based on assumptions of a lower productivity of the public sector\textsuperscript{4}. With efficient government, the government should always use SIP to completely diversify the risks associated with the public project. Diversification requires the government to hold some residual share of its own asset. Once this issue is understood, the inefficient government case is very intuitive: the government should sell more of the public assets, because private management is more efficient. Adding public sector inefficiency reduces the optimal share to be held, but this share does not fall to zero: even if the public sector is inefficient, complete privatization is not optimal.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 briefly investigates the properties of the model with lump sum taxes and introduces different privatization methods. Section 4 deals with a simple implementation of an optimal mixed privatization. Section 5 compares this policy with the solution to an actual Ramsey program. Last section concludes.

2 The model

In this section, we present a two-period model without production. We introduce state-owned property rights and public good provision, in order to focus on privatization effects.

We consider a two-period model of a closed economy, populated by \( n \) private agents indexed by \( i \in \{1, \ldots, n\} \) interacting with a government indexed by \( 0 \). In the second period, there are \( 1 + S \) exogenously determined and equally likely states of nature indexed by \( s \in \{0, \ldots, S\} \), revealed at the beginning of the period. There are two types of goods in this economy, produced and consumed in the second period. Let \( c_i \) be the private good consumption of agent \( i \) and \( g_i \) its public good consumption. As the latter is a pure public good, we set \( g_i = G \), for every \( i \).

2.1 Preferences

Agent \( i \) has preferences for his consumption of public good and of private good, which are summarized by the utility function \( U(G, c_i) \). This function is strictly increasing and concave with respect to each argument, and satisfies the following condition:

\[
U(0, 0) = 0
\]

\[\text{Among other examples, see Blanchard and Aghion [1996], Roland and Verdier [1994], Schmitz [2001] and Verdier and Winograd [1998].}\]
2.2 Endowments and technology

In the first period each private agent has a property right over a second-period stochastic endowment in private good. More precisely the endowment of agent $i$ is equal to:

$$e_i(s) = \begin{cases} 
1 & \text{if } s = i \\
0 & \text{otherwise}
\end{cases}, \quad i = 1, \ldots, n$$

The property right in the first period can be interpreted as a specific risky project, which provides a return of 1 in a specific state of nature and of 0 otherwise. In this respect there is a complete specialization and no technological diversification at all. This property right can also be interpreted as an Arrow-Debreu security that pays only in one state of nature. The assumption may look quite extreme\(^5\). However, what is crucial here is not the identity between projects and states of nature, but the fact that the different projects are imperfectly correlated and that there are risk-sharing opportunities for risk-averse agents. We could envisage replacing the relation “one project - one state of nature” by $n$ linearly independent payoff vectors (one for each agent), each individual project giving different returns in different states of nature. This would complicate the analytical solution of the model, without changing the qualitative results.

The government has the same kind of property right at the beginning of the first period, over a second-period stochastic endowment (in private good) equal to:

$$e_0(s) = \begin{cases} 
1 & \text{if } s = 0 \\
0 & \text{otherwise}
\end{cases}$$

With neither taxes nor financial markets, the only resource the government has at its disposal is this stochastic endowment. This endowment in private good is used as input and converted into public good by a specific technology in the second period. By simplicity we consider an identity production function which transforms one unit of private good into one unit of public good. We assume that the public good provision is initially not diversified across states of nature and arises only in state of nature $s = 0$. Indeed the government has no input to produce the public good in the second period if a bad state ($s > 0$) occurs. The traditional literature on privatization usually makes assumptions on the lower productivity of the public sector. We do not impose such an assumption here to justify the privatization. The initial size of the public sector (relative to the whole economy) is simply given by its weight in the initial property rights, equal to $1/(n + 1)$.

2.3 Financial markets

Shares of the private property rights (claims on the stochastic endowments) can be traded on a financial market during the first period by the private agents (this is the only economic activity during this period). Therefore the stochastic revenue of agent $i$ in the second period is given

either by the share of its own project in the portfolio or the part of others’ project bought in the first period. Without taxes or subsidies, this revenue will constitute its private good consumption.

Let \( d_{ij} \) denote agent \( i \)'s demand for the asset issued by agent \( j \). The price of this asset is \( p_j \). In consequence, \( 1 - d_{ii} \) is by definition agent \( i \) supply in terms of share of its own initial property right, sold at price \( p_i \). \( 1 - d_{ii} \) is a measure of the extent to which agent \( i \) has decided to diversify its own risk. In our world à la Arrow-Debreu, asset markets are assumed to be perfectly competitive.\(^6\)

Without privatization, the public property right cannot be exchanged on the financial market.

### 2.4 Equilibrium with no economic policy

We consider the case where \( n \leq S \). In this case, there may be no production in some states of nature, and it will be impossible to eliminate all the risk by holding a portfolio of all existing assets. However, the need for assurance can be partially achieved through financial choices. As there is a complete specialization and no technological diversification at all, only financial diversification matters.\(^7\)

Based on the above assumptions concerning the financial market, we compute the decentralized equilibrium without any economic policy such as taxation or privatization. Agent \( i \) chooses asset demands, in order to maximize its expected utility, subject to its budget constraints:

\[
\begin{align*}
\max_{d_{ij}, j=1,...,n} & \quad E[U(G(s), c_i(s))] \\
\text{s.t.} & \quad \sum_{j=1}^{n} p_j d_{ij} \leq p_i \\
& \quad c_i(s) = d_{ij} \text{ if } s = j \in \{1,...,n\}, \ 0 \text{ otherwise}
\end{align*}
\]

The first-order conditions can be written as follows:

\[
\frac{U_2(G, d_{ij})}{U_2(G, d_{ij})} = \frac{p_j}{p_{j'}}, \text{ where } j \text{ and } j' \in \{1,...,n\}
\]

where \( U_2 \) denotes partial derivatives with respect to the second argument. Besides, the market-clearing condition for the initial property rights of an agent \( j \) is:

\[
\sum_{i=1}^{n} d_{ij} = 1
\]

Obviously, as agents are perfectly symmetric (same preferences, same endowments structure), we always have a symmetric equilibrium, such that \( p_i = p_j \equiv p \), and that, not surprisingly, private agents use the financial market to smooth their private good consumption across the different states of nature as much as possible:

\[
d_{ij}^* = \frac{1}{n}, \ j = 1,...,n
\]

\(^6\)In contrast to Martin and Rey [2000], no assumption of monopoly power is made.

\(^7\)For a model stressing the duality between financial and technological diversification, see Saint-Paul [1992].
Finally, if we choose a particular asset as a numeraire, the equilibrium price $p^*$ is equal to one. The equilibrium consumption of the two types of good for the agent $i$ is given by the following table.

<table>
<thead>
<tr>
<th>State of nature</th>
<th>$G$</th>
<th>$c_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s = 0$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$s \in {1, \ldots, n}$</td>
<td>0</td>
<td>$1/n$</td>
</tr>
<tr>
<td>$s &gt; n$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1. Equilibrium consumption, with no economic policy.

Why will this equilibrium differ from the first-best? Table 1 shows that there is a twofold diversification concern. First, there is a problem of diversification across states: there is private good consumption only in states $s \in \{1, \ldots, n\}$, and no private good consumption in state $s = 0$. Similarly, there is public good provision only in one state of nature. Second, this leads to a diversification problem between goods: preferences being convex, private agents wish to consume both types of goods. This twofold imperfection is due to the lack of transfer mechanisms between private and public agents.

### 2.5 First-best

With a utilitarian criterion, a central planner would maximize the sum of the private agents’ expected utilities, under a system of resource constraints:

$$\max_{G(s), c(s)} \sum_{i=1}^n E \left[ U \left( G(s), c_i(s) \right) \right]$$

subject to:

$$\begin{cases} G(s) + \sum_{i=1}^n c_i(s) \leq 1, & \text{if } s \in \{0, \ldots, n\} \\ G(s) + \sum_{i=1}^n c_i(s) = 0, & \text{if } s > n \end{cases}$$

Clearly if $s > n$, then $c(s) = G(s) = 0$. If $s \in \{1, \ldots, n\}$, taking into account the agents’ homogeneity, the solution is given by an equivalent reduced program:

$$\max_{G(s), c(s)} E \left[ U \left( G(s), c(s) \right) \right]$$

subject to:

$$\begin{cases} G(s) + nc(s) \leq 1, & \text{if } s \in \{0, \ldots, n\} \\ G(s) + nc(s) = 0, & \text{if } s > n \end{cases}$$

The first-order conditions can be written as follows:

$$\frac{nU_1(G(s), c(s))}{U_2(G(s), c(s))} = 1 \quad \text{if } s \in \{0, \ldots, n\} \quad (1)$$

$$G(s) + nc(s) = 1 \quad \text{if } s \in \{0, \ldots, n\} \quad (2)$$

$$G(s) = nc(s) = 0 \quad \text{if } s > n$$

Equation (1) is a simple Bowen-Lindahl-Samuelson condition: the left-hand side is the marginal rate of substitution between the public good and the private good; the right-hand side is the marginal rate of transformation. Not surprisingly, this condition, jointly with equation (2),
indicates that first-best consumptions of both types of goods are perfectly smoothed across the states of nature $s = 0, ..., n$. The optimal consumption plan is given by the following table:

<table>
<thead>
<tr>
<th>State of nature</th>
<th>$G$</th>
<th>$c^*_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s \in {0, ..., n}$</td>
<td>$G^*$</td>
<td>$c^*$</td>
</tr>
<tr>
<td>$s &gt; n$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2. First-best consumption.

Nothing indicates here that the level of private good consumption must be equal to the level of public good: this distribution between both types of good depends basically on the specification of the preferences. Thus, generically, we consider that, at the first-best, the following result holds:

$$c^* \neq G^*$$

### 3 Policy instruments

In this section, we briefly investigate the properties of this economy with lump sum taxes and then introduce different privatization methods. We will discuss the impact of voucher privatization and share issue privatization (hereafter, SIP) separately.

Firstly, if we assume that the government is armed with non-distorting, state-conditioned taxes, there is always an ex-post taxation design allowing the first-best consumption plan $(G^*, c^*)$ of this economy to be reached. Lump sum taxes allow the smoothing of public good provision and private good consumption across the different states of nature. Indeed, with ex-post lump sum redistribution/taxation, the first-best allocation can be achieved by a transfer of goods between the State and the private sector in the second period: the public sector can freely choose the level of the public good and redistribute between the government and consumers. Therefore, the government can reach the first-best.

Thus, with lump sum taxes, the first-best of this economy is always reachable. As a consequence, privatization, if implemented, has no real effects, in terms of consumption and/or welfare. In other words, if the government is armed with non-distorting, state-conditioned taxes, there really is not much of a story to tell in terms of privatization story. That is precisely why we will now focus upon the tax-constrained environment, in which the government is unable to finance the public good via taxation. As we mentioned in the introduction, this is of course an extreme assumption, but it is helpful in understanding what happens in countries where the tax system is known to exhibit inefficiencies.

### 3.1 A: voucher privatization

At the beginning of the first period, the government freely distributes shares of its property right. This distribution occurs ex-ante, i.e. before financial markets open. Each one of the $n$ private agents gets $1/n$ of the stocks issued by the government. Let $\pi_A$ be the voucher privatization
level, defined as a share of the initial public property right: a private agent $i$ gets an *additional* property right over a stochastic second-period endowment (in private good) equal to:

\[
\begin{cases}
\pi_A/n & \text{if } s = 0 \\
0 & \text{otherwise}
\end{cases}
\]

In the first period, private agents can immediately trade shares of this additional property right on a financial market. Let $d_{i0}$ denote agent $i$’s demand for these additional property rights, sold by other private agents. The price of this asset is $p_0$. The first-period budget constraint of a private agent $i$ becomes:

\[
\sum_{j=0}^{n} p_j d_{ij} \leq p_i + p_0 \pi_A/n
\]

The government now has a property right over a residual second-period endowment, equal to:

\[
\begin{cases}
1 - \pi_A & \text{if } s = 0 \\
0 & \text{otherwise}
\end{cases}
\]

As the privatization consists of free-distribution, there is no privatization revenue and consequently no government budget constraint in the first period.

A partial privatization by a voucher system in which the shares are simply distributed lump sum to households accomplishes partial diversification in consumption for households, but does not diversify consumption of the public good and also reduces consumption of the public good when the public good is provided.

### 3.2 $B$: SIP and holding of a diversified portfolio

In the first period the government sells shares of its own property right on a financial market. The price of the asset issued by the government is $p_0$. Without simultaneous voucher privatization, the first-period budget constraint of a private agent $i$ is:

\[
\sum_{j=0}^{n} p_j d_{ij} \leq p_i
\]

The government has a property right over a residual endowment in private good (to be transformed in public good), equal to:

\[
\begin{cases}
1 - \pi_B & \text{if } s = 0 \\
0 & \text{otherwise}
\end{cases}
\]

In addition, as the government sells, it gets a revenue from the privatization, equal to $p_0 \pi_B$. We assume that, thanks to this first-period revenue, the government buys a diversified portfolio, which is precisely constituted by the assets sold by the private agents. Let $d_{0i}$ be the government...
demand for an asset sold by the agent \( i \), as a share of his initial property right. The government now has a first-period budget constraint, that can be written:

\[
\sum_{j=1}^{n} p_j d_{0j} \leq p_0 \pi_B \tag{4}
\]

We assume here that the government keeps its diversified portfolio until the second period and that it has an additional property right, over a stochastic second-period endowment (in private good) that can be transformed in public good.

SIP, in which the government also trades on financial markets performs diversification over both private and public good consumption.

### 3.3 Mixed privatization

The government can simultaneously implement both types of privatization. In this case, the first-period budget constraint of agent \( i \) and the government, respectively become:

\[
\sum_{j=0}^{n} p_j d_{ij} \leq p_i + p_0 \frac{\pi_A}{n}
\]

\[
\sum_{j=1}^{n} p_j d_{0j} \leq p_0 \pi_B (1 - \pi_A) \tag{5}
\]

If, according to the previous notations, \( d_{00} \) is the demand of public assets the government directs to itself, then it can be defined as follows:

\[
d_{00} \equiv (1 - \pi_A) (1 - \pi_B) \tag{6}
\]

The global privatization extent \( \pi \) is simply the following:

\[
\pi = 1 - d_{00} = 1 - (1 - \pi_A) (1 - \pi_B)
\]

The consumption of the two types of good is given by the following table:

<table>
<thead>
<tr>
<th>State of nature</th>
<th>( G )</th>
<th>( c_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s = 0 )</td>
<td>( d_{00} \equiv (1 - \pi_A) (1 - \pi_B) )</td>
<td>( d_{0i} )</td>
</tr>
<tr>
<td>( s \in {1, \ldots, n} )</td>
<td>( d_{0j} )</td>
<td>( d_{ij} )</td>
</tr>
<tr>
<td>( s &gt; n )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3. Consumption under mixed privatization.

### 4 “Naive” privatization

In this section we intend to show that a naive government can implement the first-best allocation by using mixed privatization. More precisely, it is assumed that our government is naive in the sense that it plays as a \((n + 1)\)th price-taker agent on the market, without internalizing the impact of its decisions on the agents behavior. A very simple condition is then needed to implement the first-best allocation.
4.1 Diversification across states of nature through SIP

We are first interested in the only privatization method that allows the diversification of public good provision across the different states of nature, i.e. what we have called share issue privatization followed by the holding of a diversified portfolio (type B). We consider that the government treats the variable $\pi_B$ as endogenous. We first suppose that this is the only one instrument the government has at its disposal, and we consider the level of voucher privatization as given at some level $\pi_A$, zero or not.

From definition (6), with $\pi_A$ exogenous, considering $\pi_B$ as a policy instrument is equivalent to considering $d_{00}$ as an instrument. Consequently, we rewrite $\pi_B$ as a function of $d_{00}$, and the budget constraint of the government (5) can be stated as:

$$\sum_{j=0}^{n} p_j d_{0j} \leq p_0 (1 - \pi_A)$$

In order to compute all asset demands and supplies (from both the government and private agents), the programs of a private agent $i$ and of the government can be written, respectively:

$$\max_{d_{ij}, j=0,\ldots,n} E[U(G(s), c_i(s))]$$

s.t.

$$\begin{cases} \sum_{j=0}^{n} p_j d_{ij} \leq p_i + p_0 \frac{\pi_A}{n} \\ c_i(s) \text{ given by Table 3} \end{cases}$$

$$\max_{d_{0j}, j=0,\ldots,n} \sum_{i=1}^{n} E[U(G(s), c_i(s))]$$

s.t.

$$\begin{cases} \sum_{j=0}^{n} p_j d_{0j} \leq p_0 (1 - \pi_A) \\ G(s) \text{ given by Table 3} \end{cases}$$

We then aggregate demands and supplies in market-clearing conditions for both type of assets (the assets sold by private agents and the assets sold by the government). At the symmetric general equilibrium, we get a very simple expression of the optimal level of SIP (type B), which turns out to be strictly positive, and strictly less than 100%.

**Result 1** Without efficient taxation, there is justification of some share issue privatization, because of risk-sharing issues. Formally speaking, there is an optimal level of share issue privatization $\pi_B^*$ such that:

$$0 < \pi_B^* = \frac{n}{n+1} < 1$$

Notice first that this privatization level $\pi_B^*$ does not depend on $\pi_A$. This share is the only one allowing a smoothing of private good consumption and public good provision across the states of nature $s \in \{1,\ldots,n\}$. In other words, without efficient taxation, the government should always use SIP to completely diversify the risk associated with the public project; consequently, the government should retain some share of the public project, and this share should be perfectly equal to the weight of the public sector, relative to the states of nature that can be insured.

---

*All computations are given in the Appendix A.*
Under this SIP level, in equilibrium, agent $i$’s consumption plan for the two types of good is the following:

<table>
<thead>
<tr>
<th>State of nature</th>
<th>$G$</th>
<th>$c_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s \in {0, ..., n}$</td>
<td>$\frac{1 - \pi_A}{n+1}$</td>
<td>$\frac{1 + \pi_A}{n+1}$</td>
</tr>
<tr>
<td>$s &gt; n$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4. Equilibrium consumption under the SIP level $\pi_B^*$.  

Let us compare this consumption plan with the first-best described by (3). If the first-best was also characterized by a perfect smoothing of consumption across states of nature $s = 0, ..., n$, there is no guarantee at all that the consumption plan described by table 4 matches the first-best.  

As an illustration, in the case $\pi_A = 0$ (no simultaneous voucher privatization), we get:

$$c_i(s) = G(s) = \frac{1}{n+1}, \text{ for all } s \in \{0, ..., n\}$$

In contrast, at the first-best, private good consumption level is generically different from public good consumption level.

**Result 2** Without efficient taxation, the optimal level of share issue privatization $\pi_B^*$ leads to a consumption plan which is generically different from the first-best.

**Example 1** Assuming that there is no voucher distribution ($\pi_A = 0$) and that the utility of agent $i$ has the following separable form:

$$U(G, c_i) = \beta u(G) + (1 - \beta) u(c_i)$$

where $\beta$ is the weight of the public good in the preferences, it is simple to check that the $\pi_B^*$ SIP level leads to the first-best if and only if the size of the public sector is equal to the weight of the public good in the preferences, i.e. if and only if:

$$\frac{1}{n+1} = \beta$$

If the above condition does not hold, the $\pi_B^*$ SIP level does not lead to the first-best. This particularly happens if the initial size of the public sector is greater than the weight of the public good in the preferences, i.e. under the following assumption:

$$\frac{1}{n+1} > \beta$$

Thus, the simple implementation of $\pi_B^*$ does not automatically lead to the first-best. Furthermore, we cannot actually call it a second-best solution, in the sense that this is not an allocation found by maximizing social welfare with SIP as the only instrument, subject to the restriction it can be decentralized as a competitive equilibrium. This actual second-best computation is left to section 5.

However, result 2 is not surprising: the use of only one instrument (SIP) cannot resolve simultaneously the double diversification problem i) between the public and the private goods and ii) between the different states of nature. As we shall see, a second instrument is needed for that.
4.2 Altering the size of the public sector through voucher privatization

As said above, without taxation, the implementation of the \( \pi_B^* \) SIP level generically leads to a sub-optimal consumption plan. The government needs to alter the relative size of the public sector (defined as the weight of the government in the initial property rights) to reach the first-best. Let us forget for a short while the diversification issues, to focus on the problem of public sector size. A level \( \pi_A > 0 \) for a voucher privatization (type A) simply reduces the size of the public sector to:

\[
\frac{1 - \pi_A}{n + 1}
\]

Therefore it is possible to deal with voucher distribution (type A privatization), to alter the size of the public sector. In other words, public sector privatization through voucher distribution can be examined as a mechanism for adjusting the amount of the public good, if that has been fixed at some non-optimal level.

4.3 The optimal privatization mix

Eventually we combine results 1, 2 and the intuition of the previous paragraph. The conjecture we want to prove claims that, if the size of the public sector is initially too high, then there is a privatization mix such that the economy reaches its first-best. The optimal privatization mix would be composed by:

- voucher privatization \( (\pi_A) \) to reach an optimal size of the public sector;
- a partial sale of the rest \( \pi_B (1 - \pi_A) \) to decentralize the general equilibrium and optimally smooth both types of consumption across the different states of nature.

Indeed, as previously described, privatization by a voucher system can decrease the expected size of the public sector. Such a policy can lead to a first-best outcome if combined with SIP, which is used to completely diversify both private and public good consumption. Broadly speaking voucher privatization equalizes both the subjective and objective weight of public good given respectively by the preferences and the initial influence of the State in the market, and implements the condition the market needs to automatically decentralize the first-best.

Formally, once the government knows the \( \pi_B^* \) level, the next step is a backward adjustment of the initial free distribution \( \pi_A^* > 0 \) such that the general equilibrium coincides with the first-best consumption plan. Combining the first-best Bowen-Lindahl-Samuelson condition (see equation (1)) with the above table 4, giving the consumption levels if \( \pi_B^* \) is implemented, allows a definition of the optimal level of voucher privatization \( \pi_A^* \), leading to the first-best of the economy. This optimal level \( \pi_A^* \) is such that:

\[
U_2 \left( \frac{1 - \pi_A^*}{n + 1}, \frac{1 + \pi_A^*/n}{n + 1} \right) = nU_1 \left( \frac{1 - \pi_A^*}{n + 1}, \frac{1 + \pi_A^*/n}{n + 1} \right)
\]

(10)

This leads to the following proposition.
**Proposition 1** Without efficient taxation, there is an optimal privatization mix (voucher privatization and SIP) such that the economy reaches the first-best.

This key proposition demonstrates that implementation using ex-ante lump sum redistribution via voucher property right transfers, combined with appropriate diversification-enhancing share trading by the government is equivalent to share trading with ex-post lump sum redistribution via taxation.

At this stage, we can adopt an additively separable form for preferences, in order to understand the condition under which a positive voucher privatization is needed.

**Example 2** Assuming that the utility of agent $i$ has the following form:

$$U(G,c_i) = \beta u(G) + (1 - \beta) u(c_i)$$

where $\beta$ is the weight of the public good in the preferences, condition (10) can be rewritten:

$$(1 - \beta) u'(\frac{1 + \pi_A^* / n}{n + 1}) = \beta n u'(\frac{1 - \pi_A^*}{n + 1})$$

Then, it is simple to check that a voucher privatization ($\pi_A^* \geq 0$) is needed, to reduce the size of the public sector, if and only if:

$$\frac{1}{n + 1} \geq \beta$$

i.e. if and only if the initial size of the public sector (the left-hand term) is greater than the weight of the public good in the preferences ($\beta$). If the reverse inequality holds, some nationalization is needed, in order to increase the size of the public sector.

### 4.4 Public sector inefficiency

Without efficient taxation but with efficient government as a shareholder, and conditional on the level of voucher privatization, the government should always diversify the risks associated with the public project. Diversification requires the government to hold some residual share of the public project. Once this issue is understood, the inefficient government case is very intuitive: the government should sell even more of the public property rights because private management is more efficient. Indeed, introducing public sector inefficiency in the setup without taxation leads to a trade off between two effects. On the one hand, such an inefficiency might call for the limitation of state-owned property rights. On the other hand, because of the lack of an efficient fiscal system, some maintenance of these public property rights is required to ensure consumption smoothing. Simply, adding public inefficiencies reduces the optimal share to be held, but this optimal share does not fall to zero.
5 Second-best policy

As mentioned before, the privatization program proposed above is not the solution to a Ramsey program. Formally, this Ramsey program is written as the maximization of a social welfare function, subject to the budget constraints of the individuals, the budget constraint of the government, the market-clearing conditions, and a decentralization constraint. This constraint itself comes from the first-order conditions of the individual program, and takes into account the fact that, at the decentralized equilibrium, prices ratios must equal marginal utilities ratios.\(^9\)

Of course, a Ramsey program with the two instruments available (\(\pi_A\) and \(\pi_B\)) leads to the first-best, and the conclusion of the existence of the optimal privatization mix holds, with \(\pi_A^*\) and \(\pi_B^*\) given respectively by equations (10) and (9).

Of even greater interest is the solution to the second-best program, with only one instrument (SIP, through the choice of the \(\pi_B\) level), and the comparison of this solution to the simple \(\pi_B^*\) implementation proposed above. In other words, if the government cannot act on \(\pi_A\), what is the loss of a naive privatization, compared to an actual second-best policy?

Once again, the use of only one instrument (SIP) cannot completely resolve the double diversification problem. However, at the second-best, the government does not equalize public good provision anymore (thus, private good consumption) in the \((n+1)\)th state of nature to consumption levels in the first \(n\) states, in order to reach a better allocation between private and public good in these states. The SIP level does now depend on the fixed voucher privatization level (\(\pi_A\)). In terms of welfare, this second-best policy is better than the simple implementation of \(\pi_B^*\). Thus, if, somehow or other, \(\pi_A\) is exogenously fixed at some non-optimal level, this second-best solution dominates the simple implementation of \(\pi_B^*\). Obviously, the second-best corresponds to the simple implementation of \(\pi_B^*\) if \(\pi_A\) is exogenously fixed at its optimal level \(\pi_A^*\). The second-best also corresponds to the simple implementation of \(\pi_B^*\) if \(\pi_A\) tends towards one: in this case, all of the initially public property rights are given through free distribution. Large free distribution does not leave SIP much room to maneuver.

If implementing a naive privatization clearly implies a loss in welfare, compared to a second-best policy, numerical simulations tend to prove that this loss is quantitatively small. Figure 1 below helps to compare the second-best with the naive implementation of \(\pi_B^*\). The relative difference in welfare, as a function of \(\pi_A\) (on the horizontal axis) is plotted. The relative difference in welfare (on the vertical axis) is simply defined as follows:

\[
\text{loss } L = \frac{\text{welfare at the second-best} - \text{welfare under the naive implementation of } \pi_B^*}{\text{welfare at the second-best}} \times 100
\]

In Figure 1, we present the results for a numerical configuration\(^{10}\) where the initial size of the government is larger than the weight of the public good in the preferences. The conclusion is

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\(^9\)All computations are given in the Appendix B.

\(^{10}\)It is assumed that the utility function \(U(G, c_i)\) satisfies \(U(G, c_i) = \beta G^\rho + (1 - \beta) c_i^\rho\). Parameters are fixed as follows: \(\beta = .25, \rho = .5\) and \(n = 2\).
that, even in this unfavorable case, the welfare loss resulting of the simple implementation of $\pi_B^*$ is less than 1%. More precisely, we compute that for $\pi_A = 0.01$, i.e. practically no free distribution, the relative loss due to the naive method with respect to the second-best, is only 0.26%.

Figure 1. Comparison between the second-best and the simple implementation of $\pi_B^*$: relative difference in welfare, as a function of $\pi_A$.

6 Conclusion

We have presented a two-period general equilibrium model of a closed country with state-owned property rights and public good production, to focus on privatization effects. In this framework, we have shown that if lump sum taxes are implemented, privatization has no real effects. On the other hand, if taxes are not available, there is a privatization mix (voucher privatization and share issue privatization) such that the economy reaches its first-best. If we introduce public inefficiency, there is still a justification for the maintenance of some public property rights. However the optimal degree of global privatization increases with public sector inefficiency.

The theoretical framework developed here can be applied in numerous ways. First, the model can be expanded to an international context, in order to investigate the financial effects of voucher and SIP programs in a two-country general equilibrium setup. In this case, even voucher privatization to domestic agents will have financial effects, as it introduces heterogeneity.

11 See for instance Girmens [2002].
between domestic agents and foreign ones. Consequently, the portfolio choice of both domestic and foreign agents will be altered by privatization, thus affecting private asset prices, supplies of private assets, demands for assets, market capitalization and international asset allocation strategies.

Another interesting application consists in the replacement of exogenous fixed-size projects by endogenous investment decisions in the first period (with, simultaneously, a consumption/saving decision to make), in order to connect privatization, investment and financial market development. In fact, the channel through which privatization may affect financial market development in this paper is based on the role of financial markets in achieving the need for insurance, but financial markets also facilitate such intertemporal choices as saving and investing. So, explicitly taking into account consumption-saving and investment decisions, the following questions arise:

- How does privatization influence financial markets, taking into account both insurance and intertemporal issues?
- Does privatization lead to an increase in private investment?

The answer clearly depends both on intertemporal substitution and on risk aversion, leading to the adoption of a utility specification which allows the roles played by these two distinct aspects of preferences to be isolated.\textsuperscript{12}

\textsuperscript{12}See for instance Girmens and Guillard [2002].
Diversification across states of nature through SIP

The programs of a private agent $i$ and of the government are given, respectively, by (7) and (8). Using Table 3, a household’s program can be rewritten:

$$
\max_{d_{ij}, j=0,\ldots,n} \frac{1}{n} \sum_{j=0}^{n} U(d_{0j}, d_{ij})
$$

s.t. $\sum_{j=0}^{n} p_j d_{ij} \leq p_i + p_0 \frac{\pi_A}{n}$

The first order conditions can be summarized as:

$$
\frac{U_2(d_{0j}, d_{ij})}{U_2(d_{00}, d_{i0})} = \frac{p_j}{p_0}, \ j = 0, 1, \ldots, n
$$

$$
\sum_{j=0}^{n} \frac{p_j}{p_0} d_{ij} = \frac{\pi_A}{n} + \frac{p_i}{p_0}
$$

Government implements a naive program. $\pi_A$ is given and $d_{00} = (1 - \pi_A) (1 - \pi_B)$ is determined by $\pi_B$. Using Table 3, the program of the government can be rewritten:

$$
\max_{d_{0j}, j=1,\ldots,n} \sum_{i=1}^{n} \frac{1}{n} \left( U((1 - \pi_A) (1 - \pi_B), d_{0i}) + \sum_{j=1}^{n} U(d_{0j}, d_{ij}) \right)
$$

s.t. $p_0 (1 - \pi_A) (1 - \pi_B) + \sum_{j=1}^{n} p_j d_{0j} \leq p_0 (1 - \pi_A)$

The first order conditions can be summarized as:

$$
\frac{\sum_{i=1}^{n} U_1(d_{0j}, d_{ij})}{\sum_{i=1}^{n} U_1(d_{00}, d_{i0})} = \frac{p_j}{p_0}, \ j = 0, 1, \ldots, n
$$

$$
\sum_{j=0}^{n} \frac{p_j}{p_0} d_{0j} = 1 - \pi_A
$$

Moreover, the equilibrium is characterized by:

$$
\sum_{i=0}^{n} d_{ij} = 1, \ j = 0, 1, \ldots, n
$$

This system of equations entails symmetry:

$$
d_{ij} = d_{i'j'} = d_{ij'}; \ i, i', j, j' = 1, \ldots, n
$$

$$
d_{i0} = d_{i0'}; \ i, i' = 1, \ldots, n
$$

$$
d_{0j} = d_{0j'}; \ j, j' = 1, \ldots, n
$$

$$
p_j = p_{j'}; \ j, j' = 1, \ldots, n
$$
Then:

\[
\frac{U_2 (d_{0j}, d_{ij})}{U_2 (d_{00}, d_{i0})} = \frac{p_j}{p_0} \\
\frac{nU_1 (d_{0j}, d_{ij})}{nU_1 (d_{00}, d_{i0})} = \frac{p_j}{p_0} \\
d_{i0} + np_jd_{ij} = \frac{\pi_A}{n} + \frac{p_j}{p_0} \\
d_{00} + np_jd_{0j} = 1 - \pi_A
\]

\[
d_{0j} = 1 - \sum_{i=1}^{n} d_{ij} = 1 - nd_{ij}
\]

That leads to:

\[
\frac{U_1 (1 - nd_{ij}, d_{ij})}{U_2 (1 - nd_{ij}, d_{ij})} = \frac{U_1 (1 - nd_{i0}, d_{i0})}{U_2 (1 - nd_{i0}, d_{i0})}
\]

which implies

\[
d_{i0} = d_{ij}
\]

And finally:

\[
d_{ij} = \frac{1}{n+1} \left( 1 + \frac{\pi_A}{n} \right) \\
\pi_B = \frac{n}{n+1}
\]
APPENDIX B

Second-best policy

To start, let us write the first-best Ramsey program:

\[
\max_{d_{ij}, i, j=0,1,\ldots,n} \sum_{i=1}^{n} \sum_{j=0}^{n} \frac{1}{S} U(d_{0j}, d_{ij})
\]

\[
\sum_{j=0}^{n} p_{j} d_{ij} \leq p_{0} \frac{\pi_{A}}{n} + p_{i}, \; i = 1, \ldots, n \tag{B1}
\]

\[
\sum_{j=0}^{n} p_{j} d_{0j} \leq p_{0} (1 - \pi_{A}) \tag{B2}
\]

\[
\sum_{i=0}^{n} d_{ij} \leq 1, \; j = 0, 1, \ldots, n \tag{B3}
\]

\[
\frac{p_{j}}{p_{0}} = \frac{U_{2}(d_{0j}, d_{ij})}{U_{2}(d_{00}, d_{i0})}, \; j = 1, \ldots, n \tag{B4}
\]

The objective function is a social welfare function, in which we replaced consumptions \(G\) and \(c_{i}\) by asset demands \(d_{0j}\) \((j = 0, \ldots, n)\) and \(d_{ij}\) \((j = 0, \ldots, n)\), as stated in Table 3. Constraints set (B1) is the set of the budget constraints of private agents \((i = 1, \ldots, n)\). Constraint (B2) is the government budget constraint. Constraints set (B3) describes the equilibrium on all asset markets. Finally, constraints set (B4) is a set of decentralization (or market implementation) constraints, coming from the first-order conditions of the individual programs: the government internalizes private agents’ best response to prices signals, such that at equilibrium price ratios must equal marginal utilities ratios (marginal rates of substitution). Substituting (B4) into (B1) and (B2), the program becomes:

\[
\max_{d_{ij}, i, j=0,1,\ldots,n} \sum_{i=1}^{n} \sum_{j=0}^{n} \frac{1}{S} U(d_{0j}, d_{ij})
\]

\[
\sum_{j=0}^{n} \frac{U_{2}(d_{0j}, d_{ij})}{U_{2}(d_{00}, d_{i0})} d_{ij} \leq \frac{\pi_{A}}{n} + \frac{U_{2}(d_{0i}, d_{ii})}{U_{2}(d_{00}, d_{i0})}, \; i = 1, \ldots, n \tag{B5}
\]

\[
\sum_{j=0}^{n} \frac{U_{2}(d_{0j}, d_{ij})}{U_{2}(d_{00}, d_{i0})} d_{0j} \leq 1 - \pi_{A} \tag{B6}
\]

\[
\sum_{i=0}^{n} d_{ij} \leq 1, \; j = 0, \ldots, n \tag{B7}
\]

Symmetry implies:

\[
d_{ij} = d_{i'j} = d_{ij'}; \; i, i', j, j' = 1, \ldots, n
\]

\[
d_{i0} = d_{i'0}; \; i, i' = 1, \ldots, n
\]

\[
d_{0j} = d_{0j'}; \; j, j' = 1, \ldots, n
\]
Market-clearing conditions (B7) are binding at equilibrium. With (B8), this jointly leads to:

\[ d_{0j} = 1 - nd_{ij}, j = 0, \ldots, n \quad (B9) \]

Let us also recall that:

\[ d_{00} \equiv (1 - \pi_A) (1 - \pi_B) \quad (B10) \]

We are looking for a second-best policy. With \( \pi_A \) exogenously fixed, using (B8), (B9) and (B10), the program becomes:

\[
\max_{d_{i0}, d_{ij}} U (1 - nd_{i0}, d_{i0}) + nU (1 - nd_{ij}, d_{ij})
\]

\[
0 \leq \frac{\pi_A}{n} - d_{i0} + \frac{U_2 (1 - nd_{ij}, d_{ij})}{U_2 (1 - nd_{i0}, d_{i0})} (1 - nd_{ij})
\]

\[
0 \leq \frac{1 - \pi_A}{n} \pi_B - \frac{U_2 (1 - nd_{ij}, d_{ij})}{U_2 (1 - nd_{i0}, d_{i0})} (1 - nd_{ij})
\]

Summing (B11) and (B12) (binding at equilibrium), we notice that \( \pi_B \) depends only on \( d_{i0} \):

\[ d_{i0} = \frac{1}{n} (1 - (1 - \pi_A) (1 - \pi_B)) \quad (B13) \]

The program becomes

\[
\max_{d_{i0}, d_{ij}} U (1 - nd_{i0}, d_{i0}) + nU (1 - nd_{ij}, d_{ij})
\]

\[
0 \leq \frac{\pi_A}{n} - d_{i0} + \frac{U_2 (1 - nd_{ij}, d_{ij})}{U_2 (1 - nd_{i0}, d_{i0})} (1 - nd_{ij})
\]

The solution \((d_{i0}, d_{ij})\) is implicitly given by the first-order conditions of this program, including constraint (B14), binding at equilibrium. The system (2 equations, 2 unknowns) resulting in this solution can be written:

\[
\frac{U^j}{U_{21}^0 - nU_{21}^0} = \frac{U^j}{U_{22}^0 - nU_{22}^0}
\]

\[
0 = \frac{\pi_A}{n} - d_{i0} + \frac{U^j} {U_{22}^0} (1 - nd_{ij})
\]

where:

\[ U_1^0 \equiv U_1 (1 - nd_{i0}, d_{i0}) \quad U_{21}^0 \equiv U_{21} (1 - nd_{i0}, d_{i0}) \]

\[ U_2^0 \equiv U_2 (1 - nd_{i0}, d_{i0}) \quad U_{22}^0 \equiv U_{22} (1 - nd_{i0}, d_{i0}) \]

\[ U_1^j \equiv U_1 (1 - nd_{ij}, d_{ij}) \quad U_{21}^j \equiv U_{21} (1 - nd_{ij}, d_{ij}) \]

\[ U_2^j \equiv U_2 (1 - nd_{ij}, d_{ij}) \quad U_{22}^j \equiv U_{22} (1 - nd_{ij}, d_{ij}) \]

\( \pi_B \) is eventually given by equation (B13).
References


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<th>Authors</th>
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