Partial Indexation and Inflation Dynamics: What Do the Data Say?

Jean-Guillaume SAHUC

04 – 06
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Abstract

We provide new evidence on the fit of the hybrid Phillips curve based on indexation of prices to lagged inflation and trend inflation for the Euro area and the United States over the period 1970-2002. The GMM-West estimates suggest that (i) a full indexation scheme is not data consistent whereas a partial indexation scheme allows a good fit and (ii) forgetting trend inflation induces overestimating by approximately 3-4 percent of the probability to not change its prices, for reasonable values of trend inflation.

Keywords: Phillips curve; inflation inertia; trend inflation; degree of indexation

JEL classification: E31 ; C22

\* I would like to thank Eric Jondeau, Hervé Le Bihan, Julien Matheron, Tristan-Pierre Maury, and Ferhat Mihoubi for remarks. This paper represents the views of the author and should not be interpreted as reflecting those of the Banque de France.

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1 Introduction

A recent class of dynamic stochastic general equilibrium models integrates Keynesian features, such as imperfect competition and nominal rigidities, resulting in a new view on the nature of inflation dynamics. These models are grounded in an optimizing framework where imperfectly competitive firms are constrained by costly or infrequent price adjustments. More specifically, the New Keynesian Phillips curve is based on Calvo’s (1983) formulation: when firms have the opportunity to change their price they set this price equal to the average desired price until the next opportunity arises.

However, two main issues are still problematic. First, most of the papers in the sticky-price literature are based on a log-linearization around the zero inflation steady state but unfortunately this assumption is counterfactual. Ascari (2003) has clearly shown that disregarding trend inflation is quite far from being an innocuous assumption and that results obtained by models log-linearized around a zero inflation steady state are misleading. Second, at the aggregate level, current inflation will depend on future expected inflation but not on lagged inflation. However, this specification has been criticized on the ground that it does not fit very well the econometric evidence about co-movements of real and nominal variables: according to the New Keynesian Phillips curve, inflation should be a more forward-looking than seems to be (see for example Gali et al., 2001, and Sahuc, 2002).

This paper reexamines the empirical relevance of the New Keynesian Phillips curve for the Euro area and the United States. In particular, we argue that the two last problems may be solved at once by resorting to a model where indexation on past inflation is allowed. This framework, advocated by Christiano et al. (2003), Sbordone (2003), Smets and Wouters (2003), and Woodford (2003), assumes that prices are automatically raised in accordance with some mechanical rule between the occasions on which they are reconsidered. We extend this mechanism to the case of positive trend inflation and strategic complementarity, and provide evidence of this modified hybrid Phillips curve in conducting some instrumental variable estimations.

2 The partial price indexation mechanism

The forward looking model of price setting due to Calvo (1983) is modified to allow for the possibility that firms that do not optimally set their prices
may nonetheless adjust it to keep up with the previous period increase in
the general price level. Although this seems to imply some sort of irrational
behavior of the firms since inflation data is freely available, it is often argued
that in low inflation environments this kind of behavior is normal.

In each period, a firm faces a constant probability, $1 - \phi$, of being able to
reoptimize its nominal price and chooses a price $P_t^*(z)$ that maximizes the
expected discounted sum of profits

$$E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} [P_t^*(z) X_{t,t+j} - MC_{t,t+j} (z)] \frac{Y_{t+j}(z)}{P_{t+j}},$$

subject to the sequence of demand constraints:

$$Y_{t+j}(z) = \left( \frac{P_t^*(z) X_{t,t+j}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j},$$

where $\Lambda_{t,t+j} = \beta^j \left( U' (C_t) / U' (C_{t+j}) \right)$ is the discount factor between time $t$
and $t + j$, $U' (C_{t+j})$ is the marginal utility of consumption in $t + j$, $Y_t (z)$ is the
level of output of firm $z$, $MC_{t,t+j}$ is the nominal marginal cost at $t + j$ of the
firm that optimally set prices at time $t$, $\varepsilon > 1$ is the elasticity of substitution
across goods, and

$$X_{t,t+j} = \begin{cases} \Pi_{k=0}^{j-1} \bar{\pi}^{1-\varepsilon} \pi^{\varepsilon}_{t+k} & j > 0 \\ 1 & j = 0 \end{cases}$$

(3)

The coefficient $\xi \in [0, 1]$ denotes the degree of indexation to past prices, during
the periods in which firm is not allowed to reoptimize, and the gross inflation
rate is defined by $\pi_t = P_t / P_{t-1}$.

**Remark 1** To cancel the effects of trend inflation ($\bar{\pi}$), we must assume that
the prices that cannot be reset are indexed not only partially to past inflation
rate but also partially to trend inflation. Indeed, all firms charge the same price
whatever the rate of money growth, because the updating rule coincides with
the steady state optimal pricing rule.

It follows that the aggregate price level can be expressed as:

$$P_t = \left[ \phi \left( \bar{\pi}^{1-\varepsilon} \pi_{t-1}^{\varepsilon} P_{t-1} \right)^{1-\varepsilon} + (1 - \phi) (P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.$$

(4)

Let us define the relative price by $p_t^*(z) = P_t^*(z) / P_t$, the real marginal
cost by \( m_{ct,t+j}(z) = MC_{ct,t+j}(z)/P_{t+j} \), and using the fact that \( X_{ct,t+j} = \hat{\pi}^{1-\xi}_j \left( \frac{P_{t+j-1}}{P_{t-1}} \right)^{\xi} \), the first-order condition of this problem can be expressed as

\[
E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{ct,t+j} Y_{t+j} (z) \left[ P_t^* \hat{\pi}^{1-\xi}_j \left( \frac{P_{t+j-1}}{P_{t-1}} \right)^{\xi} \frac{P_t}{P_{t+j}} - \frac{\varepsilon}{\varepsilon - 1} m_{ct,t+j}(z) \right] = 0 \quad (5)
\]

Finally, log-linearizing around a steady state with generic trend inflation the first-order condition and the law of motion (4) for the Dixit-Stiglitz price index (together with a series of manipulations), then yields a log-linear New Phillips curve with partial indexation (NPCPI) of the form,\(^1\)

\[
\hat{\pi}_t = \frac{\xi}{1 + \beta \xi} \hat{\pi}_{t-1} + \frac{\beta}{1 + \beta \xi} E_t \hat{\pi}_{t+1} + \frac{(1 - \phi \beta) (1 - \phi)}{(1 + \beta \xi) \phi (1 + \omega \varepsilon)} \overline{mc}_t \quad (NPCPI)
\]

3 Assessing the empirical properties of the hybrid Phillips curve

We next present estimates of the hybrid model with partial indexation for the Euro area and the United States. Under rational expectations, the set of orthogonality conditions is

\[
E \left\{ \left[ \hat{\pi}_t - \frac{\xi}{1 + \beta \xi} \hat{\pi}_{t-1} - \frac{\beta}{1 + \beta \xi} \hat{\pi}_{t+1} - \frac{(1 - \phi \beta) (1 - \phi)}{(1 + \beta \xi) \phi (1 + \omega \varepsilon)} \overline{mc}_t \right] \times z_t \right\} = 0 \quad (6)
\]

where \( z_t \) denotes a \( k \times 1 \) vector of relevant instruments. It includes here four lags of inflation, real marginal cost, output gap (linearly detrended log output) and short interest rates.

\(^1\) Under the hypothesis that capital is not reallocated across firms, \( m_{ct,t+j} \) is in general different from the average marginal cost at time \( t + j \). This is the reason of the presence of the term \( (1 + \omega \varepsilon) \) in \((NPCPI)\) where \( \omega \) is the output elasticity of real marginal cost for the individual firm.

In a technical appendix available from the author upon request, we detail all the intermediate steps involved in deriving these results.
3.1 Data

All data are quarterly time series over the period 1970:1-2002:4. To measure inflation we use the GDP deflator. Our measure of average real marginal cost is the log of real unit labor costs. Accordingly, we use the log deviation of real unit labor costs from its mean as a measure of $m\hat{c}_t$. Our data for the Euro area come from the updated database by Fagan et al. (2001). Unit labor costs are constructed as the ratio of compensation to employees (WIN) to GDP (YEN). Inflation is measured as the quarterly percent change in the GDP deflator (YED). The data for the U.S. come from the FRED II database. In particular, real unit labor costs are for the non-farm business sector. Figure 1 and 2 show the series used in this study.

3.2 GMM methodology

We present the limited information strategy to estimate the hybrid Phillips curve. Let define the random variable $\varepsilon_t$ such that

$$
\varepsilon_t = \left[ \hat{\pi}_t - \frac{\xi}{1 + \beta \xi} \hat{\pi}_{t-1} - \frac{\beta}{1 + \beta \xi} \hat{\pi}_{t+1} - \frac{(1 - \phi \beta)}{(1 + \beta \xi)} \frac{(1 - \phi)}{(1 + \omega \varepsilon)} m\hat{c}_t \right] \tag{7}
$$

Hansen (1982) provides conditions under which (6) can be used to consistently and efficiently estimate $(\xi, \beta, \phi)$ using Generalized Method of Moments (GMM). To discuss the procedure in our context we define the vector

$$
g_T (\xi, \beta, \phi) = \frac{1}{T} \sum_{t=1}^{T} [\varepsilon_t (\xi, \beta, \phi) \times \mathbf{z}_t].
$$

where $T$ denotes the size of the sample. We also denote the true value of $(\xi, \beta, \phi)$ by $(\xi_0, \beta_0, \phi_0)$. The vector $g_T (\xi, \beta, \phi)$ is a consistent estimator of $E [\varepsilon_t (\xi, \beta, \phi) \times \mathbf{z}_t]$. We estimate the parameter vector $(\xi_0, \beta_0, \phi_0)$ by choosing $(\xi, \beta, \phi)$ to make $g_T (\xi, \beta, \phi)$ as close as possible to zero as possible in the sense of minimizing

$$
J_T = \{g_T (\xi, \beta, \phi)\}' W_T \{g_T (\xi, \beta, \phi)\}.
$$

$W_T$ is a symmetric positive definite matrix that can depend on sample information. A given choice of $W_T$ implies that we are choosing $(\xi, \beta, \phi)$ to minimize the sum of squares of $k$ linear combinations of the elements of $g_T (\xi, \beta, \phi)$. 

5
Hansen (1982) shows that the choice of $W_T$ that minimizes the asymptotic covariance matrix of our estimator depends on the serial correlation properties of the error term $\varepsilon_t(\xi, \beta, \phi)$. If the hybrid Phillips curve is well specified, the error term is serially uncorrelated and has a moving average representation. West (1997) proposed a long-run covariance matrix estimator that is positive semidefinite by construction and that is applicable when the disturbance follows a moving average (MA) process of known order, and the innovations in this MA process have zero mean conditional on past disturbances and current and past instruments.

Assuming the error term $\varepsilon_t$ is driven by a $MA(q)$ process, it yields

$$\varepsilon_t = \eta_t + \theta_1 \eta_{t-1} + \ldots + \theta_q \eta_{t-q}$$

The $\hat{\theta}$'s and $\hat{\eta}$'s may be obtained by non linear least square applied to $\hat{\varepsilon}_t$. Then, for $t = 1, \ldots, T - q$, we can compute the weighting matrix

$$\Omega_T = \frac{1}{T-q} \sum_{t=1}^{T-q} \hat{d}_{t+q} \hat{d}_{t+q}^\prime$$

where $\hat{d}_{t+q} = (z_t + z_{t-1} \theta_1 + \ldots + z_{t-q} \theta_q) \hat{\eta}_t$.

The covariance matrix is minimized when $W_T = \Omega_T^{-1}$ and the standard errors of the optimal GMM estimator are calculated as the square roots of the diagonal elements of $\frac{1}{T} \left( \Gamma_T^\prime \Omega_T^{-1} \Gamma_T \right)^{-1}$, where $\Gamma_T = E \left( \frac{\partial [\varepsilon_t(\xi, \beta, \phi) \times z_t]}{\partial (\xi, \beta, \phi)^\prime} \right)$.

### 3.3 Empirical results

We begin by analyzing results based on reduced form of the hybrid Phillips curve. The estimated coefficients $\alpha_b = \xi / (1 + \beta \xi), \alpha_f = \beta / (1 + \beta \xi)$, and $\lambda = [(1 - \phi \beta) (1 - \phi)] / [(1 + \beta \xi) \phi (1 + \omega \varepsilon)]$ are given in Table 1. Overall, the empirical hybrid model works reasonably well in both cases. The slope coefficient on marginal cost is positive in each case, as implied by the theory. The standard errors suggest some imprecision in the point estimate, but the coefficient in each case are significantly different from zero. These estimates imply that backward looking behavior is slightly less important than forward

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2 In our study, $q = 1$.

3 We set $\varepsilon = 10$ and $\omega = 1.25$ as is conventionally assumed in the literature (see Woodford, 2003).
looking behavior in the Euro area as well as in the United States. Just like Gali et al. (2001) or Jondeau and Le Bihan (2001), we find that inflation dynamics in the Euro area appears to have a stronger forward-looking component that in the United States. It is noticed finally, that the sum of the backward and forward parameters is very close to one (but strictly lower) and that the distribution is far from being equal as it is supposed in many works.

We next estimate the structural parameter $\xi, \beta$ and $\phi$. Table 2 summarizes the results. We first impose the full indexation scheme ($\xi = 1$) considered for example in Christiano et al. (2003). The model predicts that the growth rate of inflation depends upon real marginal costs and the expected future growth rate of inflation. In this case, coefficients on past and future inflation sum to 1, and, for $\beta$ close to 1, they are approximately the same. Unfortunately this scheme is not consistent with the data since it implies values of $\beta$ that are implausible. One can even say that there is more evidence against the model for the Euro area, based on the $J$-stat. Conversely from the structural model with partial indexation, we see that all parameters are estimated with relatively small standard errors. Especially, we finds degree of inertia significant with $\xi = 0.408$ for the Euro area and $\xi = 0.639$ for the United States. Prices appear to be more flexible in the U.S. than in the Euro area, i.e the average duration of price rigidity is shorter: 3 quarters for the U.S. and 4.8 quarters for the Euro area. To impose $\beta$ to be equal to 0.99, as the theory suggests it, increases at the same time the degree of indexation and the probability to not change the price.

We now check the recommendations concerning the omission of trend inflation putted into lights by Ascari (2003) and Sahuc (2004). Ascari (2003) affirms that the omission of trend inflation involves an overestimate of the parameter $\phi$, and Sahuc (2004) claims that this omission may be neglected for a rather strong value of $\xi$. The last author derives the expression of the hybrid curve with trend inflation under strategic substitutability and shows that its presence alters the structure of the curve in two ways. First, the coefficients on past and future inflation are functions of the degree of indexation and trend inflation. Second, there is a complex additional forward-looking structure. The additional forward-looking term is very difficult to apprehend, we ignore it at first approximation in order to concentrate us on the other variables. In the present case of strategic complementarity (6) becomes
where

\[ \Phi_1 = 1 + \xi \beta \bar{\pi}(1+\omega\epsilon)(1-\xi) + (1 - \phi \bar{\pi}(1-\epsilon)(\xi-1)) \left( 1 - \bar{\pi}(1+\omega\epsilon)(1-\xi) \right) (1 - \epsilon) \beta \xi \]

and

\[ \Phi_2 = \beta \left[ \bar{\pi}(1+\omega\epsilon)(1-\epsilon) - (1 - \phi \bar{\pi}(1-\epsilon)(\xi-1)) \left( 1 - \bar{\pi}(1+\omega\epsilon)(1-\xi) \right) (\epsilon - 1) \right]. \]

Table 3 reports estimates of the structural parameters $\xi, \beta$ and $\phi$ in function of $\bar{\pi}$. We estimate the model for three values of $\bar{\pi}$: (i) the mean level of inflation over the period $0.14814$ for the Euro area and $1.009814$ for the United States, (ii) a low level of $2\%$ annually and (iii) a high level of $5\%$ annually. One clearly observes in both cases a reduction in $\phi$ when inflation is increased but this fall is all the more weak than one finds a high degree of indexation. For example, the model predicts that $\phi$ is reduced by $3.4\%$ (resp. $4.8\%$ and $2.5\%$) in the Euro area and $1.8\%$ (resp. $3.8\%$ and $9.5\%$) in the United States if annualized trend inflation is $\bar{\pi}$ (resp. $2\%$ and $5\%$).

4 Conclusion

We proposed new evidence about the properties of a hybrid Phillips curve based on partial price indexation. First, we initially cancel trend inflation in assuming that the prices that cannot be reset are indexed not only to a part of the past inflation rate but also to a part of trend inflation. Our results show that the extreme case with full indexation is data inconsistent and that the empirical model with partial indexation (and a degree of indexation around $0.5$) appears to capture the inflation dynamics for the Euro area and the United States over the period 1970-2002. Finally, since some authors claim that it can hardly be justified to not take into account trend inflation to describe and model the data of post-war inflation, we introduce it in the model and observe the theoretical awaited fall of the probability to not change the price (around $3-4\%$ for reasonable values of $\bar{\pi}$).

References


### Table 1. Reduced form estimates

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<td>$\alpha_b$</td>
<td>$\alpha_f$</td>
<td>$\lambda$</td>
<td>$J$-stat</td>
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<td>0.292</td>
<td>0.691</td>
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### Table 2. Structural estimates

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<td>$\phi$</td>
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<td>0.408</td>
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<td>0.533</td>
<td>0.99</td>
<td>0.821</td>
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### Table 3. Structural estimates under trend inflation

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<td></td>
<td>$\tilde{\pi}$</td>
<td>$\xi$</td>
<td>$\beta$</td>
<td>$\phi$</td>
</tr>
<tr>
<td>0.416</td>
<td>(0.010)</td>
<td>0.982</td>
<td>0.765</td>
<td>18.845</td>
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<tr>
<td>0.529</td>
<td>(0.085)</td>
<td>0.99</td>
<td>0.766</td>
<td>23.580</td>
</tr>
<tr>
<td>0.419</td>
<td>(0.103)</td>
<td>0.989</td>
<td>0.755</td>
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<tr>
<td>0.526</td>
<td>(0.085)</td>
<td>0.99</td>
<td>0.749</td>
<td>23.034</td>
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<tr>
<td>0.439</td>
<td>(0.110)</td>
<td>1.036</td>
<td>0.703</td>
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<tr>
<td>0.507</td>
<td>(0.086)</td>
<td>0.99</td>
<td>0.669</td>
<td>20.382</td>
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$\tilde{\pi} = (1.0148)^{\frac{1}{2}}$ for the Euro Area and $\tilde{\pi} = (1.0098)^{\frac{1}{2}}$ for the United States.
Figure 1. The Euro Area Data

Figure 2. The United States Data
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