Welfare-Theoretic Criterion and Labour Market Search

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05 - 20
Abstract

Keywords:

JEL Classifications:

*We appreciate comments and suggestions received from Sanvi Avouyi-Dovi, Marc Giannoni, Julien Mathéron and Michael Woodford. This paper represents the views of the authors and should not be interpreted as reflecting those of the European Central Bank or the Banque de France.

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1 Introduction

The consequences of labour market rigidities on employment, output and inflation constitute an issue of importance for both economists and policymakers. This is the reason why several recent papers have considered search and matching in a microfounded monetary policy model and showed that introducing these features improves the empirical performance of the standard sticky-price model in several directions (Moyen and Sahuc, 2005a, Trigari, 2004, Walsh, 2005). First, the existence of unemployed in equilibrium allows to reproduce the labour market stylized facts characterized by the Beveridge and Phillips curves. Second, labour market frictions act as a necessary complement to nominal rigidities. Third, monetary policy shocks can explain important features of labour market fluctuations.

To date, however, no work has been done to study the implications of a frictional labour market for the optimal monetary policy in a New Neoclassical Synthesis framework. Despite the development of increasingly sophisticated microfounded models, optimal monetary policy analysis remains based on ad hoc criterions. Loss functions are often assumed to depend on the variance of some key macroeconomic variables most often output and inflation with no obvious link to household utility.

In this paper, we derive a utility-based welfare criterion in a simple sticky-price model with labour market search. The loss function is then derived directly from the representative household’s utility function. This allowed policy to be evaluated with the same degree of rigor that was being used to model the economy. We are then able to consider how labour market frictions influence the criterion.

2 A simple sticky-price model with labour market search

ad hoc

...
2.1 Preferences

There is a continuum of households uniformly distributed on the unit interval. In equilibrium, some members will be unemployed while some others will be working for firms. The presence of equilibrium unemployment introduces heterogeneity in the model: each individual’s labour income differs based on his employment status. In this case, the individuals’ saving decision would become dependent on their entire employment history. To the purpose of this paper, we avoid these distributional issues by assuming that individuals may insure one another against risk of being unemployed such that a representative household have preferences defined over a composite consumption good \( (C_t) \), the employment’s rate \( (N_t) \) and hours worked \( (H_t) \) and derive utility according to the following utility function:

\[
E_0 \sum_{j=0}^{\infty} \beta^j \left[ \frac{C_{t \sigma} - N_t H_{t \varphi}}{1 - \sigma} \right]
\]

\[\beta \in (0, 1) \quad \sigma \quad \varphi \quad \theta \quad \varrho\]

\( z \in [0, 1] \)

\[
C_t = \left[ \int_0^1 C_t(z) \frac{\theta - 1}{\varphi} z \right]^{\frac{1}{\varphi}}
\]

\[
P_t = \left[ \int_0^1 P_t(z)^{1 - \theta} z \right]^{\frac{1}{1 - \theta}}
\]

2.2 The production technology

The wholesale sector.
The production technology is given by

\[ Y_{w,t} = A_t N_t H_t \]

where \( A_t \) is an exogenous stationary stochastic productivity shock.

The retail sector.

There is a continuum of monopolistic competitive retailers indexed by \( z \) on the unit interval. Retailers do nothing other than buy wholesale goods, differentiate them with a technology that transforms one unit of wholesale goods into one unit of retail goods, then re-sell them to the households.

Given that vacancy posting costs \( (\varsigma V_t) \) are expressed in terms of differentiated goods and that wholesalers demand for each retail goods is given by

\[ Y_t(z) = \mu P_t(z) P_t - \theta (C_t + \varsigma V_t) \]

the demand curve facing each retailer is:

\[ Y_t(z) = \mu P_t(z) P_t - \theta (C_t + \varsigma V_t) \]

In addition, we follow Calvo in assuming that in any given period each retailer can reset its price with a fixed probability \( 1 - \alpha \) that is independent of the time elapsed since the last price adjustment. This assumption implies that prices are fixed on average for \( \frac{1}{1-\alpha} \) periods.

Finally, the economy wide resource constraint is given by:

\[ Y_t = C_t + \varsigma V_t = Y_{w,t} D_t \]

where \( D_t = \int_0^1 \left( \frac{P_t(z)}{P_t} \right)^{-\theta} d\varsigma \) is the price dispersion expression.

2.3 Labour market matching

\[ N_{t+1} = (1 - s) N_t + M_t \]

where \( s \in [0,1] \)

\[ N_{t+1} = (1 - s) N_t + M_t \]

\[ s \]
The matching function is a very convenient hypothetical concept whose basic idea is that the recruiting effort of employers (the number of vacancies, \( V_t \)) and the number of searching workers (\( U_t = 1 - N_t \)) serve as inputs in a market matching function that generates new hires. The aggregate flow of job matches are deterministic and thus given by the following matching technology:

\[
M_t = \tilde{m} U_t^\varepsilon V_t^{1-\varepsilon}
\]

where \( \varepsilon \in (0, 1) \) \( \tilde{m} > 0 \) is a scale parameter.

The probabilities of a vacancy being filled (\( \tau_t \)) and a worker's job-finding rate (\( \varrho_t \)) are defined by

\[
\tau_t = \frac{M_t}{V_t} \quad \varrho_t = \frac{M_t}{U_t}
\]

Nash

Finally, wage and hours worked are determined by the generalized Nash-bargaining solution. Indeed, the matching between an unemployed person and a firm who coordinate each other gives rise to a surplus which must be shared between the meeting pair. This sharing takes place at the match level through a bilateral and decentralized wage/hours negotiation.

3 The utility-based welfare criterion

\[
\mathbb{W}_t = U(C_t) - V(N_t, H_t)
\]

\[
U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \quad V(N_t, H_t) = N_t \frac{H_t^{1+\varphi}}{1+\varphi}
\]

\[
V(N_t, H_t)
\]

(11)

\[
U(C_t) = \tilde{U} C \tilde{C} \left[ \dot{C}_t + \frac{1-\sigma}{2} \dot{C}_t^2 \right] + O \left( \|\zeta\|^3 \right)
\]

\[
\bar{X}
\]

\[
\dot{X}_t = \ln X_t - \ln \bar{X} \quad O \left( \|\zeta\|^3 \right)
\]

\[
V(N_t, H_t)
\]
\[ \forall (N_t, H_t) = \bar{V}_H \bar{H} \left[ \hat{y}_t - \hat{a}_t - \frac{\varphi}{1 + \varphi} \hat{n}_t + \frac{\varphi^2}{2(1 + \varphi)} \bar{n}_t^2 + \frac{1 + \varphi}{2} \hat{y}_t^2 ight] \\
- (1 + \varphi) \hat{y}_t \hat{a}_t - \varphi \hat{y}_t \hat{n}_t + \varphi \hat{n}_t \hat{a}_t + \frac{\theta_\alpha}{2(1 - \alpha)(1 - \beta \alpha)} \hat{n}_t^2 \\
+ O \left( ||\zeta||^3 \right). \]

\[ \beta^t \mathbb{W}_t = - \frac{\bar{U}_C \bar{Y}}{2} \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\theta_\alpha}{(1 - \alpha)(1 - \beta \alpha)} \bar{Y} \bar{N}^2_t \\
+ \left[ (1 - \sigma)(1 - \bar{N}) + \left( \varphi + \frac{\bar{Y}}{C} \right) \right] \frac{\bar{Y}}{CN} (\hat{y}_t - \delta \hat{y}_t^\eta - y^*)^2 \\
+ \frac{\varphi(\sigma + \varphi - 1)}{1 + \varphi} \frac{\bar{Y}}{CN} (\hat{n}_t - \delta_n \hat{n}_t^\eta - \eta^*)^2 \\
+ (1 - \sigma) \frac{\bar{V}}{C^2} (\hat{v}_t - v^*)^2 + \frac{\varphi \bar{Y}}{CN} (\hat{y}_t - \hat{n}_t)^2 - \frac{\sigma \bar{Y} \bar{V} C^2}{CN} (\hat{y}_t - \hat{v}_t^\eta)^2 \\
- \varphi \frac{(\varphi + \frac{\bar{Y}}{C})}{1 + \varphi} \frac{\bar{Y}}{CN} (\hat{n}_t - \hat{y}_t^\eta)^2 \\
+ \frac{\varphi \sigma_\delta \bar{Y} \bar{V}}{1 + \varphi \frac{\bar{Y}}{CN} (\hat{n}_t - \hat{v}_t^\eta)^2} \right\} + t.i.p. + O \left( ||\zeta||^3 \right), \]

\[ \delta_y = \frac{(\varphi + \frac{\bar{Y}}{C})}{(1 - \sigma)(1 - \bar{N}) + \left( \varphi + \frac{\bar{Y}}{C} \right)}, \]

\[ y^* = \frac{\varphi}{(1 - \Phi_y - \bar{N})}, \]

\[ \delta_n = \frac{\varphi}{(\sigma + \varphi - 1)}, \]

\[ n^* = \frac{1}{(\sigma + \varphi - 1)^2}, \]

\[ v^* = \frac{1}{(\sigma - 1)}. \]
\[
\Phi_y = \Phi_t
\]

\[
\hat{a}_t = \left( \varphi + \sigma \frac{\varepsilon}{1 + \varphi} \right) \hat{y}_t^n - \varphi \hat{n}_t^n - \left( \frac{\sigma \varepsilon}{1 + \varphi} \right) \hat{y}_t^n.
\]

\[
E_0 \sum_{t=0}^{\infty} \beta^t \mathcal{W}_t = -\frac{\bar{U}_C \bar{C}}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left( (y_t - \hat{y}_t^n - y^*)^2 + \frac{\theta \alpha}{(1 - \alpha)(1 - \beta \alpha)} \hat{\eta}_t \right) + t.i.p. + O\left( ||\zeta||^3 \right).
\]

4 Discussion

\[
\bar{V} = \bar{q}/(\bar{p} + s)
\]

\[
\bar{\Omega} = \frac{\theta_t}{(1 - \alpha)(1 - \beta \alpha)} \frac{\bar{C} \bar{y}}{\bar{v}}
\]

\[
\bar{N} = \bar{q}/(\bar{p} + s)
\]

\[
\bar{\Omega} = \frac{\theta_t}{(1 - \alpha)(1 - \beta \alpha)} \frac{\bar{C} \bar{y}}{\bar{v}}
\]

\[
\bar{N} = \bar{q}/(\bar{p} + s)
\]

\[
\bar{N} = \bar{q}/(\bar{p} + s)
\]
5 Conclusion
References

Journal of Economic Theory

Economic Modelling

Manuscript

Working Paper n° 268

Review of Economics Dynamics

Interest and prices: Foundations of a theory of monetary policy
Table 1. Variation of the relative weights as a function of $\bar{\varphi}$ and $\bar{\tau}$

| $|\lambda_{yy^*}|$ | $|\lambda_{nn^*}|$ | $|\lambda_{vn^*}|$ | $|\lambda_{yn}|$ | $|\lambda_{yn^*}|$ | $|\lambda_{ynn}|$ | $|\lambda_{nv^*}|$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\bar{\varphi}$ | $\bar{\varphi}$ | $\bar{\varphi}$ | $\bar{\varphi}$ | $\bar{\varphi}$ | $\bar{\varphi}$ | $\bar{\varphi}$ |
| $\bar{\tau}$   | $\bar{\tau}$   | $\bar{\tau}$   | $\bar{\tau}$   | $\bar{\tau}$   | $\bar{\tau}$   | $\bar{\tau}$   |

Table 2. Calibration

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<tr>
<th>$\beta$</th>
<th>$\sigma$</th>
<th>$\phi$</th>
<th>$\theta$</th>
<th>$\alpha$</th>
<th>$s$</th>
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<td>2</td>
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<td>0.01</td>
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Table 3. Values of the relative weights as a function of $\alpha$

<table>
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<th>$\lambda_{yy^*}$</th>
<th>$\lambda_{nn^*}$</th>
<th>$\lambda_{vn^*}$</th>
<th>$\lambda_{yn}$</th>
<th>$\lambda_{yn^*}$</th>
<th>$\lambda_{ynn}$</th>
<th>$\lambda_{nv^*}$</th>
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<tr>
<td>0.75</td>
<td>0.0337</td>
<td>0.0172</td>
<td>0.0001</td>
<td>-0.0002</td>
<td>-0.0230</td>
<td>0.0001</td>
<td>-0.0002</td>
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Figure 1. Effects of labour market frictions on the relative weights
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<tr>
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**2004**

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