The money demand function for the Euro area: one step beyond

Sanvi AVOUYI-DOVI, Françoise DRUMETZ & Jean-Guillaume SAHUC

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One Step Beyond*

Sanvi Avouyi-Dovi
Banque de France – Economics and Financial Research Division

Françoise Drumetz
Banque de France – Directorate Research

Jean-Guillaume Sahuc†
Banque de France – Economics and Financial Research Division

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Abstract

This article sets out to re-examine the money demand function for the euro area. Traditional specifications often yield unsatisfactory results: instability of short and long-term coefficients; relatively large differences between estimated and actual value of variables; and significant changes in the number of long-term relationships, etc. Using a standard Vector Error Correction Model, we find that the usual specification is indeed unstable. However, introducing an European equity price gives rise to a more stable system. Furthermore, recursive estimates confirm the relative stability of long-term coefficients. Estimates of the real money gap, based on the money demand equation including equity prices, point to moderate, albeit persistent, excess liquidity in the euro area in the recent years. The real money gap contains information about future inflation but this content may have diminished since 2001.

Keywords: Demand for money; stability; financial assets; substitution effect

JEL Classifications: C11, C51, E52

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1 Introduction

In recognition of the fact that monetary growth and inflation are closely related in the medium to long run, the European Central Bank (ECB) has assigned a prominent role to money in its strategy. This role for money serves two main purposes: (i) it is a tool to underpin the medium-term orientation of the ECB’s monetary policy and (ii) it allows the central bank to see beyond the impact of transitory shocks and avoids the temptation of taking an overly activist course (ECB, 2004).

However, the role of money in the strategic framework of the ECB changed in 2003. From 1999 to 2003, the monetary analysis formed the first “pillar” of the strategy, the economic analysis being the second “pillar”. Moreover, the prominent role for money was signalled by the announcement of a reference value of 4.5% for the growth of the broad monetary aggregate M3. Assumptions underlying the derivation of the reference value were reviewed annually.

An important characteristic of the reference value is that it must be consistent with the ECB’s quantitative definition of price stability and with the long-run relationship between money, prices and output.¹ The choice of M3 was based on the evidence that there was a stable money demand relationship for this aggregate and that M3 had leading indicator properties for future inflation.

In May 2003, the ECB announced a revision of its monetary policy strategy. The economic analysis became the first “pillar” and the monetary analysis the second “pillar”. Moreover, it was decided to no longer review the reference value for M3 on an annual basis because underlying medium-term trend assumptions cannot be expected to change frequently.

This modification, which was often interpreted as a “downgrading” of the reference value, occurred after two years of very strong M3 growth in excess of 4.5% year-on-year. Indeed, from 2001 to 2003, in a context of geopolitical and financial market uncertainty, monetary dynamics were influenced by an exceptionally strong preference for liquidity, followed by a return to normal, at a relatively moderate pace, of economic agents’ portfolio allocation behavior from 2004 to 2005. These developments have led the ECB to construct quantitative estimates of the portfolio shifts which have been used to correct the M3 series in order to extract the underlying signals from monetary developments for risks to price stability. M3 corrected for the estimated impact of portfolio shifts is derived by combining the information stemming from a broad range of economic indicators with evidence from a univariate time series model of M3 (ECB, 2005).

In view of these recent monetary developments in the euro area – which highlight the impor-

¹ Price stability is defined as a year-on-year increase in the Harmonized Index of Consumer Prices (HICP) for the euro area of below, but close, to 2% over the medium term.
tance of portfolio shifts – it appeared necessary to re-assess the empirical evidence regarding the money demand function for the euro area. This article adds to the literature in several respects. First, it extends early euro area studies (Dedola et al., 2001, Calza et al., 2001, Coenen and Vega, 2001, Brand and Cassola, 2004, among others) in estimating a general specification of money demand in order to test its stability. Second, this paper also considers a model in which a financial factor – the European equity price – which captures portfolio allocation behavior is introduced. In attempting to do so, we embed explicitly into the money demand model portfolio shifts which have been thus far treated in an ad hoc manner by the ECB. We perform a detailed stability analysis both of the short and long-run relationships for two modelling strategies. Finally, this article presents a measure of excess liquidity, the real money gap, which is expected to usefully complement the monitoring of M3 growth, and verifies its properties as a leading indicator for inflation.

Regarding the analytical framework, the mediocre performances of the estimates made in an univariate framework suggest using a multivariate approach. The error correction framework makes it possible to specify both the long-run relationships among the variables and their short-run behavior. This implies that, when assessing the stability of these models, it is important to distinguish between the coefficients making up the long-run relationships and those forming the short-run equations.

Our results confirm the strong sensitivity of the most general specification of money demand for the euro area. Introducing European equity prices produces a slightly more stable relationship. In addition, our estimations show a substitution effect, i.e. a negative effect of stock yields on real money holdings. The asset price effect is found to be significant and the substitution effect is in keeping with expectations. The derived real money gap appears to have been positive since 2001. This points to the presence of moderate, albeit persistent, excess liquidity in the euro area since 2001. We argue that the real money gap may be regarded as a leading indicator of inflation since the relationship between HICP inflation and the real money gap is relatively close but its predictive power has diminished over the last few years.

The remainder of this paper is organized as follows. Factors underlying monetary developments are presented in Section 2. Data and specifications are briefly described in Section 3. Since the end of the 1990s, M3 growth in the euro area has also been substantially influenced by changes in the external counterpart. Therefore, we estimated a “naive” money demand function leaving out the external counterpart of M3 so as to check whether it is a source of instability. The results are not satisfactory: notably, some coefficients have wrong signs, the system seems unstable (in terms of number of long-term relationships and coefficient stability), and some coefficients are not significantly different from zero.
Section 4 first sets out the results of the estimation of the generalized specification and those of the model including financial asset prices. An assessment of the real money gap and its cross-correlations with HICP inflation are presented in Section 5. Finally, Section 6 sets out the conclusion.

2 Factors underlying monetary developments

2.1 The monetary aggregate

The choice of the measurement of the monetary aggregate depends on one’s theoretical assumptions (Goldfeld and Sichel, 1990, Sriram, 1999): for an approach in portfolio terms, broad aggregates are generally favoured; in analyses of the transactions intermediary function of money a narrow aggregate is more likely to be used. The broad M3 aggregate is the one chosen by the ECB and therefore the most often used for euro area money demand (Dedola et al., 2001, Calza et al., 2001, Coenen and Vega, 2001, Brand and Cassola, 2004). By contrast, models of M1 or M2 money demand estimated for European countries are rare (Funke, 2001, Stracca, 2003) whereas they are applied in the United States (Baba et al., 1992, Haug and Tam, 2001, for instance).3

2.2 The scale variable

GDP is the most commonly used scale variable in empirical studies in spite of its well-known shortcomings, in particular its failure to take account of intermediate and financial transactions, as well as transfers, and its incorporation of factors that do not give rise to transactions (Goldfeld and Sichel, 1990). Other scale variables in flow have also been proposed (GNP, consumption, etc.) but they are also incomplete in nature. The only alternative in this area is provided by stock variables (wealth or permanent income, etc.). Fase and Winder (1999), for example, highlight the significant impact of wealth on demand for M2 and M3, but not on M1. By reducing to one the sum of income and wealth elasticities in their equation for M3 demand for EMU, they obtain elasticities of around 2/3 for income and 1/3 for wealth. However the stock variables are not always easy to evaluate. So, GDP is the scale variable in this paper. As a result, the GDP deflator serves both as a M3 deflator and as a basis for the calculation of inflation.

3Mention should also be made of studies based on aggregates that better reflect the greater or lesser liquidity of the different components of money.
2.3 The opportunity cost

The opportunity cost of holding money is made up of two elements, the choice of which is linked to the underlying theoretical framework (Sriram, 1999). These are the rate representing the return on an alternative asset to money and the own rate of return on money. Note that the hypothesis of the nullity of the coefficient of the intrinsic rate of return is sometimes accepted for M1: Ball (2001) accepts it while Stracca (2003) rejects it. However this hypothesis seems excessive for M3. Nonetheless, many authors use a single rate of interest in their long-run money demand equation. This is notably the case in a number of studies devoted to the European Union and the euro zone (Brand and Cassola, 2004, Fagan and Henry, 1999).

We adopt a pragmatic attitude and include the own rate, short and long-term nominal interest rates in our money demand function.

2.4 Financial factors

According to Friedman (1988), financial factors, like asset prices, have a significant impact on real money holdings and are therefore one of the potential explanatory variables of a monetary aggregate. The monetary aggregate is positively correlated with financial asset prices via a wealth effect and negatively via a substitution effect.

The wealth effect works through three channels: (a) higher asset prices lead to an increase in nominal wealth (or real wealth for a given price), which in turn results in a rise in the real wealth to real income ratio and the money holdings to income ratio; (b) the higher return expected on risky assets (equities for example) encourages economic agents to reallocate their portfolios towards safer monetary assets; (c) a rise in asset prices causes the volume of money necessary for financial transactions to increase.

The substitution effect stems from the fact that a rise in asset prices is an incentive for economic agents to switch for instance from money to equities as they would expect to obtain capital gains in the future. We will test the significance of both wealth and substitution effects.

2.5 Other explanatory variables

Inflation appears to be a potential explanatory variable for money demand. Nonetheless, there is no consensus as to its relevance as an explanatory factor involved in the long-run relationship. In addition, attempts to take account of financial innovation have been made (Goldfeld and Sichel, 1990, or Ireland, 1995). Transposing this to the case of the demand for M3 is not an easy
task. Lastly, it should be noted that, concerning the choice of international portfolios, attention is sometimes given to the possible impact of international capital mobility on money demand. This can result in the inclusion of an effective exchange rate or of foreign asset prices in the equation (Ericsson and Sharma, 1998).

3 Specifications and data

3.1 Brief review of the specifications

The money demand function used in empirical studies is written in the following general form (Ericsson, 1999):

\[ M = f(Y, r) \]

where \( M \) stands for the real monetary aggregate, \( Y \) is a variable of scale in real terms (income, transactions, wealth, etc.) and \( r \) is a vector of opportunity costs (alternative yields to the rates of return on money, yield spreads, etc.). This form of the money demand function draws both on the transactions-based approach and that based on speculation. Several specifications follow on from the function given above, but the one most frequently used in empirical studies is the semi-log linear form:

\[ m_t = \gamma_0 + \gamma_1 y_t - \gamma_2 r_t + \varepsilon_t \]

where \( m_t \) and \( y_t \) are the log of the real monetary aggregate and the real variable of scale, respectively; \( \gamma_1 \) and \( \gamma_2 \) are unknown parameters assumed to be positive and \( \varepsilon_t \) is an error term. When \( \gamma_1 = 1 \), the previous relationship is interpreted as a velocity of money equation.

A variant of this specification, derived from the maximization of an intertemporal utility function, was proposed by Stracca (2003), who obtains the following equation:

\[ m_t = \gamma_0 + \gamma_1 c_t - \gamma_2 \left( r_t^{alt} - r_t^{own} \right) + \varepsilon_t \]

with \( c_t, r_t^{alt} \) and \( r_t^{own} \) representing the log of real consumption, the return on alternative investment and the return on money respectively. The unknown parameters \( \gamma_i \) are assumed to be positive. Note that for the estimation of the demand for M1 in the euro area, Stracca (2003) used, while amending, the money demand function proposed by Ashworth and Evans (1998) which is written:

\[ m_t = \gamma_0 + \gamma_1 y_t - \gamma_2 \left( r_t^{alt} - r_t^{own} \right)^{-1} + \varepsilon_t \]
This last approach differs from the two preceding, *inter alia*, by virtue of the non-linearity introduced via the inverse of the opportunity cost. Other specifications, particularly that designated by the term double log are also available.

### 3.1.1 First specification: a generalized model

We generalize the previous semi-log specification in incorporating the inflation rate and the direct effects of the components of the opportunity costs \( r_t = (r_{t}^{own}, r_{t}^{st}, r_{t}^{lt}) \) in the money demand equation. This equation is then given by:

\[
    m_t = \gamma_0 + \gamma_1 y_t + \gamma_2 r_t^{own} - \gamma_3 r_t^{st} - \gamma_4 r_t^{lt} - \gamma_5 \pi_t + \varepsilon_t
\]

(1)

where \( m_t \) is the log of the real money stock at time \( t \), \( \pi_t \) the rate of inflation (measured by the difference of logarithm of the GDP deflator), \( y_t \) the log of real GDP, while \( r_t^{own} \), \( r_t^{st} \) and \( r_t^{lt} \) are the own rate, short and long-term nominal interest rates respectively, and finally \( \varepsilon_t \) an error term. The \( \gamma_i \geq 0 \) are the parameters to be estimated.

### 3.1.2 Second specification: a financial asset prices augmented version

Several recent studies include financial asset prices in the money demand function for the euro area. In this case, the money demand equation can be written as:

\[
    m_t = \gamma_0 + \gamma_1 y_t + \gamma_2 r_t^{own} - \gamma_3 r_t^{st} - \gamma_4 r_t^{lt} - \gamma_5 \pi_t + \gamma_6 a_t + \varepsilon_t
\]

(2)

where \( a_t \) is the log of the real price of financial assets. \( \gamma_6 \) is positive in the case of a wealth effect and negative in the case of a substitution effect.

### 3.2 Data

Analyzing behavior across an economic area requires an answer to the following question: should one work on aggregated series for the area as a whole or aggregate the individual equations estimated for each of the area’s member countries? Two problems then need to be taken into consideration (Dedola *et al.*, 2001): that arising from the aggregation bias resulting from the method of aggregation used; and that stemming from a specification bias that penalizes country-specific equations when there is a substitution between different currencies within the area. In a general context, Pesaran and Smith (1995) show that estimates for dynamic models applied to aggregated series are somewhat unsatisfactory. As a consequence, they recommend using the average of individual estimators as estimators of the model for the area as a whole.
It is not certain, however, that this general diagnosis is relevant in the case of money demand equations. Dedola et al. (2001) have shown that country-specific equations may contain relevant information for monetary policy. Thus, where there is no substitution between currencies, the aggregation of equations estimated country-by-country may be desirable. According to the same authors, the dynamics resulting from this aggregation are very close to those of the model estimated on the basis of aggregated series. We have nonetheless opted for the model estimated using aggregated data for the euro area. In so doing, we are following the findings of Fagan and Henry (1999) who conclude that there is no aggregation bias in their applications to the euro area, i.e. the forecasting errors of the aggregated model being lower that those derived from country-specific equations. In addition, we used quarterly data. The full sample is over the period 1980:1-2005:4. However we will perform the estimation over the period 1982:1-2004:4 in order: (a) to take into account the lags in the VAR system, (b) to realize static and dynamic out of sample simulations.

3.2.1 Real and monetary data

Official sources (Eurostat and ECB) were used in most cases. The M3 aggregate (ECB) available on a monthly basis since 1980, was made quarterly by taking the average value of monthly outstandings over one quarter. GDP data (Eurostat) available from 1991 were projected backwards according to the method put forward by Beyer et al. (BDH, 2001), on account of its mode of composition (aggregation with variable weighting).

The inflation rate (also from the BDH source before 1991) was calculated as the first difference of the log of the GDP deflator. The previous series were seasonally adjusted. Short and long-term nominal interest rates are obtained from the updated Area Wide Model database (Fagan et al., 2005).

The own rate is defined as the weighted average of the returns on the components of M3 (banknotes in circulation, deposits with an agreed maturity of up to two years, deposits redeemable at notice of up to three months, repurchase agreements, certificates of deposit, money market fund shares/units, and marketable debt securities issued with a maturity of up to two years). The weightings correspond to the share of the outstanding amount of each component in the total.
3.2.2 Financial data

Due to the lack of a quarterly aggregated asset price index, we have chosen an European stock price index which acts as an asset prices indicator. The European stock price index is the Eurostoxx, available from 1987 to today. It was projected backwards over the period prior to 1987 using the DAX 30 (Carstensen, 2006). The quarterly values were computed as the average of monthly values. The real index is the ratio of the back projected Eurostoxx index over the GDP deflator.

4 Empirical results

We performed the traditional stationarity tests (augmented Dickey and Fuller – ADF and Schmidt-Phillips – SP) which showed that the variables are integrated of order one (I(1)), except maybe inflation. In the case of inflation, due to the low power of the tests, we do not have a precise indication of whether it is stationary around a trend or stationary of order 1. In the remainder of the study, it is assumed to be I(1). Therefore, the M3 aggregate and its main explanatory factors are considered as I(1).

The model takes the standard form of a Vector Error Correction Model (VECM), in which the number of structural relationships between the different variables is tested. The cointegrating properties are derived using Johansen’s (1988, 1991) maximum likelihood procedure.

Assume that we can describe the dynamics of variables of model, $x_t$, by a VAR($q$) system

$$x_t = v + A_1 x_{t-1} + \ldots + A_q x_{t-q} + \varepsilon_t$$

where $\varepsilon_t$ are normally distributed Gaussian innovations. The intercept terms are collected in the $(n \times 1)$ vector $v$. By subtracting $x_{t-1}$ from both sides, this system can be written as a vector-error correction model:

$$\Delta x_t = v + \sum_{i=1}^{q-1} \Gamma_i \Delta x_{t-i} + \Pi x_{t-1} + \varepsilon_t$$

with $\Pi = -(I - \sum_{i=1}^q A_i)$ and $\Gamma_i = -(A_{i+1} + \ldots + A_q)$ for $i = 1, \ldots, q - 1$, are the matrices for the short-term coefficients.

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4 The EuroStoxx and DAX 30 indices are taken from the Datastream database.

5 This study does not include a stock market volatility index or tech stock indices, because these variables are deemed too noisy. Given the very high values of these indices at certain dates, taking them into account would be equivalent to introducing a dummy variable. Furthermore, this would only be relevant in the framework of an analysis of short-term dynamics.
Given that the variables in $x_t$ are $I(1)$, Johansen (1991) formulates the hypothesis of cointegration as a reduced rank restriction on the $\Pi$ matrix with

$$\Pi = \alpha \beta'$$

where $\alpha$ and $\beta$ are $(n \times r)$ matrices. We can interpret $\text{rank}(\Pi) = r$ as the number of stationary long-run relations while the cointegrating vector $\beta$ is determined by solving an eigenvalue problem. Thus, $\beta'x_t$ is a stationary long-run equilibrium relation with the adjustment towards the equilibrium driven by the vector of loadings $\alpha$.

Estimating the cointegrated VAR model requires setting a lag order $q$. We use the standard Akaike and Schwartz criteria (AIC and SIC) that compare the goodness of the fit of maximum likelihood estimations and correct for the loss of degrees of freedom when additional lags are added. The AIC suggests 1 lag whereas the SIC recommends to retain 2 lags for the two specifications. Since subsequent models will be heavily parameterized we favour a parsimonious specification. For this reason we follow the SIC and set $q = 2$ in the two VAR systems because the SC uses the higher penalty for extra coefficients.

Johansen’s procedure allows to test the presence and number of cointegration relationships in the system. In terms of the euro area, there is no consensus regarding the number of cointegration relations derived from the system set out above: Brand and Cassola (2000) and Coenen and Vega (2001) accept three cointegration relations while Calza et al. (2001) or Boone et al. (2004) only identify one. Furthermore, additional restriction tests can be carried out, for example testing the hypothesis of a unitary value of the elasticity of M3 aggregate to income, which makes it possible to write the money demand function as a velocity of money equation. In our specifications (separation of transactions and speculative motives), this hypothesis corresponds to the framework of the quantitative theory of money. The major breaks brought about by German reunification and Greece’s entry into the euro area have been taken into account via the introduction of two short-term dummies (1990:3 et 2001:1).

The trace test (see Table 1, panels A and B) indicates the presence of two cointegration relationships, while the maximum eigenvalue test indicates 1 relationship in the first specification and 2 for the second one. We retain 2 cointegration relationships which can be interpreted as a money demand equation and a Fisher equation. Moreover, in all the cases, the unitary elasticity of income to money demand was accepted by the likelihood ratio test, at a 5% significance level. In addition, as the coefficients of the short-term rate and the own rate are of the same magnitude (in absolute terms), we assumed them to be equal; this constraint was also accepted by the previous test. It enables us to introduce the short-term rate/own rate spread in the
money demand equation. Finally, all the constraints are not rejected around 30% for the first specification and 50% for the asset prices augmented specification.\(^6\)

### 4.1 The generalized specification

#### 4.1.1 Estimates

Each of the two relationships is identified by tests of constraints on long-run coefficients and adjustment coefficients. The first long-run relationship thus represents a demand for money in which M3 depends on real GDP, short-term rate/own rate spread and long term interest-rate, while the second relationship links the long-term interest rate and the inflation rate and can be interpreted as a Fisher equation. These two relationships are represented in normalized form in Table 2 in terms of money demand equation. Our results are positioned between those of Brand and Cassola (2004) and Coenen and Vega (2001) (3 relationships) and that of Calza et al. (2001) (1 relationship). However, a significant change in the estimation period has brought about a change in the number of cointegration relationships: the number of cointegration relationships increases with the size of the sample.

The unitary income elasticity of money demand has also been tested. The tests make it possible to not reject, at the 5% significance level, the hypothesis of a unitary GDP elasticity of money demand. These results appear interesting both from an economic and a statistical standpoint: the underlying theoretical hypotheses are not rejected by the data and the demand for money can therefore be rewritten as a velocity of money equation. In addition, the coefficient of the spread is negative as expected (−0.041) and significantly different from zero while the long-term interest rate coefficient (−0.007) is not significant. Concerning the Fisher equation, the coefficient of the long-term interest rate is positive (0.48) and highly significant.\(^7\)

The diagnostic tests on the properties of the residuals provide satisfactory results: the hypothesis of normality for residuals is accepted; we observe the absence of autocorrelation of residuals; the hypothesis of the homoscedasticity of residuals is also accepted.

In order to compare our estimates with the recent findings, we harmonized results of the different studies (Brand and Cassola, 2004; Calza et al., 2001; Coenen and Vega, 2001; and

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\(^{6}\) The exogeneity tests have been conducted in the standard manner by testing, firstly, the combined exclusion of the cointegration vectors in each of the equations and, secondly, the exclusion of each of the cointegration vectors individually in each of the equations.

\(^{7}\) We also tested the validity of all the restrictions on the coefficients based on theoretical a priori assumptions. Overall, we rejected these restrictions because the values of the coefficients were not consistent with those derived from the theory.
Gerlach and Svensson, 2003) carried out on money demand in the euro area. Panel A of Table 3 summarizes the recent estimates obtained for the euro area.8

Unlike the results produced by our study, the income elasticity of money demand is significantly greater than 1 in the studies cited in this table. It stands at between 1.1 (Coenen and Vega, 2001) and 1.5 (Gerlach and Svensson, 2003). On the other hand, the coefficients relating to interest rates are about five times lower than those resulting from our estimates. This discrepancy seems to derive from the different estimation periods and explanatory variables used in the different studies. Indeed, an estimation of the models presented in Table 3 over the period covered in this paper and with the variables used by the different authors leads to similar coefficients to ours apart from income elasticity which remains higher than 1. In this paper, we obtain a unitary income elasticity of money demand that is more consistent with the theory. There is no consensus concerning the impact of interest rates: according to Brand and Cassola, the coefficient of the long-term interest rate is significant with a right sign but short-term interest rate and own rate have no effects; however, in the other papers, including ours, a spread (short-term/own rate or short-term/long-term) is found significant.

4.1.2 Structural stability

The above-mentioned estimates may be sensitive to a change in the size of the sample. We should then test the structural stability of the money demand equation. According to the conventional methods (number of cointegration relationships, recursive estimation, stability of short-term dynamics, etc.), the generalized specification is unstable over time. Indeed, the number of long-term relationships ranges from one to five depending on the start and end dates of the sample, merely by adding a single observation (see Table 4). In the framework of this analysis, it is relatively easy to economically interpret two long-term relationships (demand for money equation and Fisher equation), but interpreting three structural relationships is more complex.

A conventional (informal) method of investigating the stability of the long-run coefficients consists of graphically examining the recursive estimates of the long-run parameters to determine whether they change significantly over time. The recursive estimates are obtained by running the model sequentially, in this case starting from a sub-sample including at least two-thirds of the observations and then extending it by one observation at a time. Figure 1 shows the

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8Unlike in Brand and Cassola’s original paper (2004), the interest rates are not divided by 400. Furthermore, all of the rates are given as a percentage. As a result, variables of the same nature are expressed in a common unit. The coefficients estimated in the different studies can therefore be compared directly.
time paths described by the recursive estimation of the long-run coefficients of the equilibrium relationships with the respective 95% confidence intervals. The coefficient associated to the yield spread varies significantly over time, in particular between 1998 and 2000 and has the wrong sign. The coefficient for the long-term interest rate displays a high value at the beginning of the period and seems to converge to zero. The results relies to a large extent on the validity of a set of theoretically-based (over) identifying restrictions on its long-run coefficients. It is, therefore, important to test also the stability of these restrictions. Figure 2 reports the results of the recursive test of statistics on the set of restrictions applied for the identification of the long-run relationships. As the figure shows, the test confirms the previous results in rejecting the set of constraints between 1998 and the end of 2000.

The stability of the long-term coefficients is linked to that of the short-term coefficients. It is therefore also important to test the validity of the stability assumption for all of the short-term coefficients, keeping the coefficients of the cointegrating vector fixed at their full sample estimates. For this purpose, three types of recursive Chow tests were performed: the 1-step ahead Chow test detects the presence of outliers while the breakpoint and predictive failure Chow tests are used to identify possible structural breaks. Figure 3 shows that three outliers are brought to light with the 1-step ahead Chow test. The last two tests point to a clear degree of instability, in the money demand equation and in the overall system since the test statistic is above the 95% critical level for several years. These results lead us to reject the assumption that the short-term coefficients are stable.

4.2 The asset prices augmented specification

4.2.1 Estimates

In this specification, all the coefficients in the money demand equation (see Table 5) are significant. In particular the coefficient for asset prices which displays the highest semi-elasticity is negative; therefore, a substitution effect prevails. All other things being equal (in this case, constant output and prices) and based on the quantitative equation of money, a fall in European stock yields would lead to a rise in real money holdings and a decline in the velocity of money. Developments on equity markets would therefore appear to be a significant explanatory factor for M3 dynamics in the euro area.

The coefficient for the long-term interest rate (−0.037) is significantly higher than the coefficient associated to the yield spread (−0.014). The private sector seems more sensitive to the most risky markets. Furthermore, we obtain a broad version of the Fisher equation: the slope
(coefficient for the long-term rate) is clearly different from zero but significantly less than one.\textsuperscript{9}

A number of recent studies have focused on the issue of including financial asset prices in the money demand function. Introducing asset prices is indeed deemed to stabilize the equilibrium relationships. As regards the euro area, these studies have mainly sought to explain the strong growth of M3 since 2001. The general climate of uncertainty and the bearish trend on stock markets may have led economic agents to reallocate their portfolios in favour of instruments within M3 in anticipation of more favorable economic conditions.

The harmonized results of the main studies carried out on financial asset prices and money demand in the euro area are summarized in panel B of Table 3.

A first set of papers highlights the transitory role of stock markets in the analysis of the monetary aggregate. For instance, Kontolemis (2002) finds that introducing asset prices (average of the DAX and CAC 50) improves the short-term relationship without affecting the stability of the equilibrium relationship.\textsuperscript{10} However, the CAC 50, which is made up of the first fifty technological stocks other than those in the CAC 40, posts large variations and can be likened to a dummy variable. In reality, the explanatory power of this variable is very weak. Besides, Bruggeman \textit{et al.} (2003) show that the impact of financial assets is not significant in the long-term monetary relationship. However, they are deemed to have a predictive power in the short run.\textsuperscript{11} Thus, according to these authors, taking asset prices into account is not essential, in particular with a view to stabilizing the money demand equation.\textsuperscript{12}

A second set of recent papers yield more clear-cut conclusions concerning the role of asset prices for the monetary developments. According to Boone \textit{et al.} (2004), the cointegration relationship which links money holdings to its usual determinants and to a household wealth variable (the weighted assets prices indexes) is stable.\textsuperscript{13} Carstensen (2006) comes to the conclusion that the deviation between the return on equity (denoted $r^a_t$) and the own rate as well

\textsuperscript{9}Like in Bruggeman \textit{et al.} (2003), the equation of the own rate of M3 can be shown as a pricing relation. As this relation does not have a microeconomic foundation, we decided not to use it. Furthermore, according to our estimations, most coefficients in this equation were not significant.

\textsuperscript{10}Similarly, Cassola and Morana (2002) explain that M3 dynamics in the euro area were governed by liquidity preference between 2001 and 2003, reflecting temporary wealth transfers from financial markets to monetary assets.

\textsuperscript{11}Bruggeman \textit{et al.} also tested the effect of a volatility indicator (defined as the conditional standard deviation of a GARCH model for weekly data) on the long-run equation. It turned out to be significant but with a weak explanatory power.

\textsuperscript{12}A second cointegration relationship appears. According to the authors, it corresponds to a pricing relation for the own rate of M3 (see footnote 9).

\textsuperscript{13}We are not comfortable with the calculation of this indicator. We, therefore, favoured a stock price index.
as the stock market volatility index (denoted “volatility”) play an important role in ensuring the stability of the money demand function: M3 growth can partly be attributed to these two variables. In addition, contrary to Boone et al., Carstensen obtains a substitution effect.

There is therefore no real consensus regarding basic equations including asset prices; in this respect, our results are more in line especially with those of the ECB. Note that the OECD brings to light a positive effect of asset prices on real money holdings. Our findings seem more realistic in the euro area context.

4.2.2 Structural stability

Whatever the sub-period, one of the two tests indicates the presence of 2 cointegration relationships except for the sub-periods beginning in 1982:4. In addition, the maximum value of the number of cointegration relationships is 3 versus 5 for the generalized specification (Table 6). Therefore, the real equity return seems to have a significant impact on the number of cointegration relationships.

The recursive estimation of the coefficients of the equilibrium relationships yields mixed results (see Figure 4): some parameters (asset prices and the slope of the Fisher equation) vary very gradually while others (yield spread or long-term rate) are more volatile. Most coefficients remain significantly different from zero. Further tests do not bring to light any problems of instability. In particular, the hypothesis of stable short-term dynamics is not rejected. In addition, despite the presence of a number of outliers (in the first quarter 2001 and the second quarter 2004 for the money demand equation), overall, the three break tests confirm the stability hypothesis (Figures 5 and 6).

4.2.3 Simulations and forecasting results

The constrained cointegration vectors (quadrants 1 and 2 in Figure 7) indicate the more or less pronounced deviations of the endogenous variables observed (real currency holdings and inflation rate) compared with their equilibrium levels. The dynamic simulations of the long-term model (quadrants 3 and 4 in Figure 7) show that the simulated values of M3 seem to be consistent with its observed values. However, the model slightly over-estimates inflation between 1995 and 2000 and under-estimates it between 2000 and 2004.\footnote{The dynamic simulations of the short-term model of the variables in level are also satisfactory. In addition to the fact that the simulated variables display the same pattern as the observed variables, there is no marked deviation between the observed and simulated values of the aggregates.}
To check the predictive properties of the asset prices augmented specification, we perform out-of-sample simulations (see Figure 8) and compare the observed and simulated values for the year 2005. The forecasts are computed as recursive out-of-sample forecasts. The model does not capture the sharp change in the evolution of inflation observed in the second quarter of 2005. However, the observed values of the main variables are comprised in the 95% confidence bounds based on the VECM model. The model strongly under-estimates the nominal variables (inflation and all the interest rates) and fits quite well the real variables (GDP, M3 and asset prices).

5 Real money gap

The real money gap (denoted RMG henceforth) is one of the excess liquidity measures currently monitored and used in both internal briefing of the ECB and in external analysis. Indeed, it reflects developments in money not explained by macroeconomic variables of the long-run money demand model. It thus contains information on money which is captured by the unexplained part of the actual monetary growth as well as the short-term monetary growth. It is defined as the deviation of the actual level of the real money stock from a particular definition of the “equilibrium” level of the real money stock. Formally, this is written in log form as:

\[ rmg_t = m_t - m_t^* \]

The equilibrium level of real money holdings (in practice, the reference level or trend) is obtained by replacing the current values of income, the own rate and opportunity costs respectively by their equilibrium or reference values in the long-term real money holdings equation. The real money gap can then be written in the following form:

\[ rmg_t = m_t - [\hat{\gamma}_0 - \hat{\gamma}_1 y_t^* + \hat{\gamma}_2 (r_{t}^{own})^* + \hat{\gamma}_3 (r_{t}^{st})^* + \hat{\gamma}_4 (r_{t}^{hl})^* + \hat{\gamma}_5 \pi_t^* + \hat{\gamma}_6 a_t^*] \]

where \( z_t^* \) represents the equilibrium value of the variable \( z_t \).

The estimated coefficients are derived from the specification of the real money holdings equation including financial asset prices. More specifically, the general expression of the real money gap takes the form:

\[ rmg_t = m_t - [1.578 - y_t^* - 0.014 [(r_{t}^{st})^* - (r_{t}^{own})^*] + 0.037 (r_{t}^{hl})^* - 0.048 a_t^*] \]

The equilibrium values of the variables are defined as trends, calculated by means of a Hodrick-Prescott filter or a quadratic trend. The results obtained using the two smoothing
methods (see Figure 9) are comparable both from a qualitative and a quantitative perspective. Priority should be given to the qualitative aspects, given the uncertainties surrounding the measurement of the equilibrium values of the variables. There are a few relatively insignificant differences, but the turning points are identical and both approaches point to the presence of excess liquidity in the euro area since 2001. This is, however, quite moderate compared with that recorded between 1991 and 1993.

The current analysis of the RMG is based on the hypothesis that a sustained deviation from the equilibrium level of the real money stock may be expected to have implications for the future inflation rate. If so, growth rates of money in excess of those needed to sustain economic growth at a non inflationary pace, may provide early information on risks to price stability in a medium to long-run perspective.

The Harmonized Index of Consumption Price (HICP) is used by the European Central Bank to assess whether price stability has been, or will be, achieved. Therefore, it seems natural to examine the information content of the real money gap with regard to HICP inflation. Indeed, the real money gap appears to be a leading indicator of HICP inflation: Table 7 highlights the strong correlation between the current RMG and the current and forward inflation rate (from 0.79 in contemporaneous to 0.54 at lead 4). According to this admittedly rough attempt to assess the leading indicator properties of the RMG, the relationship between the two variables is relatively close in the 80s and the 90s (Figure 10). However, since 2001 the information content of the RMG seems to have diminished. These results are confirmed by the calculation of the cross-correlations over the period 1982-2000: the correlations are higher (they range from 0.85 in contemporaneous to 0.55 at lead 4) than those obtained over the full sample. Notice that the correlations between the current RMG and the lagged inflation rate are also significant.

6 Conclusion

Our results confirm the strong sensitivity of the basic money demand equation for the euro area. This is mainly reflected in the instability of the parameters and the significant variation in the number of long-term relationships. Introducing European equity prices produces a slightly more stable relationship. In addition, our estimations show a substitution effect (negative effect of stock yields on real money holdings). Our equation including asset prices is in line with those recently put forward for example by the ECB and the OECD. In our study, the asset price effect is significant and the substitution effect is in keeping with expectations, whereas the OECD
highlights a wealth effect instead of a substitution effect, which is more difficult to justify.

Although caution should be exercised when interpreting the results, the real money gap appears to have been positive since 2001. This points to the presence of moderate, albeit persistent, excess liquidity in the euro area since 2001. The relationship between HICP inflation and the real money gap is relatively close. This result enables us to regard the real money gap as a leading indicator of inflation, whose predictive power seems however to have diminished in the last few years.

Our findings suggest avenues for future research: (i) determinants of money demand seem to play an important role in certain sub-periods, less so in others, for example the exceptionally strong preference for liquidity from 2001 to 2003; therefore, a natural way to proceed could be to implement a Markov-switching framework so as to detect possible regime changes and their macroeconomic determinants; (ii) money demand functions are semi-structural models in partial equilibrium; introducing money in a dynamic stochastic general equilibrium model of the euro area could considerably improve the quality of the assessment of the risks to price stability stemming from monetary developments.

References


Table 1. Cointegration tests

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<th>Panel A. Generalized model</th>
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Note: \(\dagger\) denotes adjustment for degrees of freedom as in Reimers (1992);
** (*) rejection of the null hypothesis at 1% (5%) critical level.

Table 2. Estimates of the long-term relationship
(generalized specification)

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<th>Money demand equation</th>
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Restriction test (LR): \(\chi^2(5) = 6.020 [0.304]\)
<table>
<thead>
<tr>
<th>Authors</th>
<th>Brand and Cassola</th>
<th>Calza et al.</th>
<th>Coenen and Vega</th>
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<table>
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Table 4. Number r1/r2 of cointegration relationships (generalized specification)

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r1/r2: r1 represents the number of relationships obtained using the trace test and r2 the number of relationships obtained using the maximum eigenvalue test (at a 5% significance level).

Table 5. Estimates of the long-term relationship (asset prices augmented specification)

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<th>Variables</th>
<th>Coefficients</th>
<th>Coefficients</th>
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Restriction test (LR): \( \chi^2 (10) = 9.481 [0.487] \)
Table 6. Number r1/r2 of cointegration relationship
(asset prices augmented specification)

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r1/r2: r1 represents the number of relationships obtained using the trace test and r2 the number of relationships obtained using the maximum eigenvalue test (at a 5% significance level).

Table 7. Cross-correlations of HICP inflation and Real Money Gap

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Figure 1. Recursive estimates of the long-term coefficients (generalized specification)

Demand for money

Fisher equation

25
Figure 2. Recursive test for (over) identifying restrictions (generalized specification)
Figure 3. Chow Tests (generalized specification)

System

1 step ahead CHOWs

Breakpoint CHOWs

Predictive failure CHOWs

Demand for money

1 step ahead CHOWs

Breakpoint CHOWs

Predictive failure CHOWs

27
Figure 4. Recursive estimates of the long-term coefficients (asset prices augmented specification)

Demand for money

Fisher equation

28
Figure 5. Recursive test for (over) identifying restrictions (asset prices augmented specification)
Figure 6. Chow Tests (asset prices augmented specification)

System

Demand for money
Figure 7. Dynamic simulations of the long-term relationship

Figure 8. Forecasts
Figure 9. Real money gap

![Real money gap diagram]

Figure 10. Real money gap and HICP inflation

![Real money gap and HICP inflation diagram]