Collateral Constraints, External Imbalances and Heterogeneous Agents in a Two-country World

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Abstract

In this article, we focus on current account dynamics in large open economies characterized by debt-constrained heterogeneous agents and endogenous monetary policies. We incorporate three key features that have bulked large in the New Open Macroeconomics literature: i) home bias in trade ii) price rigidities and iii) durable goods (real properties). In order to limit agents’ willingness to consume and to (partially) insure creditors against the risk of default, we incorporate collateral constraints. We show that the impatience of collateral-constrained agents can be at the roots of permanent and sustainable external imbalances. Our model has a unique and dynamically determinate steady state, which is characterized by a positive level of debt. Our framework allows us to analyze the mechanisms at the roots of the (international) transmission of shocks and to focus on the implications of the monetary policy rules.

\textit{JEL classification codes}: E52, F32, F37, F41.

\textit{Keywords}: open economy, durable goods, collateral constraints, sticky prices, simple monetary rules.

1 Introduction

During the last decades, the world economy has experienced the accumulation of significant global external imbalances. While during the 80s and the 90s great amounts of external liabilities have almost exclusively concerned the developing world\textsuperscript{1}, large amounts of external debt are nowadays a peculiar feature of developed economies. In some countries, in light of the dramatic increase in

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\textsuperscript{1}Having said that, large current account deficits have started accumulating in US starting from the 80s. See, among others, Bryant and Hooper (1998) and Bergsten (1991).
the accumulation of net external liabilities, policy makers have questioned the sustainability of current trends\(^2\). Indeed, “There is likely to be a limit to the amount of debt that a country can issue as a result of persistent deficits before investors start to worry about its ability or willingness to repay” (Mervyn King, 2005).

These considerations are not new in general equilibrium models where unsustainable trends are generally ruled out by introducing standard no-Ponzi conditions. However, no-Ponzi conditions per se do not insure creditors against the risk of default (exogenous shocks can lead the economy out from a convergent trajectory). More restrictive conditions have been thus incorporated in models addressing risky environments. For instance, thumb rules for solvency have often been imposed to developing countries and used as a benchmark for sustainability\(^3\).

In recent years, agents ability to consume has been increasingly limited by collateral constraints. The increase of collateralized debt cannot but be associated to housing finance liberalization. Indeed, in the past 30 years, the development of financial markets has been followed by a liberalization of housing finance in many industrial countries. Deregulation introduced competitive pressures from non-banking lenders. The process took different forms in different countries.\(^4\) Eventually, liberalization allowed households a better access to mortgages credit. Moreover, by allowing more agents access to collateralized credit, the share of collateral-constraint agents have increasingly boomed \(^5\).

Collateral constraints insure that in case of default, the creditor can repossess (part of) the asset. Clearly, the existence of collateral constraints creates a link between assets value and aggregate domestic debt — and thus, between fluctuations of the prices of real assets and debt levels (see Monacelli 2007, Calza et al. 2006 and Iacoviello, 2005 for some discussion).

Notice however that spillovers deriving from housing finance are not only a purely domestic issue. Indeed, the globalized structure of financial systems has increasingly allowed financial intermediaries to convert mortgages in assets. Households debt have been thus sold abroad, and have circulated around the world in the form of international assets. These considerations suggest that the evolution of both international financial markets and house finance has created a link between external assets and real asset prices — and thus, between house prices and international capital flows. In Figure 1a and Figure 1b we show the trends of real assets prices, the current account and the dynamics of exchange rates for a set of industrial countries that have experienced significant increases (decreases) in the current account balance and/or significant decreases.

\(^2\)There is now a very rich and variegated literature on US external imbalances and the sustainability of US external debt. For some discussion on UK external imbalances see among others Iliopoulos and Miller, 2007.

\(^3\)For some discussion on solvency conditions see among many others Hellman, Murdock, Stigliz, (2000); Miller and Zhang (1999). For “thumbs rules” see Williamson (1999). See also Reinhart, Rogoff and Sevastano, (2003))

\(^4\)For a survey on the developments in the housing finance, see IMF (2008).

\(^5\)In US, mortgage debt has increased from about 60% at the half of the last century to about 90% — including vehicles.
increases) in house prices. Indeed, these trends show a co-movement between house prices and the current account deficit.

In light of the above considerations, we focus on external imbalances, current account dynamics and house prices in presence of collateral constrained agents. We will in particular focus on the effects of demand shocks. Indeed, as The Economist (2008) suggests, the global economy has been recently hit by two important shocks: the one concerning the housing market and higher commodity prices. While refraining from investigating the causes at the roots of the shocks, we aim at tracking the response to the shocks of our two-country economy.

Collateral constraints are not new in the literature. In presence of durable goods, Kiotaky and Moore (1997) extend the seminal result of Becker (1980) and Becker and Fois (1987) with heterogeneous agents to the case of non-zero debt limits. Their analysis shows that the collateral plays an important role in transmitting the effects of various shocks to other sectors. Indeed, the "financial accelerator mechanism" (see Bernanke, Gertler and Gilchrist, 1999) amplifies all endogenous developments affecting the credit market and significantly affects the business cycle. Notice however, that when monetary policy controls interest rates, this same mechanism amplifies demand shocks but dampens supply shocks — working thus as a "financial decelerator" (see Iacoviello 2005).

More recently, Campbell and Hercovitz (2005) analyze the role of collateral constraints in explaining labor supply variability. They show that in presence of strict collateral requirements, all changes in wages do affect the level of labor supply; however, if requirements are loosened, variability decreases.

In this article, we extend Monacelli (2007, 2008) New Keynesian framework to a two country-world. This framework allows us to analyze the role of both price rigidities and monetary policy in presence of collateral constrained agents. Indeed, while the global dimension of the current world economy should help a gradual realignment of the current global imbalances, market imperfections could play an important role for exchange rate dynamics. As the IMF suggests in this respect, "...given the imperfect global integration of markets for goods and services and the rigidities that constrain the reallocation of resources to tradable sectors, the redistribution of world spending is likely to require considerable movements in real exchange rates." (IMF, 2007).

To our knowledge, there are no articles in the New Keynesian literature that focus both on current account and exchange rate dynamics as well as real assets prices. In a general framework, Matsuyama (1990) examines the effects of changes in government expenditure on tradables, housing subsidies and of productivity shocks on the current account. More recently, in a business cycle framework, Punzi (2007) focuses on current account and house prices dynamics — but not on exchange rates. Finally, none of these articles account for the role of monetary policy nor for price rigidities.

In our two-country world, each economy produces both tradable goods and non-tradable real properties. Tradables and houses are produced by an infinite

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6 In the euro area countries have experience diverse trends in house prices. Therefore, it is not possible to find an analogous co-movement with the exchange rate.
set of monopolistic firms. Intermediate goods are then sold to final consumers by final retailers who operate in a perfectly competitive environment. We suppose that agents are heterogeneous in their discount factor; in our framework, agents’ impatience plays an important role in explaining external debt behaviors. This hypothesis is not new in international finance. Ghironi et al. (2005) introduce heterogeneous discounting in an overlapping generation framework. More recently, Choi et al. (2008) track the dynamics of US current account introducing endogenous heterogeneous discounting.

Impatient agents are constrained in their access to credit by collateral debt limits. In particular, their ability to borrow is a positive function of the value of their real properties: the higher the value, the better credit access. Domestic borrowing is then sold abroad as an international security (i.e., uncontingent private bond) to patient agents. Eventually, our world economy will be characterized by a positive level of external debt: the stricter the collateral constraints, the lower the steady state level of debt. Clearly, all shocks that affect house prices require adjusting external debt – and thus, a current account dynamic; terms of trade play an important role as a transmission channel of country and sector specific shocks.

The article is organized as follows. In Section 2 we introduce our model and in Section 3 we analyze the steady state of the dynamics. In Section 4 we focus on the dynamics following possible stochastic shocks. Section 6 comments the main results of our analysis while the Appendix provides analytical details and the figures.

2 The model

This model is built on Monacelli (2007, 2008) and Iacoviello (2005) but is developed in a two countries setting. We consider Home and Foreign respectively (denoted by $H$ and $F$ for simplicity). Both countries are open in every ways but labor. The inhabitants of both countries have same preferences but are heterogeneous in their degree of impatience. More precisely, we assume that the representative inhabitant of country $H$ is more impatient than the one of country $F$. S/he is not a consumption smoother but her/his desire to consume is limited by a collateral constraint. For simplicity we will denote the inhabitant of country $H$ as the borrower and the one of country $F$ as the saver.

Durable goods (real properties) and tradable goods are produced in a monopolistic competition framework by domestic and foreign firms; real properties are non-tradable goods and can be used as collateral. Think for instance at the housing market: leaving tourism a part, houses are in generally owned and sold to residents. Goods are then purchased and sold to final consumers by domestic retailers, in a competitive environment. The representative retailer

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7 Our hypothesis is also consistent with Masson et al. (1994) and Henriksen (2002) who relate current account dynamics with demographic factors.
8 For more discussion, see also Buitre (1981) and Weil (1989).
9 See also Engel and Wang (2008) for some discussion.
in the housing sector in country $H$ (in country $F$), buys Home (Foreign) produced durables only and sell them to final consumers in country $H$ (country $F$) only. Analogously, the representative retailer in the tradable sector, buys both Home and Foreign produced goods to sell them to final consumers in the Home (Foreign) country only. Final consumers enjoy services coming from durables and consume tradable goods. Finally, agents can smooth their consumption by exchanging uncontingent securities on an international uncomplete market.

2.1 Final retailers

We suppose that intermediate goods are sold in both countries to final consumers by an infinite set of retailers operating in a competitive environment. Good markets in each country are segmented into the tradable and real properties sector. Real properties are produced by domestic firms and sold by retailers to domestic final consumers. Tradables are produced in both countries and can be exchanged on international good markets. We will denote by $j = T$, the representative retailer operating in the tradable sector; $j = n$, the representative retailer operating in the durable sector (real properties). The retailer $j = T$ in country $H$ (in country $F$) buys Home and Foreign produced tradables and sell them to the Home (Foreign) market. The retailer $j = n$ buys real properties produced in country $H$ (country $F$) and sell them to the Home (Foreign) market.

2.1.1 Tradables

Consider first the case of the retailer operating in the tradable sector in country $H$. S/he has access to both domestic and foreign produced goods and sell them to $H$ final consumers only. In order to reflect consumer’s preferences, we assume that the behavior of retailers is characterized by home bias. Retailers operate in a perfectly competitive environment and their basket of production is the following CES bundle:

$$Y_{T,t} = \left[ \alpha \frac{1}{\eta} Y_{h,t}^{\frac{1-\eta}{\eta}} + (1 - \alpha) \frac{1}{\eta} Y_{f,t}^{\frac{1-\eta}{\eta}} \right]^{\frac{\eta}{1-\eta}}$$

where $\alpha$ represent the weight of Home produced goods in consumers’ bundles (in presence of home bias, $\alpha > 0.5$) and $\eta$ is the elasticity of substitution between Home and Foreign produced goods. For simplicity, from now on we denote all variables referred to Home (Foreign) produced goods and prices with the pedix $h$ (pedix $f$). Retailers’ demand for respectively Home and Foreign produced goods, is the result of an optimization problem where the CES-related price index of tradables is:

$$P_{T,t} = \left[ \alpha P_{h,t}^{1-\eta} + (1 - \alpha) P_{f,t}^{1-\eta} \right]^{\frac{1-\eta}{\eta}}$$

\footnote{An alternative way to introduce home bias in our model would be to leave the choice between domestically-produced versus foreign-produced goods to consumers.}
Retailers’ intermediate demand for Home produced and Foreign produced goods (\(Y_{h,t}\) and \(Y_{f,t}\), respectively) is thus defined by:

\[
Y_{h,t} = \alpha Y_{T,t} \left( \frac{P_{h,t}}{P_{T,t}} \right)^{-\eta} \tag{2}
\]

\[
Y_{f,t} = (1 - \alpha) Y_{T,t} \left( \frac{P_{f,t}}{P_{T,t}} \right)^{-\eta} \tag{3}
\]

In country \(F\), retailers in the tradable sector behave symmetrically. This implies that the weight of Foreign produced goods on country \(F\)-CES production bundle is the same as the one of Home produced goods in country \(H\), i.e.:

\[
Y_{T,t}^* = \left[ (1 - \alpha) Y_{h,t}^{\eta^{-1}} + \alpha Y_{f,t}^{\eta^{-1}} \right]^{\frac{\eta}{1-\eta}} \tag{4}
\]

Profit maximization implies that retailers’ intermediate demand for foreign and domestic goods respectively in country \(F\) is:

\[
Y_{h,t}^* = (1 - \alpha) Y_{T,t}^* \left( \frac{P_{h,t}^*}{P_{T,t}^*} \right)^{-\eta} \tag{5}
\]

\[
Y_{f,t}^* = \alpha Y_{T,t}^* \left( \frac{P_{f,t}^*}{P_{T,t}^*} \right)^{-\eta} \tag{6}
\]

Notice however that retailers also need to choose amongst the different (infinite) varieties of heterogeneous domestic and foreign goods, \(i\). We suppose that the Home produced (Foreign produced) production basket of the representative retailer is in turn a CES bundle of a continuum of infinite varieties of goods, \(i\).

Following Obstfeld and Rogoff (1995), we reasonably assume that the elasticity of substitution amongst Home (Foreign) produced goods, is larger than the one between Home produced and Foreign goods, \(\eta\). We denote thus by \(\varepsilon\) the elasticity of substitution between single goods, \(i\). Retailers’ intermediate demand for a single variety of Home produced (Foreign) good is the result of a profit maximization program, i.e.:

\[
Y_{h,t}(i) = Y_{h,t} \left( \frac{P_{h,t}(i)}{P_{h,t}} \right)^{-\varepsilon} \tag{7}
\]

\[
Y_{f,t}(i) = Y_{f,t} \left( \frac{P_{f,t}(i)}{P_{h,t}} \right)^{-\varepsilon} \tag{8}
\]

\(\text{11 The corresponding price index is: } P_{T,t}^* = \left[ (1 - \alpha) P_{h,t}^{\eta^{-1}} + \alpha P_{f,t}^{\eta^{-1}} \right]^{1/\eta}\)
where $\varepsilon > \eta$. Analogously, in the rest of the world:

$$Y_{h,t}^{*} (i) = Y_{h,t}^{*} \left( \frac{P_{h,t}^{*} (i)}{P_{h,t}} \right)^{-\varepsilon} \quad (9)$$

$$Y_{f,t}^{*} (i) = Y_{f,t}^{*} \left( \frac{P_{f,t}^{*} (i)}{P_{f,t}} \right)^{-\varepsilon} \quad (10)$$

### 2.1.2 Durables

Consider now the housing sector. The representative retailer in country $H$ chooses a set of goods amongst an infinite continuum of domestically produced real properties (durables). Her/his demand for each differentiated good, $i$, is the result of profit maximization in a competitive environment $13$, i.e.:

$$Y_{n,t} (i) = \left( \frac{P_{n,t} (i)}{P_{n,t}} \right)^{-\varepsilon} Y_{n,t} \quad (11)$$

where $\varepsilon$ is the elasticity of substitution between single goods, $i$. $14$ Analogously, in the Foreign country, the intermediate demand of the representative retailer of real properties is:

$$Y_{n,t}^{*} (i) = \left( \frac{P_{n,t}^{*} (i)}{P_{n,t}} \right)^{-\varepsilon} Y_{n,t}^{*} \quad (12)$$

### 2.2 Optimal consumption in country $H$

Consider now the representative inhabitant of country $H$ (for simplicity, from now on we will denote her/him as the borrower). His/her utility is a positive function of his basket of consumption, $C_t$ and a negative function of his/her labor effort (his/her supply of labor, $N_t$) i.e. $15$:

$$\max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U (C_t, N_t) \right\}$$

$12$ In order to keep countries $H$ and $F$ as symmetric as possible, we assume identical elasticities of substitution across countries.

$13$ Where the CES production bundle of the retailer is: $Y_{n,t} \equiv \left( \int_0^1 Y_{n,t} (i) \frac{1}{\varepsilon} d_i \right)^{-\frac{1}{\varepsilon}}$. The associated price index is: $P_{n,t} \equiv \left( \int_0^1 P_{n,t} (i) \frac{1}{\varepsilon} d_i \right)^{-\frac{1}{\varepsilon}}$.

$14$ For simplicity, we assume that the elasticity of substitution between the infinite varieties of Home (Foreign) produced goods is the same for both sectors.

$15$ In our economy all agents have the same preferences and maximize an utility function that for analitical simplicity we assume having the following form: $U_t = \ln C_t - \left( \frac{1}{1+\varphi} \right) N_t^{1+\varphi}$.
where $\beta$ is the borrower discount factor. We do not introduce explicitly money in the utility function and we use it as the numeraire of our cashless economy à la Woodford.

The representative borrower consumes a bundle that is a CES composite of tradables and services coming from the stock of real properties. For simplicity, we assume that agents start enjoying services deriving from durables in the same period they purchase them. The borrower consumption basket is thus:

$$C_t = \left[ \gamma \frac{1}{\theta} C_{T,t}^{\theta} + (1 - \gamma) \frac{1}{\theta} C_{n,t}^{\theta} \right]^{\frac{1}{\theta}}$$

where $\gamma$ is the weight of tradables in the basket and $\theta \geq 0$ is the elasticity of substitution between durable services and tradables (services). The following assumption on preferences always needs to hold in both countries:

**Assumption 1 (preferences)** $U_i \in C^2$, $U_{iC} > 0$, $U_{iN} < 0$, $U_{iCC} < 0$, $U_{iCC} U_{iNN} > U_{iCN}$ for every $(C_i, N_i)$ such that $C_i, N_i > 0$, $i =$borrower,saver. Also, an Inada condition for consumption holds.

The individual budget constraint in real terms of tradable consumption is:

$$C_{T,t} + x_t (C_{n,t} - (1 - \delta) C_{n,t-1}) + R_{t-1} \frac{b_{t-1}}{\pi_{T,t}} \leq b_t + \frac{W_t N_t}{P_{T,t}} + \sum \Gamma$$

where $C_{T,t}$ represents tradable consumption, $P_{n,t} (C_{n,t} - (1 - \delta) C_{n,t-1})$ is the cost of durable expenditure in period $t$; $b$ are net external liabilities in real terms of tradable consumption, $B \equiv D - qD^*$ are net external liabilities in nominal terms, $D$ are home-currency domestic securities, $q$ is the nominal price of Foreign currency in terms of Home currency and $D^*$ are foreign-currency Foreign securities. Notice also that $x_t \equiv \frac{P_{T,t}}{P_{T,t-1}}$ is the relative price of durable consumption and $\pi_{T,t} \equiv \frac{P_{T,t}}{P_{T,t-1}}$ is the inflation rate in the tradable sector (the inflation rate in the durable sector is defined as $\pi_{n,t} \equiv \frac{P_{n,t}}{P_{n,t-1}}$). In practice, each period the borrower buys tradables, $C_T$, and real properties; s/he pays the interests on her/his debt $- R$ is the gross nominal interest rate factor. S/he enjoys resources coming from foreign borrowing, $B$, labor income, $W_N$ and profits, $\Gamma$. Labor is assumed mobile across sectors but not across countries; therefore, the wage is the same in each sector but not in each country.

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16 We assume also that agents cannot rent/lend houses.
17 The individual budget constraint in nominal terms is:

$$P_{T,t} C_{T,t} + P_{n,t} (C_{n,t} - (1 - \delta) C_{n,t-1}) + R_{t-1} B_{t-1} \leq B_t + W_t N_t + \sum \Gamma$$

The budget constraint is assumed to hold with equality around the deterministic steady state.
18 Notice that $b_{t-1} = \frac{B_{t-1}}{P_{T,t-1}}$.
19 See the Appendix for all details concerning the optimization program of the consumer.
Agents relative impatience (see Assumption 2) implies that the representative agent in country \( H \) is not a consumption smoother, i.e., s/he does not have an intertemporal budget constraint (for more discussion, see Section 3)\(^\text{20}\). Assuming that the inhabitants of country \( H \) are more impatient than the ones of country \( F \) is clearly a simplification. However, this simplification allows us to analyze cases where there are international differences in countries’ discount factors at an aggregate level.\(^\text{21}\) International differences in discounting are not new in the literature. Buiter (1981), Ghironi et al. (2005) and Weil (1989) introduce heterogeneous discount rates in a framework of overlapping-generations; in Choi et al. (2008) heterogeneous discount rates are an endogenous result of the model.

Borrowers’ capacity to obtain credit is limited by a collateral constraint. We suppose that households’ debt is constrained to be a share of their durables (real properties), and debt contracts are denominated in nominal terms, i.e.:

\[
b_t \leq (1 - \chi) C_n,t x_t
\]  

where \( \chi \) is the fraction of durables that cannot be used as a collateral and can be interpreted as the inverse of the loan-to-value ratio: the larger \( \chi \), the more stringent the constraint. For simplicity, we assume \( \chi \) to be an exogenous parameter of our model. The role of collateral constraints and the implications of their structure has been recently analyzed in a New-Keynesian framework by Calza et al. (2006) in a closed economy framework.\(^\text{22}\) Eventually, collateral constraints allow agents a better access to credit. Indeed, they partially ensure the creditor against the risk of default: in case of default, the creditor can always repossess (part of) the asset.\(^\text{23}\)

In our two-country world, constraint (14) reduces to a limit on international borrowing. This should not surprise the reader. Since we aim at analyzing current account dynamics, we are interested in the behavior of aggregate variables, and in the dynamics of flows (of goods and financial capital) between countries. In aggregate, the sum of national assets in each country is equal to zero. Thus, if indebted, our representative agent of each country cannot but be indebted towards his foreign counterpart only. Moreover, thanks to the globalized structure of financial systems mortgages can be easily converted in international assets. Our representative agent in each country can thus act as a financial intermediary and sold her/his (collateralized) debt abroad. In this vein, collateral constraints (and their impact) are transferred to an international dimension.

Notice finally that (14) implies also that an increase in the relative price of real properties allows agents to increase their level of debt.

\(^{20}\) See also Iacoviello, 2005.

\(^{21}\) In a two-country framework, this reduces to the case where the representative agent of one country is less patient than the one of another country.

\(^{22}\) For some discussion see also Monacelli (2006, 2007), Iacoviello (2005) and Campbell and Hercowitz (2006).

\(^{23}\) In Bernanke and Gertler (1989) they are justified by the presence of private information and limited liability.
The first order conditions of borrowers’ optimization program are:

\[ -U_{N,t} = \frac{W_t}{P_{T,t}} \]  
\[ x_t U_{T,t} = U_{n,t} + \beta (1 - \delta) E_t \{ U_{T,t+1} x_{t+1} \} + U_{T,t} \psi_t (1 - \chi) x_t \]  
\[ R_t = \left\{ \frac{(1 - \psi_t)}{\beta} \frac{U_{T,t}}{U_{T,t+1} \pi_{T,t+1}} \right\} \]  
\[ \psi_t = 1 - \beta E_t \left\{ \frac{R_t}{\pi_{T,t+1}} \frac{U_{T,t+1}}{U_{T,t}} \right\} \]

Equation (15) represents a standard consumption/leisure arbitrage. Equation (16) represents the intertemporal demand for tradable consumption relatively to durable-services consumption. In equilibrium, the value of the utility deriving to the borrower from present consumption of tradables needs to equal the value of direct utility deriving from the direct services of durables plus the value of their indirect utility, i.e.: i) the utility deriving from the possibility of selling real properties and buying durable consumption in future, \( \beta (1 - \delta) E_t \{ U_{T,t+1} x_{t+1} \} \) and ii) the marginal utility stemming from relaxing the collateral constraint, and consuming additional non-durable goods, \( U_{T,t} \psi_t (1 - \chi) x_t \). Equation (17) represents a modified Euler equation of country \( H \) — where \( \psi_t \) is the Lagrangean multiplier associated to the collateral constraint. Clearly, it reduces to the standard Euler equation whenever \( \psi_t = 0 \). \( \psi_t \) represents the marginal value of additional debt: if \( \psi_t > 0 \), the marginal utility of current consumption exceeds the one of future consumption (and thus, savings):

\[ U_{T,t} > \beta E_t \left\{ \frac{R_t}{\pi_{T,t+1}} U_{T,t+1} \right\}. \]

By marginally relaxing the collateral constraint one could have access to more current consumption and increase her/his utility (see eq. 18).

Finally, one can substitute eq. (17) in (16) and obtain:

\[ \frac{U_{n,t}}{U_{T,t}} = x_t (1 - \psi_t (1 - \chi)) - (1 - \delta) E_t \left\{ \frac{R_t}{\pi_{T,t+1}} x_{t+1} \right\} \]

Equation (19) represents the user cost of durables in terms of non-durables at time \( t \). It represents the price you pay for the flow of consumption services from durables during the period; the cost is obviously a positive function of the interest rate and the relative price of durables (for \( \psi_t \) fixed).

2.3 Exchange rates and terms of trade

In presence of home bias, the law on one price only holds for the single basket of Foreign produced and Home produced tradables, separately. In practice:
\[ P_{h,t} = q_t P^*_t \]

and

\[ P_{f,t} = q_t P^*_t \]

where \( q \) is the nominal exchange rate and \( \alpha > \frac{1}{2} \) implies \( P_{T,t} \neq q_t P^*_T \).

In addition, in equilibrium, the following relationship between Home and Foreign net external liabilities always needs to hold:

\[ B_t = q_t B^*_t \quad (20) \]

where \( B \) is Home-currency net external debt in nominal terms and \( B^* \) is the lender’s net debt in Foreign currency. Obviously, if \( B > 0 \) (and thus, \( B^* < 0 \)) borrowers are net debtor and savers are net lenders.

We finally introduce country \( H \) terms of trade and we define them in the following way:

\[ S_t = \frac{P_{f,t}}{P_{h,t}} = \frac{P^*_f}{P^*_h} \]

Symmetrically, country \( F \) terms of trade are thus:

\[ S^*_t = \frac{P_{h,t}}{P_{f,t}} = \frac{P^*_h}{P^*_f} = S_t^{-1} \]

Equation (20) can be rewritten as a function of terms of trade and real (in terms of tradables) consumption, i.e.:

\[ b^*_t = b_t \left( \frac{(1 - \alpha) S_t^{1-\eta} + \alpha}{\alpha S_t^{1-\eta} + 1 - \alpha} \right)^{\frac{1}{1-\eta}} \quad \text{ (21)} \]

### 2.4 Optimal consumption in country \( F \)

We consider now the representative agent of country \( F \). We suppose that agents in country \( F \) are more patient than agents in country \( H \). Thus, the discount rate of the borrower is strictly lower than the one of country \( F \) representative agent (the saver). Savers maximizes the following utility function:

\[ \max E_0 \left\{ \sum_{t=0}^{\infty} \mu^t U^* (C^*_t, N_t^*) \right\} \]

where \( \mu \) is savers’ discount factor \( N_t^* \) is labor and \( C_t^* \) is a CES composite good of tradables and services arising from durables. For simplicity, from now on,\[ 25 \text{Clearly, } B^* = \frac{D}{q} - D^* \]
all variables referring to the foreign country (currency) will be indexed by an asterisk. Also, Assumption 2 always need to hold:

**Assumption 2 (discounting)** \( \beta, \mu \in (0, 1) ; \beta < \mu. \)

Assumption 2 is crucial in our model and has important implications. We will analyze all implications for the steady state in Section 3. Notice for the moment that the saver is a consumption smoother and has thus an intertemporal budget constraint.

Savers’ consumption basket is composed as the one of borrowers:

\[
C^*_t = \left[ \gamma^* \frac{C^*_{T,t}^\theta}{(1-\gamma)} + \frac{1}{\theta} C^*_{n,t}^{\frac{\theta-1}{\theta}} \right]^\theta
\]

where \( C^*_n \) are services deriving from real properties in the Foreign country only and \( C^*_T \) is a basket of tradables. The budget constraint of the \( F \)-agent in nominal terms is:\(^{26}\)

\[
P^*_T C^*_{T,t} + P^*_n \left( C^*_n - (1-\delta) C^*_{n,t-1} \right) - R^*_{t-1} B^*_{t-1} \leq -B^*_t + W^*_t N^*_t + \sum_i \Gamma^*_i
\]

where \( W^*_t \) are foreign-currency wages in the Foreign country, \( R^* \) are nominal interest rates in the Foreign country and \( \Gamma^* \) are profits. Finally, \( B^* \equiv \frac{D^*}{q} - D^* \) are Foreign net external assets in Foreign currency. We can thus rewrite eq. (22) in terms of tradable Foreign consumption:

\[
C^*_{T,t} + x^*_t \left( C^*_n - (1-\delta) C^*_{n,t-1} \right) - R^*_{t-1} \frac{b^*_{t-1}}{\pi^*_{T,t}} \leq -b^*_t + W^*_t N^*_t + \sum_i \Gamma^*_i
\]

Finally, we introduce a no-Ponzi game condition on international net assets:

\[
\lim_{i \to \infty} E_t \frac{b^*_{t+i}}{\prod_{z=1}^{i} R^*_{z}} \leq 0 \tag{23}
\]

The first order conditions of savers’ optimization program are:

\[
\frac{-U^*_{N,t}}{U^*_{T,t}} = \frac{W^*}{P^*_{T,t}} \tag{24}
\]

\[
U^*_{T,t} x^*_t = U^*_{n,t} + \mu (1-\delta) E_t \{ x^*_{t+1} U^*_{T,t+1} \} \tag{25}
\]

\[
R^*_t = E_t \left\{ \frac{U^*_{T,t}}{U^*_{T,t+1}} \frac{\pi^*_{T,t+1}}{\mu} \right\} \tag{26}
\]

\(^{26}\) It holds with equality around a deterministic steady state.
Equation (24) states the standard arbitrage condition between leisure and consumption and equation (25) is an intertemporal demand equation for durables vs nondurable goods. This equation states that the value of current non-durable consumption needs to equal the value of the direct utility stemming from the services deriving from durables plus the indirect utility one can obtain by selling durables in future and consuming non-durables.

Finally, we need to introduce the relation linking interest rates in country $H$ and in country $F$. In order to obtain it, we substitute for gross external assets and liabilities in the budget constraint of country $H$ or $F$ (remember that $B ≡ D − qD^*$). Rewriting the budget constraint in real terms of tradable consumption and re-calculating the first order conditions for the borrower and/or the lender, one can easily find the needed condition. It is possible to show (see the Appendix for all details) that the following modified uncovered interest parity condition needs to hold:

$$R_t = E_t \left\{ \frac{q_{t+1}}{q_t} \right\} R_t^*$$  \hspace{1cm} (27)

The absence of arbitrage possibilities between domestic and foreign assets implies that the marginal utility of investing in Home assets is equal to the one agents obtain by investing in Foreign assets. Notice finally that given the stochastic setup of our framework and the assumption of incomplete markets, the uncovered interest parity condition only holds in expectations.

### 2.5 Production

We now set the structure of production in our two-country world. For simplicity we suppose that labor is homogeneous and mobile across sectors in the same country – but not around the world. We assume also that the representative agent in each country is also the owner of representative firms in each country. Markets in each country are segmented into tradables and durables (real properties). Firms in both sectors operate in a monopolistic competition environment.

Real properties’ producers at Home (in the Foreign country) only sell their goods to Home (Foreign) markets while tradables’ producers sell them to both countries’ retailers. We suppose that there are $i$ firms producing $i$ non perfectly substitutable durables (tradable goods). Each firm is characterized by a production function $F$, which depends on labor, $N_j$ ($j =$ tradables, durables) and a productivity shifter $A_j$, which is common for all firms within the same sector. The following proposition needs to hold:

**Assumption 3 (technology):** $F$ is homogeneous of degree 1 with $F \in C^2$, $F_N > 0$, $F_{NN} \leq 0$. Moreover $F(0) = 0$, $\lim_{N \to 0} F'(N) = +\infty$, $\lim_{N \to \infty} F'(N) = 0$. 

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2.5.1 Tradables

We first focus the attention on the tradable sector in country \( H \). Firm \( i \) production function is consistent with Assumption 3 and is defined for simplicity as:

\[
F_i(N_{h,t}) = A_{h,t} N_{h,t}(i)
\]

Firms maximize the profit function:

\[
E_0 \left\{ \sum_{t=0}^{\infty} \Lambda_{0,t} \left( Y_{h,t}(i) P_{h,t}(i) - W_t N_{h,t}(i) - \frac{\omega_T}{2} \left( \frac{P_{h,t}(i)}{P_{h,t-1}(i)} - 1 \right)^2 P_{h,t} \right) \right\}
\]

given retailers’ demand functions. \( \Lambda \) is the borrower’s stochastic discount factor\(^{27} \) and:

\[
\Lambda_{t,t+1} \equiv \frac{\Lambda_{0,t+1}}{\Lambda_{0,t}} = \beta E_t \left\{ \frac{1}{1 - \psi_t} \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{T,t}}{P_{T,t+1}} \right\}
\]

where \( \lambda \) is the borrower’s Lagrangean multiplier (i.e., the marginal utility of income) of the representative consumer in country \( H \) and

\[
\frac{\omega_T}{2} \left( \frac{P_{h,t}(i)}{P_{h,t-1}(i)} - 1 \right)^2 P_{h,t}
\]

are the firm’s costs associated to adjusting prices (menu costs); following Rotemberg and Woodford, (1997) we assume that adjustment costs are quadratic. In practice, each period firms need to optimally balance the costs arising from resetting prices and the costs associated to deviating from optimality.

Analogously, in country \( F \), firms’ stochastic discount factor is:

\[
\Lambda_{t,t+1}^* \equiv \frac{\Lambda_{0,t+1}^*}{\Lambda_{0,t}^*} = \mu E_t \left( \frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{P_{T,t}^*}{P_{T,t+1}^*} \right)
\]

where \( \lambda^* \) is the Lagrangean multiplier associated to the saver.

In both countries firms choose their optimal sequence \( \{ N_{h,t}(i), P_{h,t}(i) \} \).

Nominal and real (in terms of tradable consumption) marginal costs – \( MC \) and \( mc \) are respectively:

\[
MC_{h,t} = \frac{W_t}{A_{h,t}} \quad mC_{h,t} = \frac{W_t}{P_{h,t} A_{h,t}}
\]

\(^{27}\)Profits here are in nominal terms while the consumer max program is in real terms. That’s why we need to incorporate prices also.
In a symmetric equilibrium: \( P_{h,t}(i) = P_{h,t} \). We can thus simply solve for optimal prices. We obtain the following New Keynesian Phillips Curve:

\[
\omega_T (\pi_{h,t} - 1) \pi_{h,t} = Y_{h,t} \varepsilon \left( \frac{1 - \varepsilon}{\varepsilon} + mc_{h,t} \right) + \omega_T \beta E_t \left\{ \frac{1}{1 - \psi_t} \right\} \left( \frac{U_{T,t} + 1}{U_{T,t}} \right) \left( \frac{\alpha + (1 - \alpha) S_t^{1-\eta}}{\alpha + (1 - \alpha) S_{t+1}^{1-\eta}} \right) (\pi_{h,t+1} - 1) \pi_{h,t+1}\right\} \]

The standard optimization program of the representative agent implies that in equilibrium there cannot be gains in exchanging leisure with consumption; the non-arbitrage condition leisure/consumption needs to hold. Thus, in our case: \( \frac{-U_{N,t}}{U_{T,t}} = \frac{W_t}{P_{t,t}} \) (see eq (15)). Condition (15) can be here rewritten as:

\[
\frac{1}{U_{t,t}} = \frac{A_{h,t} - U_{N,t}}{U_{T,t}^{1-\eta}} \left( \frac{\alpha + (1 - \alpha) S_t^{1-\eta}}{\alpha + (1 - \alpha) S_{t+1}^{1-\eta}} \right) (\pi_{h,t+1} - 1) \pi_{h,t+1}\right\}
\]

therefore, substituting in (28), we obtain:

\[
mc_{h,t} = \frac{1}{A_{h,t}} \frac{-U_{N,t}}{U_{T,t}} \left( \alpha + (1 - \alpha) S_t^{1-\eta} \right)^{1-\eta} \ (30)
\]

Clearly, terms of trade affects the Phillips curve both in the form of the marginal cost, and through the discount factor. Incorporating (30) and the relation \( Y_{h,t} = A_{h,t} N_{h,t} \) in the above Philips curve we obtain:

\[
\omega_T (\pi_{h,t} - 1) \pi_{h,t} = A_{h,t} N_{h,t} \varepsilon \left( \frac{1 - \varepsilon}{\varepsilon} + \frac{-U_{N,t}}{A_{h,t} U_{T,t}} \right) \left( \alpha + (1 - \alpha) S_t^{1-\eta} \right)^{1-\eta} + \omega_T \beta E_t \left\{ \frac{1}{1 - \psi_t} \right\} \left( \frac{U_{T,t+1}}{U_{T,t}} \right) \left( \frac{\alpha + (1 - \alpha) S_t^{1-\eta}}{\alpha + (1 - \alpha) S_{t+1}^{1-\eta}} \right) (\pi_{h,t+1} - 1) \pi_{h,t+1}\right\} \]

Without price rigidities, the real marginal cost should be constant at the mark-up. Notice however that in the tradable sector, terms of trade create a wedge between the rate of substitution between consumption and leisure on the one hand, and the marginal product of labor on the other. The real marginal cost is directly affected by movements in terms of trade. This creates a scope for policy intervention whenever the policy-maker aims at optimal policies (for some discussion see Faia and Monacelli, 2007).

Analogous considerations apply for country \( F \). Marginal costs are:

\[
mc_{f,t} A_{f,t}^* = \frac{W_{f,t}^*}{P_{f,t}^*} \]

\[
mc_{f,t} = \frac{\frac{1}{A_{f,t}} \frac{-U_{N,t}}{U_{T,t}} \left( \frac{\alpha + (1 - \alpha) S_t^{1+\eta}}{\alpha + (1 - \alpha) S_{t+1}^{1+\eta}} \right)}{A_{f,t}^*} \left( \frac{\alpha + (1 - \alpha) S_t^{1+\eta}}{\alpha + (1 - \alpha) S_{t+1}^{1+\eta}} \right) (\pi_{h,t+1} - 1) \pi_{h,t+1}\right\} \]

(32)
and the NKPC is:

\[
\omega_T (\pi_{f,t}^* - 1) \pi_{f,t} = A_{f,t}^* N_{f,t}^* \left( \frac{1 - \varepsilon}{\varepsilon} + (\alpha + (1 - \alpha) S_t^{-1+\eta}) \right) - U_{N,t}^* \left( \frac{A_{f,t}^*}{U_{T,t}^*} \right) \quad (33)
\]

\[
+ \omega_T \mu E_t \left\{ \frac{U_{T,t+1}^*}{U_{T,t}^*} \left( \frac{1}{\alpha + (1 - \alpha) S_{t+1}^{-1+\eta}} \right) \right\} \left( \pi_{f,t+1}^* - 1 \right) \pi_{f,t+1}^*
\]

### 2.5.2 Durables

The price dynamics in the housing sector can be easily be calculated following the same lines of the previous section. The New Keynesian Phillips curve for durables (real properties) is:

\[
0 = Y_{n,t} \varepsilon \left( \frac{1 - \varepsilon}{\varepsilon} + mc_{n,t} \right) - \omega_n (\pi_{n,t} - 1) \pi_{n,t} + \omega_n \beta E_t \left\{ \frac{1}{1 - \psi_t} A_{n,t+1} \right\} \left( \pi_{n,t+1} - 1 \right) \pi_{n,t+1} \frac{P_{n,t+1}}{P_{n,t}}
\]

where, again:

\[
\Lambda_{t,t+1} = \frac{A_{n,t+1}}{A_{n,t}} \beta E_t \left\{ \frac{1}{1 - \psi_t} \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{T,t}}{P_{T,t+1}} \right\}
\]

and thus:

\[
0 = Y_{n,t} \varepsilon \left( \frac{1 - \varepsilon}{\varepsilon} + mc_{n,t} \right) - \omega_n (\pi_{n,t} - 1) \pi_{n,t} + \omega_n \beta E_t \left\{ \frac{1}{1 - \psi_t} \frac{U_{T,t+1}^* x_{t+1}^*}{U_{T,t}^* x_t} \right\} \left( \pi_{n,t+1} - 1 \right) \pi_{n,t+1}
\]

Also, given the arbitrage consumption-leisure, we can rewrite firms’ marginal costs in the durable sector as:

\[
mc_{n,t} = \frac{1}{A_{n,t} x_t} \frac{-U_{N,t}}{U_{T,t}^*} \quad (34)
\]

Notice that there is a wedge between the ratio of marginal utilities and the marginal cost. This wedge is created by variation in the relative price, \(x_t\). When the monetary authority aims at implementing an optimal policy, the presence of this wedge leaves the policy maker a scope for policy intervention (for some discussion see Monacelli, 2006).

Incorporating (34) and \(Y_{n,t} = A_{n,t} N_{n,t}\) in the above New-Keynesian Phillips curve, we obtain:

\[
0 = A_{n,t} N_{n,t} \varepsilon \left( \frac{1 - \varepsilon}{\varepsilon} + \frac{1}{x_t} \frac{U_{T,t+1}^* x_{t+1}^*}{U_{T,t}^* A_{n,t}} \right) \left( \pi_{n,t} - 1 \right) \pi_{n,t} + \omega_n \beta E_t \left\{ \frac{1}{1 - \psi_t} \frac{U_{T,t+1}^* x_{t+1}^*}{U_{T,t}^* x_t} \right\} \left( \pi_{n,t+1} - 1 \right) \pi_{n,t+1}
\]
Analogously, the NKPC in country $F$ is:

$$0 = A^*_{n,t}N^*_{n,t}e \left( \frac{(1 - \varepsilon)}{\varepsilon} + \frac{1}{x_t^*} \frac{-U^*_{N,t}}{U^*_{T,t}A^*_{n,t}} \right) + \omega_n \left( \pi^*_{n,t} - 1 \right) \pi^*_{n,t} + \omega_n \mu E_t \left\{ \frac{U^*_{T,t+1}}{U^*_{T,t}} \frac{x^*_{t+1}}{x^*_t} \left( \pi^*_{n,t+1} - 1 \right) \pi^*_{n,t+1} \right\}$$

where real (in terms of tradable consumption) marginal costs are:

$$mc^*_{n,t} = \frac{1}{x^*_t} \frac{-U^*_{N,t}}{U^*_{T,t}A^*_{n,t}}$$

### 2.6 Market clearing

#### 2.6.1 Home country

For markets to be cleared in country $H$, total purchases of real properties need to equal the total domestic production; they also need to account for the costs of price rigidities. We remind the reader that in this model real properties are non-tradable goods. Thus:

$$Y_{n,t} = C_{n,t} - (1 - \delta) C_{n,t-1} + \frac{\omega_n}{2} (\pi_{n,t} - 1)^2$$

Given that labor is not mobile across countries, labor market clearing implies:

$$N_n + N_h = N_t$$

and $Y_{n,t} = A_{n,t}N_{n,t}$. Therefore:

$$A_{n,t}N_{n,t} = C_{n,t} - (1 - \delta) C_{n,t-1} + \frac{\omega_n}{2} (\pi_{n,t} - 1)^2$$

Focus now on the Home sector of tradables. Market clearing requires:

$$Y_{h,t} = C_{h,t} + C^*_{h,t} + \frac{\omega}{2} (\pi_{h,t} - 1)^2$$

Notice that local retailers of tradables in country $H$ operate in a perfectly competitive environment and only sell their products to Home inhabitants (in practice, they simply act as aggregators). Therefore, the market of tradables clears when the total amount of retailers’ sales equal total Home consumption in tradables. In practice, $C_{T,t} = Y_{T,t}$. Recalling that retailers’ demand for domestically and foreign produced goods are respectively, $Y_{h,t} = \alpha Y_{T,t} \left( \frac{P_{h,t}}{P_{T,t}} \right)^{-\eta}$ and $Y_{f,t} = (1 - \alpha) Y_{T,t} \left( \frac{P_{h,t}}{P_{T,t}} \right)^{-\eta}$ the market clearing condition for the tradable sector can be rewritten first as:

$$A_{h,t}N_{h,t} = \alpha Y_{T,t} \left( \frac{P_{h,t}}{P_{T,t}} \right)^{-\eta} + (1 - \alpha) Y^*_{T,t} \left( \frac{P^*_{h,t}}{P^*_{T,t}} \right)^{-\eta} + \frac{\omega}{2} (\pi_{h,t} - 1)^2$$
and then as a function of terms of trade, i.e.:

\[
A_{h,t}N_{h,t} = \alpha C_{t,t} \left[ \alpha + (1 - \alpha) S_t^{1-q} \right] \frac{\omega}{\pi} + (1 - \alpha) C_{t,t}^* \left[ (1 - \alpha) + \alpha S_t^{1-q} \right] \frac{\omega}{\pi} + \omega_T \left( \pi_{h,t} - 1 \right)^2
\]  

(40)

2.6.2 Country F

Given the symmetric structure of our world economy, clearing conditions for country F have a symmetric structure, relatively to the ones of country H. Clearing conditions for country F are listed in the following.

Labor market clearing requires:

\[
N_n^* + N_f^* = N_t^*
\]  

(41)

Market clearing in the non-tradable sector requires:

\[
A_{n,t}^* N_n^{*,t} = C_{n,t}^* - (1 - \delta) C_{n,t-1}^* + \frac{\omega_n}{2} \left( \pi_{n,t}^* - 1 \right)^2
\]  

(42)

Finally, market clearing in the tradable sector is given by:

\[
A_{f,t}^* N_f^{*,t} = (1 - \alpha) C_{T,t} \left( \frac{P_{f,t}}{P_{T,t}} \right)^{-\eta} + \alpha C_{T,t}^* \left( \frac{P_{f,t}}{P_{T,t}} \right)^{\eta} + \frac{\omega_n}{2} \left( \pi_{f,t}^* - 1 \right)^2
\]  

that can be rewritten as a function of terms of trades:

\[
A_{f,t}^* N_f^{*,t} = (1 - \alpha) C_{T,t} \left( \alpha S_t^{q-1} + (1 - \alpha) \right) \frac{\omega}{\pi} + \alpha C_{T,t}^* \left( \frac{S_t^{q-1} (1 - \alpha) + \alpha}{\pi} \right)^{\eta} + \frac{\omega}{2} \left( \pi_{f,t}^* - 1 \right)^2
\]  

(43)

2.6.3 Budget constraints and current account

If monopolistic firms are owned by the inhabitants of the country in which they are located, the resources-expenditure balance of the borrower is given by the budget constraint, equation (24), holding with equality. Real profits (in terms of tradable consumption) are:

\[
\Gamma = Y_{n,t} P_{n,t} + Y_{h,t} P_{h,t} - W_t N_t - P_{n,t} \frac{\omega}{2} \left( \pi_{n,t} - 1 \right)^2 - P_{h,t} \frac{\omega}{2} \left( \pi_{h,t} - 1 \right)^2
\]

Therefore, substituting for real profits in the borrower’s budget constraint and substituting for (40) and (39), the resource constraint for the representative...
agent in country $H$ is:

\[
C_{T,t} (1 - \alpha) \frac{S_1^{1-\eta} \alpha + (1 - \alpha) S_1^{\alpha - 1 - \eta}}{(1 - \alpha) C_{T,t}^* \left(1 - \alpha + \alpha S_1^{1-\eta}\right)^{\frac{\eta}{1 - \eta}}} - (1 - \alpha) C_{T,t}^* \left(1 - \alpha + \alpha S_1^{1-\eta}\right)^{\frac{\eta}{1 - \eta}} = b_t - R_{t-1} \frac{b_t}{\pi_{T,t}}
\]

Eq (44) shows that the inflows of foreign resources net of interest payments (the RHS) needs to equalize the consumption of tradables at Home, net of Foreign consumption of tradables (weighted for terms of trade). If terms of trade are unitary, the inflow of foreign resources net of interests are equal to a share of domestic consumption of tradables minus the same share of tradables consumed in the Foreign country, i.e.:

\[
(C_{T,t} - C_{T,t}^*) (1 - \alpha) = b_t - R_{t-1} \frac{b_t}{\pi_{T,t}}
\]

where $(1 - \alpha)$ is the weight associated to Foreign goods (Home goods) in the Home (Foreign) basket of tradables.

Equation (44) represents a current account equation. More explicitly, we define the current account of country $H$ (in real terms of home tradable consumption) as the variation of home-currency assets (in real terms of tradable consumption), i.e.:

\[
ca_t = \left(\frac{b_t}{\pi_{T,t}} - b_t\right)
\]

\[
ca_t = x_t Y_{n,t} + Y_{h,t} \frac{P_{h,t}}{P_{T,t}} - C_{T,t} - x_t (C_{n,t} - (1 - \delta) C_{n,t-1}) - \frac{b_t}{\pi_{T,t}} (R_{t-1} - 1) - \frac{1}{P_{T,t}} \left[P_{n,t} \frac{\omega}{2} (\pi_{n,t} - 1)^2 + P_{h,t} \frac{\omega}{2} (\pi_{h,t} - 1)^2\right]
\]

clearly, by substituting $Y_{n,t}$ and $Y_{h,t}$ with (39) and (40) and equating (45) and (46) we obtain equation (44).

In the rest of the world the corresponding resource constraint is:

\[
C_{T,t}^* \frac{S_1^{\alpha - 1 - \eta} (1 - \alpha)}{\left(S_1^{\alpha - 1} (1 - \alpha) + \alpha\right)} - R_{t-1}^* \frac{b_t^*}{\pi_{T,t-1}} = -b_t + (1 - \alpha) C_{T,t} \left(\alpha S_1^{\alpha - 1} + (1 - \alpha)\right)^{\frac{\eta}{1 - \eta}}
\]

Equation (47) can be also interpreted as a current account equation of country $F$. 

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2.7 Monetary policy

The recent house prices boom and the prospect of a global downturn as a consequence of sharp softening in housing sectors, have triggered a debate over whether policy makers should respond to house prices. Conventional mainstreams agree that central bankers should respond to asset price changes only when they affect inflation, output and expectations (Mishkin, 2007). However, there are "benefits to be derived from leaning against the wind...(and)...increase(s) interest rates to stem the growth of house price bubbles and help restrain the building of financial imbalances" (IMF, 2008).28

While refraining from normative issues, we limit our analysis to the effects of stochastic shocks in presence of alternative policy stances. In our framework, we assume that exchange rates are completely flexible and policy makers do not engage in any specific exchange rate policy. This leaves policy makers three possible targets: durable goods inflation, tradable goods inflation and/or domestically produced goods inflation. In the following, when focusing on the dynamics of the adjustment, we will evaluate the effects of targeting different basket of goods. For the moment we simply suppose that each policy maker react both to durables’ and tradables’ inflation according to the following Taylor rules:

\[
\frac{R_t}{\hat{R}} = \left( \frac{\pi_{h,t}}{\pi_h} \right)^{\Phi_{1,h}} \left( \frac{\pi_{n,t}}{\pi_n} \right)^{\Phi_{2,h}} \tag{48}
\]

\[
\frac{R^*_t}{\hat{R}^*} = \left( \frac{\pi_{f,t}^*}{\pi_f^*} \right)^{\Phi_{1,f}} \left( \frac{\pi_{n,t}^*}{\pi_n^*} \right)^{\Phi_{2,f}} \tag{49}
\]

In a two country setup, nominal determinacy requires \(\Phi_1\) and/or \(\Phi_2\) to be sufficiently large; we assume that the monetary policy is set to assure that sufficient conditions for nominal determinacy hold (see Benigno and Benigno, 2001)

2.8 Equilibrium conditions

For each monetary policy in country \(H\) and \(F\), the equilibrium of our world economy is defined by (13) and (14) holding with equality in each period (see discussion in the following section), (16), (17), (25), (24), (26), (27) and the no-Ponzi game condition, (23). In the tradable production sector, (31) and (33) need to hold while in the durables production sector (35) and (36). Market clearing is insured by (38)-(44) and (20). Finally purchasing parity conditions need to hold.

28 For more discussion on house prices and monetary policy targets see Borio and White (2004), Bordo and Jeanne (2002).
3 Steady state

In this section we first focus on the qualitative features and the dynamic properties of the steady state. Once we have proved the existence of a "well behaving equilibrium", it is possible to calculate it analytically.

3.1 Dynamic properties of the steady state

In order to focus on the existence of a determinate steady state and on its features we first shift our attention on the (modified) steady-state Euler equations of our model.

Consider first equations (71) and (26) at the steady state. Notice that in steady state, \( \pi_T = dq \pi_T^* \) always needs to hold — where \( dq \) is the steady-state depreciation rate of the nominal exchange rate (see eqs (??) and (??)). Also, long-run values of tradable inflation need to coincide with the target of the monetary policy for tradables in the Home and in the Foreign country, respectively. We can thus pin down the the long-run value of \( \psi \), i.e.:

\[
\psi = 1 - \frac{\beta}{\mu} \tag{50}
\]

implying that

\[
1 > \psi > 0 \tag{51}
\]

whenever \( 0 < \frac{\beta}{\mu} < 1 \). Notice however that since \( 1 < \beta < \mu < 0 \), inequality (51) always holds. Eventually, Assumption 2 reduces to the Becker (1980) and Becker and Foias (1987, 1994) condition (see the following Proposition).

**Proposition 1** Under assumptions 1-3 and a monetary policy that insures nominal determinacy, if the system is sufficiently close to the deterministic steady state then \( b_t = (1 - \chi) C_{n,t} x_t \) for every \( t \geq 0 \).

**Proof.** The formal proof is in Becker (1980) and Becker and Foias (1987, 1994) with zero-borrowing constraints. In order to ensure the existence of a "dominant consumer" in this model, we need to focus on (modified) Euler equations. Given that the saver is a consumption smoother, the ratio of her/his steady-state marginal utilities is equal to one, i.e.:

\[
\frac{U_{T,t}^*}{U_{T,t+1}^*} = R_t^{*} \frac{\mu}{\pi_{T,t+1}^*} = 1 \tag{52}
\]

at the contrary, equation (18) shows that whenever at the steady state \( 0 < \psi < 1 \),

\[
0 < 1 - \beta \frac{R_t}{\pi_{T,t+1}} \frac{U_{T,t+1}^*}{U_{T,t}^*}
\]
and the borrower is thus the "dominated consumer". Indeed, Assumption 2 ensures that $0 < \psi < 1$. \[\blacksquare\]

Proposition 1 implies that in our framework the borrower is always debt-constrained. Indeed, given that the Lagrangean multiplier, $\psi$, is positive, the constraint must be binding.\[29\]

In addition, Proposition 1 states that there exists an unique steady state, which is characterized by a non-zero level of external liabilities. The steady state is also dynamically determinate. Indeed, as in the standard model of Becker (1980), the steady state is determined by the Euler equation of the patient agent and therefore does not depend on initial conditions. It follows that the steady state of our system is not characterized by unit roots.

Proposition 1 also allows us to introduce the following corollary:

**Corollary 2** If Proposition 1 holds, the dynamics of our two-country economy with heterogeneous agents and imperfect markets is not characterized by unit roots.

The above Corollary needs some comments. Indeed, the pioneer analysis of Obstfeld and Rogoff (1995) shows that when markets are incomplete, the steady state of an open economy is generally subject to unit roots. This means that the steady state depends on initial values (i.e., initial external assets/liabilities) and transient shock have long-run effects.

In a two country framework, Corsetti and Pesenti (2001) introduce an unitary elasticity of substitution between domestic and foreign goods. This specification provides full risk sharing, renders securities redundant and implies a zero current account for every period. In turn, a zero current account refrains transitory shocks from having long-run effects.\[30\] Alternatively, indeterminacy and non-stationarity of the steady state are ruled out by Cavallo and Ghironi (2000); in their model zero steady-state liabilities are an endogenous result. More recently, still in a framework of overlapping generations, Ghironi et al. (2005) extend this result to the case of non-zero long run external liabilities.\[31\] This result should not surprise the reader; empirical evidence (see Lane and Milesi-Ferretti, 2001, 2002) show that non-zero long-run external assets/liabilities are a common phenomenon. Moreover, Schmitt-Grohe and Uribe (2003) provide a detailed analysis on different methods to rule out non-stationarity in a small-economy framework that allows for non-zero steady-state external liabilities. The proposed modifications to the standard model aim at inducing stationarity of the equilibrium dynamics: they make the steady-state independent of initial conditions.\[32\] However, when stationarity is induced by portfolio adjustment costs or interest rate premia, long-run assets are determined exogenously;

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\[29\] See also Iacoviello (2005) for additional discussion.  
\[30\] See also, among others, Cole and Obstfeld (1991) . For a literature review see Lane (2001).  
\[31\] See also Buiter (1981) and Weil (1989).  
\[32\] See also Kollman (2001).
indeed there is an exogenous level around which the adjustment function is centered: an exogenous level of debt, in presence of portfolio adjustment costs. An exogenous centering of the function, when there is an interest rate premium.

In our model, we extend Becker (1980) seminal result to the case of collateral-constrained agents in a two-country framework. In a model à la Becker, the steady state is determined by the Euler equation of the "dominant consumer" (the patient agent) and the long-run level of the debt is an endogenous result of the model. Moreover, the steady state only depends on the parameters of the model. This result implies that (sufficiently small) stochastic shocks do not have long-run effects and the steady state is not subject to unit roots dynamics.

Notice finally that these results do not depend neither on nominal rigidities neither on the introduction of durables.

3.2 Analytical solution

We now explicitly calculate the steady state of our model. Long term inflation levels are defined by the target of the monetary policy (we assume that \( \pi^*_n = \pi_n = \pi^*_T = \pi_T = 1 \)) and the saver’s discount rate pins thus down both the real rate of return, \( RR = \frac{1}{\beta} \) and \( \psi = 1 - \frac{1}{\beta} \), as in Monacelli (2006).

In steady state, the price rigidities à la Rotemberg are no more at stake. Therefore, the steady state of our framework coincides with the flexible prices steady state. Marginal costs are thus equal to the mark up. Assuming for simplicity the same mark-up for all sector in both countries,

\[
mc_n = mc_h = mc_T^* = mc_n^* = \frac{\varepsilon - 1}{\varepsilon} \quad (53)
\]

Supposing for analytical simplicity that the elasticity of substitution between tradables and nontradables, \( \theta \), is unitary, the consumption aggregator assumes a Cobb-Douglas specification; conditions (53), (30) (32) (34), (25), (37) and (16) allow us to pin down the durable and non-durable level of consumption both at Home and in Foreign and relative prices \( x \) and \( x^* \):

\[
C_n = \left( 1 - \gamma \right) \frac{e_1}{vN^\rho} a_1 \quad (54)
\]

\[
C_T = \frac{\varepsilon - 1}{\varepsilon} \frac{\gamma}{vN^\rho} \left[ \alpha + (1 - \alpha) S^{1-\eta} \right] \frac{1}{1 - \eta} \quad (55)
\]

\[
x = \left[ \alpha + (1 - \alpha) S^{1-\eta} \right] \frac{1}{1 - \eta} \quad (56)
\]

\[
C_n^* = \left( 1 - \gamma \right) \frac{e_1}{vN^\rho} a_2 \quad (57)
\]

\(^{33}\)Punzi (2007) incorporates collateral constraints and agents’ discount rates heterogeneity in a two country business cycle model. In her model however, collateral constraints and agents heterogeneity only concern the intra country dymentation of the model. (i.e., collateral constraints do not cross countries). In her framework, the steady-state value of external debt is eventually pinned down by standard portfolio adjustment costs.

\(^{34}\)We will keep this simplification during the simulation of our model.
\[ x^* = \left[ \alpha + (1 - \alpha) S^{\eta - 1} \right]^{\frac{1}{1 - \eta}} \]  

(58)

\[ G_T^* = e_1 \frac{\gamma}{vN^*} [\alpha + (1 - \alpha) S^{\eta - 1}]^{\frac{1}{1 - \eta}} \]  

(59)

where \( e_1 = \frac{\epsilon - 1}{\eta} \), \( a_1 = 1 - \beta (1 - \delta) - \psi (1 - \chi) \) and \( a_2 = 1 - \mu (1 - \delta) \). Substituting (54) and (56) in (14) we obtain the steady-state level for net external debt in Home:

\[ b = (1 - \chi) \frac{(1 - \gamma) e_1}{a_1} \left[ \alpha + (1 - \alpha) S^{1 - \eta} \right]^{\frac{1}{1 - \eta}} \]  

(60)

and the one in Foreign:

\[ b^* = b \left( \frac{(1 - \alpha) S^{1 - \eta} + \alpha}{\alpha S^{1 - \eta} + 1 - \alpha} \right)^{\frac{1}{1 - \eta}} \]

Substituting (54) and (56) in (38), (39) and (40) we pin down steady-state levels for \( N_h \), \( N_n \) and \( N \), i.e.:

\[ N_h = \alpha e_1 \frac{\gamma}{vN^*} \left[ \alpha + (1 - \alpha) S^{1 - \eta} \right]^{-1} \]  

(61)

\[ + (1 - \alpha) e_1 \frac{\gamma}{vN^*} \left[ (1 - \alpha) + \alpha S^{1 - \eta} \right]^{\frac{1}{1 - \eta}} \]

\[ N_n = \delta \frac{(1 - \gamma) e_1}{vN^*} a_1 \]  

(62)

\[ N = N_h + N_n \]  

(63)

The terms of trade, \( S \), are pinned down by substituting all above steady-state values in (44).

An analogous procedure allows us to find all Foreign steady-state values, i.e.:

\[ N_n^* = \delta e_1 \frac{\gamma}{a_2 vN^*} \]  

(64)

\[ N_f^* = (1 - \alpha) e_1 \frac{\gamma}{vN^*} \left( \alpha S_q^{q - 1} + (1 - \alpha) \right)^{\frac{1}{1 - \eta}} \]  

(65)
4 Dynamics and quantitative insights

The recent swings in house prices and their dramatic consequences have triggered an hot debate on the implications for the global economy. Moreover, the current scenario of increasing commodity prices represent an additional challenge for policy makers. Indeed, as The Economist (2008) suggests, the global economy has recently received 2 shocks: the one concerning the housing market and higher commodity prices.

In this section, we aim at analyzing the impact of demand shocks on our two-country economy. While refraining to enter the debate on the causes of shocks, we will limit our analysis to the dynamics of the adjustment following exogenous demand shocks.

4.1 Calibration

In order to have a quantitative insight of the dynamics of the model, we proceed by simulating the response of our economy to stochastic shocks. Our parameterization is consistent with the recent literature and is based in particular on Monacelli (2007, 2008), Obstfeld and Rogoff (1995, 2000, 2004) and Monacelli and Faia (2006).

Quarterly discount factors are set respectively to $\mu = 0.99$ and $\beta = 0.98$. The structure of the model implies that the real interest rate of the $F$ country is pinned down by the discount factor of the dominant consumer, and thus is equal to $\frac{1}{\mu}$.

We stick on Monacelli (2006) benchmark value for houses depreciation rate and we set $\delta = (0.025)^{1/4}$, which is consistent an annual depreciation between 1.5% and 3%. Analogously, we adopt the average loan-to-value ratio on home mortgages for the period 1952-2005 in U.S. and let $\chi = 0.25$.

We calibrate $\gamma$ by assuming that the share of durable spending on total spending in Home, $\left(\frac{\delta C_n}{C_T}\right)$, is equal to 0.2. This is consistent with U.S. data on spending. We calibrate $v$ by assuming that the steady state level of labor in $F$ is one third of one unit of time, consistently with European data; the inverse elasticity of labor supply is assumed to be equal to 3.

The elasticity of substitution amongst single varieties (for each sector in each country) is set equal to 8. This implies a mark-up of about 15%. The elasticity of substitution between the basket of home and foreign goods is reasonably assumed to be lower than 8 and is set equal to 2. Moreover, the share of Home (Foreign) good consumption in the Home (Foreign) country, $\alpha$, is set equal to 0.7.

In our benchmark parametrization, we follow the standard parametrization on sticky prices adjustment and we assume a frequency of four quarters for tradable goods. We finally assume that the prices of durables are flexible.

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35 Caroll and Samwick (1997) estimate discount factors to be in a range between 0.91 and 0.99.
36 See Bils and Klenow (2004) and Monacelli (2007) for some recent discussion on the fre-
4.2 Debt constraints

In presence of agents’ heterogeneity in the degree of impatience, debt constraints are a key element to assure the unicity and the dynamic determinacy of the steady state. Debt constraints are not new in international finance. Indeed, thumb rules for solvency have been often used as a benchmark to evaluate the sustainability of external debt (see, among others, Williamson, 1999). In our framework, debt constraints are an endogenous function of the value of the value of real assets. We assume that debt contracts are stipulated in nominal terms, so as to proxy reality. Thus, as highlighted by Monacelli (2007, 2008) and Iacoviello (2005), all shocks entailing an increase in inflation may imply a flow of wealth from the lender to the borrower. To the contrary, all shocks entailing a decrease in inflation may imply a beneficial wealth transfer for the lender. In our two country world, the transmission of wealth effects is due to the exchange rate and the terms of trade.

4.3 Demand shocks

We now focus the attention on the possible effects of two different demand shocks: i) a positive shock in preferences for housing in the debtor country, $H$ (so as to proxy the demand shock for houses in countries characterized by great amount of net external liabilities); a positive shock on preferences for tradables in country $F$ (so as to proxy the global increase in the demand for commodities at the roots of the jump in commodities prices).

4.3.1 Demand shock in the housing sector

Suppose now that country $H$ is affected by a positive shock in consumers’ preferences such that its inhabitants desire to buy more houses. Even if the reason at the roots of a change in preferences are not clear, it seems that the current rise in housing prices could be attributed to an increase in housing demand. While refraining from investigating the reason at the roots of a change in preferences, we focus on the insights of our model following a positive demand shock for houses.

For simplicity, we suppose that the shock has a log-normal distribution such that:

$$p_{n,t} = \rho_n p_{n,t-1} + u_t$$

$$u_t \sim (iid)$$

where we let $\rho_n = 0.85$, following the calibration of Iacoviello (2005).

We see that in response to an increase in households’ appetite for real properties, the financial accelerator is at work. The increase in the stock of real

\footnotetext[37]{The utility function is thus: $\max E_0 \{ \sum_{t=0}^\infty \beta^t U_t (e^{p_{n,t}} C_{n,t}, C_{T,t}, N_t) \}$}
assets – and thus, in houses relative price – entails a better access to credit. Since agents in country $H$ are impatient, they use all their collateral to obtain credit. In turn, this allows them to consume tradables and to further increase their stock of houses: external debt increases on impact and continues to gradually accumulate before the shock is absorbed. The accumulation of collateral is accommodated by a gradual decrease in the user cost of durables. By inspecting equation (19), one can see that the user cost of durables is a positive function of both the interest rate and the relative price of durables. Both relative prices and interest rates jumps on impact and decrease gradually. In absence of nominal rigidities, the interest rate and the level of inflation are determined by the Taylor rule (together with the modified Euler equation of the borrower).

The increase in external debt is accommodated by an increase in imports, and thus, by a surge of a current account deficit. In particular, the surge in consumption of tradables entails an increase in Home produced goods consumption and in imports of Foreign goods (of a smaller extent, given home bias). The increase in tradable consumption is indeed stimulated by the fall in terms of trade. In absence of nominal rigidities, given that labor is mobile across sectors, the real marginal costs need to be constant in each period. In our case, given an identical mark-up for all sectors, it always need to hold:

$$\frac{1}{x_t} \frac{-U_{N,t}}{U_{T,t} A_{n,t}} = \frac{(\varepsilon - 1)}{\varepsilon} = \frac{1}{A_{h,t}} \frac{-U_{N,t}}{U_{T,t}} \left[ \alpha + (1 - \alpha) S_1^{1-\eta} \right]^{-1}$$

As you can see, the increase in tradable consumption is enhanced by the increase in the relative price of durables. On the other hand, the increase in the relative price of durables entails an improvement in the terms of trade. Eventually this will transmit the effect of the shock to country $F$.

The fall in terms of trade plays a double role in response to the shock: on the one hand, by making Foreign consumption cheaper, it accommodates borrowers’ impatience and willingness to increase current consumption. On the other hand, it enhances lenders’ accumulation of Foreign currency assets and thus, their investment incomes – we remind the readers that the lenders are consumption smoothers. Notice in particular that (for reasonable values of the parameters), the amount of Foreign external assets is a negative function of terms of trade, according to (21). The fall in $S$ has thus a positive impact on Foreign external assets (see Figure 2). Moreover, this effect is also stimulated by the initial appreciation of the nominal exchange rate through revaluation effects of lenders’ external assets.

The current account deficit is absorbed as a consequence of the gradual decrease in borrowers’ tradable consumption on the one hand; and the decrease in borrowers’ ability to access to credit, on the other. Notice that the current account and the relative price of houses move in opposite directions.

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38 For a detailed discussion see also Monacelli (2008).
39 Clearly, qualitative results would hold also with different mark-up.
40 For a more detailed discussion on the trasmission of shocks, see the following section.
41 Revaluation effects are here not quantitatively significant. Indeed, we do not consider gross external assets/liabilities.
Nominal rigidities In Figure 3, we show the effect of the above shock in presence of different types of price rigidities. We compare the dynamics of variables when tradables have a 4 quarter frequency in price adjustment (case 1 in Figure 3), with the case of price flexibility (case 2) and the case where only house prices have a 4 quarter frequency in price adjustment (case 3).

Figure 3 shows that the scenario characterized by price flexibility is intermediate between the other two. The main effect of price rigidities lies in their impact on the relative price durables-tradables. When the effect on $x$ is positive, agents do not afford to buy large amounts of durables but borrowing is sustained by the increase in the value of the collateral. Moreover, there is also a (quantitatively small) substitution effect that goes in favour of purchases of tradables – see Figure 3.

If tradables are sticky, the relative price of durables increases more than when prices are flexible because the price of tradables does not immediately adjust to the expansionary effect of the shock. The increase in $x$ accommodates agents willingness to consume tradables; however, since the prices of durables are kept flexible, it still needs to hold:

$$1 - U_{x,t} = \varepsilon - 1.$$ 

In the tradable-goods sector inflation is now pinned down by the New Keynesian Phillips curve, eq. (29). Therefore, the terms of trade and the relative price of durables are no more directly linked; still, they continue to move inversely. Following the jump in $x$, terms of trade improve but in a smaller extent (indeed, the price of Home produced goods increase more slowly). Given the increase in the user cost of durables, agents buy a smaller amount of real properties. Having said that, the increase in the value of real assets allows them to continue borrowing so as to consume tradables. Moreover, the smaller decrease in the terms of trade prompts agents to substitute Foreign consumption with Home consumption so that the initial current account deficit is of a smaller extent. Having said that, the effects of price rigidities are not quantitatively relevant for the dynamics of the current account. Notice finally that the current account and the relative price of durables move in opposite directions.

Monetary policy We now focus on the role of the monetary policy. We choose to assume the perspective of the Home policy maker; we aim at investigating the effects of the choice of the targets of the Taylor rule on the dynamics of our collateral-constrained open economy. Given the structure of our two-country two-sector economy policy makers in each country can choose alternative targets: tradable goods inflation, Home-produced goods inflation and durable-goods inflation. In this exercise we assume for simplicity that the policy maker does not aim at stabilizing output. Clearly, as remarked by Iacoviello (2005) in a similar framework, output targeting may be a source of possible policy trade-offs in this framework.

We consider the following alternative simple monetary rules:
According to rule (66) (i.e., scenario 2 in the simulations), the monetary authority stabilize inflation of Home produced goods (i.e., $\Phi_2,h = 0$ and $\Phi_1,h$ is large). According to rule (67), scenario 3, the policy maker targets inflation both in the housing sector and in the tradable sector. Notice however that this specification implies that the monetary authority directly targets also the inflation of imported goods; in this way, s/he directly responds to shocks that may be imported from abroad and exchange rate swings. Finally, when the policy maker follows rule (68), scenario 4, s/he responds to tradable inflation but disregards the trends of durable-goods prices.

Figure 7 shows that when the policy maker implements rule (68) terms of trade are best stabilized and most of the adjustment is carried by the relative price of durables, $x$. Indeed, by stabilizing the price of durables, the relative price of houses is allowed to jump higher. Notice also that in response to the shock, interest rates react less and thus, the exchange rate appreciates (an then depreciates) in a smaller extent. The impact of rule (68) on the current account deficit is quantitatively analogous of the one of the other rules. Indeed, both the positive effect of lower interest payments (due to relatively lower interest rates) and the stronger substitution effect between Home and Foreign goods (due to a smaller decrease $S$) are offset by the negative effect of the increase of aggregate tradable consumption (due to a stronger increase in $x$).

Notice finally that in both cases where the monetary policy targets also house prices, interest rates need to react stronger and the exchange rate experiences larger swings. Notice however that this effect has not a quantitatively significant effect on the transmission of the shock in country $H$. As in Iacoviello (2005) and Monacelli (2007), targeting house prices does not significantly alter the adjustment dynamics following shocks.

### 4.3.2 Demand shock for tradables in country F

The two-country structure of our model allows us to analyze the transmission of shocks from one country to the other. In this section we focus on the transmission of a positive demand shock from country $F$ to $H$; this exercise may provide some useful insights in light of the current scenario of global increase in the demand for commodities.

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$R_t = \left( \frac{\pi_{h,t}}{\pi_h} \right)^{\Phi_1,h}, \Phi_1,h \rightarrow \infty$ (66)

$R_t = \left( \frac{\pi_{T,t}}{\pi_h} \right)^{\Phi_1,h} \left( \frac{\pi_{n,t}}{\pi_n} \right)^{\Phi_2,h}$ (67)

$R_t = \left( \frac{\pi_{T,t}}{\pi_h} \right)^{\Phi_1,h}, \Phi_1,h \rightarrow \infty$ (68)
Suppose that country $F$ is affected by positive demand shock for tradables. For simplicity, we suppose that the shock has a log-normal distribution such that:

\[
pt_t = \rho_T pt_{t-1} + u_t
\]

where we let $\rho_T = 0.85$, as above.\footnote{The utility function is thus: $\max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U_t \left( C_{n,t}^*, e^{pt,t}, C_{T,t}^*, N_t^* \right) \right\}$}

Focusing first on $F$, we see that, as expected, the positive shock triggers a strong increase in tradable consumption. If prices are flexible, real marginal costs in each sector need to be equal to the mark up. This needs to hold in each period. We have assumed for simplicity the same mark-up for all sectors\footnote{If mark up are not identical, results wouldn’t qualitatively change.}, therefore:

\[
\frac{1}{x_t^*} \frac{U_{N,t}^*}{e^{pt,t} U_{T,t}^* A_{n,t}^*} = \frac{(\varepsilon - 1)}{\varepsilon} = \left( \alpha + (1 - \alpha) S_t^{1+\eta} \right) \frac{1}{\varepsilon} \frac{U_{N,t}^*}{A_{T,t}^* e^{pt,t}}
\]

In a closed economy, the term $\left( \alpha + (1 - \alpha) S_t^{-1+\eta} \right)^{\frac{1}{\varepsilon}}$ would be equal to $1.\footnote{See Monacelli (2007).}$ In this case, a positive demand shock would not allow a variation in the relative price $x^*$. In our framework, both $x^*$ and $S$ represent a wedge and are allowed to accommodate the shock moving proportionally. Figure 4 shows that the preference shock for tradables makes relative prices, $x^*$ decrease – together with the terms of trade, $S$.\footnote{The dampening effect of terms of trade in country $F$ can be interpreted as follows. In country $F$, the Taylor rule triggers a jump in interest rates. This makes current consumption more expensive and gives the lenders an incentive to accumulate more assets. However, given that assets depend on borrowers’ collateral constraint, the only way to partially offset the fall in borrowers’ ability to borrow is a variation in $S$ (indeed, $b^*$ is a positive function of $S$).} In particular, the decrease of both variables leaves consumption and labor less scope for jumping – dampening thus the expansionary effect on tradable consumption. Eventually, a decrease in $x^*$ also dampens the substitution effect between durables and non-durables following the shock in preferences. Indeed, given that lenders are not collateral constrained, all shocks affecting the real price of durables entail a substitution between goods – see the arbitrage equation, (25).

The variation in terms of trade transmits the shock to country $H$. Focusing now on country $H$, we see that the variation of $S$ implies a variation of $x$ in an opposite way with respect to $x^*$,

\[
\frac{1}{x_t} \frac{U_{N,t}}{U_{T,t}^* A_{n,t}^*} = \frac{(\varepsilon - 1)}{\varepsilon} = \frac{1}{A_{h,t}^*} \frac{U_{N,t}}{U_{T,t}^*} \left[ \alpha + (1 - \alpha) S_t^{1+\eta} \right]^{\frac{1}{\varepsilon}}
\]

The decrease in $S$ triggers an increase in tradable consumption, $C_T$. Eventually, the shock is also transmitted to the relative price, $x$, which entails a stronger
incentive not to accumulate debt but enhances the value of the collateral, i.e., the value of borrower’s wealth) and moves opposite to $S$. On impact, the decrease in $S$ has a positive effect on borrowers’ income. Indeed, on impact, tradable consumption increases more than labor in country $H$ (in the tradable sector) but borrowers experience a current account surplus. The central bank reacts thus strongly to keep this expansionary effect under control; then, interest payments together with the increase in terms of trade entail a current account deficit (even if tradable consumption decreases more than labor in the tradable sector) which is slowly absorbed.

If prices are flexible, the dynamics of interest rates and inflation can be pinned down by combining the modified Euler equations with the Taylor rules. Thus, the nominal exchange rate accommodates the stance of both policies. Given that the simple monetary rules used by policy makers are not efficient, inflation is allowed to jump.

Notice finally that the shock gradually carries a positive wealth effect to country $F$. Indeed, even if external debt falls, the reaction of the monetary policy together with the trend of terms of trade allow country $F$ to accumulate interest rate incomes (see Figure 4).

The shock in preferences makes lenders increase their tradable consumption on impact but deteriorates lenders’ terms of trade — entailing a current account deficit for country $F$. Then, the increase of interest rate incomes, together with the gradual decrease of tradable consumption (and thus, Home produced consumption) assures a current account surplus for country $F$. The current account deficit for country $H$ (surplus for country $F$) is eventually balanced when the effect of the variation in interest rate incomes and the increase of country $F$ tradable consumption are absorbed.

**Nominal rigidities** In order to investigate the transmission of the shock in presence of nominal price rigidities we compare the dynamics of the variables in the following scenarios: flexible prices (case 1 in Figure 5a, 5b); nominal rigidities for Home tradables only (case 2 in Figure 5a, 5b); nominal rigidities for Foreign tradables only (case 3 in Figure 5a, 5b); nominal rigidities for both Home and Foreign tradables (case 4).

Figure 5 shows that at a qualitative level, the impact of the shock in country $F$ depends on the existence of nominal rigidities in country $F$: the dynamics of case 1 (3) are indeed analogous of those of case 2 (4).

If prices are rigid in country $F$, the dynamics of the relative price of durables, $x^*$ and of terms of trade, $S$ are still linked. However, agents expectations and sectoral inflation create a wedge between the dynamics of the two variables — following the New Keynesian Philips curve, eq. (33). A positive demand shock for tradables in country $F$ entails on impact a stronger fall in relative prices, $x^*$ but a smaller decrease in terms of trade, $S$. This damps the substitution

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47 Keeping everything else fixed, the user cost of durables increases (see eq (19)).

48 Eventually the shock has a negative impact on aggregate labor in country $H$. Clearly, the quantitative impact of the shock on both $U_{T,t}$ (and thus, $C_T$) and $U_{N,t}$ (and thus $N$) depends on the parameters of the utility function.
effect between tradables and durables, on impact. Tradable inflation in both
countries does not jump and is determined by the New Keynesian Philips curve.

Notice finally that the effect of the shock on the relative price of durables in
country $H$ depends on price rigidities in country $H$ (see Figure 5a). The shock
will be eventually transmitted on tradable and durable consumption through
the relative price of durables, $x$: the higher the relative price, the smaller the
amount of purchased houses (see Figure 5b).

**Monetary policy** We now focus on the role of the monetary policy. We
continue assuming the perspective of the Home policy maker and we analyze
the effects of a demand shock for tradables when the policy maker implement
rules (48), (66)-(68), respectively.

Figure 6 show that the adjustment dynamics (in country $H$) following a
demand shock for tradables (in country $F$) are affected by different policy rules,
according to the mechanisms analyzed in Section 4.3.3. As expected, when
rule (68) is implemented, terms of trade are stabilized and large part of the
adjustment is carried by a jump in the relative price of houses, $x$. This entail a
stronger increase in tradable consumption and a stronger fall in the consumption
of durables. Notice finally that when the policy maker targets house prices also,
interest rates react more strongly, entailing, on impact, a stronger exchange rate
appreciation. In turn, higher interest rates entail larger interest payments and
thus, a larger current account deficit during the adjustment.

5 Concluding remarks

We have focused on the current account dynamics of a two-country world pop-
ulated by heterogeneous agents in their degree of impatience. We have shown
that if the inhabitants of country $H$ are more impatient than the ones of the
Foreign country, and their willingness to consume is limited by a collateral con-
straint, we can extend Becker (1980) and Becker and Foias (1987) seminal result
to an open economy dimension. Indeed, given that the $H$-inhabitants are not
consumption smoothers, they always prefer to borrow as much as possible and
the collateral constraint is binding in each period. In the long run, the collateral-
constrained country is characterized by a positive amount of external debt and
a balanced current account; non-zero liabilities are thus an endogenous result of
our model.

We have then analyzed the effect of demand shocks and their international
transmission. When the debtor country is affected by a positive demand shock
for housing, the increase in house prices makes the collateral constraint less
tight so as to allow for an increase in both durable and tradable consumption.
The increase in tradable consumption is accommodated by an improvement in
terms of trade that is at the roots of a current account deficit. The nominal
exchange rate appreciates on impact and gradually depreciates so as to enhance
the current account adjustment; moreover, the current account and the relative
price of durables move in opposite directions. Notice finally that the dynamics
of the adjustment can be affected by price rigidities as soon as they have an impact on both the relative price of durables and terms of trade; however, price rigidities (and the monetary policy stance) do not significantly affect current account dynamics.

We also have shown that when the rest of the world (country $F$) is affected by a positive demand shock for tradable goods the shock is transmitted to the Home country through its effect on terms of trade. Indeed, borrowers enjoy the increase in the value of their collateral – due to the surge of house prices – that enhances consumption; on impact, country $H$ experience a current account surplus that is reversed as soon as interest payments accumulate. Moreover, house relative prices and the current account move in opposite directions. Finally, nominal rigidities affect the dynamics of the adjustment through the terms of trade and the relative price of durables.

6 Appendix

6.1 The complete optimization program of the consumer.

The borrower

Utility function:

$$\max E^0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \right\}$$

Complete budget constraint, in nominal terms:

$$P_{T,t} C_{T,t} + P_{n,t} (C_{n,t} - (1 - \delta) C_{n,t-1}) + R_{t-1} d_{t-1} - q_t R^*_t d^*_t + D_t - q_t D^*_t + W_t N_t + \sum \Gamma$$

where $D$ are the bonds issued at Home in Home currency and $D^*$ are bonds issued in the Foreign country in Foreign currency. The individual budget constraint in real terms of tradable consumption is:

$$C_{T,t} + x_t (C_{n,t} - (1 - \delta) C_{n,t-1}) + R_{t-1} \frac{d_{t-1}}{P_{T,t}} - q_t R^*_t d^*_t = d_t - q_t d^*_t + P_{n,t} + W_t N_t + \sum \Gamma$$

Using price index definitions, and the law of one price, (69) can be rewritten as:

$$C_{T,t} + x_t (C_{n,t} - (1 - \delta) C_{n,t-1}) + R_{t-1} \frac{d_{t-1}}{P_{T,t}} - q_t R^*_t d^*_t = d_t - q_t d^*_t + \frac{P_{n,t}}{P_{T,t}} + W_t N_t + \sum \Gamma \frac{P_{T,t}}{P_{T,t}}$$

$$= d_t - \frac{(1 - \alpha) + \alpha S^1_{t-1}}{\alpha + (1 - \alpha) S^1_{t-1}} + W_t N_t + \sum \Gamma \frac{P_{T,t}}{P_{T,t}}$$

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The collateral constraint is:
\[
d_t - d_t^* \left[ \frac{(1 - \alpha) + \alpha S_{t}^{1-n}}{\alpha + (1 - \alpha) S_{t}^{1-n}} \right] \leq (1 - \chi) C_{n,t} x_t
\]

First order conditions:

a) Arbitrage leisure/consumption:
\[
U_{T,t} = \lambda_t
\]

b) Arbitrage tradable consumption/durable services:
\[
x_t U_{T,t} = U_{n,t} + \beta (1 - \delta) E_t \{ U_{T,t+1} x_{t+1} \} + U_{T,t} \psi_t (1 - \chi) x_t
\]

c) Modified Euler equation
\[
R_t = E_t \left\{ \frac{U_{T,t}}{U_{T,t+1} \pi_{T,t+1}} \right\} \frac{(1 - \psi_t)}{\beta}
\]

b) Optimal condition for foreign securities:
\[
R_t^* = E_t \left\{ \frac{U_{T,t}}{U_{T,t+1} \pi_{h,t+1}} \right\} \frac{q_t}{q_{t+1}} \pi_{h,t+1} \left[ \frac{\alpha + (1 - \alpha) S_{t+1}^{1-n}}{\alpha + (1 - \alpha) S_{t}^{1-n}} \right] \left( 1 - \psi_t \right)
\] (70)

where \( \pi_{T,t} = \pi_{h,t} \left[ \frac{\alpha + (1 - \alpha) S_{t+1}^{1-n}}{\alpha + (1 - \alpha) S_{t}^{1-n}} \right] \). Eq. (70) can thus be rewritten as:
\[
R_t^* = E_t \left\{ \frac{U_{T,t}}{U_{T,t+1} q_{t+1} \pi_{T,t+1}} \right\} \frac{1 - \psi_t}{\beta}
\] (71)

Equations (69) and (71) imply that the following non-arbitrage condition needs to hold, i.e.:
\[
R_t = E_t \left\{ \frac{q_{t+1}}{q_t} \right\} R_t^*
\]
Figure 1a: Trends in the nominal and effective exchange rate (BIS database)
Figure 1b. House prices and current account (Ecowin database)
Figure 2: Demand shock for real properties in $H$. 49

49 $\Phi_{1,h} = \Phi_{2,h} = \Phi_{1,f} = \Phi_{2,f} = 1$. rb refers to interest payments from the borrower while rbstar refers to interest incomes for lenders; all prices are here flexible.
Figure 3: Demand shock for real properties in $H$, different price rigidities.$^{50}$

$^{50}$Φ$_{1,h}$=Φ$_{2,h}$ = Φ$_{1,f}$=Φ$_{2,f}$ =1. Variables indexed with 1 refer to the case of price rigidities in the tradable sector; variables indexed with 2 refer to the case of price flexibility in both sectors; variables indexed with 3 refer to the case of price rigidities in the housing sector.
Figure 4: Effect of a preference shock for tradable consumption in country $F$, flexible prices.\textsuperscript{51}

\[ \Phi_{1,h} = \Phi_{2,h} = \Phi_{1,f} = \Phi_{2,f} = 1; \] flexible prices in both countries. $rb$ refers to interest payments from the borrower while $rbstar$ refers to interest incomes for lenders.

\textsuperscript{52} $\Phi_{1,h} = \Phi_{2,h} = \Phi_{1,f} = \Phi_{2,f} = 1; $ case 1 refers to flexible prices; nominal rigidities for Home tradables only refer to case 2; nominal rigidities for Foreign tradables only refer to case 3; nominal rigidities for both Home and Foreign tradables refer to case 4.

Figure 5a: Effect of a preference shock in country $F$, different price rigidities.

\[ \Phi_{1,h} = \Phi_{2,h} = \Phi_{1,f} = \Phi_{2,f} = 1; \] flexible prices in both countries. $rb$ refers to interest payments from the borrower while $rbstar$ refers to interest incomes for lenders.

\textsuperscript{52} $\Phi_{1,h} = \Phi_{2,h} = \Phi_{1,f} = \Phi_{2,f} = 1; $ case 1 refers to flexible prices; nominal rigidities for Home tradables only refer to case 2; nominal rigidities for Foreign tradables only refer to case 3; nominal rigidities for both Home and Foreign tradables refer to case 4.
Figure 5b: Effect of a preference shock for tradable consumption in country $F$, different price rigidities.\textsuperscript{53}

\textsuperscript{53}$\Phi_{1,h}=\Phi_{2,h} = \Phi_{1,f}=\Phi_{2,f} =1$; case 1 refers to flexible prices; nominal rigidities for Home tradables only refer to case 2; nominal rigidities for Foreign tradables only refer to case 3; nominal rigidities for both Home and Foreign tradables refer to case 4).

Figure 6: Demand shock for tradables and Taylor rules.\textsuperscript{54}

\textsuperscript{54}Nominal rigidities for tradables; $\Phi_{1,f}=\Phi_{2,f} =1$ always. Case 1 refers to $\Phi_{1,h}=\Phi_{2,h} =1$,  

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7 References

References


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