Do We Need Handshakes to Cooperate in Buyer-Supplier Relationships?

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Do we need handshakes to cooperate in buyer-supplier relationships?¹

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Abstract

Based on differences in production costs, McLaren (1999) [“Supplier relations and the market context: a theory of handshakes”, Journal of International Economics 48, 121-138] demonstrates that informal ‘handshake’ arrangements foster cooperation in buyer-supplier relationships, compared to formal contractual arrangements. This may explain international differences in procurement practices, such as American vs. Japanese. However, McLaren’s result holds under particular assumptions about production costs. Allowing for more traditional assumptions in procurement practices, such as relationship-specific investment costs and renegotiable contracts, we find in contrast that formal contractual arrangements may induce more cooperation than handshakes.

Keywords: Incomplete contracts, relationship-specific investments, cooperation.

JEL Classification : K12, L22, C7.

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1 Introduction

Business research has opposed Western to Japanese buyer-supplier arrangements. Western arrangements have been considered as formal relying on detailed written contracts, while Japanese arrangements have been depicted as informal relying on far less detailed contracts. Moreover, “an emphasis has been placed on the idea of Japanese supplier relationships as ‘partnerships,’ or ‘cooperative’ arrangements, in contrast to ‘antagonistic’ supplier relations in the West” (McLaren, 1999: 122-23).\(^\text{4}\) In a theoretical contribution, McLaren (1999) shows that these international differences in buyer-supplier arrangements can emerge in a simple economic model, without resorting to cultural or attitude differences. Based on differences in production costs, he demonstrates that an informal ‘handshake’ arrangements fosters cooperation compared to a formal contractual arrangement. The point of our paper is to show first that McLaren’s result holds under particular assumptions about production costs and second that firms do not necessarily need handshakes to cooperate. A formal contract may also promote cooperation.

Why contracting does not foster cooperation in McLaren (1999)? The author models a procurement relationship, in which a buyer may commission a supplier to produce a tailor-made input. Two procurement arrangements are specified. The first arrangement is formal. Parties sign an unbreakable fixed-price contract, which specifies a date of delivery of the input and a price. The second is informal. Parties agree verbally, without signing a prior contract, that the supplier will produce the input, while the payment would be worked out later through bargaining.

Before producing the input, i.e. ex ante, the supplier can reduce the cost of the tailor-made input by making two types of process investments, called joint and autonomous respectively. First, joint investments are relationship-specific. They require an explicit cooperation between the buyer and the supplier. Such investments are frequently observed across industries. The automobile is a textbook example (Asanuma, 1989; Nishiguchi, 1994). Car manufacturers establish close cooperations with suppliers to design parts of the final product.\(^\text{5}\) They coordinate tasks, share information and meet each other. This cooperation is typically linked to joint investments, which are the source of productivity improvements over time. Second, autonomous investments, that the supplier can undertake on its own, are not relationship-specific. This first particular assump-

\(^\text{4}\) Characterizing Japanese arrangements as informal and cooperative and Western arrangements as formal and antagonistic is of course a coarse generalization. In fact the difference is blurred, since Western firms have adopted Japanese practices (see for instance Cusumano and Takeishi, 1991). It seems however that Japanese and Western practices tend to differ in key areas such as quality control and price determination (e.g. meeting targets, reducing prices over time).

\(^\text{5}\) For instance, with the introduction of the airbag systems car manufacturers initiated cooperations with plastic subcontractors to redesign the dashboard and bear the additional weight of the airbag.
tion of McLaren (1999) about production costs implies that investment costs are not sunk ex post. Parties can pay an additional cost to recover the ex ante investment costs and adapt the tailor-made input for an alternative buyer. The second particular assumption about production cost concerns the additional cost of adaption. The cost structure assumed by the author ensures that when parties sign a contract the intended buyer will purchase in equilibrium the input. Under these two assumptions, the unbreakable fixed-price contract gives optimal incentives for supplier’s autonomous investments. On the other hand, the externality of the joint investment is not internalized and “the supplier will do virtually no joint investment, because it knows that the buyer will have no incentive to do follow-up work” (Mclaren, 1999). Consequently, the contract has no value to foster cooperation. In contrast, the informal arrangement provides suboptimal incentives to make autonomous investments; but there will be cooperation since the supplier can always get the buyer to share the costs ex post.

Why contracting might foster cooperation? We allow here for more traditional assumptions in procurement practices by considering autonomous relationship-specific investments and renegotiable fixed-price contracts. As presumed by Mclaren (1999: 125), this relaxation blurs the distinction between formal and informal arrangements in interesting ways.

We consider that supplier’s autonomous investments are relationship-specific. Part of these investments is spent to reduce the cost of producing the buyer’s customized input. The relationship-specific nature of both autonomous and joint investments rules out external market for the input. Thus, a credible non-modification clause of the initial contract implies that the default point value is zero. Since specific investments are sunk, the break-up of the contract would generate a loss of value, which gives optimal incentives to invest. As a result, it can be shown that if parties credibly commit not to renegotiate their initial fixed-price contract, optimal cooperation can be reached, by implementing a game of messages which discloses the relevant information to a third party.6

Empirically, courts are however reluctant to enforce contracts with strict non-modification clauses (see Schwartz and Scott, 2003). Moreover, we may be concerned with the credibility of the commitment: why do parties commit to execute the no trade threat point when they know that there are positive gains from trade. We thus follow the incomplete contract literature and allow for a renegotiable fixed-price contract (see e.g. Aghion et al., 1994; Che and Hausch, 1999; Chung 1991; Edlin and Reichelstein, 1996). However, when allowing for renegotiation, we find that efficiency is not attainable. Nevertheless, we find that contracting is valuable to promote cooperation and provides the parties an advantage over an informal arrangement. The reasoning here is the following. Cooperation requires

6 This efficiency result, obtained by relaxing the non-specificity of the autonomous investments in McLaren (1999), is demonstrated and available as supplementary material on http://jose.desousa.univ.free.fr/research/sup.htm.
that both parties contribute to the joint investment. Assuming, reasonably, that each contribution yields marginally more to its contributor than to its partner, this induces the parties to increase their own contribution to the joint investment. As a result, this incentive improves their status quo position and renders contracting valuable. Thus, we find that formal arrangements may dominate informal arrangements in encouraging joint investments.

To some extent our results are in line with the empirical literature on Japanese buyer-supplier relationships. First, parties trade in customized parts, which require autonomous and joint relationship-specific investments by the supplier (see Asanuma, 1985a,b, 1989; Aoki, 1988, Head et al., 2004, Qiu and Spencer, 2002 and Spencer and Qiu, 2001). Second, Japanese practices of procurement rely on formal contractual arrangements (Asanuma, 1985a,b, Nishiguchi, 1989). Based on a survey of automobile manufacturers in Japan, Cusumano and Takeishi (1991) find that the average contract length of a manufacturer-supplier relationship is about 3.2 years corresponding more or less to a model life-cycle. On the other hand, the average duration of their business relationship exceeds ten years. Thus, for each new cycle model, parties sign a new contract. They do not rely simply on handshakes and ex post bargaining. Third, these contractual arrangements give room for renegotiation. Parties write basic renegotiable contracts establishing basic rules covering a range of items including price determination, payment, delivery, property rights, materials supply and quality issues (Nishiguchi, 1989). Finally, these contractual arrangements give room for cooperation. Contracts set a target price for each input produced. Then, buyers cooperate and help suppliers to reach their targets (Cusumano and Takeishi, 1991; see also Nishiguchi, 1989).

The rest of the paper is organized as follows. In the next section, we present the model, a simple two-stage game between a buyer and a supplier. This model departs from Mclaren’s model in two respects: we introduce autonomous relationship-specific investments and renegotiability of the contract. In section 2, we establish two benchmark outcomes to compare our results: the first-best and the ex post bargaining (without an initial contract). In section 4, we show that a formal fixed-price contract arrangement may foster cooperation. Finally, in section 5, we conclude.

## 2 Model

We consider a basic two-stage procurement model between a buyer $b$ and a supplier $s$. The buyer procures an input from the supplier. There are two simple ways of procuring the input: a formal or an informal arrangement. In the formal setting, parties design a renegotiable fixed-price contract. In the informal arrangement, parties bargain ex post over the terms of trade without an initial contract. The sequence of moves slightly differs according to the chosen arrangement.


If parties sign a fixed-price contract in the first stage, they specify ex ante a fixed monetary transfer \( (t \in \mathbb{R}) \) of the buyer to the supplier for a fixed quantity of input \( (q \in \mathbb{R}^+) \). This initial allocation is enforceable by the court and ensures to the parties a status quo payoff. Contract terms are unforced in the second stage, unless they are renegotiated. In that case, parties share ex post the surplus from renegotiation according to their bargaining strength.

The sequence of events is slightly different, when parties choose the informal arrangement. In the first stage, parties agree verbally on the quantity of input. In the second stage, they bargain the terms of trade and determine the payment.

Autonomous and joint investments are not contractible and made simultaneously in the first stage. They are relationship-specific, which rule out outside options and the possibility to adapt the input for an alternative buyer (see above).

### Payoff functions and the nature of investments

Let \( v(q, j_b, j_s) \) denote the buyer’s gross value of procuring the good \( q \in \mathbb{R}^+ \) and \( c(q, a, j_b, j_s) \) the supplier’s gross monetary cost of producing \( q \). Valuations are determined by relationship-specific investments. Let \( a \in \mathbb{R}^+ \) be the level (and cost) of autonomous investments made by the supplier. Let \( j_b \in \mathbb{R}^+ \) and \( j_s \in \mathbb{R}^+ \) be the level (and cost) of the joint investment contributions made by each party, respectively.\(^7\) Throughout, we make the following assumptions.

**Assumption 1** \( v \) and \( c \) are continuously differentiable in all arguments.

**Assumption 2** \( v(q, j_b, j_s) \geq 0 \) is increasing in all arguments and strictly concave. For all \( q > 0 \) and \((j_b, j_s) \in \mathbb{R}^2_+\), it satisfies:

\[
\begin{align*}
\lim_{q \to 0} v_1(q, j_b, j_s) &= \infty, \quad \lim_{j_b \to 0} v_2(q, j_b, j_s) = \infty, \quad \lim_{j_s \to 0} v_3(q, j_b, j_s) = \infty; \\
\lim_{q \to \infty} v_1(q, j_b, j_s) &= 0, \quad \lim_{j_b \to \infty} v_2(q, j_b, j_s) = 0, \quad \lim_{j_s \to \infty} v_3(q, j_b, j_s) = 0.
\end{align*}
\]

**Assumption 3** \( c(q, a, j_b, j_s) \geq 0 \) is increasing in \( q \), decreasing in investments and strictly convex. For all \( q > 0 \) and \((a, j_b, j_s) \in \mathbb{R}^3_+\), it satisfies:

\[
\begin{align*}
\lim_{q \to 0} c_1(q, a, j_b, j_s) &= 0, \quad \lim_{a \to 0} c_2(q, a, j_b, j_s) = -\infty, \\
\lim_{j_b \to 0} c_3(q, a, j_b, j_s) &= -\infty, \quad \lim_{j_s \to 0} c_4(q, a, j_b, j_s) = -\infty. \\
\lim_{q \to \infty} c_1(q, a, j_b, j_s) &= \infty, \quad \lim_{a \to \infty} c_2(q, a, j_b, j_s) = 0, \\
\lim_{j_b \to \infty} c_3(q, a, j_b, j_s) &= 0, \quad \lim_{j_s \to \infty} c_4(q, a, j_b, j_s) = 0.
\end{align*}
\]

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\(^7\)Another way to model cooperation would be to assume that autonomous investments generate also direct externalities to the partner as in Che and Hausch (1999). A restriction however is that such autonomous investments render direct benefits to the investor’s partner without involving a joint work.
Concavity and convexity of assumptions 2 and 3 imply decreasing returns for both parties. Moreover, the marginal return of the first produced unit and the first invested unit is infinite. This implies strict positive levels of production and investment. Asymptotically, the marginal return on production and investment is null. It follows that production and investment cannot be infinite.

**Assumption 4**

\[ \forall (j_b, j_s) \in \mathbb{R}_+^2 \ v(0, j_b, j_s) = 0, \quad \text{and} \quad \forall (a, j_b, j_s) \in \mathbb{R}_+^3 \ c(0, a, j_b, j_s) = 0. \]

Assumption (4) says that when \( q = 0 \) both valuations do not depend on the level of investments. Since there is no outside market for investments, this assumption suggests that investments are relationship-specific (Chung, 1991: 1034).

**Assumption 5** The cross derivatives of \( v(q, j_b, j_s) \) and \( c(q, a, j_b, j_s) \) satisfy

\[ v_{i\ell}(q, j_b, j_s) > 0 \quad \text{and} \quad c_{i\ell}(q, a, j_b, j_s) < 0 \quad \text{for all} \quad i \neq \ell. \]

Assumption (5) says that investments are complementary.

**Assumption 6**

\[ v_2(q, j_b, 0) = 0, \quad \text{and} \quad c_3(q, a, 0, j_s) = 0. \]

Assumption (6) stipulates that the marginal return of the joint investment is null when only one party is contributing.

### 3 Benchmark outcomes

We establish two useful benchmarks, the first-best and the no-contracting outcome, against which later results about contracting may be compared.

#### 3.1 The first-best outcome

The first-best corresponds to the solution of the integrated firm program, which internalizes the effects of investment. The maximization program of the integrated firm is separable. In a first step, we determine the optimal quantity \( q^* \) given the investment levels. Then, we determine the investment levels given the optimal quantity.

Let \( \pi \) denote the maximum gross joint surplus, such that:

\[ \pi(a, j_b, j_s) = \max_{q \geq 0} [v(q, j_b, j_s) - c(q, a, j_b, j_s)]. \]

According to the optimality condition \( v_1 = c_1 \), we obtain

\[ q^* = q^*(a, j_b, j_s) \quad (1) \]
the quantity equalizing the marginal benefit to the marginal cost, therefore
\[ \Pi(a, j_b, j_s) = v(q^*, j_b, j_s) - c(q^*, a, j_b, j_s). \]

The net joint surplus of investments \( S(a, j_b, j_s) \) is given by:
\[ S(a, j_b, j_s) = \Pi(a, j_b, j_s) - a - j_b - j_s, \]
with \( \Pi(a, j_b, j_s) \) strictly concave since \( v(\cdot) \) is concave and \( c(\cdot) \) convex. The efficient investments are such that \((a^*, j_b^*, j_s^*) \in \arg \max_{a,j_b,j_s} \Pi(a, j_b, j_s) - a - j_b - j_s.\)

Given the assumptions on \( v \) and \( c \), \((a^*, j_b^*, j_s^*)\) are unique and satisfy a system of first-order conditions (FOCs):
\[ \Pi_1(a^*, j_b^*, j_s^*) - 1 = 0, \quad (2) \]
\[ \Pi_2(a^*, j_b^*, j_s^*) - 1 = 0, \quad (3) \]
\[ \Pi_3(a^*, j_b^*, j_s^*) - 1 = 0, \quad (4) \]

3.2 The (informal) no-contracting outcome

We now consider the no-contracting game. Let’s recall the sequence of events. Ex ante, parties agree verbally, without a prior contract, that the supplier will produce the input. Ex post, parties share the surplus according to their exogenous bargaining positions.\(^8\) Considering this sequence, we retain the subgame-perfect equilibrium as the equilibrium solution concept (Selten, 1975). The game is thus solved by backward induction. The optimal quantity \( q^* \in \mathbb{R}^+ \) and the monetary transfer \( t \in \mathbb{R} \) are determined in the second stage, while investments are realized in the first stage to maximize utility functions.

At the second stage, the negotiation outcome on \( q \) and \( t \) is solution of a Nash bargaining process, with \( \mu \in [0, 1] \) the supplier’s bargaining strength:
\[ \max_{t,q} [v(q, j_b, j_s) - t]^{1-\mu} [t - c(q, a, j_b, j_s)]^\mu. \]

Therefore \( q^* = q^*(a, j_b, j_s) \), is implicitly determined by
\[ v_1(q^*, j_b, j_s) = c_1(q^*, a, j_b, j_s), \]
and
\[ t(a, j_b, j_s) = (1 - \mu)c(q^*, a, j_b, j_s) + \mu v(q^*, j_b, j_s). \]

At the first stage, the buyer and the supplier maximize their surplus:

- for the buyer:
\[ U_b = v(q^*, j_b, j_s) - (1 - \mu)c(q^*, a, j_b, j_s) - \mu v(q^*, j_b, j_s) - j_b; \]

\(^8\)In an incomplete contract framework, it does not seem reasonable to assume that bargaining positions may be endogenously determined ex ante and enforced.
• for the supplier:

\[ U_s = (1 - \mu)c(q^*, a, j_b, j_s) + \mu v(q^*, j_b, j_s) - c(q^*, a, j_b, j_s) - a - j_s. \]

Given the optimal produced quantity \( q^* \), determined by equation (1) in both the first-best and the no-contracting outcome, we rewrite the above surplus using \( \Pi(a, j_b, j_s) \). It follows that parties make the investment levels of the no-contracting outcome \((\hat{a}, \hat{j}_b, \hat{j}_s)\) satisfying

\[ (\hat{j}_b) \in \arg \max_{j_b} (1 - \mu)\Pi(a, j_b, j_s) - j_b, \]

\[ (\hat{a}, \hat{j}_s) \in \arg \max_{a, j_s} \mu \Pi(a, j_b, j_s) - a - j_s, \]

and the following system of FOCs:

\[
\begin{align*}
\frac{\partial U_b}{\partial j_b} &= (1 - \mu)\Pi_2(\hat{a}, \hat{j}_b, \hat{j}_s) - 1 = 0, \\
\frac{\partial U_s}{\partial a} &= \mu \Pi_1(\hat{a}, \hat{j}_b, \hat{j}_s) - 1 = 0, \\
\frac{\partial U_s}{\partial j_s} &= \mu \Pi_3(\hat{a}, \hat{j}_b, \hat{j}_s) - 1 = 0.
\end{align*}
\]

Since \( \mu \in [0, 1] \), efficiency cannot be achieved. This may be explained as follows. Investments are made ex ante, while the surplus is shared ex post according to the bargaining positions. The payment \( t \) is determined independently of the investments made and thus externalities cannot be internalized.

What are the consequences of such an inefficiency? It can be shown that parties will invest less than the socially optimal level.\(^9\)

**Proposition 1** Under assumptions (1) to (6), the absence of contracting prior to investing in specific assets induces under-investments, such that: \( \hat{a} < a^* \), \( \hat{j}_b < j_b^* \) and \( \hat{j}_s < j_s^* \).

**Proof** See appendix.

### 4 Contracting and cooperation

We have seen that parties do not reach efficiency by simply bargaining ex post the terms of trade, without a prior contract. Now suppose that parties sign a simple renegotiable fixe-price contract which specifies a fixed monetary transfer \( (\tilde{T} \in \mathbb{R}) \) of the buyer to the supplier for a fixed quantity of good \( (\tilde{q} \in \mathbb{R}_+) \). Two questions arise. First, does the signing of this simple renegotiable contract make it possible to achieve efficiency? Second, failing that, does contracting offer a better outcome than the no-contracting game? If not, the contract has no value and the optimal contract is the “no contract”.

\(^9\)Note that overinvesting is also a possible and inefficient outcome.
4.1 The contracting outcome

With regard to the first question, we find in a simple way that contracting does not make it possible to reach the first-best. This is not much surprising and can be shown formally using the same equilibrium solution concept as in the no-contracting outcome.

Let’s first define the (gross) renegotiation surplus \((RS)\), available ex post as:

\[
RS = \Pi(a, j_b, j_s) - [v(q, j_b, j_s) - c(q, a, j_b, j_s)].
\]

At the second stage, we assume a Nash bargaining process on \(q\) and \(t\), solution of

\[
\max_{t,q} \left[ v(q, j_b, j_s) - t - v(q, j_b, j_s) + \tilde{t} \right]^{1-\mu} \times \left[ t - c(q, a, j_b, j_s) - \tilde{t} + c(q, a, j_b, j_s) \right]^{\mu}.
\]

We obtain \(q^* = q^*(a, j_b, j_s)\) implicitly determined by

\[
v_1(q^*, j_b, j_s; \theta) = c_1(q^*, a, j_b, j_s),
\]

and

\[
t(a, j_b, j_s) = (1 - \mu) [c(q^*, a, j_b, j_s) - c(q, a, j_b, j_s)] + \mu [v(q^*, j_b, j_s) - v(q, j_b, j_s)] + \tilde{t}.
\]

The first stage objectives to be maximized are:

- for the buyer:

\[
U_b = v(q, j_b, j_s) - \underbrace{t - (1 - \mu)RS - j_b}_{A} - \underbrace{\tilde{t}}_{B}.
\]

\((A)\) is the buyer’s payoff given by the initial contract. It represents the buyer’s status quo position. \((B)\) is the payoff from the renegotiation process, depending on the buyer’s bargaining strength \((1 - \mu)\).

- for the supplier:

\[
U_s = \underbrace{\tilde{t} - c(q, a, j_b, j_s)}_{C} + \underbrace{\mu RS - a - j_s}_{D}.
\]

\((C)\) is the supplier’s cost given by the initial contract. It represents the supplier’s status quo position. \((D)\) is the payoff from the renegotiation process, depending on the supplier’s bargaining strength \(\mu\).

Parties make the investment levels of the contracting outcome \((\tilde{a}, \tilde{j}_b, \tilde{j}_s)\), satisfying

\[
(\tilde{j}_b) \in \arg \max_{j_b} v(q, a, j_b, j_s) - \tilde{t} + (1 - \mu)RS - j_b,
\]

\[
(\tilde{a}, \tilde{j}_s) \in \arg \max_{a, j_s} \tilde{t} - c(q, a, j_b, j_s) + \mu RS - a - j_s.
\]
Lemma 1

1. If \( A \subset B \) implies that contracting with renegotiation does not make it possible to achieve writing a contract is valuable, \( i.e. \) we fail to achieve efficiency with contracting. However, we may wonder whether contracting or no-contracting?

\[ \forall \]

Proof

See appendix.

4.2 Contracting or no-contracting?

We fail to achieve efficiency with contracting. However, we may wonder whether writing a contract is valuable, \( i.e. \) if contracting offers a better outcome than no-contracting. Simple comparison of the no-contracting FOCs (5 - 7) with the contracting FOCs (8 - 10) shows that there is no obvious result regarding the improving effect of contracting.

The bargaining position \( (\mu) \) plays an important role to derive more precise results about the comparison between contracting and no-contracting outcomes. Before proceeding to the formal comparison, we consider some critical values of the parameter \( \mu \) and two useful lemmas. Then, we work out the comparison.

Let’s first define the sets \( \mathcal{A} \) and \( \mathcal{B}, \forall q, a, j_b, j_s: \)

\[ \mathcal{A} = \{ k \in [0, 1] /(kv_3(q, j_b, j_s) + (1 - k)c_4(q, a, j_b, j_s) \leq 0 \} , \]

\[ \mathcal{B} = \{ k \in [0, 1] /kv_3(q, j_b, j_s) + (1 - k)c_4(q, a, j_b, j_s) \leq k\pi_3(a, j_b, j_s) \} . \]

Let \( \overline{\mu} = \sup \mathcal{A} \) and \( \overline{\mu}^* = \sup \mathcal{B} \). These number exist; if \( k = 0 \), the above inequalities, used to define \( \sup \mathcal{A} \) and \( \sup \mathcal{B} \), reduce to \( c_4(q, a, j_b, j_s) \leq 0 \), which is satisfied \( \forall q, a, j_b, j_s \). Note also that \( \overline{\mu} \) and \( \overline{\mu}^* \) verify \( \overline{\mu} \leq \overline{\mu}^* \). This follows from the fact that \( \mathcal{A} \subset \mathcal{B} \). The following useful lemma can now be stated.

Lemma 1

1. If \( \mu < \overline{\mu} \), then: \( \mu v_3(q, j_b, j_s) + (1 - \mu)c_4(q, a, j_b, j_s) \leq 0, \forall q, a, j_b, j_s. \)

2. If \( \mu < \overline{\mu}^* \), then: \( \mu v_3(q, j_b, j_s) + (1 - \mu)c_4(q, a, j_b, j_s) \leq \mu \pi_3(a, j_b, j_s), \forall q, a, j_b, j_s. \)

Proof

See appendix.

We now define two other critical values of \( \mu \). Consider the sets \( \mathcal{C} \) and \( \mathcal{D}, \forall q, a, j_b, j_s: \)

\[ \mathcal{C} = \{ k \in [0, 1] /kv_2(q, j_b, j_s) + (1 - k)c_3(q, a, j_b, j_s) \geq 0 \} , \]

\[ \mathcal{D} = \{ k \in [0, 1] /kv_2(q, j_b, j_s) + (1 - k)c_3(q, a, j_b, j_s) \geq -(1 - k)\pi_2(a, j_b, j_s) \} . \]
Let define $\mu = \inf \mathcal{C}$ and $\mu^* = \inf \mathcal{D}$. These numbers again exist; if $k = 1$, the two inequalities, used to define $\mathcal{C}$ and $\mathcal{D}$, become $v_2(q, j_b, j_s) \geq 0$, which is satisfied $\forall q, j_b, j_s$. We get $\mathcal{D} \subset \mathcal{C}$, consequently $\mu \geq \mu^*$. A second useful lemma can now be stated.

**Lemma 2**  
1. If $\mu > \underline{\mu}$, then: $\mu v_2(q, j_b, j_s) + (1-\mu)c_3(q, a, j_b, j_s) \geq 0, \forall q, a, j_b, j_s$.  
2. If $\mu > \mu^*$, then: $\mu v_2(q, j_b, j_s) + (1-\mu)c_3(q, a, j_b, j_s) \geq -(1-\mu)\pi_2(a, j_b, j_s) \forall q, a, j_b, j_s$.

**Proof** The proof is the same than the one of Lemma 1.

Using Lemmas 1 and 2, we now determine the value of the simple fixed-price contract:

**Proposition 2** Suppose that assumptions (1) to (6) hold. If $\mu < \overline{\mu}$ and if $\mu \in [\underline{\mu}, \overline{\mu}]$, then the no-contracting outcome generates a general under-investment in comparison with the contracting outcome. It follows that $\hat{a} < \tilde{a}, \hat{j}_b < \tilde{j}_b$ and $\hat{j}_s < \tilde{j}_s$.

**Proof** See appendix.

The value of the fixed-price contract lies on the condition that $\mu < \overline{\mu}$. The following reasonable assumption allows this condition to be satisfied.

**Assumption 7** For all $(q, a, j_b, j_s) \in \mathbb{R}^4_+$, the following inequalities are satisfied:  
$v_2(q, j_b, j_s) > -c_3(q, a, j_b, j_s)$ and $v_3(q, j_b, j_s) < -c_4(q, a, j_b, j_s)$.

Each party contributes to the joint investment. However, marginal returns of each contribution are not necessarily symmetrical. It seems reasonable to assume that each contribution to the joint investment yields marginally more to its contributor than to its partner. Assumption (7) specifies this idea. Using this assumption, we establish more precise results about the value of the simple fixed-price contract.

**Corollary 1** Suppose that assumptions (1) to (7) hold. We get:

1. $\mu$ and $\overline{\mu}$ satisfy the inequality $\underline{\mu} \leq \frac{1}{2} \leq \overline{\mu}$;  
2. if $\mu \in [\underline{\mu}, \overline{\mu}]$, then the no-contracting outcome generates under-investments compared to the contracting outcome. It follows that $\hat{a} < \tilde{a}, \hat{j}_b < \tilde{j}_b$ and $\hat{j}_s < \tilde{j}_s$.  

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Proof From assumption (7), we easily deduce:

\[ \frac{1}{2}v_2(q, j_b, j_s) + \frac{1}{2}c_3(q, a, j_b, j_s) > 0 \quad \text{and} \quad \frac{1}{2}v_3(q, j_b, j_s) + \frac{1}{2}c_4(q, a, j_b, j_s) < 0. \]

It follows that \( \mu \geq \frac{1}{2} \) and \( \mu \leq \frac{1}{2} \). The rest is an application of proposition (2).

Contracting appears to be valuable because the realization of the joint investment exhibits direct effects for both parties and improve their status quo position. This effect is missing when parties do not write an initial contract.

Results of propositions (1) and (2) do not allow to discriminate between contracting and no-contracting outcomes in terms of welfare. They state that no-contracting leads to under-investments compared to both the first-best and the contracting solutions. But, we could not infer a clear ranking between the first-best and the (suboptimal) contracting outcome, since over-investment is also a suboptimal solution. Yet, if \( q \) is lower than a certain limit, it is possible to specify that contracting moves closer to the first-best.

To achieve this result, let’s first remark that investment levels in the contracting case \((\hat{\alpha}, \hat{j}_b, \hat{j}_s)\) only depend on \( q \) and not on \( \overline{q} \) (see proof of proposition 2). We also observe that when \( q = 0 \), the contracting solution becomes a special case of the no-contracting outcome. We thus get \( \tilde{\alpha}(q) = \hat{\alpha}, \tilde{j}_b(q) = \hat{j}_b \) and \( \tilde{j}_s(q) = \hat{j}_s \). By continuity, when \( q \) is positive but sufficiently low, we get by application of propositions 1 and 2: \( \hat{\alpha} < \tilde{\alpha}(q) < a^*, \hat{j}_b < \tilde{j}_b(q) < j_b^* \) and \( \hat{j}_s < \tilde{j}_s(q) < j_s^* \). Given assumptions (1) to (6), total surplus satisfy \( \tilde{S}(\hat{\alpha}, \tilde{j}_b, \tilde{j}_s) < \tilde{S}(\tilde{\alpha}(q), \tilde{j}_b(q), \tilde{j}_s(q)) < S^*(a^*, j_b^*, j_s^*) \). There are thus contracts (characterized by a fixed positive quantity lower than a certain limit) allowing to improve the global surplus compared to the no-contracting case.

5 Conclusion

In this paper, we analyzed two simple ways in which the input could be procured: (1) a renegotiable fixed-price contract and (2) a bargaining of the terms of trade without a prior contract. We found that both arrangements fail to achieve efficiency and to undertake optimal joint investment. A direct implication of this result is that a process of vertical integration, with a unified direction, provides optimal incentives to cooperate.

We also aimed to compare directly the contracting and the no-contracting solutions. Under reasonable assumptions, we found that the contracting solution induces larger autonomous and joint investments compared to no-contracting. Moreover, fixing by contract a positive but sufficiently low quantity, ensures at

\[ S(a, j_b, j_s) = \Pi(a, j_b, j_s) - a - j_b - j_s. \]

\[ S(a, j_b, j_s) = \Pi(a, j_b, j_s) - a - j_b - j_s. \]

\[ S(a, j_b, j_s) = \Pi(a, j_b, j_s) - a - j_b - j_s. \]
least a minimal *status quo* and is welfare improving. Thus, handshakes are not always needed to promote cooperation.

### A Proof of proposition 1

It is worth noting that assumption (5) implies that $\Pi_l > 0$ for all $i \neq l$.

Consider now the following problem:

$$
\max_{a, j_b, j_s} \Pi(a, j_b, j_s) - \left( \lambda + \frac{1 - \lambda}{\mu} \right) a - \left( \lambda + \frac{1 - \lambda}{1 - \mu} \right) j_b - \left( \lambda + \frac{1 - \lambda}{\mu} \right) j_s.
$$

(11)

with $\lambda \in [0, 1]$.

The first order conditions are:

$$
\Pi_1(a, j_b, j_s) = \lambda + \frac{1 - \lambda}{\mu},
$$

$$
\Pi_2(a, j_b, j_s) = \lambda + \frac{1 - \lambda}{1 - \mu},
$$

$$
\Pi_3(a, j_b, j_s) = \lambda + \frac{1 - \lambda}{\mu}.
$$

The maximization problem (11) has a unique solution, i.e. $a(\lambda)$, $j_b(\lambda)$ and $j_s(\lambda)$.

Note that $a(1) = a^*$, $j_b(1) = j_b^*$, $j_s(1) = j_s^*$ and $a(0) = \hat{a}$, $j_b(0) = \hat{j}_b$, $j_s(0) = \hat{j}_s$.

Define

$$
V(\lambda) = \Pi(a(\lambda), j_b(\lambda), j_s(\lambda)) - \left( \lambda + \frac{1 - \lambda}{\mu} \right) a(\lambda)
$$

and

$$
W(\lambda) = \Pi(a(\lambda), j_b(\lambda), j_s(\lambda)) - \left( \lambda + \frac{1 - \lambda}{1 - \mu} \right) j_b(\lambda) - \left( \lambda + \frac{1 - \lambda}{\mu} \right) j_s(\lambda) - V(\lambda).
$$

We necessarily have $W(\lambda) \leq 0$ and $W(\lambda_0) = 0$. Thus, the function $W(\lambda)$ attains a maximum at $\lambda = \lambda_0$. At this point, the first and second order optimality conditions are necessarily satisfied, we thus have $W''(\lambda_0) = 0$ and $W'''(\lambda_0) < 0$.

The first and second derivatives of $W(\lambda)$ are:

$$
W''(\lambda) = - \left( 1 - \frac{1}{\mu} \right) a(\lambda) - \left( 1 - \frac{1}{1 - \mu} \right) j_b(\lambda) - \left( 1 - \frac{1}{\mu} \right) j_s(\lambda) - V'(\lambda),
$$

$$
W'''(\lambda) = - V''(\lambda).
$$

The first order condition gives:

$$
W'(\lambda) = - \left( 1 - \frac{1}{\mu} \right) a(\lambda) - \left( 1 - \frac{1}{1 - \mu} \right) j_b(\lambda) - \left( 1 - \frac{1}{\mu} \right) j_s(\lambda) - V'(\lambda) = 0.
$$

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Consider any values of $q, a, j_b, j_s$. We get

\[ g(\mu) = \nu_3(q, j_b, j_s) + (1 - \mu) c_4(q, a, j_b, j_s). \]

We get $g'(\mu) = \nu_3(q, j_b, j_s) - c_4(q, a, j_b, j_s) > 0$. It follows that $g(\mu) < g(\tilde{\mu}) \leq 0, \forall \mu \in [0, \tilde{\mu}]$. 

**B Proof of lemma 1**

Consider any values of $q, a, j_b, j_s$.

1. Define $g(\mu) = \nu_3(q, j_b, j_s) + (1 - \mu) c_4(q, a, j_b, j_s)$. We get $g'(\mu) = \nu_3(q, j_b, j_s) - c_4(q, a, j_b, j_s) > 0$. It follows that $g(\mu) < g(\tilde{\mu}) \leq 0, \forall \mu \in [0, \tilde{\mu}]$. 

The above expression holds for any $\lambda_0$. The first derivative of $V(\lambda)$ is then:

\[
V'(\lambda) = - \left( 1 - \frac{1}{\mu} \right) a(\lambda) - \left( 1 - \frac{1}{1 - \mu} \right) j_b(\lambda) - \left( 1 - \frac{1}{\mu} \right) j_s(\lambda).
\]

We deduce the expression of the second derivative of $V(\lambda)$:

\[
V''(\lambda) = - \left( 1 - \frac{1}{\mu} \right) a'(\lambda) - \left( 1 - \frac{1}{1 - \mu} \right) j_b'(\lambda) - \left( 1 - \frac{1}{\mu} \right) j_s'(\lambda).
\]

The second order condition $W''(\lambda_0) < 0$ also holds for any $\lambda_0$. We thus have $W''(\lambda) < 0$ for all $\lambda$. Consequently:

\[
W''(\lambda) = - \left( 1 - \frac{1}{\mu} \right) a'(\lambda) - \left( 1 - \frac{1}{1 - \mu} \right) j_b'(\lambda) - \left( 1 - \frac{1}{\mu} \right) j_s'(\lambda) < 0.
\]

Given that $\mu \in [0, 1]$, we necessarily have $a'(\lambda) > 0$ or $j_b'(\lambda) > 0$ or $j_s'(\lambda) > 0$. Suppose for example that $a'(\lambda) > 0$ and differentiate the first order conditions with respect to $\lambda$ to obtain:

\[
a'(\lambda) \Pi_{11} + j_b'(\lambda) \Pi_{12} + j_s'(\lambda) \Pi_{13} = 1 - \frac{1}{\mu} < 0,
\]

\[
a'(\lambda) \Pi_{21} + j_b'(\lambda) \Pi_{22} + j_s'(\lambda) \Pi_{23} = 1 - \frac{1}{1 - \mu} < 0,
\]

\[
a'(\lambda) \Pi_{31} + j_b'(\lambda) \Pi_{32} + j_s'(\lambda) \Pi_{33} = 1 - \frac{1}{\mu} < 0.
\]

The cross derivatives $\Pi_{ij}$ being negative, we get:

\[
j_b'(\lambda) \Pi_{22} + j_s'(\lambda) \Pi_{23} = 1 - \frac{1}{1 - \mu} - a'(\lambda) \Pi_{21} < 0,
\]

\[
j_b'(\lambda) \Pi_{32} + j_s'(\lambda) \Pi_{33} = 1 - \frac{1}{\mu} - a'(\lambda) \Pi_{31} < 0.
\]

It immediately follows that:

\[
\begin{pmatrix} j_b'(\lambda) & j_s'(\lambda) \end{pmatrix} \begin{pmatrix} \Pi_{22} & \Pi_{23} \\ \Pi_{32} & \Pi_{33} \end{pmatrix} \begin{pmatrix} j_b'(\lambda) \\ j_s'(\lambda) \end{pmatrix} = j_b'(\lambda) \left( 1 - \frac{1}{1 - \mu} - a_b'(\lambda) \Pi_{31} \right) + j_s'(\lambda) \left( 1 - \frac{1}{\mu} - a_s'(\lambda) \Pi_{41} \right) < 0.
\]

We necessarily have $j_b'(\lambda) > 0$ or $j_s'(\lambda) > 0$. Suppose for example that $j_b'(\lambda) > 0$. The same argument shows that $j_s'(\lambda) > 0$. 

\[\]
2. Define \( g(\mu) = \mu v_3(q, j_b, j_s) + (1 - \mu) c_4(q, a, j_b, j_s) - \mu \pi_3(a, j_b, j_s) \).

We get \( g'(\mu) = v_3(q, j_b, j_s) - c_4(q, a, j_b, j_s) - \pi_3(a, j_b, j_s) \).

Suppose that \( q, a, j_b, j_s \) are

(a) such that \( v_3(q, j_b, j_s) - c_4(q, a, j_b, j_s) - \pi_3(a, j_b, j_s) > 0 \), then \( g(\mu) < g(\bar{\mu}^*) \leq 0, \forall \mu \in [0, \bar{\mu}^*] \).

(b) such that \( v_3(q, j_b, j_s) - c_4(q, a, j_b, j_s) - \pi_3(a, j_b, j_s) < 0 \).

We get \( g(0) = c_4(q, a, j_b, j_s) \leq 0 \) and \( g'(\mu) < 0 \). Thus, \( g(\mu) \leq 0, \forall \mu \geq 0 \).  

C Proof of proposition 2

Let \((\bar{a}, \bar{j}_b, \bar{j}_s) \in \mathbb{R}_+^4\) be the investment levels of the contracting outcome, solutions of the following first-order conditions:

\[
(1 - \mu)\Pi_2(\bar{a}, \bar{j}_b, \bar{j}_s) = 1 - \mu v_2(\bar{q}, \bar{j}_b, \bar{j}_s) + (1 - \mu) c_4(\bar{q}, \bar{a}, \bar{j}_b, \bar{j}_s),
\]

\[
\mu \Pi_1(\bar{a}, \bar{j}_b, \bar{j}_s) = 1 + (1 - \mu) c_2(\bar{q}, \bar{a}, \bar{j}_b, \bar{j}_s),
\]

\[
\mu \Pi_3(\bar{a}, \bar{j}_b, \bar{j}_s) = 1 + \mu v_3(\bar{q}, \bar{j}_b, \bar{j}_s) + (1 - \mu) c_4(\bar{q}, \bar{a}, \bar{j}_b, \bar{j}_s).
\]

where \((\bar{a}, \bar{j}_b, \bar{j}_s) \in \mathbb{R}_+^4\) are the investment levels of the no-contacting solution:

\[
(1 - \mu)\Pi_2(\bar{a}, \bar{j}_b, \bar{j}_s) = 1; \quad \mu \Pi_1(\bar{a}, \bar{j}_b, \bar{j}_s) = 1; \quad \mu \Pi_3(\bar{a}, \bar{j}_b, \bar{j}_s) = 1.
\]

Using Lemmas 1 and 2, it can be easily shown that for all \( \mu \in [\bar{\mu}, \bar{\mu}] \)

\[
\Pi_1(\bar{a}, \bar{j}_b, \bar{j}_s) - \Pi_1(\bar{a}, \bar{j}_b, \bar{j}_s) = \frac{1 - \mu}{\mu} v_2(\bar{q}, \bar{a}, \bar{j}_b, \bar{j}_s) < 0,
\]

\[
\Pi_2(\bar{a}, \bar{j}_b, \bar{j}_s) - \Pi_2(\bar{a}, \bar{j}_b, \bar{j}_s) = \frac{\mu}{1 - \mu} v_2(\bar{q}, \bar{a}, \bar{j}_b, \bar{j}_s) + (1 - \mu) c_3(\bar{q}, \bar{a}, \bar{j}_b, \bar{j}_s) < 0,
\]

\[
\Pi_3(\bar{a}, \bar{j}_b, \bar{j}_s) - \Pi_3(\bar{a}, \bar{j}_b, \bar{j}_s) = \frac{\mu}{\mu} v_3(\bar{q}, \bar{j}_b, \bar{j}_s) + (1 - \mu) c_4(\bar{q}, \bar{a}, \bar{j}_b, \bar{j}_s) < 0,
\]

The rest of the proof is similar to the one of the under-investment result in the no-contacting outcome (see proposition 1).  

References


