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Risk Management and Public Policies: How prevention challenges monopolistic insurance markets

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Abstract

Using a principal-agent framework, we extend the insurance monopoly model (Stiglitz, 1977) to self-insurance opportunities. Relying on a two-part tariff contract as an analytical tool, we show that an insurance monopoly can achieve the same equilibrium as a competitive insurer. However, in the monopoly situation, the insurer captures all the insurance market surplus. Yet, compared to a monopoly market with insurance *only,* self-insurance opportunities act as a threat to the insurer, resulting in a cut of the insurer's market power and an increase in the policyholders' welfare. Moreover, within our principal-agent framework, we show that while insurance and self-insurance are substitutes, compulsory self-insurance, and compulsory insurance have non-equivalent effects. Although compulsory self-insurance reduces the market size of the insurer, it has no impact on the policyholder's well-being. On the other hand, mandatory insurance favors the insurer and makes policyholders worse off. The implications of these public policies are discussed.

Keywords: self-insurance, insurance, monopoly, compulsory insurance, public regulation.

Classification JEL: D86, D42, G22

Introduction

A key feature of many risks (aging, climate change, chronic diseases) is that our societies have little control over their occurrence frequencies. The rise of these risks strengthens interest in preventive actions. Understanding the interplay between insurance coverage and prevention opportunities (self-protection and self-insurance) is fundamental to optimizing risk management.[1] The article investigates the impact of insurance and self-insurance obligations on the insurer/insured relationship using a principal-agent model. The presence of self-insurance opportunities makes the policyholder's threat of market exit more likely and plausible. It acts as a countervailing power to the monopoly power, given the well-known substitutability property between insurance and self-insurance (Ehrlich & Becker, 1972). This property suggests that an increase (resp. reduction) in insurance price enhances (resp. discourages) the demand for prevention. This property has been extensively documented, both theoretically ((Courbage, 2001) with dual choice theory, (Kelly & Kleffner, 2003) in the case of an insurance monopoly, and (Brunette et al., 2020) in the presence of ambiguity with or without asymmetric information) and empirically (Carson et al., 2013; Kousky et al., 2014), but also experimentally (Pannequin et al., 2020).

The property of substitutability between insurance and self-insurance raises the question of the impact of self-insurance opportunities on the insurance market. Several papers show that the presence of self-insurance changes the equilibrium in the insurance market, whether it is competitive or monopolistic, under symmetric or asymmetric information. Crainich (2017) shows that in a competitive insurance market, some health insurance market equilibria with adverse selection are altered by self-insurance opportunities such as genetic testing. Anderberg (1999) reveals that a linear self-insurance technology affects the functioning of a Rothschild-Stiglitz competitive insurance market and shows that self-insurance opportunities might reduce the consumers' welfare. Kelly and Kleffner (2003) find that in the context of an insurance monopoly, self-insurance opportunities increase the consumer's elasticity of demand and lower the optimal rate charged by the monopolist.[2]

To manage these increasing risks, the main policies to be considered are either insurance or prevention obligations. Compulsory self-insurance reduces the size of the loss and controls its maximum amount. It makes it possible for an insurer to homogenize risks and, as a result, estimate actuarial premiums more reliably. One of the justifications for insurance obligation is that it mitigates the negative externalities caused by non-insured individuals. Risk lovers choose not to cover their losses.

In the same way, some risk-averse individuals do not find the insurance contract attractive enough to take out an insurance policy. Empirical (Browne & Hoyt, 2000) and experimental (Corcos et al., 2017; Slovic et al., 1977) works show that in many cases (natural disasters – even with subsidized prices, flooding, earthquakes) individuals do not buy insurance when they do not *have* to. In the same way, numerous experimental (Corcos et al., 2021; Shapira & Venezia, 2008) and empirical studies (Sydnor, 2010) point to the prevalence of all-or-nothing insurance behavior. Adverse selection considerations can also justify insurance obligations

[1] Since Ehrlich and Becker (1972), self-protection refers to all expenditure that reduces the probability of loss and self-insurance involves all expenditure devoted to reducing the extent of damages in the event of an accident.

[2] Our approach is more general than Kelly and Kleffner's (insurance monopoly with self-insurance opportunities): we consider two contractual parameters (p and C) instead of the single unit insurance price p.

(Akerlof, 1970; Rothschild & Stiglitz, 1978): an average insurance premium is likely to crowd low risks out from the market by overcharging their premiums.
On the other hand, compulsory insurance can discourage prevention investment (Pannequin et al., 2020). Moreover, by threatening the insurer with market exit, self-insurance opportunities could cut the insurer's profit, compared to the context where only insurance is available. This raises the question of whether insurance obligations could harm policyholders by bringing back the insurer's monopoly power.
Comparing the effects of insurance or prevention obligations and knowing whether the substitutability property between insurance and self-insurance extends to their legal commitments would help inform and guide public policy. Is it equivalent for the insured and the insurer to impose insurance or a self-insurance obligation? In other words, does the insurance obligation have symmetrical effects to those of the self-insurance obligation? The literature has yet to address the impacts of these policies on the policyholders' welfare and the insurer's profit.
In this paper, we investigate the public policy implications of compulsory insurance and compulsory self-insurance when both insurance and self-insurance are available. Using a principal-agent framework, we model the fundamental relationship between the insurer and the insured. The article extends the standard insurance monopoly model with symmetric information (Stiglitz, 1977) to the case where the policyholder can use both insurance and self-insurance. As in Ehrlich and Becker (1972), we assume self-insurance technology has decreasing returns. Instead of using the standard specification of insurance contracts $\{P, I\}$ – where P is the insurance premium and I is the indemnity - we substitute, without loss of generality, two-part insurance contracts with a unit insurance price p and a fixed cost C. Thus, an insurance contract is written as $\{P = pI + C, I\}$, which has no impact on the characterization of insurance market optima.[3] As shown in this paper, this type of insurance contract is an effective analytical tool for understanding the effect of public policies on insurance markets. Moreover, this contractual device is standard in real life. For example, in health insurance, insureds typically pay a fixed annual amount and have to supplement it with fees proportional to their medical consumption. More generally, since insurance technology involves fixed costs added to actuarial premiums, this pricing seems like an eligible approximation.
In this context, we show that, compared to the situation with insurance only, self-insurance opportunities weaken the monopoly power and enable the policyholder to claw back a portion of the rent. Self-insurance opportunities represent a threat to insurers: they reduce their influence on the risk management process by lowering the insurance market share compared to the situation without self-insurance opportunities. Moreover, we show that while insurance and self-insurance are substitutes, compulsory self-insurance, and compulsory insurance have non-equivalent effects. Compulsory self-insurance reduces the market power of the insurer and, yet, has no impact on the policyholder's well-being. On the other hand, compulsory insurance favors the insurer while policyholders are worse off.
In section 1, we extend Stiglitz's (1977) insurance model to characterize the equilibrium of a monopoly and a competitive insurance market with self-insurance opportunities and a two-part tariff. Within this framework, section 2 analyzes the effects of two public policies on the

[3] Since any insurance contract $\{P, I\}$ can be rewritten as {P=pI+C, I} by correctly specifying the unit price p and the fixed cost C.

insurer's profit and equilibrium: insurance and self-insurance obligations. Section 3 discusses the policy issues arising from our main results and concludes.

1 Non-linear pricing: Two-part tariff insurance equilibrium in the presence of self-insurance opportunities

Within our framework, policyholders face a probability *q* of losing a portion *x* of their initial wealth W_0. Their final wealth is $W_1 = W_0$ if no loss occurs and $W_2 = W_0 - x$ if it occurs. Policyholders can invest in a self-insurance technology and a two-part insurance contract to reduce their risk exposure.

A. ISI General framework

The use of self-insurance (SI) alone will define policyholders' reservation utility.

1 Defining the reservation utility

The function $SI(e)$ represents the self-insurance technology. Investing an amount *e* in self-insurance technology reduces the size of the loss by an amount $SI(e)$. The returns of the self-insurance technology $SI(e)$ are decreasing: $SI'(e) > 0$, $SI''(e) < 0$.[4] The final wealth with self-insurance opportunities only is represented as:

$$\begin{cases} W_{1SI} = W_0 - e \\ W_{2SI} = W_0 - x - e + SI(e) \end{cases} \tag{1}$$

The subscript *SI* refers to the situation where only self-insurance opportunities are available. W_1 and W_2 stand respectively for the no-loss and loss state.

We characterize the policyholder's preferences with the utility function *U(W),* strictly increasing and concave $(U'(W) > 0, U''(W) < 0)$. When insurance is not available, individuals choose the level of self-insurance, e_{SI}, that maximizes their expected utility $EU(e)$:

$$e_{SI} = \underset{e}{\mathrm{ArgMax}} EU(e) \tag{2}$$

where $EU(e) = (1-q)U(W_0 - e) + qU(W_0 - x - e + SI(e))$.
e_{SI} provides the minimal expected utility $EU(e_{SI})$ – reservation utility – obtained by the policyholders if they decide not to use insurance coverage.

2 Policyholders EU when using both I and SI

Policyholders may also use insurance to cover their losses. In our two-state world, we generally model an insurance contract through a couple $\{P, I\}$, where P corresponds to the

[4] A necessary condition for a strictly positive investment in self-insurance is that there be at least one strictly positive value e_s such that $SI'(e_s) > 1$. The purchase of an additional unit of self-insurance must generate a gain (in terms of reducing loss x) of more than one unit, as this gain is random (Henriet & Rochet, 1991).

insurance premium and I to the indemnity. However, and without loss of generality, we chose to rely on a two-part insurance tariff, involving a unit insurance price *p* and a fixed cost *C*, to ease the analytic presentation of our results. Therefore, to characterize efficient solutions, we specify insurance contracts as $\{P = pI + C, I\}$ instead of considering simply the couples $\{P, I\}$. As this approach does not change the set of feasible contracts, both specifications would converge to the exact optimal solutions. Moreover, the two contractual parameters (*p*, *C)* are instrumental in stressing the main features of market equilibria.

In addition, we assume that transaction costs are negligible and that the insurer is risk neutral.[5] We characterize the insurance contract as follows: [6]

$$\begin{cases} 0 \leq I \leq x \\ P = pI + C \end{cases} \tag{3}$$

When policyholders use both insurance policy and self-insurance, their random wealth is equal to $W_{1\,ISI}$ or $W_{2\,ISI}$, depending on whether a loss occurred during the period:

$$\begin{cases} W_{1\,ISI} = W_0 - pI - C - e \\ W_{2\,ISI} = W_0 - pI - C - e - x + I + SI(e) \end{cases}$$

The *ISI* subscript now refers to the *ISI context* where both insurance and self-insurance opportunities are available. The policyholder's expected utility becomes:

$$EU(I, e) = (1 - q)U(W_0 - pI - C - e) + qU(W_0 - pI - C - e - x + I + SI(e)) \tag{4}$$

B. ISI Competitive equilibrium

As competition induces the insurer to maximize the policyholder's well-being under a non-zero profit constraint, the following program characterizes the equilibrium:[7]

$$\begin{cases} \underset{p,C,I,e}{Max} \quad (1 - q)U(W_0 - pI - C - e) + qU(W_0 - pI - C - e - x + SI(e) + I) \\ s.t. \quad E(\pi) = (p - q)I + C \geq 0 \end{cases} \tag{5}$$

Solving the program (see Appendix I) leads to the following equilibrium (the subscript *ISI* refers to the *ISI context,* while the exponent *c* stands for the (competitive) market situation):

[5] This is the hypothesis generally used in the literature. It is based on the law of large numbers (verified by the fact that the insured client base is very large, or, more realistically, the availability of reinsurance opportunities at a negligible cost). Kelly and Kleffner (2003) underline the fact that insurance companies may be risk averse, for example in the context of earthquake insurance.

[6] We implicitly assume that insurers do not observe the self-insurance behavior of their policyholders. As a result, the insurance premium, as modeled, is independent of the prevention choices made by policyholders. However, due to the nature of self-insurance, the opposite hypothesis ($0 \leq I \leq x - SI(e)$ instead of $0 \leq I \leq x$) would lead to the same results (see (Brunette et al., 2019)).

[7] The presence of the fixed cost C could justify the use of a participation constraint on the policyholder in the insurance market. As it is not saturated at the equilibrium, we omitted it.

$$\begin{cases} p_{ISI}^{c} = q \\ C_{ISI}^{c} = 0 \\ I_{ISI}^{c} = x - SI(e_{ISI}^{c}) \\ SI'(e_{ISI}^{c}) = \dfrac{1}{q} \end{cases}$$

Proposition 1: *Under competition, the insurer's profit is nil. The unit price of insurance is actuarial ($p_{ISI}^{c} = q$), and the fixed cost is nil ($C_{ISI}^{c} = 0$). The policyholder is fully covered and equalizes the marginal returns of insurance and self-insurance ($1/p_{ISI}^{c} = SI'(e_{ISI}^{c})$).*

C. ISI Monopoly equilibrium

As in the previous section, the insurance premium of the monopoly is equal to $P = pI + C$. The profit expectation for this insurance policy is equivalent to $E(\Pi) = (p - q)I + C$.
In this setting, the insurance monopoly equilibrium with self-insurance opportunities results from a two-stage game whereby the insurer specifies a price *p* and a fixed cost *C*. Then, given the insurance pricing, the policyholders choose *I* and *SI(e) levels* that maximize their expected utility.
An insurer in a monopoly position can anticipate the policyholder's behavior. For the policyholder to purchase an insurance (participation constraint *PC*), the insurer must guarantee a level of welfare at least equivalent to what the policyholder could obtain with self-insurance alone, $EU(e_{SI})$.
To characterize this equilibrium, we first determine the pairs of contractual parameters (*p, C*) and the policyholder's decisions (*I, e*), maximizing the monopoly's expected profit. Then, we show that choosing a specific pair of values for (*p, C*) is sufficient to realize the monopoly's optimum. The optimization problem is as follows:[8]

$$\begin{aligned} &\underset{p,C,e,I}{Max} \quad (p - q)I + C \\ &s.t. \quad (1 - q)U(W_{1\,ISI}) + qU(W_{2\,ISI}) \geq EU(e_{SI}) \end{aligned} \qquad \text{(PC)}$$

The optimum is described below (see Appendix II):

$$\begin{cases} I_{ISI} = x - SI(e_{ISI}) \\ SI'(e_{ISI}) = \dfrac{1}{q} \\ W_{1\,ISI} = W_{2\,ISI} = W_0 - p_{ISI}(x - SI(e_{ISI})) - C_{ISI} - e_{ISI} \\ p_{ISI} \text{ and } C_{ISI} \text{ such that } U(W_{1\,ISI}) = EU(e_{SI}) \end{cases}$$

The monopoly's profit maximization requires the insured to be fully covered and the marginal return on *SI* equal 1/q. The insurer simultaneously sets the unit insurance price and the fixed cost to saturate the participation constraint: $U(W_0 - p(x - SI(e_{ISI})) - C_{ISI} - e_{ISI}) =$

[8] The characterization of the best take-it-or-leave-it contract would lead to the following optimization problem: $\max_{P,I,e}\{P - qI : qU(W_0 - P - e + SI(e) - x(e) + I) + (1 - q)U(W_0 - P - e) \geq EU(e_{SI})\}$ and would result in the same solution.

$EU(e_{SI})$. Therefore, all pairs (p_{ISI}, C_{ISI}) respecting this constraint are compatible with the achievement of the optimum since they result in the same insurance premium P_{ISI}^{m}.[9]

Moreover, the insurer expects the policyholders to equalize marginal returns on both insurance and self-insurance $(SI'(e) = \frac{1}{p})$ and to choose the insurance coverage that optimizes their expected utility (see FOC (α) in Appendix II). For the values of *I* and *e* obtained in the insurer's profit maximization to be an equilibrium, they must result from the maximization of the policyholders' expected utility, where p_{ISI}^{m} and C_{ISI}^{m} are the values enforced by the insurer. The following optimization problem describes the policyholder's reaction to a two-part tariff (p, C):

$$\begin{cases} \underset{I,e}{Max} & (1-q)U(W_0 - pI - C - e) + qU(W_0 - pI - C - e - x + SI(e) + I) \\ s.t. & (1-q)U(W_0 - pI - C - e) + qU(W_0 - pI - C - e - x + SI(e) + I) \geq EU(e_{SI}) \quad (PC) \end{cases}$$

Solving the policyholders' program (see Appendix II), we find that when the unit price of insurance is actuarial $(p_{ISI}^{m} = q)$, the policyholders' optimum is full insurance coverage as long as the fixed cost *C* does not deter them from insurance. The self-insurance expenditure is such that the marginal returns of *SI* and *I* are equal. Therefore, the equilibrium resource allocation is as follows:

$$\begin{cases} p_{ISI}^{m} = q \\ I_{ISI}^{m} = x - SI(e_{ISI}^{m}) \\ SI'(e_{ISI}^{m}) = \dfrac{1}{q} \\ W_{1\,ISI} = W_{2\,ISI} = W_0 - p_{ISI}^{m}(x - SI(e_{ISI}^{m})) - C_{ISI}^{m} - e_{ISI}^{m} \\ C_{ISI}^{m} \text{ such that } U(W_{1\,ISI}) = EU(e_{SI}) \end{cases}$$

Proposition 2: *In a monopoly insurance market, the policyholder is fully covered and equalizes the marginal returns of insurance and self-insurance. The optimal two-part tariff involves an actuarial unit price ($p_{ISI}^{m} = q$) and a fixed cost C_{ISI}^{m} saturating the participation constraint.*

Corollary 1*: To achieve the ISI monopoly optimum through a take-it-or-leave-it contract, the insurer needs only to propose the optimal contract: (P_{ISI}^{m}, I_{ISI}^{m}).*[10] *However, with the two-part tariff, the indemnity is not imposed but incentivized by p_{ISI}^{m} and C_{ISI}^{m}.*

An essential feature of the two-part tariff lies in its incentive properties. If the monopoly optimizes the two contractual parameters, the insured will invest optimally in self-insurance and buy comprehensive insurance coverage. Compared to the take-it-or-leave-it contract, this

[9] Indeed, the insurance premium is the same for all pairs (p_{ISI}, C_{ISI}) saturating (PC): $P_{ISI}^{m} = W_0 - U^{-1}(EU(e_{SI})) - e_{ISI}^{m}$.

[10] Using the FOC (β) of the policyholder's problem developped in the second part of Appendix II, it is straightforward that the policyholder would be induced to choose the right investment e_{ISI}^{m}, and the take-it-or-leave-it equilibrium would meet all optimal conditions.

scenario seems more realistic since the policyholder can generally negotiate the extent of the insurance coverage in real life.

Competitive and monopoly ISI equilibria

According to Kelly and Kleffner (2003), policyholders invest more in self-insurance activities in a monopolistic insurance market than in a competitive market. However, as they only rely on one contractual parameter, the unit insurance price, they do not characterize efficient insurance contracting. Therefore, their result is no longer valid in our more general setting. As the optimal two-part contract relies on an actuarial unit insurance price, it deters the policyholders from reducing their insurance demand and increasing their self-insurance expenditure. Our monopoly equilibrium is efficient: it results in the same levels of insurance and self-insurance as in a context of pure and perfect competition.

Corollary 2: *Monopoly and competitive markets result in the same self-insurance and insurance coverages. The only difference lies in the fixed cost (or insurer's profit), which is not nil for the monopoly.*

D. Graphical analysis of ISI competitive and monopoly equilibria

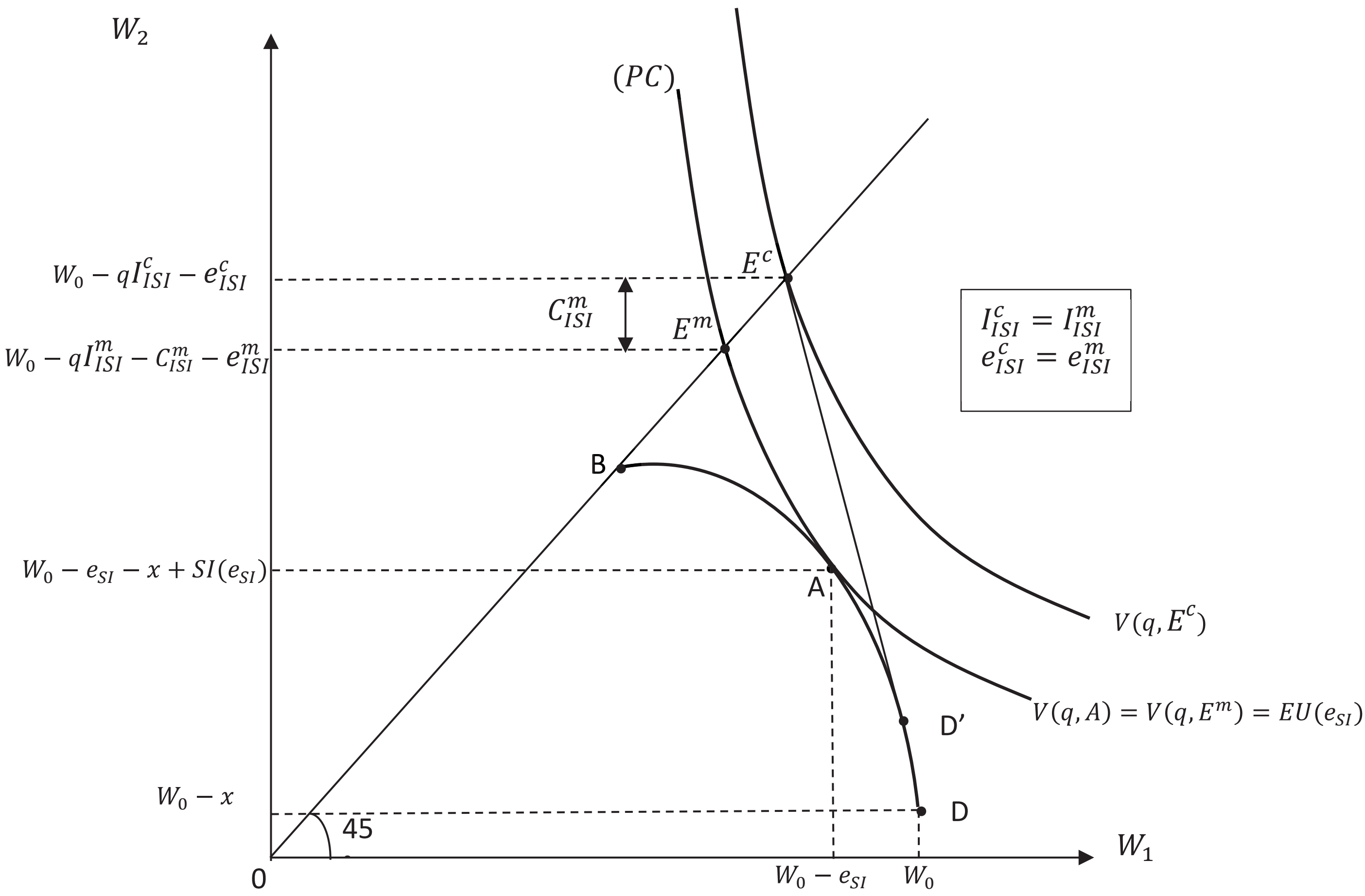

Figure 1: Monopoly and competitive equilibria (two-part tariff p, C)

Figure 1 represents both competitive and monopoly equilibria in the plane (W_1, W_2). In this plane, the 45° line gathers all full insurance points. Point D corresponds to the endowment point of coordinates $(W_1, W_2) = (W_0, W_0 - x)$. Policyholder's preferences are represented through indifference curves, denoted $V(q, W)$, which, given the loss probability, connect all wealth outcomes $W = (W_1, W_2)$ that generate the same expected utility. Due to risk aversion, any indifference curve is convex compared to the origin; moreover, its slope, at any point $W = (W_1, W_2)$ of the quadrant is equal to $-\frac{(1-q)U'(W_1)}{qU'(W_2)}$.
The curve DB represents self-insurance opportunities, i.e., all the feasible wealth allocations given the self-insurance technology. As the marginal returns of this technology are decreasing, DB is concave, and its slope, evaluated at any self-insurance effort e, is equal to $1 - SI'(e)$.[11]
As previously shown, when the decision-maker only invests in self-insurance, the best expected utility is given by $EU(e_{SI})$. In figure 1, this self-insurance investment corresponds to a move from point D to point A.[12] Thus, the indifference curve through point A, $V(q, A)$, represents the participation constraint (PC): any insurance contract implying a shift below this curve will trigger an insurance market exit; otherwise, any shift above (or on) $V(q, A)$, will be acceptable. As point A represents a welfare improvement compared to the endowment point (D), any combination of Insurance and Self-insurance should lie above $V(q, A)$.

In the plane (W_1, W_2), starting from any point $W = (W_1, W_2)$, any insurance contracting with (p, C) can be represented through a move along a line with slope $-\frac{(1-p)}{p}$ towards the 45° line. This shift is combined with a translation proportional to C towards the origin.[13]

In Figure 1, point E^c represents the competitive equilibrium. The policyholder combines both risk management tools:

- a self-insurance investment represented by the move from D to D', along DB. The tangency point D' corresponds to the equalization of marginal returns of insurance and self-insurance.
- an insurance investment corresponding to the move from D' to E^c.

As the competition premium is actuarial ($p_{ISI}^c = q, C_{ISI}^c = 0$), the slope of the insurance line is equal to $-\frac{(1-q)}{q}$. D' is characterized by the tangency between DB and the insurance line: $1 - SI'(e_{ISI}^c) = -\frac{(1-q)}{q} \Leftrightarrow SI'(e_{ISI}^c) = \frac{1}{q}$. Then, the residual loss ($x - SI(e_{ISI}^c)$) is fully covered in the insurance market.
On the other side, point E^m represents the monopoly equilibrium. Facing the actuarial unit price ($p_{ISI}^m = q$), the policyholder provides the same self-insurance effort ($SI'(e_{ISI}^m) = \frac{1}{q}$,

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[11] Following a one unit decrease in e, wealth W_1 increases by one unit while the decrease in wealth W_2 amounts to $1 - SI'(e)$.
[12] Point A corresponds to the tangency point between the DB curve and the highest indifference curve $V(q, W)$. Therefore, at point A, $-\frac{(1-q)U'(W_1)}{qU'(W_2)} = 1 - SI'(e)$.
[13] Starting from point D $(W_0, W_0 - x)$ and assuming $(p > 0, C = 0)$, insurance opportunities are described by the following equation: $(1-p)W_1 + pW_2 = W_0 - px$. Indeed, point D and the full insurance point $(W_0 - px, W_0 - px)$ belong to this line. Adding a positive fixed cost to the insurance pricing uniformly affects the two wealth states; this results in a shift (proportional to C) of the previous insurance line, towards the origin, and according to the direction of the 45° line.

therefore, $e_{ISI} \equiv e_{ISI}^m = e_{ISI}^c$) and voluntarily buys the same full insurance coverage ($I_{ISI}^m = x - SI(e_{ISI}^m)$). Again, the solution is characterized by a shift from D to D', then from D' to E^c, but the third shift, from E^c to E^m, corresponds to the insurer's profit ($\pi_{ISI}^m = C_{ISI}^m$). The insurer captures all the market rent compatible with the participation constraint $V(q, A)$.

2 Public policies

The previous section provides a framework for analyzing the effects of policy measures on prevention behavior. Using the two-part pricing, we assess the impact of two basic public policies: insurance obligation versus self-insurance obligation. Our analysis highlights that self-insurance opportunities challenge the monopoly's market power.

A. Compulsory self-insurance $I\overline{SI}$

When self-insurance is compulsory, the insurance monopoly maximizes its profit under the policyholder's participation constraint (PC) and the legal constraint of prevention (LC):

$$\begin{cases} \underset{p,C,e,I}{Max} \quad (p-q)I + C & \\ \quad s.t. \quad (1-q)U(W_{1\,I\overline{SI}}) + qU(W_{2\,I\overline{SI}}) \geq EU(e_{SI}) & (PC) \\ \qquad\qquad e \geq \bar{e} & (LC) \end{cases}$$

Where $\bar{e}$ stands for the legal self-insurance obligation and the subscripts $I\overline{SI}$ for the compulsory self-insurance context, where insurance is also available. The no-loss state wealth and the loss state wealth are given by:

$$W_{1\,I\overline{SI}} = W_0 - pI - C - e, \text{ and } W_{2\,I\overline{SI}} = W_0 - pI - C - e - x + SI(e) + I$$

To focus on the realistic values of $\bar{e}$ we assume that $\bar{e} > e_{ISI}^m$. Otherwise, with $\bar{e} \leq e_{ISI}^m$, constraint (LC) would not be binding, and we would return to the previous *ISI* monopoly equilibrium. However, we also assume $\bar{e} \leq e_{SI}$, since above this threshold, the policyholder's well-being[14] and the size of the insurance market would decrease.

The solution is detailed in Appendix III. First, solving the monopoly's optimization problem, we find that the best insurance contract involves full insurance coverage ($I_{I\overline{SI}} = x - SI(e_{I\overline{SI}})$) while and $e_{I\overline{SI}} = \bar{e} > e_{ISI}^m$. Besides, all admissible pairs of values $(p_{I\overline{SI}}, C_{I\overline{SI}})$ should respect the (PC) constraint:

$$U(W_0 - p_{I\overline{SI}}[x - SI(\bar{e})] - C_{I\overline{SI}} - \bar{e}) = EU(e_{SI}).$$

Second, taking into account the policyholder's reaction to a two-part tariff (p, C), we find that the unit insurance price should be actuarial ($p_{I\overline{SI}}^m = q$) to induce full insurance coverage of the policyholder. At equilibrium, the policyholder overinvests in self-insurance ($e_{I\overline{SI}}^m = \bar{e} > e_{ISI}^m$) but condition (PC) remains defined by the same target value ($EU(e_{SI})$), which guarantees the same well-being to the policyholder. As a result, comparing the final wealth of the two contexts of monopoly equilibrium (ISI vs. $I\overline{SI}$), it is straightforward that $C_{I\overline{SI}}^m < C_{ISI}^m$ and that $C_{I\overline{SI}}^m$ is decreasing in $\bar{e}$ whenever (LC) is binding (See Appendix III).

[14] With $\bar{e} > e_{SI}$, (PC) becomes $(1-q)U(W_{1\,I\overline{SI}}) + qU(W_{2\,I\overline{SI}}) \geq EU(\bar{e})$, with $EU(\bar{e}) < EU(e_{SI})$. Then, at equilibrium, (PC) sets the policyholder's welfare equal to $EU(\bar{e})$.

Therefore, the monopoly equilibrium subject to the legal constraint (LC) is characterized by the following optimal values:

$$\begin{cases} p_{I\overline{SI}}^m = q \\ e_{I\overline{SI}}^m = \bar{e} > e_{ISI}^m \text{ and } SI'\left(e_{I\overline{SI}}^m\right) = SI'(\bar{e}) < \dfrac{1}{q} \\ I_{I\overline{SI}}^m = x - SI(\bar{e}) \\ W_{1\,I\overline{SI}} = W_{2\,I\overline{SI}} = W_0 - qI_{I\overline{SI}}^m - C_{I\overline{SI}}^m - \bar{e} \\ C_{I\overline{SI}}^m \text{ such that } U(W_{1\,I\overline{SI}}) = EU(e_{SI}) \text{ but } C_{I\overline{SI}}^m < C_{ISI}^m \end{cases}$$

Unexpectedly, the restrictive legislation on *SI* only impacts the insurer's welfare. The "reservation" prevention e_{SI} and the utility $EU(e_{SI})$ it provides are not affected by the self-insurance obligation. Therefore, *SI* opportunities remain a threat to the monopoly and represent a baseline for its pricing. The insurer sets a two-part tariff to capture the remaining surplus while ensuring the policyholder's participation in the insurance market. Maximizing its profit leads to the same unit (actuarial) price as in the ISI context (without the legal clause) $p_{I\overline{SI}}^m = p_{ISI}^m = q$. On the other hand, the fixed cost is lower $C_{I\overline{SI}}^m < C_{ISI}^m$. Moreover, $C_{I\overline{SI}}^m$ decreases as $\bar{e}$ increases. The self-insurance obligation challenges the insurance monopoly by cutting into its market share and profit (given by the fixed cost).

Proposition 3: *In a monopoly insurance market with compulsory self-insurance, the policyholder is fully covered but over-self-insured. The marginal return of self-insurance is lower than that of insurance, but this inefficiency only affects the insurer's profit. The optimal two-part tariff still involves an actuarial unit price ($p_{I\overline{SI}}^m = q$), but the fixed cost $C_{I\overline{SI}}^m$ is lower than in the context with no self-insurance obligation ($C_{I\overline{SI}} < C_{ISI}$, assuming $\bar{e} > e_{ISI}^m$).*

Graphical analysis

We assume that the law requires the individual to invest an amount at least equal to $\bar{e}$ in self-insurance, such that $e_{SI} \geq \bar{e} > e_{ISI}^m$ (Figure 3). Following this obligation, the policyholder invests in self-insurance on curve DB until point D", which is above the optimal self-insurance investment achieved at point D'. From point D", full insurance coverage at fair unit price q would lead the policyholder at point $E_{I\,\overline{SI}}^c$, the competitive equilibrium under self-insurance obligation. Since the participation constraint $(V(q, A) = EU(e_{SI}))$ is the same as in the *ISI* context, the monopoly equilibrium lies at the same point E^m. As in Figure 1, the only difference between $(E_{I\,\overline{SI}}^c)$, the competitive equilibrium under self-insurance obligation, and the monopoly equilibrium (E^m) is the fixed cost (or profit) $C_{I\overline{SI}}^m$. However, this cost is lower than C_{ISI}^m, the profit achieved at the no-(LC) monopoly equilibrium (see Figure 1). As the monopoly equilibrium point E^m is the same in both contexts (without and with (LC)), the well-being of the insured remains unchanged. On the monopoly's side, as the extent of the insurance market decreases with $\bar{e} > e_{ISI}^m$, the profit decreases. Finally, the only effect of a self-insurance regulation is, at worst, to reduce the insurer's profit. As soon as the legal requirement is inefficient ($\bar{e} > e_{ISI}^m$), any loss in the risk management surplus (Insurance and self-insurance) only affects the monopoly's profit.

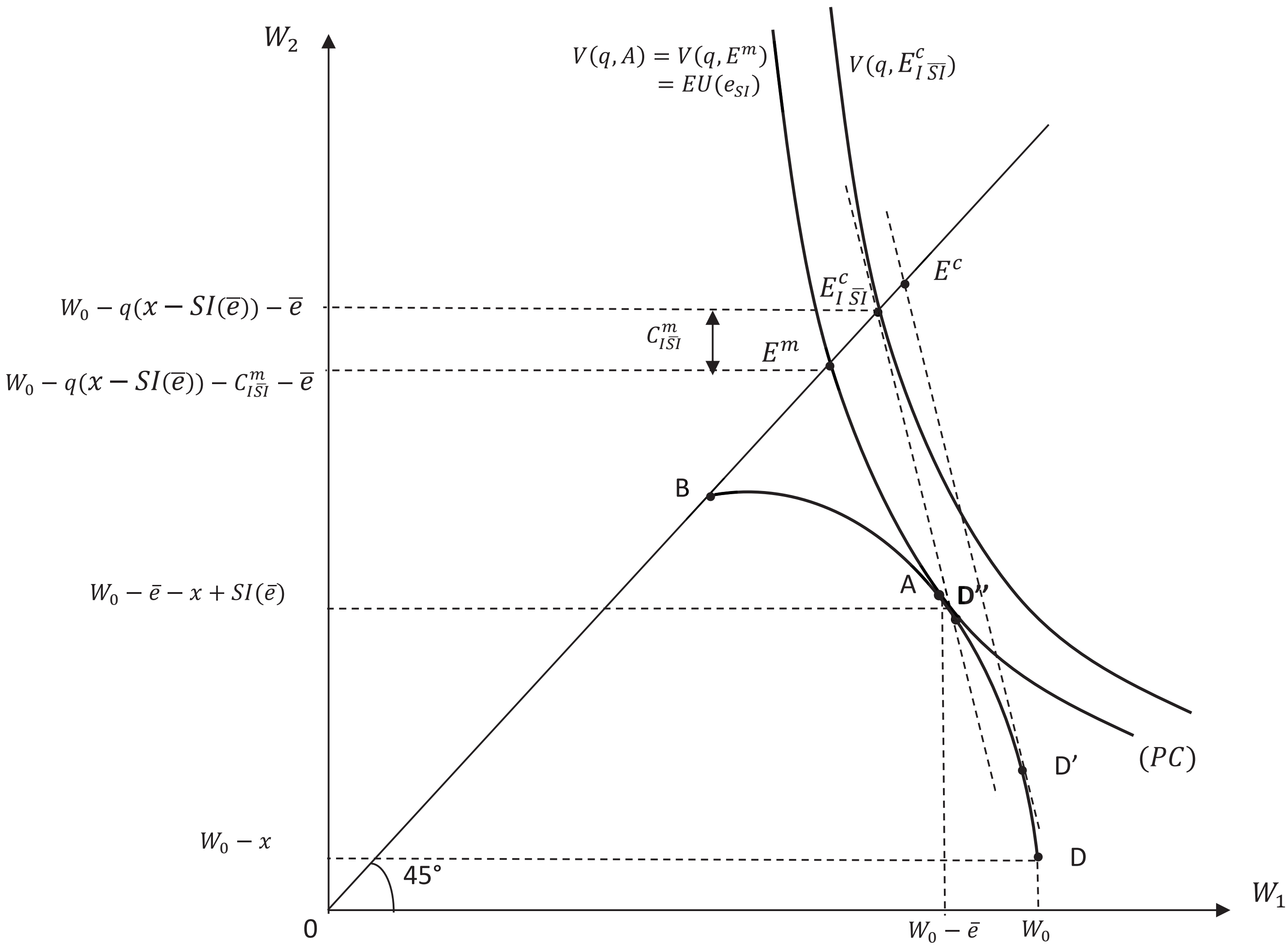

*Figure 2: Monopoly equilibrium with compulsory self-insurance **($\bar{e} > e_{ISI}^m$)***

B. Compulsory insurance $\bar{I}SI$

The model with compulsory insurance is identical to the monopoly model (ISI) presented in section 1.C. The only difference lies in the reference point defining the participation constraint, which is altered by the fact that exit from the insurance market is now illegal. It makes the threat of market exit costly for the individual (risk of fines, jail). In case of market exit, the best self-insurance investment e'_{SI} is now the solution of the following program:

$$e'_{SI} = \underset{e}{\text{ArgMax}}\, EU(e) = (1-q)U(W_0 - e) + qU(W_0 - x - e + SI(e) - C_{illegal})$$

We assume that non-compliance with the insurance obligation (absence of an insurance contract) only becomes apparent in the event of a loss. Individuals then face the probability p of paying a fine $C_{illegal}$ which reduces their wealth if a loss occurs. With $C_{illegal} > 0$ and given the strict concavity of $U(W)$, $EU(e'_{SI}) < EU(e_{SI})$. Moreover, since the individual feels poorer and more risk-averse in the presence of an additional cost $C_{illegal}$, it is straightforward: $e'_{SI} > e_{SI}$ since the demand for self-insurance increases with risk aversion (Dionne & Eeckhoudt, 1985).

The following program characterizes the insurer's optimum: [15]

$$\begin{cases} \underset{p,C,e,I}{Max} \quad (p-q)I + C \\ s.t. \quad (1-q)U(W_{1\,\bar{I}SI}) + qU(W_{2\,\bar{I}SI}) \geq EU(e'_{SI}) \end{cases} \qquad (PC')$$

Where the no-loss state wealth and the loss state wealth are:

$W_{1\,\bar{I}SI} = W_0 - pI - C - e$, and $W_{2\,\bar{I}SI} = W_0 - pI - C - e - x + SI(e) + I$

The participation constraint $EU(e'_{SI})$, is the expected utility of an individual not using insurance and facing a situation of illegality. As the threat of exit from the market is weaker, the insurer's profit increases.

Compared to the ISI context, only the participation constraint (PC) is modified. Results are straightforward and are provided in Appendix II (with (PC') instead of (PC)):

$$\begin{cases} p^m_{\bar{I}SI} = q \\ SI'(e_{\bar{I}SI}) = \dfrac{1}{q} \\ I_{\bar{I}SI} = x - SI(e_{\bar{I}SI}) \\ W_{1\,\bar{I}SI} = W_{2\,\bar{I}SI} = W_0 - q(x - SI(e_{\bar{I}SI})) - C_{\bar{I}SI} - e_{\bar{I}SI} \\ C_{\bar{I}SI} \text{ such that } U(W_0 - q(x - SI(e_{\bar{I}SI})) - C_{\bar{I}SI} - e_{\bar{I}SI}) = EU(e'_{SI}) < EU(e_{SI}) \end{cases}$$

[15] An alternative (or complementary) approach to the compulsory insurance problem could consist in adding a quantitative insurance requirement such that $I_{\bar{I}SI} \geq \bar{I}$. However, as previously shown in the ISI context, the unconstrained monopoly equilibrium results in a full insurance coverage. Therefore, for this constraint to be binding at equilibrium, it would be necessary that $\bar{I} > I_{ISI} = x - SI(e_{ISI})$. Then the constraint $I_{\bar{I}SI} \geq \bar{I}$ would entail an unrealistic situation of over-insurance. So, we discarded the quantitative option and stressed the penal dimension of compulsory insurance.

As a result, $C_{\bar{I}SI} > C_{ISI}$, proving that the insurer's expected profit is greater when insurance is compulsory.

Proposition 4: *In a monopoly insurance market with compulsory insurance, the policyholder is fully covered ($I_{\bar{I}SI} = I_{ISI}$) and equalizes the marginal returns of insurance and self-insurance ($e_{\bar{I}SI} = e_{ISI}$), as in the ISI context. However, while the optimal two-part tariff still involves an actuarial unit price ($p^m_{\bar{I}SI} = q$), the fixed cost is higher $C^m_{\bar{I}SI} > C^m_{ISI}$. The cost of illegality ($C_{illegal} > 0$) weakens the participation constraint and triggers a positive (negative) redistributive effect for the insurer (insured).*

Compulsory insurance, even partial, strengthens the market power of the insurer. This is true as soon as the obligation level is strictly positive. As policyholders face a cost of illegality if they refuse to take out insurance, they lose their full ability to threaten the insurer using self-insurance opportunities. However, the demands for self-insurance and insurance are identical to those of the ISI monopoly.

Graphical analysis

In Figure , we show that the insurance obligation challenges the *ISI* monopoly equilibrium initially achieved at the point E^m. Non-compliance with the law is now sanctioned and detected with probability q (i.e., when the loss state occurs). In this event, the individual faces an additional cost $C_{illegal}$ which decreases the wealth W_2. Therefore, in the no-insurance context, the endowment point moves from D to $D_{\bar{I}}$, and the curve $D_{\bar{I}}B_{\bar{I}}$ refers to the same self-insurance technology as DB, but from point $D_{\bar{I}}$. Then, the new participation constraint (PC') corresponds to the indifference curve $V(q, E^m_{\bar{I}\,SI}) = EU(e'_{SI})$ through point $A_{\bar{I}}$, at the tangency between $D_{\bar{I}}B_{\bar{I}}$ and the highest possible indifference curve.
If the individual participates in the insurance market, self-insurance opportunities start from point D, and the best risk management combination is achieved at the point E_C (full insurance coverage at fair price q and efficient self-insurance investment). However, the new monopoly equilibrium lies on the indifference curve $V(q, E^m_{\bar{I}\,SI}) = EU(e'_{SI})$ at point $E^m_{\bar{I}\,SI}$. The insurance obligation weakens the exit threat of the policyholder, which, on the insurer's side, results in a higher profit: $C^m_{\bar{I}SI} > C^m_{ISI}$.

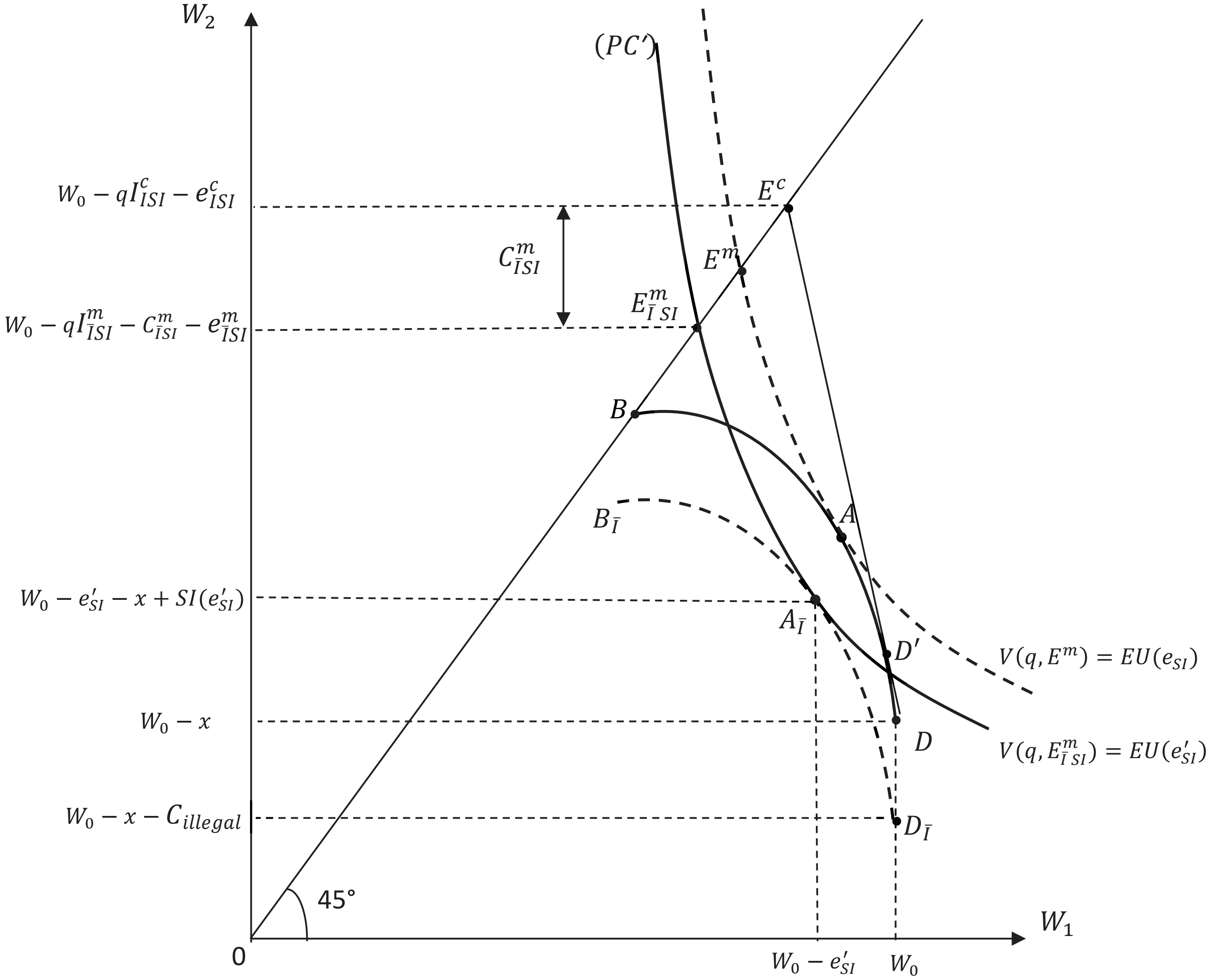

Figure 3: Monopoly equilibrium with mandatory insurance

3 Discussion and Conclusion

This article analyzes the equilibrium characteristics of insurance markets when the government implements insurance and self-insurance obligations. It focuses on the impact of self-insurance on the pricing and profit of the monopoly. Figure 4 below represents the welfare consequences of the different market contexts. The ordinate measures the policyholder's welfare as a function of *e*, without insurance ($EU(e/I = 0) = (1-p)U(W_0 - e) + qU(W_0 - e - x + SI(e))$) and with a full insurance coverage ($U(e/I = x - SI(e)) = U(W_0 - q(x - SI(e)) - e)$).[16] The difference between the two curves, $\Delta_I(e)$, represents the policyholder's welfare gain generated by the insurance market. As shown in Appendix IV,

[16] Both curves are strictly concave; $U(e/I = x - SI(e))$ achieves its maximum at e_{ISI} while $EU(e/I = 0)$ achieves it at e_{SI}.

$\Delta_I(e)$ diminishes with *e,* but the market exit threat guarantees at least the welfare $EU(e_{SI})$ to the insured. Figure 4 summarizes our main conclusions:

- Without any self-insurance opportunity ($e = 0$), the monopoly captures the whole insurance rent R_I ;
- When a self-insurance technology is available, the policyholder benefits from a positive rent R_{ISI}^p in the insurance market, while the insurer captures only the share R_{ISI}^i of the market surplus;
- Insurance rents decrease for both parties if the government enforces a self-insurance obligation (with $e_{ISI} \leq \bar{e} \leq e_{SI}$), but the policyholder's welfare is still the same; for $\bar{e} > e_{SI}$ – assuming to simplify that e_{max} corresponds to a full coverage of the risk – the insurer captures all the market surplus, but the welfare of both parties decreases.

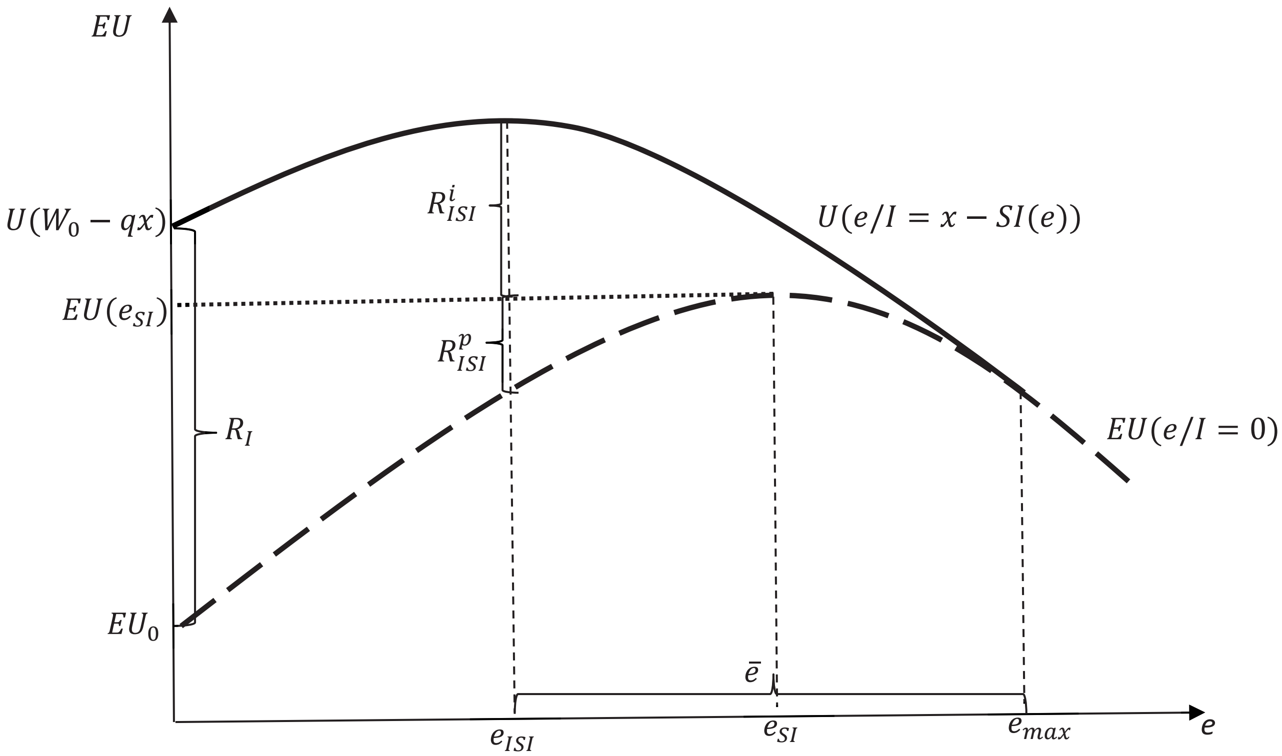

Figure 4: sharing of the insurance market surplus depending on the context

Figure 4 also enables us to figure out the consequences of mandatory insurance involving an additional cost $C_{illegal}$. The basic effect passes through a downward shift of the curve $EU(e/I = 0)$. It increases the insurance market surplus ($\Delta_I(e/C_{illegal}) > \Delta_I(e)$) and the market power of the insurer.

How self-insurance opportunities challenge insurer's market power

Insurance Monopoly vs. Insurance and self-insurance monopoly (ISI)

Our model highlights the bargaining power that self-insurance provides the policyholder. As can be seen in Figure 4, the two extremes are the "I" context of monopoly with insurance alone and the context of competition when both coverage tools are available (ISI^c), or with insurance alone (I^c). They provide the highest (C_I)) and lowest ($C_{ISI^c} = C_{I^c} = 0$) market power to the insurer, respectively. Indeed, as shown in Appendix IV, the maximum profit achievable, $C_I(e)$, decreases in e. We then infer that ISI, $\bar{I}SI$ and $I\bar{SI}$ contexts provide the insurer with intermediate market power where $C_{\bar{I}SI} > C_{ISI} > C_{I\bar{SI}}$.

In the ISI context, although the unit price is the same (and equal to q) in all contexts, the fixed cost (and thus the monopoly profit) C_{ISI} is lower than C_I (see Appendix IV). Compared to context I, the insurer's market share in context ISI is reduced by the share now covered by self-insurance. All other things being equal, the existence of self-insurance opportunities challenges the market power of the insurance monopoly by strengthening the policyholder's threat of insurance market exit. Compared to the situation with insurance only, as it represents an additional opportunity, self-insurance improves both self-insurance and insurance policyholders' surplus. Moreover, self-insurance opportunities raise the likelihood of risk retention and therefore the policyholder's participation constraint. The insurer has no choice but to consider this threat and give up a portion of the surplus to the policyholder by bearing a premium reduction.

The non-equivalence principle of Insurance and self-insurance obligations

In the context of coverage obligations, self-insurance obligations are inefficient but leave some power back to the insured. In contrast, insurance obligations do not jeopardize market efficiency but reinforce the insurer's market power.

Compulsory Self-insurance leaves some power back to the policyholders

Self-insurance obligations intensify the constraint on the monopoly, which experiences a further reduction in profit if the level required of self-insurance exceeds the level freely achieved in the ISI context ($e_{SI} > \bar{e} > e_{ISI}^m$). In this case, the SI obligation leaves the insured's welfare unchanged compared to the ISI context. Still, it reduces the insurer's profit due to the shrinking of the insurance market (reduction of its market share to the benefit of SI). Therefore, although inefficient, a self-insurance obligation does not affect the policyholder’s welfare. Its only effect is to reduce the insurer’s profit.

Compulsory insurance weakens the threat

Compulsory insurance does not challenge market efficiency: policyholders equalize insurance and self-insurance marginal returns. However, it triggers a redistribution from the policyholder to the insurer. Although the insured can still use self-insurance to mitigate the insurer's predatory behavior, the insurance obligation weakens the market exit threat. The insurance obligation introduces a penal sanction in the event of non-participation in the market. It results that the reservation utility of the policyholder, $EU(e'_{SI})$, decreases with C_{illegal}, the cost of illegality. However, the reservation utility of the policyholder sets the power-sharing between the insurer and the insured. As a consequence, the insurer captures a higher surplus than in the situation without insurance obligation: $C_{\bar{I}SI} > C_{ISI}$. Thus, the insurer’s profit grows with C_{illegal} the cost of illegality. It could even exceed the amount of rent existing in a market with insurance opportunities alone ($C_{\bar{I}SI} > C_I$) by capturing part of the surplus resulting from the self-insurance opportunities.

The status of illegality

Unexpectedly, insurance obligations can foster non-participation in the insurance market. Indeed, as they make it possible for the insurer to charge a higher fixed cost, they may discourage people from being insured. By contrast, self-insurance obligations do not have this drawback. The effectiveness of the first units of self-insurance guarantees participation in the *SI* market. Even when the prevention requirement exceeds a threshold (e_{ISI}), the policyholders are not enticed to leave the self-insurance market. Their welfare is not affected and remains constant as the insurer adjusts its rate (downwards) so that the self-insurance obligation provides the insureds with at least their reservation utility (participation constraint).

*Only the **existence** of (insurance) obligation matters*

The mere existence of an insurance obligation (whatever its amount) makes the power shift to the insurer because of the cost of illegality. According to our modeling, a binding quantitative insurance requirement ($\bar{I} > I_{ISI}$) is unrealistic and not considered (as the policyholder would be over-insured). On the other hand, a binding self-insurance obligation ($e_{SI} > \bar{e} > e_{ISI}^m$) only affects the insurer's profit and the equilibrium. If the constraint is not binding ($\bar{e} \leq e_{ISI}^m$), the equilibrium reached in the ISI context is preserved, and neither party is affected by the self-insurance obligation. If the self-insurance requirement is too high ($\bar{e} > e_{SI}$), both parties are penalized (see Figure 4 and Appendix IV).

Public policies

The substitution property between insurance and self-insurance does not lead to an equivalence between insurance and self-insurance obligations. This non-equivalence principle calls for measures that promote self-insurance.
Despite its potential inefficiency, a policy of self-insurance obligation (with $e_{SI} > \bar{e} > e_{ISI}^m$) does not affect the policyholder's well-being. Its only effect is to reduce the insurer's profit. This feature is particularly relevant for a regulator who has to set a legal norm for self-insurance. If policyholders have heterogeneous self-insurance opportunities, the public policy lesson could be the following: setting a relatively high level for $\bar{e}$ will induce a high level of prevention in the population, while its adverse effects will focus on the insurer. However, the self-insurance obligation must not be excessive. It should not achieve the threshold beyond which the effects on well-being are harmful to both parties (i.e. $\bar{e} > e_{SI}$).

On the contrary, to the extent that they reinforce the monopoly market power, insurance obligations should be limited to situations in which they reduce negative externalities. Although they do not introduce allocative inefficiency, insurance obligations entail a counter-redistributive effect. For this reason, a policy that aims to maintain competition in the insurance market to limit insurers' profits seems relevant. The cost of illegality cannot be a lever for public policy to reduce monopoly power. Increasing the cost of illegality to limit negative externalities would allow the monopoly to raise its fixed cost and harm the policyholder. On the other hand, reducing the cost of illegality would increase non-participation in the insurance market.

Eventually, regardless of the context (with or without insurance or self-insurance obligations), promoting access to prevention by reducing its price (through subsidies or improved technology) reduces the monopoly power by increasing the policyholders' reserve utility - and thus the strength of their threat to exit the market. Such policies should be implemented.

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5 Appendices

Appendix I: The competitive equilibrium

In a competitive setting, insurance companies are induced to maximize policyholders' expected utility under a non-negative profit condition. The following optimization problem characterizes the competitive equilibrium:

$$\begin{cases} \underset{I,p,C,e}{Max} & (1-q)U(W_{1\,ISI}) + qU(W_{2\,ISI}) \\ s.t. & E(\pi) = (p-q)I + C \geq 0 \end{cases}$$

Where $W_{1\,ISI} = W_0 - pI - C - e,$ and $W_{2\,ISI} = W_0 - pI - C - e - x + SI(e) + I,$ correspond respectively to the no-loss state wealth and the loss state wealth. If γ is the Lagrange multiplier associated with the profit constraint, the Lagrange function can be written as:

$$L(I, \mathrm{e}, p, C; \gamma) = (1-q)U(W_{1\,ISI}) + \mathrm{q}\ \mathrm{U}(W_{2\,ISI}) + \gamma[(p-q)I + C]$$

The first-order conditions, where U_i' represents the marginal utility in state *i* ($U_i' = U'(W_{i\,ISI})$), while $EU' = (1-q)U_1' + qU_2'$ stands for the marginal expected utility, are calculated below:

$$(i)\ \frac{\partial L}{\partial I} = -(1-q)pU_1' + (1-p)qU_2' + \gamma(p-q) = 0$$

$$(ii)\ \frac{\partial L}{\partial p} = I[-(1-q)U_1' - qU_2' + \gamma] = 0$$

$$(iii)\ \frac{\partial L}{\partial C} = -(1-q)U_1' - qU_2' + \gamma = 0$$

$$(iv)\ \frac{\partial L}{\partial e} = -EU' + SI'(e)qU_2' = 0$$

From conditions (ii) and (iii), and since $I > 0$, we get that $\gamma = EU' > 0$. Substituting this expression in condition (i), we find that $W_1 = W_2$ and p = q. As $\gamma > 0$, the non-negativity profit condition is saturated, and C = 0. Then, simplifying condition (iv), we show that policyholders equalize the marginal returns of self-insurance and insurance: $\mathrm{SI}'(e_{ISI}^c) = \frac{1}{q}$.

Under perfect competition, the equilibrium contract provides full insurance coverage at fair pricing:

$(p_{ISI}^c, C_{ISI}^c, I_{ISI}^c) = \left(q, 0, x - SI(e_{ISI}^c)\right).$

The final wealth is the same in both states of the world:

$$W_{1\,ISI} = W_{2\,ISI} = W_0 - q(x - SI(e_{ISI}^c)) - C_{ISI}^c - e_{ISI}^c.$$

Appendix II The monopoly equilibrium

We first determine the pairs of contractual parameters (p, C) and the policyholder's decisions (I, e), maximizing the monopoly's expected profit. Then, we show that choosing a specific pair of values for (p, C) is sufficient to induce the optimal decisions for (I, e) and the equilibrium's achievement.

1. The following optimization program characterizes the optimum of an insurance monopoly relying on a two-part tariff:

$$\begin{cases} \underset{p,C,I,e}{Max} \quad (p-q)I + C \\ s.t. \quad (1-q)U(W_{1\,ISI}) + qU(W_{2\,ISI}) \geq EU(e_{SI}) \qquad\qquad (PC) \end{cases}$$

The no-loss state wealth and the loss state wealth are respectively equal to:

$W_{1\,ISI} = W_0 - pI - C - e$, and $W_{2\,ISI} = W_0 - pI - C - e - x + SI(e) + I$

The participation constraint (PC) is defined by reference to the maximum expected utility, $EU(e_{SI})$, achievable with the sole self-insurance opportunities when the decision-maker leaves the insurance market.

Thereby, $EU(e_{SI}) = \underset{e}{Max}(1-q)U(W_0 - e) + qU(W_0 - e - s + SI(e))$.

The Lagrange function corresponding to the monopoly optimum, where γ>0 refers to the Lagrange multiplier, can be written as:

$$\begin{aligned} L(p,C,I,e;\gamma) = (p-q)I + C \\ &+ \gamma[(1-q)U(W_0 - pI - C - e) + qU(W_0 - pI - C - e - x + SI(e) + I) \\ &- EU(e_{SI})] \end{aligned}$$

The first-order conditions are calculated below:

$$(i)\frac{\partial L}{\partial p} = I[1 - \gamma EU'] = 0$$

$$(ii)\frac{\partial L}{\partial C} = 1 - \gamma EU' = 0$$

$$(iii)\frac{\partial L}{\partial I} = (p-q) + \gamma[-p(1-q)U'_1 + (1-p)qU'_2] = 0$$

$$(iv)\frac{\partial L}{\partial e} = \gamma[-EU' + SI'(e)qU'_2] = 0$$

Resorting to conditions (i) and (ii), we get that $\gamma = \frac{1}{EU'}$. By substitution in the equation (iii), and owing that the marginal utility is strictly decreasing, we find that the insurer imposes a full insurance contract such that: $EU' = U'_1 = U'_2 \Leftrightarrow W_{1\,ISI} = W_{2\,ISI} \Leftrightarrow I_{ISI} = x - SI(e_{ISI})$. Condition (iv) implies that: $-1 + SI'(e)q = 0$.

Finally, the optimal solution involves full insurance coverage ($I_{ISI} = x - SI(e_{ISI})$), while the policyholder's self-insurance effort should equalize its marginal return to the actuarial unit insurance price: $SI'(e_{ISI}) = \frac{1}{q}$. All pairs of values (p_{ISI}, C_{ISI}), compatible with (PC), are admissible: optimal contractual parameters are such that $U(W_0 - p_{ISI}[x - SI(e_{ISI})] - C_{ISI} - e_{ISI}) = EU(e_{SI})$. All these pairs result in the same insurance premium.

2. To find if a specific pair (p^m_{ISI}, C^m_{ISI}) guarantees the achievement of the insurer's optimum, we analyze the policyholder's behavior in this pricing context.

When facing a two-part tariff (p, C), the policyholder solves the following problem:

$$\begin{cases} \underset{I,e}{Max} \quad (1-q)U(W_1) + qU(W_1) \\ s.t. \quad (1-q)U(W_1) + qU(W_2) \geq EU(e_{SI}) \quad (PC) \end{cases}$$

Where $W_1 = W_0 - pI - C - e$ and $W_2 = W_0 - pI - C - e - x + SI(e) + I$, while $EU(e_{SI})$ represents the highest expected utility achievable when the decision-maker leaves the insurance market.

The Lagrange function of this problem, with $\theta \geq 0$ as a multiplier, is the following:

$$L(I, e; \theta) = (1-q)U(W_1) + qU(W_1) + \theta[(1-q)U(W_1) + qU(W_2) - EU(e_{SI})\,]$$

We deduce the first-order conditions:

$$(\alpha)\frac{\partial L}{\partial I} = (1+\theta)[-p(1-q)U'(W_1) + (1-p)qU'(W_2)] = 0$$

$$(\beta)\frac{\partial L}{\partial e} = (1+\theta)[-EU' + SI'(e)qU'_2] = 0$$

These conditions describe the policyholder's reaction to a tariff (p, C). From equation (α), it is straightforward that the only unit price compatible with full insurance coverage is the actuarial price: $p^m_{ISI} = q$. Then, condition (β) confirms that the policyholder implements the right self-insurance effort: $SI'(e^m_{ISI}) = \frac{1}{q}$. Rewriting condition (PC), we find that the policyholder accepts the insurance contract if C^m_{ISI} is such that $U(W_0 - q[x - SI(e^m_{ISI})] - C^m_{ISI} - e^m_{ISI}) = EU(e_{SI})$. Therefore, the pair $(p^m_{ISI} = q, C^m_{ISI})$ implements the monopoly's optimum.

Finally, with $p^m_{ISI} = q$, the monopoly's profit is equal to C^m_{ISI}.

Appendix III Monopoly equilibrium and compulsory self-insurance $I\bar{S}I$

Assuming a legal constraint (LC), the self-insurance investment $e_{I\bar{S}I}$ has to achieve, at least, the legal threshold $\bar{e}$ such that $\bar{e} < e_{SI}$. Therefore, the monopoly solves the following problem (where $e \equiv e_{I\bar{S}I}$):

$$\begin{cases} \underset{p,C,e,I}{Max} & (p-q)I + C \\ s.t. & (1-q)U(W_{1\,I\bar{S}I}) + qU(W_{2\,I\bar{S}I}) \geq EU(e_{SI}) \qquad (PC) \\ & e_{I\bar{S}I} \geq \bar{e} \qquad (LC) \end{cases}$$

Denoting γ and δ the respective Lagrange multipliers for (PC) and (LC), the Lagrange function is equal to the following expression:

$$L(p,C,I,e;\gamma,\delta) = (p-q)I + C + \gamma[(1-q)U(W_{1\,I\bar{S}I}) + qU(W_{2\,I\bar{S}I}) - EU(e_{SI})] + \delta[e_{I\bar{S}I} - \bar{e}]$$

Where the no-loss state wealth and the loss state wealth are respectively equal to: $W_{1\,I\bar{S}I} = W_0 - pI - C - e$, and $W_{2\,I\bar{S}I} = W_0 - pI - C - e - x + SI(e) + I$

The first-order conditions are calculated below:

$$(i)\frac{\partial L}{\partial p} = I[1 - \gamma EU'] = 0$$

$$(ii)\frac{\partial L}{\partial C} = 1 - \gamma EU' = 0$$

$$(iii)\frac{\partial L}{\partial I} = (p-q) + \gamma[-p(1-q)U_1' + (1-p)qU_2'] = 0$$

$$(iv')\frac{\partial L}{\partial e} = \gamma[-EU' + SI'(e)qU_2'] + \delta = 0$$

As previously shown in Appendix II, conditions (i) to (iii) characterize an optimal solution involving full insurance: $W_{1\,I\bar{S}I} = W_{2\,I\bar{S}I} \Leftrightarrow I_{I\bar{S}I} = x - SI(e_{I\bar{S}I})$. From equation (iv'), it is straightforward:

- $e_{I\bar{S}I} = e_{ISI}$ when $\delta = 0$,
- while $SI'(e_{I\bar{S}I}) < \frac{1}{q}$ and $e_{I\bar{S}I} = \bar{e} > e_{ISI}$ when $\delta > 0$.

The 1st case refers to the no-(LC) solution. The 2nd case is the relevant one: the policyholder overinvests in self-insurance ($e_{I\bar{S}I} > e_{ISI}$) but condition (PC) guarantees the same well-being to the policyholder. Indeed, all admissible pairs of values $(p_{I\bar{S}I}, C_{I\bar{S}I})$ should respect the (PC) constraint:

$$U(W_0 - p_{I\bar{S}I}[x - SI(e_{I\bar{S}I})] - C_{I\bar{S}I} - e_{I\bar{S}I}) = EU(e_{SI}).$$

On the policyholder's side, due to the legal constraint (LC), the optimization problem becomes:

$$\begin{cases} \underset{I,e}{Max} & (1-q)U(W_1) + qU(W_1) \\ s.t. & (1-q)U(W_1) + qU(W_2) \geq EU(e_{SI}) \qquad (PC) \\ & e \geq \bar{e} \qquad (LC) \end{cases}$$

The Lagrange function of this problem, with $\varepsilon \geq 0$ the additional multiplier for (LC), is written as:

$L(I,e;\theta) = (1-q)U(W_1) + qU(W_1) + \theta[(1-q)U(W_1) + qU(W_2) - EU(e_{SI})\,] + \varepsilon[e - \bar{e}]$

We deduce the first-order conditions:

$$(\alpha)\frac{\partial L}{\partial I} = (1+\theta)[-p(1-q)U'(W_1) + (1-p)qU'(W_2)] = 0$$

$$(\beta')\frac{\partial L}{\partial e} = (1+\theta)[-EU' + SI'(e)qU_2'] + \varepsilon = 0$$

Again, from the condition (α), and to enforce full insurance coverage, the only acceptable candidate for the unit price is $p_{I\bar{S}I}^{m} = q$. Then, using the condition (β') and focusing on the relevant case where legislation is binding ($\bar{e} > e_{ISI}$ and $\varepsilon > 0$), we find that: $SI'\left(e_{I\bar{S}I}^{m}\right) < \frac{1}{q}$.

Finally, the highest acceptable fixed cost $C_{I\bar{S}I}^{m}$ is defined by the saturation of (PC):

$$U\left(W_0 - q\left[x - SI\left(e_{I\bar{S}I}^{m}\right)\right] - C_{I\bar{S}I}^{m} - e_{I\bar{S}I}^{m}\right) = EU(e_{SI}).$$

Therefore, the pair $\left(p_{I\bar{S}I}^{m} = q, C_{I\bar{S}I}^{m}\right)$ implements the constrained monopoly's optimum. However, while the policyholder's well-being is not affected by (LC), the insurer's profit decreases: $C_{I\bar{S}I}^{m} < C_{ISI}^{m}$.

Indeed, since the final wealth of the policyholder is the same in both cases (without and with (LC)), we get:

$$qSI(e_{ISI}^{m}) + C_{ISI}^{m} + e_{ISI}^{m} = qSI\left(e_{I\bar{S}I}^{m}\right) + C_{I\bar{S}I}^{m} + e_{I\bar{S}I}^{m}$$

Since $SI(e)$ is increasing and $e_{I\bar{S}I}^{m} = \bar{e} > e_{ISI}^{m}$, it is straightforward that $C_{I\bar{S}I}^{m} < C_{ISI}^{m}$ and $C_{I\bar{S}I}^{m}$ is decreasing in $\bar{e}$ whenever (LC) is binding.

It is straightforward to investigate the case where $e_{I\bar{S}I}^{m} = \bar{e} > e_{SI}$. Indeed, the self-insurance requirement deteriorates the (PC) constraint by substituting $EU(\bar{e})$ for $EU(e_{SI})$. Therefore, when $\bar{e}(> e_{SI})$ increases, policyholders' welfare decreases. On the insurer's side, the profit ($C_{I\bar{S}I}^{m}$) diminishes (see Appendix IV).

Appendix IV: Effect of the policyholder's investment in self-insurance on the insurance monopoly profit

First, on a monopoly insurance market – without *SI* – we implicitly characterize the insurer's rent through the difference between the policyholder's well-being resulting from the full insurance coverage at a fair price (*p=q*) and the no-insurance initial situation:

$$\Delta_I = U(W_0 - qx) - [(1-q)U(W_0) + qU(W_0 - x)]$$

As (PC) binds at the monopoly equilibrium, it is possible to replace the second term of Δ_I. Then, we obtain $\Delta_I = U(W_0 - qx) - U(W_0 - qx - C_I)$, where the fixed cost C_I corresponds to the monopoly's profit.
Introducing an investment in self-insurance (with e such that $SI'(e) > 1$ and $0 \leq e \leq e_{max}$ where $e_{max} > e_{SI}$) [17], the previous difference becomes a function of e:

$$\Delta_I(e) = U(W_0 - q(x - SI(e)) - e) - [(1-q)U(W_0 - e) + qU(W_0 - e - x + SI(e))] \quad \text{(i)}$$

From the policyholder's point of view, this difference characterizes the maximum rent achievable through the insurance market. Again, relying on the constraint (PC), we get:

$$\Delta_I(e) = U(W_0 - q(x - SI(e)) - e) - U\big(W_0 - q\big(x - SI(e)\big) - e - C_I(e)\big) \quad \text{(ii)}$$

where the maximum profit (or fixed cost), $C_I(e)$, is an implicit function of e.

Suppose we ignore the exit market threat of the policyholder. In that case, this equation corresponds to the well-being difference between the full insurance situation (at fair price q) and the no-insurance situation, but this time, in the presence of self-insurance opportunities. This difference equals the policyholder's surplus in a competitive insurance market. It complements the well-being gain resulting from the investment in self-insurance. Deriving the equation (i) with respect to e, we obtain:

$$\begin{aligned}\Delta_I{}'(e) = {} & [qSI'(e) - 1]U'\big(W_0 - q\big(x - SI(e)\big) - e\big) + (1-q)U'(W_0 - e) \\ & - q[SI'(e) - 1]U'(W_0 - e - x + SI(e))\end{aligned}$$

Rearranging this equation, we get the following:

$$\begin{aligned}\Delta_I{}'(e) = {} & q[SI'(e) - 1]\big[U'\big(W_0 - q\big(x - SI(e)\big) - e\big) - U'(W_0 - e - x + SI(e))\big] \\ & + (1-q)\big[U'(W_0 - e) - U'\big(W_0 - q\big(x - SI(e)\big) - e\big)\big] < \mathbf{0}\end{aligned}$$

This expression is negative with $W_0 - e > W_0 - q\big(x - SI(e)\big) - e > W_0 - e - x + SI(e)$, since the marginal utility is decreasing. Therefore, the maximum for the policyholder's surplus on the insurance market reduces with e.

[17] To ease the mathematical exposition, we assume that e_{max} provides a full coverage against the risk and crowds out the insurance market.

We now show that the maximum fixed cost the insurer can charge decreases also when the *SI* investment increases. At the monopoly equilibrium, ignoring the exit market threat of the policyholder, the maximum profit achievable ($C_I(e)$), for any *e*, is characterized as follows:

$$U\big(W_0 - q\big(x - SI(e)\big) - e - C_I(e)\big) = (1-q)U(W_0 - e) + qU(W_0 - e - x + SI(e)) \quad \text{(iii)}$$

Assuming that the inverse utility function $U^{-1}(.)$ is well-defined, we can express $C_I(e)$:

$$C_I(e) = W_0 - q\big(x - SI(e)\big) - e - U^{-1}[(1-q)U(W_0 - e) + qU(W_0 - e - x + SI(e))]$$

Deriving this expression with respect to *e*, we obtain:

$$C_I{}'(e) = qSI'(e) - 1 + \left[\frac{(1-q)U'(W_0 - e) - q(SI'(e) - 1)U'(W_0 - e - x + SI(e))}{U'\big[U^{-1}[(1-q)U(W_0 - e) + qU(W_0 - e - x + SI(e))]\big]}\right]$$

For any investment *e,* the certainty equivalent of the right-hand side of equation (iii), $W^*(e)$, is defined as follows:

$$U(W^*(e)) = (1-q)U(W_0 - e) + qU(W_0 - e - x + SI(e))$$

where $W_0 - e - x + SI(e) < W^*(e) < W_0 - e$.

Replacing the argument of $U^{-1}(.)$ by $U(W^*(e))$ in the expression of $C_I{}'(e)$, we get:

$$C_I{}'(e) = qSI'(e) - 1 + \left[\frac{(1-q)U'(W_0 - e) - q(SI'(e) - 1)U'(W_0 - e - x + SI(e))}{U'(W^*(e))}\right]$$

Then, rearranging this expression, we finally obtain:

$$C_I{}'(e) = q(SI'(e) - 1)\left[1 - \frac{U'(W_0 - e - x + SI(e))}{U'(W^*(e))} + (1-q)\left[\frac{U'(W_0 - e)}{U'(W^*(e))} - 1\right]\right]$$

Since the marginal utility is decreasing and $W_0 - e - x + SI(e) < W^*(e) < W_0 - e$, it is straightforward that $C_I{}'(e) < 0$.

Therefore, even in the absence of the market exit threat, the potential profit of the monopoly reduces with *e*:

- For any *e* such that $0 < e \leq e_{ISI},\ C_I > C_I(e) > C_{ISI}^m = C_I(e_{SI})$;
- For $\bar{e} \geq e_{SI}$, in the context of self-insurance obligation, the insurer's profit decreases in $\bar{e}$ and is equal to $C_{I\bar{S}I}^m = C_I(\bar{e})$.