WORKING PAPERS

N°3

April 2023

Limited Commitment, Social Control and Risk-Sharing Coalitions in Village Economies

Juan Daniel Hernandez, Fernando Jaramillo, Hubert Kempf, Fabien Moizeau, Thomas Vendryes

CEPS
Centre for Economics at Paris-Saclay

école normale supérieure paris—saclay
université evry val d'essonne
université PARIS-SACLAY
université PARIS-SACLAY
Maison des Sciences de l'Homme PARIS-SACLAY

Limited Commitment, Social Control and Risk-Sharing Coalitions in Village Economies *

Juan Daniel Hernández† Fernando Jaramillo‡ Hubert Kempf§
Fabien Moizeau¶ Thomas Vendryes‖

March 29, 2023

Abstract

The need to insure against idiosyncratic income risk leads to the formation of risk-sharing groups in village economies where formal financial markets are absent. We develop a theoretical model to address the impact of limited commitment and social control on the extent of informal risk-sharing when agents are induced to form such risk-sharing coalitions. Social control increases the prospect of the future punishment of present defectors and thus mitigates the absence of commitment. A defection-proof core-partition exists, is unique, and is homophilic. Riskier societies may not be more segmented and may not pay a higher cost for insurance. A higher social control leads to a less segmented society but does not necessarily lead to a lower price for sharing risk. We provide evidence, based on data on Thai villages, that consumption smoothing conforms with our theoretical result of homophily-based coalitions and that social control contributes to a lesser segmentation of a society.

Keywords: Risk Sharing, Informal Insurance, Group Formation, Social Control, Risk Heterogeneity, Homophily, Dyadic Models, Thailand.

JEL Classification: C71, D81, O12, O17

*Acknowledgment: Fernando Jaramillo thanks Alianza EFI-Colombia Cientifica grant with code 60185 and FP44842-220-2018. We would like to thank participants of the 2022 International Conference on Development Economics (AFEDEV # 1, CERDI, Clermont-Ferrand, France) and the DESIR seminar (CES, Université Paris 1 Panthéon - Sorbonne, Paris, France) for their useful comments and remarks.

†Université Paris Saclay, Ecole Normale Supérieure Paris-Saclay, CEPS. email: juan.hernandez_colmenares@ens-paris-saclay.fr

‡Universidad del Rosario, Bogota, Colombia, email: fernando.jaramillo@urosario.edu.co.

§Université Paris Saclay, Ecole Normale Supérieure Paris-Saclay, CEPS. email: hubert.kempf@ens-paris-saclay.fr

¶Université de Rennes, CNRS, CREM-UMR6211, F-35000 Rennes, France, email: fabien.moizeau@univ-rennes.fr.

‖Université Paris Saclay, Ecole Normale Supérieure Paris-Saclay, CEPS. email: thomas.vendryes@ens-paris-saclay.fr

1 Introduction

Risk-sharing in "village economies", that is, in economies without financial markets and more generally in economies with insufficient or imperfect access to financial markets, is a central tenet of collective well-being. A way to overcome the absence of financial markets is to share risk within a group by forming informal risk-sharing groups. In this paper, we study the composition of risk-sharing groups under the assumption of limited commitment, and under the presence of social control constraining individual behaviors. There is a limited commitment when individuals are able to defect; that is when they are not obliged to strictly fulfill their obligations toward the coalition they belong to.

The trade-off between the expected benefits and costs of accepting a new member is modified by the inability to enforce a redistribution scheme aiming at sharing risk among members. However, social control understood as an informal capacity to punish defectors, can be seen as a way to limit the phenomenon of defection: A more effective social control should allow to alleviate the negative consequences of limited commitment through defection and lead to more efficient protection against idiosyncratic shocks. The present paper aims to explore this intuition.

[Daniel's suggestion]

We first proceed to the theoretical analysis of the problem. We consider a village formed of risk-averse individuals. These individuals differ with respect to their exposure to risk: variances of the idiosyncratic shock affecting individuals' income differ. Individuals can be ranked according to this measure of risk exposure. In the absence of financial markets, the only way they can be insured against these shocks is through the voluntary formation of risk-sharing coalitions based on a risk-sharing rule applied to the realized incomes of its members. Individuals solely depend on social solidarity to be protected against risk. For a given risk-sharing rule, heterogeneity among individuals plays a crucial role in forming risk-sharing groups.

Once income shocks are realized, individuals can walk out from the coalition they belong to and retain their individual income without any sharing. However, a defector can be "punished" through a trigger strategy in the form of informal and imperfect social control. Once he has defected, he may be forced into social autarky (inability to belong to any risk-sharing coalition). As we aim to study the influence of social control on the pattern of risk-sharing coalitions, we assume that the enforcement of sanctions is hampered by a social inability to perfectly monitor defection. We capture the degree of social control by the probability of applying the trigger strategy to a defector.

We prove that there exists a partition of defection-proof coalitions belonging to the core. That is, a partition formed according to full and complete unanimity about membership, named the DP-core partition. The coalitions in the DP-core partition are consecutive, that is, formed according to a homophily principle: individuals with similar exposure to risk tend to join the same coalition, and the larger the difference in risk exposure between two individuals, the less likely they are to belong to the same coalition. The defection-proof condition which may limit the extent of a coalition depends on the intensity of social control. We prove that a riskier society (based on the criterion of second-order risk dominance) is not necessarily more segmented: its partition includes a smaller number of risk-sharing coalitions. We measure the global cost of providing protection by means of the aggregate risk-premium which is the average of the differing risk premia levied on its members in each coalition. A riskier society may not pay a higher risk-premium as some coalitions of a given rank may be larger in the riskier society and better share risk. We then analyze the impact of our index of social control. A higher social control leads to a less segmented society in the sense that the number of risk-sharing coalitions is lower: the threat of punishment reduces the incentives to defect. Finally, we show that there is a non-monotonous relationship between social control and the aggregate risk premium paid within a given society to insure against risk.

On the empirical side, we use the well-known Townsend Thai data set to illustrate our theoretical results. We rely more specifically on the "Monthly Rural Surveys", which followed approximately 45 households in 16 Thai villages, from 1997 to the present. We extract information on the level of idiosyncratic income risk for each household, on the correlation of consumptions for each dyad of households in each village, and, finally on the level of social control at the village level. We proceed to two types of exercises based on these data.

First, we proceed to econometric exercises with the aim of supporting various propositions obtained in the theoretical section. Our econometric estimations are in line with the result of the consecutivity property of the partition, and show evidence of homophily in the following sense: applying a dyadic approach, the correlation between individual consumption is negatively related to the difference in idiosyncratic income risk between two individuals.

Moreover, consistent again with our theory, a social control proxy affects positively the correlation between consumptions for individuals at the same distance in terms of idiosyncratic income risk: for pairs of agents belonging to different villages, but at the same distance in terms of idiosyncratic risk, this consumption correlation tends to be higher in villages with higher social control.

Second, we proceed to simulations of the theoretical setups to illustrate our theoretical propositions and show that some counterintuitive results are easily obtained. From the estimated village distributions of individuals' exposure to risk, we generate 16 hypothetical villages with 3000 inhabitants in each village and compute the DP-core partitions arising in each village. Compared to full commitment, our simulations highlight that limited commitment can drastically increase the number of risk-sharing coalitions and limit the extent of risk-sharing. A riskier society can share risk more efficiently as some risk-sharing coalitions become larger and less heterogeneous. Furthermore, the benefit of a larger social control in checking defection may be overcome by its consequences in the coalition formation process. Overall, a larger social control may lead to a larger aggregate risk premium as shown in our simulations.

Related literature From a theoretical point of view, the paper is related to the strand of literature on risk-sharing and group formation under limited commitment. Genicot and Ray (2003) study the impact of limited commitment on the stability of risk-sharing coalitions. They consider coalition-proof coalitions, that is to say, coalitions being robust to deviations by subgroups, themselves passing the same robustness test, and show that the requirement of coalition-proofness limits the size of the entire group. Following Genicot and Ray (2003), Fitzsimons *et al.* (2018) use simulations to show that the relationship between group size and risk sharing is theoretically ambiguous. Bold (2009) solves for the optimal dynamic risk-sharing contract in the set of coalition-proof equilibria. Building on these previous works, Bold and Broer (2021) show that the quantitative version of a model with endogenous groups using the ICRISAT dataset on Indian villages replicates the observed degree of risk sharing and predicts small stable risk-sharing coalitions. Cole *et al.* (2020) develop a model of coalition-proof risk-sharing coalitions when beliefs on future cooperation may not coordinate. They show that for medium to low values of the parameter capturing the ability to coordinate beliefs, coalition-proof risk arrangements exist. For high values of the parameter, stable risk sharing does not exist.

We depart from these works in three respects. First, we study the formation of a partition of the society into risk-sharing coalitions as in Jaramillo *et al.* (2015). Second, we focus on the heterogeneity of idiosyncratic risks as the force limiting the size of risk-sharing coalition. An alternative option would be to assume heterogeneity in risk preferences (Dubois, 2006; Chiappori *et al.*, 2014; Laczó, 2015). Third, we retain the assumption of individual autarky as punishment in the case of defection but assume that this punishment is not systematic but depends on the degree of social control.

There is evidence that in village economies, informal risk-sharing arrangements are based on social norms of solidarity and redistribution (Platteau and Abraham, 2002). Deviating from social norms of sharing can trigger powerful sanctions such as banishment (Platteau, 2006). Bold & Dercon (2014) document the role of fines based on Ethiopian data, along with a variety of punishment mechanisms. More generally, the role of social control in societies is well established (Chriss, 2022).

On the empirical side, we face the same two general issues that characterize the risk-sharing literature. The first one is the issue of identifying risk-sharing networks. Only two dedicated surveys propose an exhaustive mapping of society into its risk-sharing relationships, based on direct information, from individuals or households themselves, on who shares risk with whom: one carried out in the Philippines in 1994-1995 (see Fafchamps and Lund, 2003; Fafchamps and Gubert, 2007), the other in Tanzania in 2000 (and used for example by De Weerdt and Dercon, 2006; De Weerdt and Fafchamps, 2011; Comola and Fafchamps, 2014; Henderson and Alam, 2021). Other studies relied on partial mapping, from a sample of households in a community (as for example Hoang *et al.*, 2021, on Vietnam). But, in the absence of direct and exhaustive information on risk-sharing networks, most of the existing literature, since the seminal work of Townsend (1994), investigates the structure, modalities, and extent of risk-sharing through its potential manifestation in the co-movements of individual and aggregate variables such as consumption and income.[1] We will follow this methodology here and use the correlation of consumption between two households in a given village as indicating whether they share risk or not. The second empirical issue is that it is difficult to find arguably exogenous sources of variation in the factors that might affect the structure and extent of risk-sharing networks.[2] In our case, the empirical relationships that we will find between social control and dyadic consumption correlations are consistent with our theoretical predictions, but cannot be taken as definitive proof of the causality mechanisms actually at play. We also follow some previous articles (e.g. Coate and Ravallion, 1993; Foster and Rosenzweig, 2001; Bold, 2009) in using simulations to bypass to some extent the imperfection of the data and the difficulties in terms of identification.

In brief, the characterization of the links between (i) the risk characteristics of a society, (ii) the degree of social control and (iii) the extent of risk sharing resulting from the segmentation into risk-sharing coalitions without commitment has not been addressed up to now.[3]

The plan of the paper is as follows. In section 2, we set up our theoretical model of a village economy and the risk-sharing coalition formation game with limited commitment. In section 3, we analyze and characterize the defection-proof core partition. Section 4 is devoted to comparative statics exercises. Section 5 exposes the empirical consequences of

[1]The literature is far too large to be exhaustively listed and described here, but recent and/or important works include Kinnan (2022); Bold and Broer (2021); Bourlès *et al.* (2021); Árpád Ábrahám and Laczó (2018); Laczó (2015); Chiappori *et al.* (2014); Dubois *et al.* (2008); Ligon *et al.* (2002); Cochrane (1991).

[2]To the best of our knowledge, for example, only two experiments have been carried out on this topic: Attanasio *et al.* (2012) in Colombia, and Meghir *et al.* (2019) who carried out a RCT in Bangladesh.

[3]Other dimensions of risk-sharing not directly related to our paper are the role of patience (Kocherlakota, 1996), altruism (Foster and Rosenzweig, 2001), migration (De Weerdt and Hirvonen, 2016; Morten, 2019; Meghir *et al.*, 2019), self-insurance through storage (Ligon *et al.*, 2000; Árpád Ábrahám and Laczó, 2018), asymmetric information (Ligon, 1998; Karaivanov and Townsend, 2014; Kinnan, 2022). Models of network formation also tackle the risk sharing issue (Bramoullé and Kranton, 2007; Bloch *et al.*, 2008). As they consider ex ante identical individuals, they do not examine how heterogeneity shapes the architecture of networks and limits the extent of risk sharing.

limited commitment based on the Thai dataset. First, econometric estimations aimed at supporting our theory are presented; second, simulations based on these data are performed and show for given distributions of risk exposure how a society is segmented into several risk-sharing coalitions under commitment as well as in the absence of commitment and how this segmentation is impacted by social control. Section 6 concludes.

2 The Framework

2.1 Agents

A society $\mathbf{I}$ is formed by N agents, indexed by $i = 1, 2, ...N-1, N$. Agents are infinitely lived and risk averse. At each period t, each agent i is endowed with income $y_{i,t}$. We suppose that agents are heterogenous with respect to their exposure to risk. Each agent faces an idiosyncratic income risk denoted by $\varepsilon_{i,t}$ which is i.i.d. across individuals and time and normally distributed, i.e. $\varepsilon_{i,t} \sim \mathcal{N}(0, \sigma_{\text{i}}^2)$. Society $\mathbf{I}$ is characterized by a risk schedule $\overrightarrow{\sigma}^2 = \{\sigma_1^2, \sigma_2^2, ..., \sigma_N^2\}$. Agents are indexed so that: $i' < i \Leftrightarrow \sigma_{i'}^2 < \sigma_i^2$. The lower the index, the less risky the agent. The risk schedule captures the risk exposure heterogeneity within society $\mathbf{I}$. There is also a global shock common to the whole society denoted by ν_t which is i.i.d. across time and normally distributed: $\nu_t \sim \mathcal{N}(0, \sigma_\nu^2)$.

The income of agent i is defined as :

$$y_{i,t} = \omega_i + \nu_t + \varepsilon_{i,t} \tag{1}$$

with ω_i the deterministic part of income which is assumed constant over time. We denote by $\epsilon_t \equiv (\varepsilon_{1,t}, ..., \varepsilon_{N,t}, \nu_t)$ the state of nature at date t. There is perfect information, that is, the distribution of $y_{i,t}$ whatever i is common knowledge and the realizations of shocks are perfectly observed by all agents when they occur. We assume that financial markets are missing and thus agents cannot be insured against idiosyncratic shocks through financial contracts. Consumption goods are not storable.

In any period t, a non-empty subset $S_{j,t}$ of $\mathbf{I}$ is called a coalition. In each period t, agents have the possibility to form coalitions valid for this period in order to share risk according to a given transfer scheme. A set of coalitions $\mathcal{P}_t = \{S_{1,t}, ..., S_{j,t}, ..., S_{J,t}\}$ for $j = 1, ..., J$ is called a partition of $\mathbf{I}$ if (i) $\bigcup\limits_{j=1}^{J} S_{j,t} = \mathbf{I}$ and (ii) $S_{j,t} \bigcap S_{j',t} = \emptyset$ for $j \neq j'$. In any period t, any agent i belongs to one and only one risk-sharing coalition denoted by $S_{j,t}(i)$. A sequence of partitions from t onwards is denoted by $\mathcal{S}_t = \{\mathcal{P}_t, \mathcal{P}_{t+1}, ...\}$.

An agent i who belongs to coalitions $S_{j,t+\tau}(i), \tau = 0, 1, ...$, in a sequence $\mathcal{S}_t$ has an expected lifetime utility:

$$U_i(\mathcal{S}_t) = \mathbb{E}_t\left[\sum_{\tau=0}^{\infty} \delta^{\tau} u_i(S_{j,t+\tau}(i))\right] \tag{2}$$

where $\mathbb{E}_t$ is the rational expectations operator applied at t, $\delta \in (0, 1)$ is the common discount factor, $u_i(S_{j,t+\tau}(i))$ is the instantaneous utility of consumption enjoyed in coalition $S_{j,t+\tau}(i)$.

A risk-sharing coalition is a coalition of agents applying a given risk sharing rule. We consider the following non-contingent informal insurance rule: within a risk-sharing coalition, there is an equal sharing between its members of the sum of the stochastic components of individual incomes.This rule is the sole informal insurance arrangement at disposal of the agents at any period of time, irrespective of the state of nature. In this respect, we depart from the literature on the dynamic risk-sharing arrangements (see for example Ligon *et al.*, 2002; Bold, 2009; Cole *et al.*, 2020). The risk-sharing rule assumed here is referred as the *mutual insurance* rule. This rule leads to optimal risk sharing within each coalition $S_{j,t}(i)$ (see equation (24) in Appendix A.3). It also allows us to highlight the role played by both size and risk heterogeneity in the formation of risk-sharing coalitions. Denoting by $n_{j,t}$ the coalition size of $S_{j,t}$, the utility function is assumed to be CARA and writes:

$$u_i(S_{j,t}(i)) = -\frac{1}{\alpha} e^{-\alpha\left[\omega_i + \nu_t + \frac{\sum\limits_{k \in S_{j,t}(i)} \varepsilon_{k,t}}{n_t}\right]} \tag{3}$$

where α denotes the absolute risk aversion parameter.

As we assume a CARA utility function and that shocks are distributed normally, the Arrow-

Pratt formula leads to the instantaneous utility:

$$\mathbb{E}_t\left[u_i\left(S_{j,t}\left(i\right)\right)\right] = -\frac{1}{\alpha}e^{-\alpha\left[\omega_i-\frac{\alpha}{2}\sigma_\nu^2-\alpha\frac{\sum\limits_{k\in S_{j,t}(i)}\sigma_k^2}{2n_t^2}\right]} \tag{4}$$

Hereafter, we assume that the normal distribution of shocks is truncated over an interval $(\underline{\varepsilon},\overline{\varepsilon})$ but this interval is sufficiently large so that this approximation holds.

Given (4), the risk premium associated with coalition $S_{j,t}\left(i\right)$, denoted by $\pi\left(S_{j,t}\left(i\right)\right)$, is defined as follows:

$$\pi\left(S_{j,t}\left(i\right)\right) \equiv \frac{\sum\limits_{k\in S_{j,t}(i)}\sigma_k^2}{2n_t^2}. \tag{5}$$

[4]

2.2 Defection and punishment

We assume limited commitment in the following way: In period t, once coalitions have been formed and the realization of the stochastic components observed, agents may defect from their risk-sharing coalitions so as not to share their ex post realized income. We only consider individual defection and ignore defection by a subset of agents belonging to the initial risk-sharing coalition.[5]

Once a defection has been observed, society may retaliate through punishment: In period t, with probability p, $p\in[0,1]$, a defecting individual i belonging to a coalition $S_{j,t}\left(i\right)$ is punished and left in autarky forever from $t+1$ onwards. With probability $1-p$, he is free to form a coalition with agents of society $\mathbf{I}$ in period $t+1$. Various frictions prevent the implementation of punishment: lack of proper tools for punishment, inability to properly identify shirkers, imperfect observability of individual deviation, political factors, etc. Thus

[4] We assume that individuals do not differ with respect to their risk aversion. We acknowledge that there is empirical support of heterogeneity of risk preferences (Chiappori *et al.*, 2014; Laczó, 2015). Risk aversion heterogeneity can be considered instead of heterogeneity in risk in the model with commitment (Jaramillo *et al.*, 2015). Applying our methodology would yield similar results at the expense of additional and unnecessary complexities.

[5] In this respect we depart from models studying risk-sharing arrangements immune to coalitional deviations (Bold, 2009; Bold and Broer, 2021).

the ability to punish a defecting agent is imperfect. We refer to this ability as "social control" and we interpret p as the measure of social control. The higher is p, the higher is the social control. When $p = 1$, there is perfect social control, when $p = 0$, there is no social control at all. Although one could object that the punishment of permanent reversion to autarky is a costly strategy that is not coalition-proof (Genicot and Ray, 2003; Bold, 2009), there is also evidence that individuals strongly punish those who have violated the adopted social norm, even when punishment appears to be costly (see for example Bowles and Gintis (2000) for a review on evidence of social reciprocity motives). We take p as a structural exogenous parameter. As it is, this assumption is sufficient to explore the role of social control on the formation of risk-sharing coalitions.

Once shocks are realized, any agent makes her choice about defecting or not based on the comparison between the benefit of defecting now, given the expected future cost of being left in autarky and the benefit of not defecting. The latter benefit for an agent i belonging to $S_{j,t}(i)$ is given by :

$$\mathcal{U}_i\left(S_{j,t}(i), \mathcal{S}_{t+1}\right) = u_i\left(S_{j,t}(i)\right) + \delta U_i\left(\mathcal{S}_{t+1}\right) \tag{6}$$

where $U_i(\mathcal{S}_{t+1})$, respectively $u_i(S_{j,t}(i))$, is given by (2), respectively (3). The value of defecting for agent i from the risk-sharing arrangement in her coalition $S_{j,t}(i)$ is given by:

$$\mathcal{U}_i^d\left(S_{j,t}(i), \mathcal{S}'_{t+1}\right) = u_i\left(\{i\}\right) + \delta\left[pU_i\left(\{i\}\right) + (1-p)\,U_i\left(\mathcal{S}'_{t+1}\right)\right] \tag{7}$$

where $u_i(\{i\})$ is the instantaneous utility obtained consuming $y_{i,t}$ when she defects in period t, formally $u_i(\{i\}) = -(1/\alpha)\,e^{-\alpha[w_i+\nu_t+\varepsilon_{i,t}]}$. With probability p, agent i is ostracized forever, while with probability $(1-p)$ she expects to belong to risk-sharing coalitions in the subsequent periods of her life. [6] This leads us to offer the following:

Definition 1. In any period t, a risk-sharing coalition $S_{j,t}$ is **defection-proof** when

$$\mathcal{U}_i\left(S_{j,t}(i), \mathcal{S}_{t+1}\right) \geq \mathcal{U}_i^d\left(S_{j,t}(i), \mathcal{S}'_{t+1}\right), \forall i \in S_{j,t}(i),$$

[6] It is relevant to note that our social control parameter is the opposite of the social capital parameter used in Cole *et al.* (2020).

or equivalently, using (6) and (7),

$$\delta U_i\left(\mathcal{S}_{t+1}\right)-\delta\left[pU_i\left(\{i\}\right)+\left(1-p\right)U_i\left(\mathcal{S}'_{t+1}\right)\right]\geq u_i\left(\{i\}\right)-u_i\left(S_{j,t}\left(i\right)\right),\forall i\in S_{j,t}\left(i\right). \tag{8}$$

According to this definition a defection-proof risk-sharing coalition is immune to deviation by any individual participating to the risk sharing-arrangement in $S_{j,t}\left(i\right)$. For a coalition to be defection-proof (a DP-coalition), according to (8), the instantaneous net gain obtained from defecting must be lower than the discounted expected gain from continued participation. Individuals must be patient enough (with a sufficiently high discount factor) for coalitions to be DP. Hereafter, we define a DP-partition as a partition formed of DP-coalitions.

3 The DP-core partition

In any period, individuals investigate the prospect of forming risk-sharing coalitions as insurance mechanisms against external shocks based on the mutual insurance rule. Coalitions are formed before the realization of idiosyncratic shocks. These coalitions are valid for one period only. Once the shock is known, some agents may defect from their coalition (when it is not a singleton) and do not abide to the risk-sharing rule. Typically, if the realization of the shock is high enough, it may be rational (from an individualistic and selfish point of view) to defect and reap all the immediate gains of this shock despite the possibility of being punished in the future. Intra-coalition transfers among non-defectors are then completed and finally any coalition disbands. A risk-sharing coalition is not a long-term arrangement as it is valid for one period only. Formally, the sequence of events is as follows:

1. In period t:
   (a) Agents form risk-sharing coalitions, applying the punishment strategy towards defectors in period $t-1$ with probability p, and a partition of society is obtained.
   (b) Idiosyncratic shocks are realized.
   (c) Agents choose between sharing risk according to the mutual insurance rule in the coalition or defecting from the insurance arrangement and not sharing their ex

post realized income.

2. The game is repeated in period $t+1$,...

In line with this sequence of events we use the following

Definition 2. A **DP-core partition** in period t, denoted by $\mathcal{P}_t^*$, satisfies the two following conditions:

(i) **the no-blocking condition:** for any agent i belonging to a coalition $S_{j,t}^* \in \mathcal{P}_t^*$ in t for a given sequence $\mathcal{S}_{t+1}$,

$$\nexists \mathcal{L}_t \text{ which is DP and such that } \forall i \in \mathcal{L}_t, \mathbb{E}_t \left[u_i \left(\mathcal{L}_t \right) \right] > \mathbb{E}_t \left[u_i \left(S_{j,t}^* \right) \right].$$

(ii) **the defection-proof condition**: any coalition $S_{j,t}^* \in \mathcal{P}_t^*$ is DP.

According to this definition, a DP-core partition is such that there exists no blocking coalition that would make all its members better off. Moreover, any blocking $\mathcal{L}_t$ must pass the defection-proof test. The no-blocking condition relates to the formation stage of coalitions and thus the partition of the society before the shocks are realized. The defection-proof condition relates instead to the stability of these coalitions under no commitment: once the realization of shocks are known, no one decides to defect from her chosen coalition. In other words, a DP-core partition is a set of optimal coalitions subject to the DP constraint.

Solving the coalition-formation game and looking for a sequence of DP-core partitions $\mathcal{S}_t^* = \{\mathcal{P}_t^*, ..., \}$ we offer the following

Proposition 1. *There exists a constant DP-core partition $\mathcal{P}^* = \mathcal{P}_t^*$, that is, its elements are identical over time: $S_{j,t}^* = S_j^*, \forall t, j$.*

Proof. See Appendix A.1. □

Proposition 1 shows the existence of coalitions that satisfy both the no-blocking and defection-proof conditions. Given that the insurance arrangement is the mutual insurance and agents have the same risk preferences, all agents rank the different coalitions similarly. Hence, the coalition-formation satisfies the common ranking property implying the non-emptiness of the core partition (Farrell and Scotchmer, 1988; Banerjee *et al.*, 2001). Due to the absence of any state variable and because the mutual insurance rule used in (3) is non-state-contingent, the coalition-formation game remains static and is repeated at each period of time. As no one defects in t and thus is ostracized, the coalition-formation problem in $t+1$ remains the same as in t and leads to the same outcome $\mathcal{P}_{t+1}^* = \mathcal{P}_t^*$. Hence, the DP-core partition in period t is the same as the DP-core partition in $t+1$ and the sequence of DP-core partitions is a set of identical partitions.

Given Proposition 1, plugging (4) into (2), the utility level obtained by agent i in S_j^* can be expressed as follows

$$U_i\left(S_j^*\right) = -\frac{\delta}{1-\delta}\frac{1}{\alpha}e^{-\alpha\left[\omega_i - \frac{\alpha}{2}\sigma_\nu^2 - \alpha\frac{\sum\limits_{k\in S_j^*}\sigma_k^2}{2n_j^{*2}}\right]}. \tag{9}$$

The risk premium $\pi\left(S_j^*\right) \equiv \sum\limits_{k\in S_j^*}\sigma_k^2/2n_j^{*2}$ can be assimilated to the value of coalition S_j^* membership for any of its members. All agents in a given coalition bear an identical risk premium due to the assumption of identical risk-aversion. The risk premium increases with members' idiosyncratic risks and decreases with the size of the coalition. Precisely, while the arrival of a new member in the coalition has the same impact on the size of the coalition whatever her risk exposure, her impact on the increase of the sum of the coalition's total risk depends on her type. Hence, the formation of a risk-sharing coalition relies on a trade-off between size and risk exposure heterogeneity.

For each coalition S_j, the agent with the lowest variance among members of S_j is called the "head" of coalition S_j, her index being denoted by h_j. The agent with the highest variance is called the "pivotal agent" and her index denoted by q_j. Proposition 1 implies that for any coalition S_j a necessary and sufficient condition to be DP is that its head h_j does not defect in any state of nature. Hence, using equation (8), any S_j of size n_j is DP if and only if

$$\frac{\delta}{1-\delta}pe^{\frac{\alpha^2}{2}\sigma_\nu^2}\left(e^{\frac{\alpha^2}{2}\sigma_{h_j}^2} - e^{\frac{\alpha^2}{2n_j^2}\sum\limits_{k\in S_j}\sigma_k^2}\right) > e^{-\alpha\underline{\nu}}\left(e^{-\alpha\frac{\left(n_j-1\right)\underline{\varepsilon}+\hat{\varepsilon}(n_j)}{n_j}} - e^{-\alpha\hat{\varepsilon}}\right) \tag{10}$$

with $\hat{\varepsilon}(n_j) = \min\left[n_j \log n_j / (\alpha(n_j - 1)) + \underline{\varepsilon}, \overline{\varepsilon}\right]$.[7] This condition considers the state of nature which renders coalition S_j the most vulnerable to deviation: the head h_j experiences a high positive shock $\hat{\varepsilon}(n_j)$ while the other $n_j - 1$ members incur the most adverse cost $\underline{\varepsilon}$. If (10) holds, then coalition S_j is DP. In Appendix A.1, we show that for any coalition S_j there always exists a size n_j such that S_j satisfies (10). It is trivial to check that singletons are DP. We write (10) in a more compact way

$$C_1 X\left(n_j; \sigma^2_{h_j}\right) \geq C_2 Y(n_j). \tag{11}$$

with $C_1 \equiv (\delta/(1-\delta))\, p e^{\frac{\alpha}{2}\sigma^2_\nu}$, $X\left(n; \sigma^2_{h_j}\right) \equiv e^{(\alpha^2/2)\sigma^2_h} - e^{\left(\alpha^2/2n^2\right)\sum\limits_{z\in S}\sigma^2_z}$, $C_2 \equiv e^{-\alpha\underline{\nu}}$, and $Y(n) \equiv e^{-\alpha\hat{\varepsilon}(n)}\left(e^{-\alpha\frac{(n-1)(\underline{\varepsilon}-\hat{\varepsilon}(n))}{n}} - 1\right)$.

Focusing on the properties of the DP-core partition, we prove the following

Proposition 2. *At any date t, the DP-core partition $\mathcal{P}^* = \left\{S^*_1, ..., S^*_j, ..., S^*_J\right\}$ is characterized by the following properties:*

(i) *All coalitions S^*_j in the DP-core partition $\mathcal{P}^*$ are consecutive.*

(ii) *$\mathcal{P}^*$ is generically unique.*

Proof. See Appendix A.2. □

According to this proposition, any coalition S^*_j in the DP-core partition is composed by individuals who are close to each other on the risk ladder: an homophily principle drives the coalition-formation stage. Hence a consecutive coalition is comprised of all the individuals with a risk exposure between $\sigma^2_{h_j}$ and $\sigma^2_{q_j}$ that is $S^*_j = \left\{h_j, h_j + 1, ..., q^*_j\right\}$. The consecutivity property highlights that the risk exposure heterogeneity plays a key role in the formation of risk-sharing coalitions. Further, any coalition $S^*_j = \left\{h_j, h_j + 1, ..., q^*_j\right\}$ in the DP-core partition satisfies both the no-blocking and the defection-proof conditions. The pivotal agent is given by $q^*_j = \min\left\{q^{NB}_j, q^{DP}_j\right\}$ where q^{NB}_j minimizes the risk premium of the consecutive

[7] The assumption of a truncated normal distribution allows us to derive (10) as the worst state of nature $\underline{\varepsilon}$ is finite.

coalition $S_j^{NB} = \left\{h_j, h_j + 1, ..., q_j^{NB}\right\}$

$$q_j^{NB} = \arg\min \pi \left(S_j^{NB}\right)$$

and q_j^{DP} defines the largest size n_j^{DP} such that the consecutive coalition $S_j^{DP} = \left\{h_j, h_j + 1, ..., q_j^{DP}\right\}$ is DP. From (11), q_j^{DP} is such that

$$C_1 X \left(n_j^{DP}; \sigma_{h_j}^2\right) \geq C_2 Y \left(n_j^{DP}\right)$$

and

$$C_1 X \left(n_j^{DP} + 1; \sigma_{h_j}^2\right) < C_2 Y (n_j^{DP} + 1).$$

In Appendix A.2, we show that when $q_j^{NB} < q_j^{DP}$ the coalition S_j^{NB} is also DP so that $q_j^* = q_j^{NB}$. Otherwise, the coalition S_j^{DP} is among the DP-coalitions the one that obtains the lowest risk premium, hence $q_j^* = q_j^{DP}$.[8] In other words, either the size (and thus membership, given the consecutiveness property) of a coalition S_j^* belonging to a DP-core partition minimizes the risk premium or the defection-proof condition binds before reaching the minimum of $\pi \left(S_j^{NB}\right)$. Given the consecutivity property, we adopt the following convention that lower indexed coalitions are formed of less risky agents: for any S_j^* and $S_{j'}^*$, $j' > j$, and any $i \in S_j^*$, and $i' \in S_{j'}^*$, $\sigma_i^2 < \sigma_{i'}^2$. However, in contrast to the commitment case (Jaramillo *et al.*, 2015), when a coalition S_j^* of the DP-core partition binds the defection-proof condition, it may generate a higher risk premium, equivalently a lower payoff, than a higher indexed coalition $S_{j'}^*$ with $q_{j'}^* = q_{j'}^{NB}$. In other words, under no commitment, the coalitions comprising the less risky individuals do no more provide lower risk premium than coalitions with riskier agents.

Notice that our coalition formation game is similar to Farrell and Scotchmer (1988) who show that the core partition is generically unique. Further, as in Genicot and Ray (2003), if agents are identical with respect to their risk exposure, the grand coalition would be in the core but could be non-DP, because agents after the shocks become heterogenous and can be incited to defect, given the limited social control.

[8] Nothing prevents from having several sizes n satisfying the defection-proof inequalities for the consecutive coalition with head h_j, $C_1 X \left(n; \sigma_{h_j}^2\right) \geq C_2 Y (n)$ and $C_1 X \left(n + 1; \sigma_{h_j}^2\right) < C_2 Y (n + 1)$. In the Appendix, we show that the size n mostly preferred by members is the largest one.

4 Risk, social control and segmentation

In this section, we aim to study the impact of risk heterogeneity and social control on the pattern of risk-sharing coalitions and the extent of risk sharing.

4.1 Risk and Segmentation

We draw from the previous section that the income shocks shape the pattern of risk management implemented through the formation of risk-sharing coalitions. If the characteristics of risk are modified, we expect that the partition of society into such coalitions varies.

In order to better understand the impact of risk on the DP-core partition, we focus on two issues:

1. Is a riskier society more or less segmented into risk-sharing coalitions?
2. How does risk affect the "cost of protection" paid by a society through its partitioning?

To answer theses questions, we consider two societies $\mathbf{I}$ and $\widetilde{\mathbf{I}}$ differing in their risk exposure distributions. In order to concentrate on risk, we make the following assumption:

A1 The distribution of ε_i associated to $\mathbf{I}$ second-order stochastically dominates (hereafter "SS-dominates") the distribution of $\widetilde{\varepsilon}_i$ associated to $\widetilde{\mathbf{I}}$ for every $i = 1, ..., N$.

According to Assumption A1, we characterize $\mathbf{I}$ as "less risky" than $\widetilde{\mathbf{I}}$: agents belonging to $\widetilde{\mathbf{I}}$ face more risk than agents in $\mathbf{I}$.

With respect to the first issue, we use the following

Definition 3. $\widetilde{\mathbf{I}}$ is more segmented than $\mathbf{I}$ if the number of coalitions in the DP-core partition associated with $\widetilde{\mathbf{I}}$ is larger than the number of coalitions in the DP-core partition associated with $\mathbf{I}$.

Our criterion of segmentation captures the idea that, for any agent, the chance to share risk among a larger coalition is reduced in a more segmented society. On average, a more segmented society is partitioned into smaller coalitions and this impacts on measures of risk sharing. Most empirical studies test optimal risk sharing considering that conditional expectation of individual consumption equals $\mathbb{E}\left(c_{it}|\,\frac{Y_t^{\mathbf{I}}}{N}, y_{i,t}\right) = \kappa_i + \beta_i \frac{Y_t^{\mathbf{I}}}{N} + \zeta_i y_{it}$ with $Y_t^{\mathbf{I}} \equiv \sum_{i=1}^{N} y_{i,t}$. This equation is obtained assuming CARA utility function (Townsend, 1994). Nonetheless, the coefficient ζ_i depends on social segmentation as $\lim_{N \longrightarrow +\infty} \overline{\zeta}_{\mathbf{I}} \equiv \left(\sum_{i=1}^{N} \zeta_i\right)/N \simeq (J-1)/N$ (see Appendix A.3). Hence, if individuals decide to pool risk in smaller groups than the whole society $\mathbf{I}$ then $J > 1$ implying that $\lim_{N \longrightarrow +\infty} \left(\sum_{i=1}^{N} \zeta_i\right)/N \neq 0$. Most empirical studies assume that the relevant unit to test for efficient risk sharing is the grand coalition. This assumption may be inaccurate and may explain why the null hypothesis $\overline{\zeta}_{\mathbf{I}} = 0$ is rejected.

With respect to the second issue about the cost of protection for society, we use the concept of "aggregate risk premium" developed by Jaramillo *et al.* (2015), defined in the following

Definition 4. The aggregate risk premium is the average of the risk premia paid within each coalition S_j of a partition $\mathcal{P}$:

$$\overline{\pi}(\mathcal{P}) = \frac{1}{N} \sum_{i=1}^{N} \pi_i = \frac{\alpha}{2N} \sum_{j=1}^{J} \left(\frac{\sum_{k \in S_j} \sigma_k^2}{n_j}\right)$$

Given these assumptions and definitions, we offer the following

Proposition 3. *Consider two societies* $\mathbf{I}$ *and* $\widetilde{\mathbf{I}}$ *satisfying (A1).*

(i) $\widetilde{\mathbf{I}}$ *is not necessarily more segmented than* $\mathbf{I}$.

(ii) *The aggregate risk premium* $\overline{\pi}\left(\widetilde{\mathcal{P}}\right)$ *associated to the DP-core partition in* $\widetilde{\mathbf{I}}$ *is not necessarily higher than* $\overline{\pi}(\mathcal{P})$ *associated to* $\mathbf{I}$.

Proof. See Section 5.3. □

(i) in Proposition 3 states that a riskier society can be less segmented than a less risky society. This implies that a coalition $\widetilde{S}_j^*$ can be larger than S_j^*. This stems from the fact that the increase of risk between societies $\mathbf{I}$ and $\widetilde{\mathbf{I}}$ reshapes the entire pattern of coalitions. Some coalitions with the same index j may actually be bigger in the riskier society than in the less risky society if they are comprised of individuals who are close in the risk ladder.

(ii) states that, using the aggregate risk premium, a riskier society does not necessarily

devote more resources for risk-sharing purposes than a less risky society. This is due to the change in the DP-core partition implied by higher risk. It may be such that actually some larger coalitions appear in the partition of the riskier society so that the risk premium paid by some individual so as to be protected by the assumed risk-sharing arrangement decreases. Hence *(ii)* highlights that the way risk is managed in a given society must be assessed in the light of the whole partition of the society into coalitions.

4.2 Social control and segmentation

Social control modifies the incentives to form a risk-sharing coalition and thus affects the way society manages the risks faced by individuals. Here we assume that social control drives the capacity to retaliate against a defector and punish her through autarky: Social control is captured by the probability of ex post punishment, p. We are interested in the impact of an increase in social control over the segmentation of society in risk-sharing coalitions and the cost of protection as given by the aggregate risk premium.

We answer these questions in the following

Proposition 4. *For a given risky society* $\mathbf{I}$*,*

(i) *the number of coalitions belonging to the DP-core partition is weakly decreasing with* p *and* δ*.*

(ii) *For* $\widetilde{p}$ *and* p *given with* $\widetilde{p} < p$*, the aggregate risk premium associated to* $\widetilde{p}$ *is not always bigger than the one associated to* p*.*

Proof. See Appendix A.4. □

(i) states that an increase in social control leads to a less segmented society in the sense that the number of coalitions is weakly decreasing. We do not prove that any coalition of a given index is larger when p increases. A higher propensity to detect and punish defection makes the punishment threat more effective and induces less risky agents to accept more heterogeneity within the first coalition. The head of the second coalition is a riskier agent who coalesces with much riskier agents which induces a smaller risk-sharing coalition. Hence,

subsequent risk-sharing coalitions may be smaller in the more controlled society. However, we prove that this phenomenon cannot be so large as to induce an increase in the number of coalitions in the DP-core partition.

(ii) states that the aggregate price for protection, as measured by the aggregate price premium is not necessarily larger in a society with a lower social control. This is due to the fact that some coalitions may become larger in the less controlled society: new coalitions' heads may face a lower heterogeneity in the subsequent risk ladder and accept belonging to larger coalitions. Said differently, and contrarily to what suggest intuition and common sense, increasing social control in order to reduce the number of defectors may not decrease the cost of insurance paid by a society.

In brief, these two propositions prove what is gained in adopting a general view on the entire partition of a given society. Some effects which seem intuitively correct are reversed when the endogenous partition based on risk-sharing coalitions is taken into account.

5 Empirical Evidence

Our theoretical model makes predictions about the relationships between idiosyncratic income risks, social control, the formation of risk-sharing coalitions within a community and the associated risk premia. A thorough empirical validation of these theoretical predictions would imply a complete mapping of inter-individual risk-sharing arrangements within a set of different communities and relate the formation of these arrangements to individual-level income risk exposures, community-level distributions of risks and levels of social control. This would require detailed information on risk-sharing networks, similar to the one used, for example, by Fafchamps and Lund (2003) and Fafchamps and Gubert (2007) on rural Philippines, or De Weerdt and Dercon (2006) and Comola and Fafchamps (2014) on rural Tanzania. Our aim in this present research being mainly theoretical, such an in-depth empirical investigation is beyond the scope of this paper. On the empirical side, our more modest objective is to provide with empirical evidence consistent with and illustrative of some of our main theoretical predictions. More specifically, based on the Townsend Thai

Projet panel data (further described below), we find empirical evidence (section 5.2) that coalitions are based on homophily (proposition 2.i) and that social control contributes to a lesser segmentation of a society (proposition 4.i). Simulations calibrated on the same data offer in section 5.3 further evidence consistent with our theoretical predictions, namely that a riskier society is not necessarily more fragmented, nor does it necessarily pay a higher aggregate risk premium (proposition 3), and also that a lower level of social control is not always associated with a higher aggregate risk premium (proposition 4.ii).

5.1 Data

We use data from the well-known and publicly-available Townsend Thai Project.[9] Started in 1997, this project probably provides with the longest running panel data set in a developing country, with extensive information at the individual, household and community levels. For this research, we use the initial baseline survey of 1997 (the "Big Survey") and the Townsend Thai Monthly Rural Surveys, which started in August 1998. As for the 1997 baseline survey, we use information at the household level: 2880 households were interviewed, 15 for each of the 192 villages selected in 48 *tambons* ("counties" - *ie* 4 villages per *tambon*), themselves located in 4 *changwats* ("provinces" - *ie* 12 *tambons* per *changwat*). As for the following Monthly Rural Surveys, approximately 45 households were followed in 16 villages selected among the 192 of the baseline survey, *ie* 4 villages in one *tambon* in each of the 4 *changwats* selected for the initial survey.

These high-frequency panel data, over a very long time span, provide with detailed information on households' incomes and consumption, that allows us to precisely estimate incomes' variability, thus idiosyncratic risks, and how they relate to consumption's variability. The questionnaire also includes questions about the behaviours of lenders and borrowers that we can use to investigate - at least indirectly - the role of social control, as discussed in our theoretical model.

However, the scope and ambition of our empirical investigation will be limited by the fact,

[9] `http://townsend-thai.mit.edu/`. See also Townsend *et al.* (2013) and Townsend (2016) for a description of the project, and of the related data and researches.

as mentioned above, that these surveys do not offer direct information on inter-individual risk-sharing networks. In addition, only 16 villages and about 45 households per village are surveyed for the panel data, limiting cross-villages and within-village variation in the observations.

We will restrict our empirical analysis to households for which we have all the relevant data for the whole time span considered, thus balancing the panel to avoid in particular potential issues of endogeneity stemming from attrition and the potential selection of households in and out of the surveyed sample. We use data for the time period January 1999 to December 2012 (165 months).[10] In the end, we use information on 605 households (out of the 720 targeted by the Monthly Rural Surveys of the Townsend Thai Project), approximately evenly distributed in the 4 changwats and 16 villages surveyed.

5.2 Econometric Evidence

5.2.1 Objectives

We first provide econometric evidence on two empirical claims derived from our model:

1. The homophily principle: The further away two households are apart in terms of idiosyncratic income risk, the less likely they are to belong to the same risk-sharing coalition, and so the lower the correlation of their consumptions should be. This assertion follows from the consecutivity property of the DP-core partition stated in Proposition 2 *(i)*.

2. The impact of social control: A higher social control (weakly) decreases the number of risk-sharing coalitions in a village, thus increasing the likelihood that two households at a given distance in terms of risk exposure belong to the same coalition and perfectly share risk. This assertion follows from Proposition 4 *(i)*.

[10]The survey started in 1997, and has continued beyond 2012, but we restrict ourselves to the period Jan. 1999 to Dec. 2012, for which information we need from the Income Statement Survey module is available and comparable.

5.2.2 Variables

Dependent variable: consumption correlation $r^2_{c_i,c'_i\in v}$ Our independent variable is a measure of consumption correlation between two households, denoted $r^2_{c_i,c'_i\in v}$ for the dyad constituted by households i and i' in village v. In Appendix A.5, we explicitly compute the correlations of the consumptions for a dyad of households i and i' in the village v, $r^2_{c_i,c_{i'}\in v}$, respectively when these households belong to the same risk-sharing coalition and when they do not. In the former case, the correlation of their consumptions should be high, and even, in our theoretical case of perfect risk-sharing within a coalition, equal to unity. In the latter case, this correlation should be lower, but note that it is not generally nil, and that it depends in particular, on the one hand, at least indirectly, on the levels of the households' idiosyncratic income risks σ_i^2 and $\sigma_{i'}^2$, and on the other hand, directly, on the level of village-level aggregate income risk σ_ν^2 .

Practically, we use the household-level variable "consumption" computed following the methodology of Samphantharak and Townsend (2009),[11] from August 1998 to December 2012.

Then, for all possible dyads of surveyed households i and i' in a village v, we compute the (Spearman) correlation of their consumption over the whole time span. Spearman (1904) correlation coefficients are preferred to Pearson ones as they do not require the assumption that the data are bivariate normal and because they are less sensitive to outlying values (Croux and Dehon, 2010). Coefficients of correlation that are not statistically significant at the 10% threshold are considered as nil and treated as zeros.

Independent variable 1: Idiosyncratic income risk distance $|\sigma_i^2 - \sigma_{i'}^2|$ Our first and main independent variable of interest is the distance in terms of idiosyncratic income risk faced by different agents in a community.

Firstly, each household's income is computed, following the methodology of Townsend (2017)

[11] Very practically, it is the variable IS5 of the Townsend Thai Monthly Survey Household Financial Accounting database, available at `https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/LJDPJM&widget=dataverse@harvard` and described in the User's Manual: "Financial Account Manual Instruction.pdf" available at the same url. This variable includes depreciation of household assets and land improvement, consumption of household production, consumption expenditure (food and non-food) and insurance premium.

who treat households as corporate financial units. In a nutshell, following this approach, a household's net operating income represents revenues from production activities and the household's assets, minus production costs, depreciation, interest and taxes. This provides with a measure y_{it} of net income for household i in month t.

Secondly, the important variable in our theoretical model (see equation (1)) is each agent's idiosyncratic income risk, σ_i^2, that is to say the variance of ε_{it}, the income residual, at each point in time, that cannot be accounted for by the agent's deterministic part of income and by global shocks common to the whole society. These ε_{it} are empirically computed as the residuals of the following equation, estimated separately for each household i:

$$y_{it} = \alpha_i + \gamma y_{vt} + \varepsilon_{it} \tag{12}$$

where y_{vt} is the average net operational income of all other agents in the village v at time t. The constant α_i accounts for the deterministic part of household i's income while y_{vt} accounts for aggregate village-level shocks. The variance σ_i^2 of the residuals ε_{it} is taken as the measure of each agent's idiosyncratic income risk exposure. This measure is normalized by applying the inverse hyperbolic sine transformation, as it approximates the natural logarithm while allowing to retain non-strictly-positive values (Bellemare and Wichman, 2020).

Thirdly and finally, we measure the distance, in terms of risk exposure, between two households i and i' by the absolute value of the difference between their (ihs transformed) income risk exposures: $|\sigma_i^2 - \sigma_{i'}^2|$.

Independent variable 2: Social Control SC_v Our second main variable of interest is the level of social control in a village, that is, the intensity of the threat faced by agents who defect from the coalition arrangement they belong to and choose not to share their ex post realized income with other members of the coalition. In our theoretical model, this threat, denoted by p, is the probability that a defecting agent will be punished by being excluded from any future risk-sharing coalition and left in autarky. We approximate this probability by the proportion of borrowers in a given village v who declare, in the baseline survey of 1997, that they will face a sanction if they do not properly repay their loans. This proxy is not an exact measure of our theoretical variable of social control, as it concerns only borrowing - whereas mutual risk sharing arrangements can take other forms than financial transfers - and declared sanctions do not always correspond to complete future autarky, a simplifying but practically unrealistic theoretical assumption.

Note here that as we take the average for each village at the beginning of the survey, we have no within-village but only cross-village variation - itself limited, as we have only 16 villages in our data.

Control variables The Townsend Thai Project dataset offers an extremely rich array of potential control variables, as it provides with detailed information on household characteristics and behaviors - like labour allocation for example - each month over a very long time span (January 1999 to December 2012 for the data we actually use). In order to alleviate to some extent endogeneity issues, as households are likely to change their behaviors over time depending on the mechanisms of risk sharing and the level of social control in their respective village, we focus on information available at the very onset of the monthly panel survey. We run a lasso procedure on all variables available for the very first month, and end up with the following control variables: numbers of household members (total and adults), average age and education and maximum education among household members. The vector of characteristics is denoted by z_i for household i. Following the methodology of Fafchamps and Gubert (2007), and as the dyadic relationship we investigate here is undirectional ($r^2_{c_i,c_{i'}} = r^2_{c_{i'},c_i}$), we include these controls as the absolute value of the difference for a dyad of households: $|z_i - z_{i'}|$ for households i and j, and we propose estimates of our models with and without level effects $(z_i + z_{i'})$.

Note that all these variables are measured over the whole time period, which means that we assume stability over time for all the phenomena of interest: in particular the households' income generating process (and the associated idiosyncratic risk), the level of social control in village and their respective partition into risk-sharing coalitions. Households' initial characteristics used as controls are similarly constant over the period. This is quite a strong assumption but our theoretical model predicts that the partition of a village community into risk-sharing coalitions remains stable over time, given the stability of village-level social control and of households' idiosyncratic risks. Moreover, to compute the latter, we need a long enough time span and large enough number of observations to disentangle the deterministic part of household's incomes from unexpected shocks. Hence our choice is to consider the whole time period covered by the data (1999-2012) as a single period from the point of view of the mechanisms at play - but as a consequence, empirically, we do not capture with precision the potential changes in the underlying variables of interest and our estimates will provide a rather blunt picture of a probably much finer reality.

5.2.3 Specification

The first prediction of our model that we aim to empirically illustrate is Proposition 2 *(i)*, that all coalitions S_j^* in the DP-core partition P^* are consecutive, that is to say that coalition formation is driven by a homophily principle: a coalition gathers agents who are close in terms of risk-exposure. This should translate into the empirical observation that the further away two households are apart in terms of idiosyncratic risk exposure, the lower should be the correlation of their consumptions. Following the methodology of Fafchamps and Gubert (2007), the equation we estimate is:

$$r^2_{c_i,c_{i'}\in v} = \beta|\sigma_i^2 - \sigma_{i'}^2| + \gamma_1(\sigma_i^2 + \sigma_{i'}^2) + \gamma_2\sigma_i^2 + \gamma_3|z_i - z_{i'}| + \gamma_4(z_i + z_{i'}) + \alpha_v + \epsilon_{ii'} \quad (13)$$

Where i and i' denote two households in a village v, $r^2_{c_i,c_{i'}\in v}$ the correlation of their consumptions, σ_i^2 and $\sigma_{i'}^2$ their respective idiosyncratic income risk, and $|\sigma_i^2 - \sigma_{i'}^2|$ the distance thereof. Following Fafchamps and Gubert (2007), we use the absolute value of the difference of the independent variables of interest, as the dyadic relationship here is undirectional ($r^2_{c_i,c_{i'}} = r^2_{c_{i'},c_i}$) and symmetry requires that regressors satisfy $\beta X_{ii'} = \beta X_{i'i}$. For the same reason, we include the absolute difference in households's controls $|z_i - z_{i'}|$. And, again following Fafchamps and Gubert (2007), we provide estimates with and without level effects, $(\sigma_i^2 + \sigma_{i'}^2)$ and $(z_i + z_{i'})$. We also provide estimates with the level of idiosyncratic risk of one member of the dyad, σ_i^2, to account for the effect of the position of the households in the risk-ladder of their village. Village-level fixed effects α_v are included to account for structural differences between villages.[12] $\epsilon_{ii'}$ denotes the error term, the dyadic nature thereof is taken into account in empirical estimates.

Our main prediction is that the coefficient β in equation (13) is negative: the closer two households i and i' are, in terms of risk exposure, the more likely they are to belong to the same risk-sharing coalition, and thus the more likely the correlation of their consumptions is to be high.

The second prediction of our model that we aim to empirically illustrate is derived from proposition 4 *(i)*, which states that social control contributes to a lesser segmentation of a society. This implies that ceteris paribus, and in particular for the same distance in terms of risk exposure $|\sigma_i^2 - \sigma_{i'}^2|$, two households i and i' in a village v are more likely to belong to the same risk-sharing coalition (and thus display a high level of consumption correlation) when the level of social control SC_v is high in village v. The equation we estimate is then:

$$r^2_{c_i,c_{i'}\in v} = \beta_1|\sigma_i^2-\sigma_{i'}^2|+\beta_2 SC_v\times|\sigma_i^2-\sigma_{i'}^2|+\gamma_1(\sigma_i^2+\sigma_{i'}^2)+\gamma_2\sigma_i^2+\gamma_3|z_i-z_{i'}|+\gamma_4(z_i+z_{i'})+\alpha_v+\epsilon_{ii'} \tag{14}$$

Where we add, on top of the variables included in equation (13), our variable SC_v proxying social control in a village, interacted with the distance between two households in terms of idiosyncratic risk $SC_v \times |\sigma_i^2 - \sigma_{i'}^2|$. As before, we expect the coefficient β_1 to be negative, that is to say the higher the distance in terms of risk exposure, the lower the correlation of consumptions of households i and i', but now we also expect this effect of the distance to be lessened by village-level social control: the higher the level of this social control, the larger the risk-sharing coalitions, the lesser the (negative) effect of the distance. The coefficient β_2 should then be positive.

Due to their dyadic structure, observations are not independent, and to obtain consistent standard errors for all estimations, we rely on the method of Fafchamps and Gubert (2007), who themselves extend the method developed by Conley (1999) to deal with spatial correlation of errors.

[12] Note that, unlike Fafchamps and Gubert (2007), we do not include a set $w_{ii'}$ of characteristics of the relationship between households i and i' itself, as we do not have such information.

Note that the precision of our empirical investigation with respect to the underlying mechanisms at play in terms of income generating processes, risk sharing behaviors and consumption correlations, and social control, is limited by two strong assumptions. First, and as mentioned above, all variables are measured over the whole time period, and thus considered as constant, as in our theoretical framework. Second, and again following our theoretical framework, we assume that our regressors, and in particular our two main variables of interest, distance in terms of risk exposure and social control, are exogenous. Our theoretical model shows how distance in terms of risk exposure and social control - themselves independent of each other - affect in a causal way risk sharing behaviors and thus consumption correlations. But practically, these variables might be related in several other ways. The patterns of consumption correlation between different households might affect these households' choices of activities and thus their income generating processes, as well as the mechanisms of social control. All of these variables might also depend on underlying characteristics, for example social, cultural and/or economic features that we do not observe. These concerns are somewhat - but not fully - alleviated by some of our methodological choices. First, we include village-level fixed effects, to account for time-invariant local characteristics. Second, our idiosyncratic income risk variable is built in such a way (see equation 12) that it should measure purely idiosyncratic and unexpected shocks, net of time-invariant household characteristics (α_i), temporal effects common to all and village-level income (y_{vt}). Third, for our control variables and our measure of social control, we take information from the 1997 baseline survey or from the very first month of monthly panel re-survey, and so these measures should be less affected by the income generating processes and consumption patterns that are measured afterwards.

5.2.4 Results

Cross-village evidence The results of the estimates of equations (13) and (14) across all sixteen villages are presented in Tables 1 and 2. Following Fafchamps and Gubert (2007), we use the OLS estimator and correct the standard errors to account for the dyadic structure of the observations. While we run the estimation on all villages, we of course restrict dyadic relationships to households belonging to the same village.

INSERT TABLES 1 AND 2 - SEE APPENDIX B.1.1

The results show that, as expected, the coefficients β and β_1 (associated with the dyadic distance in terms of risk exposure - see equations (13) and (14)) are significantly negative, while the coefficient β_2 (associated with social control) is positive. In both tables, in column (1), we only take into account our main variable(s) of interest: distance in income risk $|\boldsymbol{\sigma_i^2} - \boldsymbol{\sigma_{i'}^2}|$ in Table 1, to which we add the interaction with social control SC_v (*ie* probability of sanction in case of default) in Table 2. Without any further variables, coefficients β and β_1 are negative, and significantly so, at least at the 5% levels. All the same, without any further variables taken into account, in column (1) of Table 2, the coefficient β_2 (associated with the interaction between village-level social and the dyadic distance in terms of risk exposure - see equation (14)) is positive and statistically significant at the 5% level. In the following columns, we progressively add more variables: difference in control variables $|z_i - z_{i'}|$, sum of idiosyncratic risks $(\sigma_i^2 + \sigma_{i'}^2)$ and of control variables $(z_i + z_{i'})$ to account for level effects, the individual level of idiosyncratic income risk (σ_i^2), and, finally, village fixed effects α_v. The estimated coefficients remain remarkably stable, and statistically significant.

This is consistent with our theoretical predictions, respectively, that households that are closer in terms of risk exposure are more likely to belong to the same risk-sharing coalition and thus to display a high correlation of their consumption, and that a higher social control contributes to a lesser segmentation of a society. Note that, in Table 2, the two coefficients of interest β_1 and β_2 are of the same order of magnitude. That would mean that when social control is at its highest possible level, with a probability equal to one of sanction in case of default, the sum of both effects would be roughly equal to zero, meaning that the distance between two households in terms of idiosyncratic income risk does not affect the correlation of their consumption - implying that all households belong to the same big risk-coalition: the community is not partitioned anymore into several coalitions.

By-village evidence The results presented above gather observations for all sixteen villages in our dataset. Even if dyads are restricted to be formed between pairs of households within the same village, this approach still constrains the effect of the pairwise distance in terms of risk exposure, measured by β and β_1, and the effect of social control, measured by β_2 to be the same in all villages. To allow for more flexibility, we carry out exactly the same estimation based on equation (13) but village by village, first without any and then with all controls.[13] This yields an estimate of our parameter of interest for each of the 16 villages, $\hat{\beta}_v$. In Figures 1 and 2, these estimated coefficients are plotted against the level of social control for each village, respectively when no control variables and all control variables are taken into account in the village-level estimations of equation (13).

INSERT FIGURES 1 AND 2 - SEE APPENDIX B.1.2

These figures provide with some visual evidence that in many villages, the pairwise distance in terms in risk exposure $|\sigma_i - \sigma_{i'}|$ is negatively correlated to consumption correlation $r^2_{c_i,c_{i'}}$. This result is in line with our theoretical prediction that households tend to share risk with other households that are close to them in terms of idiosyncratic income risk. The line of best fit is also reported and appears to be increasing, indicating that on average, in villages where the level of social control is lower, the correlation of the consumptions of a dyad of households is more affected by the distance in terms of idiosyncratic income risk for this same dyad of households. It is consistent with our theoretical prediction that more social control allows on average for larger risk-sharing coalitions, and thus decreases the impact of idiosyncratic income risk distance. For villages with the highest possible level of social control, that is to say, a probability equal to one to be sanctioned in case of default, this impact is actually around zero which echoes the results we found in Table 2. However this line of best fit should be taken with much caution as it is drawn for only 16 observations, with, then, an extremely limited statistical power.

5.3 Simulations

Based on the Thailand Townsend Thai Project database, we proceed to simulations of the partitioning of each village economy into risk-sharing coalitions and illustrate Propositions 3 and 4 of the theoretical setup. The simulations are based on the 16 villages of the Thailand

[13]Equation (14) cannot be estimated for each single village, as our measure of social control varies across villages, but not within each village.

Townsend Thai Project database. We estimate the variance of idiosyncratic shocks (our index of risk) affecting each inhabitant and the kernels of the density functions of the logarithm of the variances of each village, using the Epanechnikov method. Using these empirical kernels, we generate 16 hypothetical villages of identical size, drawing 3000 values of the logarithm of variances for each village. We work with adjusted sigmas defined as follows $\hat{\sigma}_i^2 = \sigma_i^2/\overline{\sigma}^2$ for all i with $\overline{\sigma}^2$ being the mean of the 3000 draws for each village. It implies that the mean of the adjusted sigmas is equal to 1 so as to abstract from scale issues.[14] The kernel distributions of the $\log \sigma_i$ are depicted in Figure 3. Villages differ with respect to risk heterogeneity. While some distributions are unimodal (villages #765, #2753, #4961), other are rather multimodal (villages #763, #2751, #5353). Further, some distributions are right-skewed (villages #4957, #5353, #5354), others are left-skewed (villages #4960, #5360).

INSERT FIGURE 3 - SEE APPENDIX B.2

For each 3000-large village, we derive the partition associated with commitment and its counterpart under no commitment. Simulations of the DP-core partitions are obtained using the equilibrium properties described in Section 3. The pivotal agent of the first coalition is determined by selecting the global maximum of the function $X\left(n_1; \sigma_{h_1}^2\right)$ subject to the constraint $C_1 X\left(n_1; \sigma_{h_1}^2\right) \geq C_2 Y(n_1)$. For this purpose, we compute the $X\left(n_1; \sigma_{h_1}^2\right)$ value of the coalition with head $h_1 = 1$ for any size n_1 satisfying condition $C_1 X\left(n_1; \sigma_1^2\right) \geq C_2 Y(n_1)$. The size $\hat{n}_1$ with the highest $X\left(n_1; \sigma_1^2\right)$ value is chosen among such a set of coalitions. Given $\hat{n}_1$, h_2 is deduced. The same algorithm is applied using the functions $X\left(n_2; \sigma_{h_2}^2\right)$and $Y\left(n_2; \sigma_{h_2}^2\right)$, and recursively, all q_j^* are deduced.

The parameters used in the simulations are described below. For the sake of simplicity, the aggregate risk is assumed ti be null, $\sigma_v^2 = 0$. Based on Barro and Sala-i-Martin (2004), we assume the yearly discount factor denoted by δ_y to be equal to 0.98. This implies that the monthly discount factor is $\delta = (\delta_y)^{1/12} = 0.998$. The idiosyncratic shock $\varepsilon_{i,t}$ is distributed over the interval $[\underline{\varepsilon}, \overline{\varepsilon}]$ with $\underline{\varepsilon} = -2 \times (0.95\sigma_N^2)$ and $\overline{\varepsilon} = 2 \times (0.95\sigma_N^2)$. These bounds guarantee that the truncated normal distribution of the idiosyncratic shocks is close to the

[14]Simulation programs are available upon request.

normal distribution. All possible shocks are within the 95% confidence interval of the normal distribution of the top 5% riskiest agent. For less risky agents, the approximation is more accurate. For example, in village #5359 which we focus on below, $\underline{\varepsilon} = -2.75$, and $\overline{\varepsilon} = 2.75$. Further, the absolute risk aversion parameter is normalized to $\alpha = 1$ as well as the means of σ_i^2 at the village level. Notice that the function $X\left(n_j; \sigma_{h_j}^2\right)$ depends on $\alpha^2\sigma_i^2$ while the function $Y(n_j)$ depends on $\alpha\varepsilon_i$. We use $p = 1$, which corresponds to perfect social control: any defection is perfectly detected, and punishment strategy is applied with certainty. Yet, perfect social control does not systematically deter defection: the shock experienced by an agent i may be so large that she chooses defection despite the certainty of subsequent punishment.[15]

First, we compare the number of coalitions characterizing the partition with and without commitment for each village. We also compute the two associated measures of risk sharing, that is, the associated aggregate risk premium and the average coefficient $\overline{\zeta}_{\mathbf{I}}$. Under commitment, according to Proposition 2, coalitions are consecutive and the pivotal agent of any coalition S_j^{NB} minimizes the coalition risk premium, that is $q_j^{NB} = \arg\min \pi\left(S_j^{NB}\right)$. Under the limited commitment case, coalitions are consecutive with pivotal agents given by $q_j^* = \min\left\{q_j^{NB}, q_j^{DP}\right\}$. Table 3 displays our results. Even though incentives to defect under the limited commitment case are at their lowest (we set $p = 1$), we observe that the associated patterns of risk sharing differ. First, we find that under no commitment the number of coalitions increases in all the villages. For 14 villages, the number of coalitions increase by more than 100 fold (from 72 times for village #5359 to 297 times for village #5353).[16] Villagers share risk in much smaller coalitions. This is not surprising: the lifting of the full commitment assumption induces the less risky agents to become extremely sensitive to the risk of defection of their immediate follower. We then observe that, except for village #762, both the aggregate risk premium as well as $\overline{\zeta}_{\mathbf{I}}$ increase. Note that for village #762, the partition with commitment is such that the largest coalitions are the ones with less risky villagers

[15]We made various simulations changing parameters values and obtained the same qualitative results. We do not present them for sake of brevity. They are available upon request.

[16]Of course, the purpose of the simulations is to proceed to some comparative static exercises and highlight the sign of variation of the variables of interest. The figures obtained by the simulations have no interest per se.

while without commitment the riskier individuals share risks in the largest coalitions. This explains why the aggregate risk premium is smaller without commitment.

INSERT TABLE 3 - SEE APPENDIX B.2

Second, in order to study the impact of increasing risk through the partitioning of society, we proceed as follows. For each village, we multiply the *calibrated* sigmas by a positive integer.[17] We thus obtain two distributions of σ satisfying (A1). We aim at illustrating the counter-intuitive results contained in Propositions 3 and 4, focusing on village #5359. Figure 4 displays the risk schedule (in logs) for this village, that is, the ranking of variances faced by villagers from the lowest to the largest. This illustrates the large heterogeneity existing in a given village.

INSERT FIGURE 4 - SEE APPENDIX B.2

In the absence of commitment, village#5359 is partitioned into 575 risk-sharing coalitions in the DP partition. The 570th coalition includes the largest number of villagers (1089) and provides the lowest risk premium which is approximately equal to $12 * 10^{-5}$ of the median village risk premium obtained in coalition $j = 27$. The residual coalition is formed by the 10 riskiest villagers which explains why the risk premium is so high (9.02). Clearly, the risk premium schedule is not monotonously increasing in j. This reflects that for some coalitions the DP condition is binding (e.g. at least coalitions $j = 489$ and $j = 523$). By contrast, under full commitment, the number of coalitions shrinks to $J = 8$. The 3th coalition gathers 1095 villagers and the residual second coalition just 10. The risk premium is equal to $12 * 10^{-6}$ for the first coalition and to 9.02 for the last one.

INSERT FIGURE 5 - SEE APPENDIX B.2

For the chosen set of calibration parameters, village #5359 displays the counter-intuitive results highlighted in Propositions 3 and 4. First, as depicted in Figure 6, for any p, the partition obtained with the estimated distribution of risks is more segmented than the one

[17]For a given risk schedule $\overrightarrow{\sigma}^2 = \left\{\hat{\sigma}_1^2, \hat{\sigma}_2^2 ..., \hat{\sigma}_N^2\right\}$, we define the risk ratios for any two individuals i and $i+1$, $\lambda_i \equiv \hat{\sigma}_{i+1}^2 / \hat{\sigma}_i^2$, we denote by Λ the risk-ratio schedule $\Lambda = \{\lambda_1, \lambda_2, ..., \lambda_{N-1}\}$. The positive integer ι defines the new risk ratios λ_i' such that $\lambda_i' = \iota \lambda_i$ for any i. We choose $\iota = x$ so that ${\hat{\sigma}'_N}^2$ is 1.2 times bigger than $\widehat{\sigma}_N^2$. The number 1.2 is arbitrary.

arising with an SS-dominated distribution of risks. For instance, when $p = 0.1$, the number of coalitions falls from 2058 to 1981 with the riskier simulated distribution of risks (Proposition 3 *(i)*). Although there is a rise in absolute idiosyncratic risks, the change in coalitions' heads again may lead to a flattening of the subsequent remaining risk schedule. Thus some villagers are closer in the risk ladder and strive to share risk in larger coalitions. Notice that as $\overline{\zeta}$ depends positively on J it decreases with p. Further, Figure 6 shows the significant impact of the limited commitment assumption. Whereas the SS-dominated society $\widetilde{\mathbf{I}}$ generates a higher number of risk-sharing coalitions (determined by the no-blocking condition) under commitment, the reverse is true in the absence of commitment, due to the binding effect of the DP-condition.

INSERT FIGURE 6 - SEE APPENDIX B.2

Second, as Figure 7 shows, the riskier and less segmented partition does not necessarily devote more resources for risk-sharing purposes than the partition obtained with the estimated distribution of risks (Proposition 3 *(ii)*). When $p = 0.1$, the aggregate risk premium increase from 0.087 to 0.092 in the riskier society. However, the riskier society incurs a lower aggregate risk premium at a higher social control level, that is, $p = 0.7$. Moreover, the aggregate risk premium happens to be non-monotonic with respect to p (Proposition 4 *(ii)*), confirming that a higher social control does not imply a lower cost of protection against risk. Again, we observe a critical impact of the limited commitment assumption as the aggregate risk premium may be lower in the SS-dominated society $\widetilde{\mathbf{I}}$ under no commitment.

INSERT FIGURE 7 - SEE APPENDIX B.2.

6 Conclusion

When financial markets are insufficient, improper or absent, individuals resort to non-market arrangements for getting some protection against hazards: exchanges of services or informal groupings based on some redistribution rules. These groupings are made fragile by the capacity to renege on previous promises to be part of a collective arrangement. This paper tackles the issue of the outcome of informal risk-sharing coalitions with limited commitment.

In particular, it explores the role played by social control on the formation of these risk-sharing coalitions and on their effectiveness in sharing risk. Social control is understood here as any disciplinary devices existing at the level of a society, supported by mores, habits or institutions which constrain individual behavior. In particular social control mechanisms are assumed to be used so as to punish defectors from a risk-sharing coalition and thus mitigate the incentives to defect. Precisely, we consider village economies where financial markets are absent altogether and villagers resort to the formation of risk-sharing coalitions to be protected against idiosyncratic shocks. As these coalitions are based on voluntary agreements, a village (a society) may include a plurality of such coalitions. Social control exerted within a village is assumed to force into social autarky in subsequent periods any defector to a coalition she pledged to join in a given period. The heterogeneity between households is related to their exposure to idiosyncratic risk: households in a given society are ranked according to the increasing variance of their idiosyncratic shock.

We first search for the partition of society into risk-sharing coalitions based on voluntary behavior in the absence of commitment. We define a defection-proof core partition as the partition of society which fulfills both a core equilibrium condition and a defection-proof condition. We prove that such a partition exists and is generically unique. It is based on homophily principles: agents form coalitions based on proximity. Moreover, it depends on social control as the defection proof condition may bind.

Turning to comparative statics exercises, we show that the riskier society is not necessarily more segmented, when segmentation is captured by the number of coalitions. A society with a higher degree of social control (capacity to punish defectors) is likely to be less segmented according to the same criterion. However, a higher social control can lead to a higher aggregate risk premium.

Empirically, the assumption of limited commitment in villages is more easily acceptable than its opposite. However, sociologists and anthropologists alike point to the pervasive presence of informal social control mechanisms (Chriss, 2022). It is therefore enticing to look for the empirical consequences of social control over consumption smoothing and how the degree of social control affects the consumption smoothing pattern in a non-financial society. Turning to this matter, we provide some empirical evidence which are consistent with our theoretical results based on a large dataset obtained from a vast enquiry on sixteen Thai villages. We obtain that the correlation between households' consumption reflects homophily and this effect is more pronounced when social control is lower.

Simulations illustrate our theoretical propositions and in particular show that some counterintuitive results are easily obtained. They also highlight the consequence of assumptions made about the capacity to fully commit or not. Moreover, they show the sensitivity of the DP-core partition to changes in these parameters. In particular a change in the social control parameter appears to drastically alter the partitioning of the society into risk-sharing coalitions.

Theoretically, our enquiry is founded on several assumptions which can be relaxed. First we could look to another source of heterogeneity among villagers: income inequality, as was done in Jaramillo *et al.* (2003) or different risk aversions. Second, we could mitigate the assumption of no access to financial markets and instead assume that there is an partial and imperfect access to financial markets. Finally we could assume different (and less stringent) punishment patterns. Studying such changes is fully compatible with the logic of our enquiry and would represent a welcome continuation of our paper, allowing us to better understand how societies deal with risk which cannot always and perfectly be managed by financial markets.

Empirically, a precise verification of our theoretical results would imply a complete description of risk-sharing groups in several different communities and a linking between these partitionings with the modalities and level of social control, as well as with the distribution of income risks.

A Proofs

A.1 Proof of Proposition 1

1. First we prove the following: *If a sequence of DP core partitions exists, its elements are identical over time.*

Assume by contradiction a sequence of DP core partitions denoted by $\mathcal{S}_t$ where an individual i belongs to the DP-coalition $S_t^*(i)$ at date t and to a different DP-coalition $S'_{t+\tau}(i) = S'(i)$ afterwards. We drop the j index for sake of brevity. The expected lifetime utility writes:

$$U_i(\mathcal{S}_t) = \mathbb{E}_t\left[u_i\left(S_t^*(i)\right)\right] + \mathbb{E}_t\left[\sum_{\tau=1}^{\infty} \delta^\tau u_i\left(S'(i)\right)\right]$$

Since individual shocks are i.i.d. and the mutual insurance rule is implemented for any period of time:

$$\mathbb{E}_t\left[\sum_{\tau=1}^{\infty} \delta^\tau u_i\left(S'(i)\right)\right] = \frac{\delta}{1-\delta}\mathbb{E}_t\left[u_i\left(S'(i)\right)\right]$$

with for any $\tau = 1, ...,$ and

$$u_i\left(S'(i)\right) = -\frac{1}{\alpha}e^{\left(-\alpha w_i - \alpha \nu_{t+\tau} - \alpha \frac{\sum_{k\in S'(i)} \varepsilon_{k,t+\tau}}{n'(i)}\right)},$$

$n'(i)$ denoting the size of $S'(i)$.

Given the risk sharing rule is non-state-contingent, the indirect utility is not time-dependent. As $S'(i)$ is assumed to satisfy the no-blocking condition, we must have that

$$\mathbb{E}_t\left[u_i\left(S'(i)\right)\right] > \mathbb{E}_t\left[u_i\left(S_t^*(i)\right)\right].$$

This contradicts the fact that $S_t^*(i)$ satisfies the no-blocking condition and that $\mathcal{S}_t$ is a sequence of DP-core partitions.

2. *Existence*

Consider a partition constituted solely of singletons. Given Definition (1), it is easy to check that any singleton is DP. Thus, the set of DP partitions is not empty.

For any two DP-coalitions S and S', $S \neq S'$, of size n respectively n', in $\mathcal{S}$ and $\mathcal{S}'$, S and S' remaining the same over time, we have for any i in S, and any i in S'

$$U_i(S) = \frac{1}{1-\delta}\left(-\frac{1}{\alpha}e^{-\alpha\left[\omega_i - \frac{\alpha}{2}\sigma_\nu^2 - \alpha\frac{\sum_{k\in S}\sigma_k^2}{2n^2}\right]}\right) \gtrless U_i(S') = \frac{1}{1-\delta}\left(-\frac{1}{\alpha}e^{-\alpha\left[\omega_i - \frac{\alpha}{2}\sigma_\nu^2 - \alpha\frac{\sum_{k\in S'}\sigma_k^2}{2n'^2}\right]}\right),$$

or equivalently,

$$\frac{\sum_{k\in S}\sigma_k^2}{2n^2} \lesseqgtr \frac{\sum_{k\in S'}\sigma_k^2}{2n'^2}.$$

Hence, for any DP-coalitions S and S', all individuals i in either S or S' are unanimous about the preference ordering of the coalitions. This is the common ranking property that implies the non-emptiness of the core partition.

The following Lemma proves the existence of DP-core partitions.

Lemma 1. Any coalition S_j is DP if and only if the head h_j of S_j, such that $\sigma_{h_j}^2 \equiv \min\{\sigma_z^2\}_{z\in S_j}$, does not defect in any state of nature, for all j.

We denote by n_j the cardinal of S_j.

1. Necessary condition. Definition (1) requires that in order to be defection-proof, a coalition must be such that none of its members, in particular h_j, has any interest to defect.

2. Sufficient condition. Let us prove that if the head does not want to defect in any state of nature from a coalition S_j, no other member wants to defect in any state of nature.

Plugging (3) and (9) into (8), leads, for any agent i in S_j, to

$$\frac{\delta}{1-\delta}pe^{\frac{\alpha^2}{2}\sigma_\nu^2}\left(e^{\frac{\alpha^2}{2}\sigma_z^2} - e^{\frac{\alpha^2}{2n_j^2}\sum_{k\in S_j}\sigma_k^2}\right) \geq e^{-\alpha\nu_t}\left(e^{-\alpha\frac{\sum_{k\in S_j}\varepsilon_{k,t}}{n_j^*}} - e^{-\alpha\varepsilon_{i,t}}\right),\ \forall t. \tag{15}$$

By definition of the "head" h_j, we have for any i in S_j:

$$e^{\frac{\alpha^2}{2}\sigma_i^2} - e^{\frac{\alpha^2}{2n_j^2}\sum_{k\in S_j}\sigma_k^2} \geq e^{\frac{\alpha^2}{2}\sigma_{h_j}^2} - e^{\frac{\alpha^2}{2n_j^2}\sum_{k\in S_j}\sigma_k^2}. \tag{16}$$

Therefore if $\frac{\delta}{1-\delta} p e^{\frac{\alpha^2}{2}\sigma_\nu^2}\left(e^{\frac{\alpha^2}{2}\sigma_{h_j}^2} - e^{\frac{\alpha^2}{2n_j^2}\sum_{k\in S_j}\sigma_k^2}\right) \geq e^{-\alpha\nu_t}\left(e^{-\alpha\frac{\sum_{k\in S_j}\varepsilon_{k,t}}{n_j}} - e^{-\alpha\varepsilon_{i,t}}\right)$ for any t, this is true for all $i \in S_j$.

Let us focus on the scenario that makes the most likely the defection of h_j. Let us study the RHS of (15) in the case of the "worse state of nature" from the point of view of i: for a given ε_i the other members of S_j suffer from the lowest idiosynchratic shocks and the lowest common shock. Hence:

$$f(\varepsilon_i) \equiv e^{-\alpha\frac{\left(n_j-1\right)\underline{\varepsilon}+\varepsilon_i}{n_j^*}} - e^{-\alpha\varepsilon_i} > e^{-\alpha\frac{\sum_{k\in S_j}\varepsilon_k}{n_j}} - e^{-\alpha\varepsilon_i}. \tag{17}$$

The derivative of $f(\varepsilon_i)$ is :

$$f'(\varepsilon_i) = \alpha e^{-\alpha\varepsilon_i}\left[1 - \frac{1}{n_j}e^{\frac{\alpha\left(n_j-1\right)}{n_j}(\varepsilon_i-\underline{\varepsilon})}\right].$$

Let us define ε^{max} such that $f'(\varepsilon^{max}) = 0$, i.e.:

$$n_j^* = e^{\left(\frac{\alpha\left(n_j^*-1\right)}{n_j^*}(\varepsilon^{max}-\underline{\varepsilon})\right)} \Leftrightarrow \frac{n_j\log(n_j)}{\alpha(n_j-1)} + \underline{\varepsilon} = \varepsilon^{max}.$$

$\varepsilon^{max}$is bigger than $\underline{\varepsilon}$ as $n_j\log(n_j)/\alpha(n_j-1) > 0$. As $f'(\varepsilon_i) > 0$ for any $\varepsilon_i \in [\varepsilon, \varepsilon^{max}]$, the maximum value of $f(\varepsilon_i)$ over the support of ε_i is obtained for $\hat{\varepsilon} = \min[\varepsilon^{max}, \bar{\varepsilon}]$. Then, given (17)

$$f(\hat{\varepsilon}) > f(\varepsilon_i) > e^{-\alpha\frac{\sum_{k\in S_j}\varepsilon_{k,t}}{n_j}} - e^{-\alpha\varepsilon_{i,t}}.$$

The head of S_j does not defect in the worse case for her when

$$\frac{\delta}{1-\delta}pe^{\frac{\alpha^2}{2}\sigma_\nu^2}\left(e^{\frac{\alpha^2}{2}\sigma_{h_j}^2}-e^{\frac{\alpha^2}{2n_j^2}\sum\limits_{k\in S_j}\sigma_k^2}\right)>e^{-\alpha\underline{\nu}}f\left(\hat{\varepsilon}\right)>e^{-\alpha\underline{\nu}}f\left(\varepsilon_{h_j}\right)>e^{-\alpha\underline{\nu}}\left(e^{-\alpha\frac{\sum\limits_{k\in S_j}\varepsilon_{k,t}}{n_j}}-e^{-\alpha\varepsilon_{i,t}}\right) \tag{18}$$

If h_j does not defect in her worse state of nature, she will not defect in any state of nature. If in any given period, h_j does not defect, all other members of S_j do not defect given (16). This completes the proof of Lemma 1.

Finally, according to (18), a coalition S_j is defection-proof if and only if:

$$\frac{\delta}{1-\delta}pe^{\frac{\alpha}{2}\sigma_\nu^2}\left(e^{\frac{\alpha^2}{2}\sigma_{h_j}^2}-e^{\frac{\alpha^2}{2n_j^2}\sum\limits_{z\in S}\sigma_z^2}\right)\geq e^{-\alpha\underline{\nu}}e^{-\alpha\hat{\varepsilon}}\left(e^{-\alpha\frac{\left(n_j-1\right)\left(\underline{\varepsilon}-\hat{\varepsilon}\right)}{n_j}}-1\right).$$

Letting $X\left(n_j;\sigma_{h_j}^2\right)\equiv e^{(\alpha^2/2)\sigma_{h_j}^2}-e^{\left(\alpha^2/2n_j^2\right)\sum\limits_{z\in S_j}\sigma_z^2}$, $Y(n_j)\equiv e^{-\alpha\hat{\varepsilon}(n_j)}\left(e^{-\alpha\frac{\left(n_j-1\right)\left(\underline{\varepsilon}-\hat{\varepsilon}(n_j)\right)}{n_j}}-1\right)$, $C_1\equiv(\delta/(1-\delta))\,pe^{\frac{\alpha}{2}\sigma_\nu^2}$, $C_2\equiv e^{-\alpha\underline{\nu}}$, $\hat{\varepsilon}(n_j)=\min\left[n_j\log n_j/(\alpha(n_j-1))+\underline{\varepsilon},\overline{\varepsilon}\right]$, it can be written as follows

$$C_1X\left(n;\sigma_{h_j}^2\right)\geq C_2Y(n). \tag{19}$$

Notice that for $n=1$ this condition is satisfied as $X\left(1;\sigma_1^2\right)=Y(1)=0$: singletons are always DP. There exists at least a size n that satisfies (18). This ends the proof of existence.

A.2 Proof of Proposition 2

(i) Consecutiveness

Let us first provide

Lemma 2. Consider a DP-coalition S_j of size n_j, which comprises i but not i' where $h_j<i'<i$. Then the coalition $S_{j'}\equiv S_j\backslash\{i\}\cup\{i'\}$ is DP.

Proof. Consider a coalition S_j which comprises i but not i' with $h_j<i'<i$. According to

Lemma 1, S_j is DP when it satisfies (18), that is:

$$\frac{\delta}{1-\delta} p e^{\frac{\alpha^2}{2}\sigma_\nu^2}\left(e^{\frac{\alpha^2}{2}\sigma_{h_j}^2} - e^{\frac{\alpha^2}{2n_j^2}\sum\limits_{k\in S_j}\sigma_k^2}\right) > e^{-\alpha\underline{\nu}}\left(e^{-\alpha\frac{\left(n_j-1\right)\underline{\varepsilon}+\hat{\varepsilon}}{n_j}} - e^{-\alpha\hat{\varepsilon}}\right).$$

Consider coalition $S_{j'}$=$\{S_j \setminus \{i\}\} \cup \{i'\}$. Coalitions $S_{j'}$ and S_j have the same size. Since $\sigma_{h_j}^2$< $\sigma_{i'}^2 < \sigma_i^2$, we get that :

$$\frac{\delta}{1-\delta} p e^{\frac{\alpha}{2}\sigma_\nu^2}\left(e^{\frac{\alpha^2}{2}\sigma_{h_j}^2} - e^{\frac{\alpha^2}{2n_j^2}\left(\sum\limits_{z\in S_{j'}}\sigma_z^2\right)}\right) > \frac{\delta}{1-\delta} p e^{\frac{\alpha}{2}\sigma_\nu^2}\left(e^{\frac{\alpha^2}{2}\sigma_{h_j}^2}\alpha - e^{\frac{\alpha^2}{2n_j^2}\left(\sum\limits_{z\in S_j}\sigma_z^2\right)}\right) >$$
$$e^{-\alpha\underline{\nu}}\left(e^{-\alpha\frac{\left(n_j-1\right)\underline{\varepsilon}+\hat{\varepsilon}}{n_j}} - e^{-\alpha\hat{\varepsilon}}\right)$$

Therefore $S_{j'}$ is DP. $\square$

Let us assume that $\mathcal{P}^*$ contains a coalition S^* which is not consecutive, i.e. which comprises i but not i' where $h < i' < i$. Let us define a new coalition S=$\{S^* \setminus \{i\}\} \cup \{i'\}$. Since $i' < i \iff \sigma_i^2 > \sigma_{i'}^2$, we get from (4) that for any i

$$\mathbb{E}_{\mathrm{t}}\left[u_i\left(S^*\right)\right] = -\frac{1}{\alpha}e^{-\alpha\left[\omega_i - \frac{\alpha}{2}\sigma_\nu^2 - \frac{\alpha^2}{2n^{*2}}\sum\limits_{k\in S^*}\sigma_k^2\right]} < \mathbb{E}_{\mathrm{t}}\left[u_i\left(S\right)\right] = -\frac{1}{\alpha}e^{-\alpha\left[\omega_i - \frac{\alpha}{2}\sigma_\nu^2 - \frac{\alpha^2}{2n^{*2}}\sum\limits_{k\in S}\sigma_k^2\right]}$$

From Lemma 2, S is also DP and thus is a welfare-augmenting coalition. This implies that S^* cannot be in the DP core partition $\mathcal{P}^*$.

(ii) Uniqueness

We consider the consecutive coalition S of size n with the head h. According to (18), it is defection-proof if and only if:

$$C_1 X\left(n; \sigma_{h_j}^2\right) \geq C_2 Y(n). \tag{20}$$

Consider $Y(n)$ and we consider that n is a continuous variable. We have:

$$Y'(n) \equiv -\alpha\hat{\varepsilon}'(n)e^{-\alpha\hat{\varepsilon}(n)}\left(-1+\frac{1}{n}e^{-\alpha\frac{(n-1)(\underline{\varepsilon}-\hat{\varepsilon}(n))}{n}}\right)+e^{-\alpha\hat{\varepsilon}(n)}\left(-\alpha\frac{(\underline{\varepsilon}-\hat{\varepsilon}(n))}{n^2}e^{-\alpha\frac{(n-1)(\underline{\varepsilon}-\hat{\varepsilon}(n))}{n}}\right)$$

with $\hat{\varepsilon}'(n) = 0$ when $\varepsilon^{max} > \bar{\varepsilon} = \hat{\varepsilon}(n)$ and $-\alpha\hat{\varepsilon}'(n)e^{-\alpha\hat{\varepsilon}(n)}\left(-1+\frac{1}{n}e^{-\alpha\frac{(n-1)(\underline{\varepsilon}-\hat{\varepsilon}(n))}{n}}\right) = 0$ when $\hat{\varepsilon}(n) = \varepsilon^{max} < \bar{\varepsilon}$ since the envelope theorem applies. Hence,

$$Y'(n) = e^{-\alpha\hat{\varepsilon}(n)}\left(-\alpha\frac{(\underline{\varepsilon}-\hat{\varepsilon}(n))}{n^2}e^{-\alpha\frac{(n-1)(\underline{\varepsilon}-\hat{\varepsilon}(n))}{n}}\right) > 0, \forall n. \tag{21}$$

We thus conclude that $Y(n)$ is an increasing function of n.

In the absence of commitment, the core partition is generically unique (Farrell and Scotchmer, 1988). The possible existence of two core partitions with two different "first-ranked" coalitions generating the lowest risk premium is non-generic in the following sense: a slight perturbation of idiosyncratic risks would modify each coalition's risk premium and make impossible that these two first-ranked coalitions generate the same payoff. This argument can be generalized to any coalitions in the core partitions generating the same level of risk premium. Hence, in the absence of commitment, the core partition is generically unique.

We assume that $X(n)$ is strictly concave with size n and reaches its maximum $\hat{n}$ when the most risky agent of the consecutive coalition S, denoted by q, satisfies the two following inequalities:

$$\sigma_q^2 \leq (2\hat{n}-1)\sum_{k\in S\setminus\{q\}}\frac{\sigma_k^2}{(\hat{n}-1)^2} \tag{22}$$

and

$$\sigma_{q+1}^2 > (2\hat{n}+1)\sum_{k\in S}\frac{\sigma_k^2}{\hat{n}^2}. \tag{23}$$

Let us consider S_1 the consecutive coalition of size n_1 comprised of the less risky individuals of society **I**. Let us show that the coalition S_1 that belongs to the DP core partition is unique. First, notice that $n_1 = 1$ fulfills this condition as $X(1;\sigma_1^2) = Y(1) = 0$: singletons are DP. $X(n;\sigma_1^2)$ reaches its maximum at $\hat{n}_1$ which is given by (22) and (23). $Y(n)$ is increasing. If $C_1X(n;\sigma_1^2) < C_2Y(n)$ whatever $n > 1$, then the defection-proof condition is not satisfied for any coalition with $n > 1$ and the only DP-coalition is the singleton.

Second, let us consider cases where there are some values of $n_1 > 1$ such that the defection-proof condition is satisfied. When the defection-proof condition is satisfied for $n_1 > \hat{n}_1$, with $\hat{n}_1$ the size satisfying inequalities (22) and (23). Given the characteristics of $X(n; \sigma_1^2)$ and $Y(n)$, there can be at most one value of $n_1 > \hat{n}_1$such that

$$C_1 X\left(n; \sigma_1^2\right) \geq C_2 Y(n)$$

and

$$C_1 X\left(n+1; \sigma_1^2\right) < C_2 Y(n+1).$$

This implies that $\hat{n}_1$ verifies $C_1 X(\hat{n}_1; \sigma_1^2) \geq C_2 Y(\hat{n}_1)$ and is DP. As $\hat{n}_1$ also maximizes the utility of individuals belonging to S_1, we deduce that $\hat{n}_1$ is in the DP core coalition. Consider now that the defection-proof condition is satisfied for several $n_1 < \hat{n}_1$, and let us focus on the highest value of n_1 denoted by $\overline{n}_1$. We then must consider the two following cases:

1. If $C_1 X(\overline{n}_1; \sigma_1^2) \geq C_2 Y(\overline{n}_1)$ and $C_1 X(\overline{n}_1 + 1; \sigma_1^2) < C_2 Y(\overline{n}_1 + 1)$, we get $C_1 X(\hat{n}_1; \sigma_1^2) < C_2 Y(\hat{n}_1)$ and $\hat{n}_1$ is not DP. As $\overline{n}_1$ is located on the upward leg of $C_1 X(n)$ it generates a higher ex ante utility than the other n_1 satisfying the defection-proof condition. Hence, $\overline{n}_1$ satisfies the conditions for a DP-coalition in the core partition.

2. If $C_1 X(\overline{n}_1 - 1; \sigma_1^2) < C_2 Y(\overline{n}_1 - 1)$ and $C_1 X(\overline{n}_1; \sigma_1^2) \geq C_2 Y(\overline{n}_1)$, we get $C_1 X(\hat{n}_1; \sigma_1^2) > C_2 Y(\hat{n}_1)$ and $\hat{n}_1$ is DP and again satisfies the condition for a DP-coalition in the core partition.

There is therefore always a unique DP core coalition S_1^* including agent 1. Reasoning on $\mathbf{I} \backslash S_1^*$, and replicating the reasoning for a coalition including the lowest ranked agent in $\mathbf{I} \backslash S_1^*$, and so on, we obtain a unique DP core partition.

A.3 Optimal risk sharing and regression coefficients

We first present as a benchmark the case of risk-sharing within a given society when resources are allocated by a benevolent planner. Individuals differ with respect to exposure to risk and also risk aversion. There is commitment.

Let the state of nature be denoted by $\epsilon_t^{\mathbf{I}} = (\varepsilon_{1,t}, \ldots, \varepsilon_{j,t}, \ldots \varepsilon_{N,t}, \nu_t)$. We will denote by $Y_t^{\mathbf{I}}(\epsilon_t^{\mathbf{I}})$ the aggregate level of resources at date t:$\epsilon_t \equiv (\varepsilon_{1,t}, ..., \varepsilon_{N,t}, \nu_t)$ the state of nature at date t.

$$Y_t^{\mathbf{I}} = \sum_{i=1}^{N} \omega_i + \sum_{i=1}^{N} \varepsilon_{i,t} + N\nu_t$$

Following Townsend (1994), the planner's program can be expressed as follows:

$$\max_{\{c_{it},\}} U = \sum_{i=1}^{N} \mu_i \left(-\mathbb{E}_0 \left[\frac{1}{\alpha_i} \sum_{t=1}^{T} \delta^{t-1} e^{-\alpha_i c_{i,t}} \right] \right)$$

subject to the following feasibility constraint at each date t:

$$c_t^{\mathbf{I}} \equiv \sum_{i=1}^{N} c_{it} \leq Y_t^{\mathbf{I}}$$

where μ_i, $i = 1, ..., N$, denotes the non-negative Pareto weight attached to agent i. It turns out that Pareto-optimal consumptions can be written as follows[18]:

$$c_{it} = \frac{1}{\alpha_i} \left[\ln \mu_i - \frac{\sum_{k \in \mathbf{I}} \frac{\ln \mu_k}{\alpha_k}}{\sum_{k \in \mathbf{I}} \frac{1}{\alpha_k}} \right] + \frac{\frac{1}{\alpha_i} N}{\sum_{k \in \mathbf{I}} \frac{1}{\alpha_k}} \frac{Y^{\mathbf{I}}}{N}, \ \forall i \in \mathbf{I}. \tag{24}$$

The mutual insurance rule corresponds to the case when $\ln \mu_i = \alpha_i \omega_i$ for any i.

Given (24) the conditional expectation of individual consumption used by most of the literature when testing for the perfect risk sharing hypothesis:

$$\mathbb{E}(c_{i,t} | \frac{Y_t^{\mathbf{I}}}{N}, y_{i,t}) = \kappa_i + \beta_i \frac{Y_t^{\mathbf{I}}}{N} + \zeta_i y_{i,t} \tag{25}$$

where the formulas of β_i and ζ_i are obtained by using properties of conditional expectations

[18] If we considered a production sector and leisure choice, formulas of Pareto-optimal consumptions would not be affected if separable utility functions are assumed (see Townsend, 1994).

of multivariate normal distributions (Ramanathan, 1993):

$$\beta_i = \frac{cov\left(\frac{Y_t^{\mathbf{I}}}{N}, c_{it}\right) var\left(y_{it}\right) - cov\left(y_{it}, c_{it}\right) cov\left(\frac{Y_t^{\mathbf{I}}}{N}, y_{it}\right)}{var\left(\frac{Y_t^{\mathbf{I}}}{N}\right) var\left(y_{it}\right) - \left[cov\left(\frac{Y_t^{\mathbf{I}}}{N}, y_{it}\right)\right]^2} \tag{26a}$$

$$\zeta_i = \frac{cov\left(y_{it}, c_{it}\right) var\left(\frac{Y_t^{\mathbf{I}}}{N}\right) - cov\left(\frac{Y_t^{\mathbf{I}}}{N}, c_{it}\right) cov\left(\frac{Y_t^{\mathbf{I}}}{N}, y_{it}\right)}{var\left(y_{it}\right) var\left(\frac{Y_t^{\mathbf{I}}}{N}\right) - \left[cov\left(\frac{Y_t^{\mathbf{I}}}{N}, y_{it}\right)\right]^2}. \tag{26b}$$

Given (24), some straightforward computations lead to the following

$$\beta_i = \frac{\frac{1}{\alpha_i}}{\sum_{k\in\mathbf{I}} \frac{1}{\alpha_k}} \frac{(N\sigma_\nu^2 + \frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N})\left(\sigma_\nu^2 + \sigma_i^2\right) - \left(N\sigma_\nu^2 + \sigma_i^2\right)\left(\sigma_\nu^2 + \frac{\sigma_i^2}{N}\right)}{\left(\sigma_\nu^2 + \sigma_i^2\right)\left(\sigma_\nu^2 + \frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2}\right) - \left(\sigma_\nu^2 + \frac{\sigma_i^2}{N}\right)^2}$$

$$\zeta_i = \frac{\frac{1}{\alpha_i}}{\sum_{k\in\mathbf{I}} \frac{1}{\alpha_k}} \frac{\left(N\sigma_\nu^2 + \sigma_i^2\right)\left(\sigma_\nu^2 + \frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2}\right) - (N\sigma_\nu^2 + \frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N})\left(\sigma_\nu^2 + \frac{\sigma_i^2}{N}\right)}{\left(\sigma_\nu^2 + \sigma_i^2\right)\left(\sigma_\nu^2 + \frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2}\right) - \left(\sigma_\nu^2 + \frac{\sigma_i^2}{N}\right)^2}.$$

Hence,

$$\beta_i = \frac{\frac{1}{\alpha_i}}{\sum_{k\in\mathbf{I}} \frac{1}{\alpha_k}} N$$

and

$$\zeta_i = 0.$$

Second, let us now assume that optimal risk sharing takes place in subset $S \subset \mathbf{I}$, with $n \equiv card(S) < N$. It turns out that Pareto-optimal consumptions can now be written as follows

$$c_{it} = \frac{1}{\alpha_i}\left[\ln\mu_i - \frac{\sum_{k\in S}\frac{\ln\mu_k}{\alpha_k}}{\sum_{k\in S}\frac{1}{\alpha_k}}\right] + \frac{\frac{1}{\alpha_i}n}{\sum_{k\in S}\frac{1}{\alpha_k}} \frac{\sum_{k\in S}(\omega_k + \varepsilon_{kt} + \nu_t)}{n}, \text{ for } i \in S. \tag{27}$$

If we still consider (25) and use the latter expression of c_{it} to compute (26a) and (26b), it turns out that the coefficients β_i and ζ_i are equal to:

$$\beta_i = \frac{\frac{1}{\alpha_i}}{\sum_{k\in S} \frac{1}{\alpha_k}} \frac{(n\sigma_\nu^2 + \frac{\sum_{k\in S}\sigma_k^2}{N})\left(\sigma_\nu^2 + \sigma_i^2\right) - \left(n\sigma_\nu^2 + \sigma_i^2\right)\left(\sigma_\nu^2 + \frac{\sigma_i^2}{N}\right)}{\left(\sigma_\nu^2 + \sigma_i^2\right)\left(\sigma_\nu^2 + \frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2}\right) - \left(\sigma_\nu^2 + \frac{\sigma_i^2}{N}\right)^2} \tag{28}$$

$$\zeta_i = \frac{\frac{1}{\alpha_i}}{\sum_{k\in S}\frac{1}{\alpha_k}} \frac{\left(n\sigma_\nu^2+\sigma_i^2\right)\left(\sigma_\nu^2+\frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2}\right)-\left(n\sigma_\nu^2+\frac{\sum_{k\in S}\sigma_k^2}{N}\right)\left(\sigma_\nu^2+\frac{\sigma_i^2}{N}\right)}{\left(\sigma_\nu^2+\sigma_i^2\right)\left(\sigma_\nu^2+\frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2}\right)-\left(\sigma_\nu^2+\frac{\sigma_i^2}{N}\right)^2} \tag{29}$$

If we concentrate on ζ_i we get easily a simplified expression highlighting the role of social segmentation. Let us assume $\sigma_\nu^2 = 0$ and dividing by σ_ν^2 we get

$$\zeta_i = \frac{\frac{1}{\alpha_i}}{\sum_{k\in S}\frac{1}{\alpha_k}} \frac{\frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2}-\frac{\sum_{k\in S}\sigma_k^2}{N}\frac{1}{N}}{\frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2}-\frac{\sigma_i^2}{N^2}}.$$

Assuming $\frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2}-\frac{\sigma_i^2}{N^2}\backsimeq\frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2}$ and summing over all individuals of the coalition S_j we have

$$\sum_{i\in S_j}\zeta_i \backsimeq \frac{\frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2}-\frac{\sum_{k\in S}\sigma_k^2}{N^2}}{\frac{\sum_{m\in\mathbf{I}}\sigma_m^2}{N^2}}.$$

Summing over all coalitions S of the partition $\mathcal{P}$ of society $\mathbf{I}$ yields

$$\overline{\zeta} = \frac{\sum_{j=1}^{J}\sum_{i\in S}\zeta_i}{N} \backsimeq \frac{J}{N}-\frac{1}{N}\backsimeq 0.$$

A.4 Proof of Proposition 4

For a given risk schedule $\overrightarrow{\sigma}^2 = \{\sigma_1^2, \sigma_2^2, ..., \sigma_N^2\}$, we define the risk ratios as follows

$$\lambda_{j,i} \equiv \frac{\sigma_i^2}{\sigma_j^2}.$$

Adopting the convention that, for any two individuals i and $i+1$, $\lambda_i \equiv \sigma_{i+1}/\sigma_i$, we denote by Λ the risk-ratio schedule $\Lambda = \{\lambda_1, \lambda_2, ..., \lambda_{N-1}\}$.

A.4.1 *(i)*

First, we prove the following

Lemma 3. For a given society $\mathbf{I}$, let us consider two DP-coalitions, $S = \{h, h+1, ..., q(h)\}$ and $S' = \{h', h'+1, ..., q(h')\}$ with $h < h'$, $q(h)$ and $q(h')$ are the pivotal individuals $q(h) = \min\left\{q^{DP}(h), q^{NB}(h)\right\}$ and $q(h') = \min\left\{q^{DP}(h'), q^{NB}(h')\right\}$. We have the following four cases:

1- $q(h) = q^{DP}(h)$ and $q(h') = q^{DP}(h')$. Denoting by $\lambda_{h,q(h)} \equiv \sigma^2_{q(h)}/\sigma^2_h$, if for any h

$$\lambda_h - 1 - \frac{\lambda_{h,q(h)} - 1}{(1 + q(h) - h)^2} > 0 \tag{30}$$

then $q(h) < q(h')$.

2- $q(h) = q^{NB}(h)$ and $q(h') = q^{NB}(h')$, then $q(h) < q(h')$.

3- $q(h) = q^{DP}(h)$ while $q(h') = q^{NB}(h')$ then $q(h) < q(h')$.

4- $q(h) = q^{NB}(h)$ while $q(h') = q^{DP}(h')$ then $q(h) < q(h')$ under (30).

This lemma proves that coalitions defined by successive heads cannot be nested but overlap or are disjoint.

Case 1: $q(h) = q^{DP}(h)$ *and* $q(h') = q^{DP}(h')$. For sake of clarity, let us use the following notations $v(z) \equiv \sigma^2_z$, $n(h,q) \equiv 1 + q - h$, $\pi(h,q) \equiv \sum_{z=h}^{q} v(z) / (n(h,q))^2$. Let us introduce the following

$$\Psi(h,q) \equiv X(h,q) - \frac{C_2}{C_1} \cdot Y(n(h,q)). \tag{31}$$

with $X(h,q) \equiv e^{\alpha^2/2 \cdot v(h)} - e^{\alpha^2/2 \cdot \pi(h,q)}$, $Y(n(h,q)) \equiv e^{-\alpha\hat{\varepsilon}(n(h,q))} \left(e^{-\alpha \frac{(n(h,q)-1)(\underline{\varepsilon} - \hat{\varepsilon}(n(h,q)))}{n(h,q)}} - 1 \right)$ and $\widehat{\varepsilon}(n) \equiv \min[((n \log n)/(\alpha(n-1))) + \underline{\varepsilon}, \overline{\varepsilon}]$.

We first show that $Y(n(h,q))$ is concave. Let us consider that n is a continuous variable and, using (21), let us compute $Y''(n)$:

$$Y''(n) = e^{-\alpha\hat{\varepsilon}(n)} e^{-\alpha \frac{(n-1)(\underline{\varepsilon} - \hat{\varepsilon}(n))}{n}} \left[\hat{\varepsilon}'(n) \left(\alpha^2 \frac{(\underline{\varepsilon} - \hat{\varepsilon}(n))}{n^3} + \frac{\alpha}{n^2} \right) + \left(\frac{2\alpha}{n^3} (\underline{\varepsilon} - \hat{\varepsilon}(n)) + \frac{\alpha^2}{n^4} (\underline{\varepsilon} - \hat{\varepsilon}(n))^2 \right) \right]$$

with $\hat{\varepsilon}'(n) = 0$ if $\widehat{\varepsilon}(n) = \overline{\varepsilon}$ and $\hat{\varepsilon}'(n) = (1/(\alpha(n-1)))[1 - ((\log n)/(n-1))]$ if $\widehat{\varepsilon}(n) = ((n \log n)/(\alpha(n-1)))$.

We can rearrange $Y''(n)$ as follows. If $\widehat{\varepsilon}(n) = \overline{\varepsilon}$ and $\hat{\varepsilon}'(n) = 0$ then

$$Y''(n) = e^{-\alpha\hat{\varepsilon}(n)} e^{-\alpha \frac{(n-1)(\underline{\varepsilon} - \hat{\varepsilon}(n))}{n}} (\underline{\varepsilon} - \overline{\varepsilon}) \frac{\alpha}{n^3} \left(2 + \frac{\alpha}{n} (\underline{\varepsilon} - \overline{\varepsilon}) \right).$$

Let us concentrate on the last term in the above expression, $2 + (\alpha/n)(\underline{\varepsilon} - \overline{\varepsilon})$. As we assume that $\widehat{\varepsilon}(n) = \overline{\varepsilon} \Longleftrightarrow ((n \log n)/(\alpha(n-1))) + \underline{\varepsilon} > \overline{\varepsilon}$, hence this amounts that $2 + (\alpha/n)\left(\underline{\varepsilon} - \overline{\varepsilon}\right) > 2 - \log n/(n-1)$. It can be easily checked that $2 - \log n/(n-1)$ is positive for any $n \in [1, +\infty[$. Hence, when $\widehat{\varepsilon}(n) = \overline{\varepsilon}$ and $\hat{\varepsilon}'(n) = 0$, $Y''(n) < 0$.

Now consider the case $\widehat{\varepsilon}(n) = n \log n/\alpha(n-1)$. Then $\hat{\varepsilon}'(n) = (1/\alpha(n-1))[1 - (\log n/(n-1))]$ and we can rearrange $Y''(n)$ and get

$$Y''(n) = e^{-\alpha\hat{\varepsilon}(n)} e^{-\alpha\frac{(n-1)(\underline{\varepsilon}-\hat{\varepsilon}(n))}{n}} \frac{1}{n^2(n-1)} \left[1 - \frac{2\log n}{n-1} - 2\log n + n\left(\frac{\log n}{n-1}\right)^2\right].$$

It can be easily checked that the expression in brackets is negative for any $n \in [2, +\infty[$, implying that $Y(n)$ is concave for any $n \in [2, +\infty[$. Taking now n as a discrete variable we can say that $Y(n) - Y(n-1) > Y(n+1) - Y(n)$ for any $n = 2, 3, ..., N$.

Second, consider that the pivotal agents $q(h)$, $q(h')$ are such that they bind the DP condition associated to their respective coalition, then

$$\Psi(h, q(h)) \geq 0 \text{ and } \Psi(h, q(h) + 1) < 0. \tag{32}$$

and for $q(h')$ we have

$$\Psi(h', q(h')) \geq 0 \text{ and } \Psi(h', q(h') + 1) < 0. \tag{33}$$

We aim to provide the sufficient conditions so that when $h' > h$ then $q(h') > q(h)$. We adopt the following notations for the following partial differences $\triangle_q \Psi(h, q(h)) \equiv \Psi(h, q(h) + 1) - \Psi(h, q(h))$, $\triangle_h \Psi(h, q(h)) \equiv \Psi(h + 1, q(h)) - \Psi(h, q(h))$, $\triangle_{qq} \Psi(h, q(h)) \equiv \triangle_q \Psi(h, q(h) + 1) - \triangle_q \Psi(h, q(h))$, $\triangle_q X(h, q(h)) \equiv X(h, q(h) + 1) - X(h, q(h))$, $\triangle_h X(h, q(h)) \equiv X(h + 1, q(h)) - X(h, q(h))$.

When the coalition $S = \{h, h + 1, ..., q(h)\}$ is DP, $q(h)$ is such that

$$\triangle_q \Psi(h, q(h)) = \triangle_q X(h, q(h)) - \frac{C_2}{C_1} \cdot [Y(1 + n(h, q(h))) - Y(n(h, q(h)))] < 0. \tag{34}$$

Given $Y\left(n(h,q)\right)$ is concave, then (34) implies

$$\begin{aligned} \triangle_q X\left(h,q\left(h\right)\right) &< \frac{C_2}{C_1}\cdot\left[Y\left(1+n\left(h,q\left(h\right)\right)\right)-Y\left(n\left(h,q\left(h\right)\right)\right)\right] \\ &< \frac{C_2}{C_1}\cdot\left[Y\left(n\left(h,q\left(h\right)\right)\right)-Y\left(n\left(h,q\left(h\right)\right)-1\right)\right]. \end{aligned} \tag{35}$$

Since $n\left(h+1,q\right)=n\left(h,q\right)-1$ for a given q, we have $Y\left(n(h+1,q(h))\right)=Y\left(n(h,q(h))-1\right).$ Hence,

$$\triangle_h \Psi\left(h,q\left(h\right)\right)=\triangle_h X\left(h,q\left(h\right)\right)-\frac{C_2}{C_1}\cdot\left[Y\left(n(h+1,q(h))\right)-Y\left(n(h,q(h))\right)\right]$$

can be written as follows

$$\triangle_h \Psi\left(h,q\left(h\right)\right)=\triangle_h X\left(h,q\left(h\right)\right)+\frac{C_2}{C_1}\cdot\left[Y\left(n(h,q(h))\right)-Y\left(n(h,q(h))-1\right)\right].$$

Further, we aim to find a sufficient condition so that

$$\triangle_h \Psi\left(h,q\left(h\right)\right)=\triangle_h X\left(h,q\left(h\right)\right)+\frac{C_2}{C_1}\cdot\left[Y\left(n(h,q)\right)-Y\left(n(h,q)-1\right)\right]>0,\forall h \tag{36}$$

as it would imply

$$\Psi\left(h',q\right)>\Psi\left(h,q\right). \tag{37}$$

If (37) holds it would imply that the coalition $\left(h',q\left(h\right)\right)$ satisfies the DP condition, which prevents from getting $q\left(h'\right)<q\left(h\right).$ Given (36) and (35) we have

$$\begin{aligned} \triangle_h \Psi\left(h,q\left(h\right)\right)=&\triangle_h X\left(h,q\left(h\right)\right)+\frac{C_2}{C_1}\cdot\left[Y\left(n(h,q)\right)-Y\left(n(h,q)-1\right)\right] \\ >&\triangle_h X\left(h,q\left(h\right)\right)+\triangle_q X\left(h,q\left(h\right)\right). \end{aligned}$$

So that a sufficient condition for $\triangle_h \Psi\left(h,q\left(h\right)\right)>0$ is to have

$$\triangle_h X\left(h,q\left(h\right)\right)+\triangle_q X\left(h,q\left(h\right)\right)>0.$$

We approximate $\triangle_h X\left(h,q\left(h\right)\right)$ and $\triangle_q X\left(h,q\left(h\right)\right)$ by their derivatives. To this aim, we consider that population is a continuum of households (i being a continuous variable) and that an individual i is characterized by the risk $\nu(i)$. Hence the risk premium writes

$$\pi(h,q)=\frac{\int_h^q \nu(z)\,dz}{(n(h,b))^2}$$

and $\triangle_h\, X(h,q(h))$ is approximated by

$$X_h(h,q)=e^{\alpha^2/2\cdot v(h)}v'(h)-e^{\alpha^2/2\cdot\pi(h,q)}\pi_h(h,q)$$

while $\triangle_q\, X(h,q(h))$ is approximated by

$$X_q(h,q)=-e^{\alpha^2/2\cdot\pi(h,q)}\pi_q(h,q).$$

Hence,

$$X_h(h,q)+X_q(h,q)=e^{\alpha^2/2\cdot v(h)}v'(h)-e^{\alpha^2/2\cdot\pi(h,q)}\pi_h(h,q)-e^{\alpha^2/2\cdot\pi(h,q)}\pi_q(h,q)>0\Rightarrow$$

$$X_h(h,q)+X_q(h,q)>0\Leftrightarrow e^{\alpha^2/2\cdot(v(h)-\pi(h,q))}v'(h)-\pi_h(h,q)-\pi_q(h,q)>0\Leftrightarrow$$

where

$$\pi_h(h,q)=-\frac{v(h)}{(n(h,q))^2}+2\frac{\int_h^q v(z)\,dz}{(n(h,q))^3}$$

and

$$\pi_q(h,q)=\frac{v(q)}{(n(h,q))^2}-2\frac{\int_h^q v(z)\,dz}{(n(h,q))^3}.$$

Since $v(h)>\pi(h,q)$, a sufficient condition to get $X_h(h,q)+X_q(h,q)>0$ is

$$v'(h)-\pi_h(h,q)-\pi_q(h,q)>0$$

which approximates

$$v(h+1)-v(h)-\frac{v(q)-v(h)}{(1+q-h)^2}>0$$

and is equivalent to

$$\lambda_h - 1 - \frac{\lambda_{h,q} - 1}{(1 + q - h)^2} > 0.$$

This condition amounts to saying that the growth rate of idiosyncratic risk between h and q, $\lambda_{h,q+1} - 1$ must not be too high, in other terms the distribution of idiosyncratic risks must not be too heterogenous.

Case 2: $q(h) = q^{NB}(h)$ and $q(h') = q^{NB}(h')$. Consider two coalitions, $S = \left\{h, h+1, ..., q^{NB}(h)\right\}$ and $S' = \left\{h', h'+1, ..., q^{NB}(h')\right\}$ with $h < h'$ and suppose by contradiction that $q(h) > q(h')$. Equivalently, $S' \subset S$. Since $n(h', q(h')) < n(h, q(h))$, S is consecutive and $q(h) = q^{NB}(h)$, we have

$$\pi(\{h, ..., h + n(h', q(h')) - 1\}) > \pi(\{h, h+1, ..., h + n(h, q(h)) - 1\}) = \pi(\{h, h+1, ..., q(h)\}). \tag{38}$$

Moreover, as $h < h'$, we have

$$\pi(\{h', ..., h' + n(h', q(h')) - 1\}) = \pi(\{h', ..., q(h')\}) > \pi(\{h, ..., h + n(h', q(h')) - 1\}) \tag{39}$$

We deduce from (38) and (39) that

$$\pi(\{h', ..., q(h')\}) > \pi(\{h, ..., q(h)\}).$$

Members in S' are accepted in a coalition S generating a lower risk premium. This contradicts the initial assumption that $S' = \{h', h'+1, ..., q(h')\}$ satisfies the no blocking condition.

Case 3: $q(h) = q^{DP}(h)$ and $q(h') = q^{NB}(h')$. As $q^{DP}(h) < q^{NB}(h)$ for any h, and from Case 2 above $q^{NB}(h) < q^{NB}(h')$, we deduce that $q(h) < q(h')$.

Case 4: $q(h) = q^{NB}(h)$ and $q(h') = q^{DP}(h')$. As $q^{NB}(h) < q^{DP}(h)$ and from Case 1 above we have $q^{DP}(h) < q^{DP}(h')$, it turns out that $q(h) < q(h')$.

This ends the proof of Lemma 3.

A.4.2 *(ii)*

Let us consider $P^* = \{S_1^*, ..., S_j, ..., S_J^*\}$ the DP-core partition associated to $\mathbf{I}$. We study an increase of p from p to $p > \widetilde{p}$. We denote by $q_j^*(h;p)$ the pivotal agent of coalition S_j^* with head h and given p. Hence, we have from Proposition 2 that $q_1^*(1;p)$ the pivotal agent of S_1^* is given by

$$q_1^*(1;p) = \min\left\{q_1^{DP}(1;p), q_1^{NB}(1;p)\right\}.$$

※ Consider $q_1^*(1;p) = q_1^{NB}(1;p) < q_1^{DP}(1;p)$,

(i) If $q_1^*(1;\widetilde{p}) = q_1^{NB}(1;\widetilde{p})$ then from conditions (22) and (23), $q_1^*(1;\widetilde{p}) = q_1^{NB}(1;\widetilde{p}) = q_1^{NB}(1;p)$.

(ii) If $q_1^*(1;\widetilde{p}) = q_1^{DP}(1;\widetilde{p}) < q_1^{NB}(1;\widetilde{p})$ then from conditions (22) and (23), we deduce $q_1^*(1;\widetilde{p}) = q_1^{DP}(1;\widetilde{p}) < q_1^{NB}(1;\widetilde{p}) = q_1^{NB}(1;p)$.

※ Consider $q_1^*(1;p) = q_1^{DP}(1;p) < q_1^{NB}(1;p)$. Using (22) and (23) we deduce that $q_1^*(1;\widetilde{p}) = q_1^{DP}(1;\widetilde{p}) < q_1^{NB}(1;\widetilde{p}) = q_1^{NB}(1;p)$. Moreover from the DP condition (11), we have $q_1^*(1;\widetilde{p}) = q_1^{DP}(1;\widetilde{p}) < q_1^{DP}(1;p)$. We thus deduce that $q_1^*(1;\widetilde{p}) \leq q_1^*(1;p)$. From Lemma 3, we have : $q^*(q_1^*(1;p)+1;p) \geq \widetilde{q}^*(\widetilde{q}_1^*(1;\widetilde{p})+1;\widetilde{p})$. Iterating this reasoning leads to Proposition 4. The argument is similar when considering an increase of δ.

A.4.3 *(iii)* A Simple Example

A society $\mathbf{I}$ is formed by 4 agents. For a given p, let us assume that the DP core partition is such that $\mathcal{P}^* = \left\{S_{1,2}^*, S_{3,4}^*\right\}$ with $S_{1,2}^*$ DP-binding so that it satisfies

$$C_1 X\left(2;\sigma_1^2;p\right) > C_2 Y(2), \tag{40}$$

$$\sigma_2^2 \leq 3\sigma_1^2 \text{ and } \sigma_3^2 > \left(\sigma_1^2+\sigma_2^2\right)\frac{5}{4}. \tag{41}$$

and $S_{3,4}^*$ satisfies both following inequalities

$$C_1 X\left(2;\sigma_3^2;p\right) > C_2 Y(2)$$

$$\sigma_4^2 \leq 3\sigma_3^2.$$

The associated aggregate risk premium is

$$\overline{\pi}\left(\mathcal{P}^{*}\right)=\frac{\alpha}{8}\left[2\frac{\sigma_1^2+\sigma_2^2}{4}+2\frac{\sigma_3^2+\sigma_4^2}{4}\right].$$

With $p'<p$, the DP core partition becomes $\mathcal{P}'^{*}=\left\{S_1^*,S_{2,3,4}^*\right\}$. It must satisfy the following inequalities amounting to say that (i)$\{1,2\}$is no more DP, (ii) $\{2,3,4\}$is DP, (iii) individual 3 is accepted by 2 and (iv) individual 4 is accepted by 2 and 3.

- (i) $C_1X\left(2;\sigma_1^2;p'\right)<C_2Y\left(2\right)$
- (ii) $C_1X\left(3;\sigma_2^2;p'\right)>C_2Y\left(3\right)$
- (iii) $\sigma_2^2>\left(\sigma_2^2+\sigma_3^2\right)/4$
- (iv) $sigma_4^2>5\left(\sigma_2^2+\sigma_3^2\right)/4$

$(i)\ C_1X\left(2;\sigma_1^2;p'\right)<C_2Y\left(2\right),(ii)C_1X\left(3;\sigma_2^2;p'\right)>C_2Y\left(3\right)\ (iii)\sigma_2^2>\left(\sigma_2^2+\sigma_3^2\right)/4$ and$(iv)\ \sigma_4^2>5\left(\sigma_2^2+\sigma\right.$

The associated aggregate risk premium is

$$\overline{\pi}\left(\mathcal{P}'^{*}\right)=\frac{\alpha}{8}\left[\sigma_1^2+3\frac{\sigma_2^2+\sigma_3^2+\sigma_4^2}{9}\right].$$

Notice that

$$\overline{\pi}\left(\mathcal{P}'^{*}\right)<\overline{\pi}\left(\mathcal{P}^{*}\right)$$

is equivalent to

$$\sigma_1^2<\frac{\sigma_2^2+\sigma_3^2+\sigma_4^2}{6}.$$

Given that

$$C_1X\left(n;\sigma_h^2;p\right)\equiv\frac{\delta}{1-\delta}pe^{\frac{\alpha}{2}\sigma_\nu^2}\left(e^{\left(\alpha^2/2\right)\sigma_h^2}-e^{\left(\alpha^2/\left(2n^2\right)\right)\sum_{z\in S}\sigma_z^2}\right)$$

and

$$C_2Y\left(n\right)\equiv e^{-\alpha\hat{\varepsilon}(n)-\alpha\underline{\nu}}\left(e^{-\alpha\frac{(n-1)(\underline{\varepsilon}-\hat{\varepsilon}(n))}{n}}-1\right)$$

with

$$\hat{\varepsilon}(2) \equiv \min\left[2\log 2/\left(\alpha\right) + \underline{\varepsilon}, \overline{\varepsilon}\right] = 2\log 2/\alpha + \underline{\varepsilon}$$

and

$$\hat{\varepsilon}(3) = 3\log 3/2\alpha + \underline{\varepsilon}.$$

We have to find the λ_i and the p satisfying the following set of 10 inequalities:

$$\frac{\delta}{1-\delta} p e^{\frac{\alpha}{2}\sigma_\nu^2} e^{\frac{\alpha^2}{2}\sigma_1^2}\left[1 - e^{\frac{\alpha^2}{2}\sigma_1^2\left(\frac{1+\lambda_1}{2^2}-1\right)}\right] \geq e^{-(2\log 2+\alpha\underline{\varepsilon})-\alpha\underline{\nu}}, \tag{42}$$

$$\lambda_1 < 3, \tag{43}$$

$$\lambda_1\lambda_2 > (\lambda_1 + 1)\,\frac{5}{4}. \tag{44}$$

$$\lambda_3 < 3 \tag{45}$$

$$\frac{\delta}{1-\delta} p e^{\frac{\alpha}{2}\sigma_\nu^2} e^{\frac{\alpha^2}{2}\sigma_3^2}\left[1 - e^{\frac{\alpha^2}{2}\sigma_3^2\left(\frac{1+\lambda_3}{2^2}-1\right)}\right] \geq e^{-(2\log 2+\alpha\underline{\varepsilon})-\alpha\underline{\nu}} \tag{46}$$

$$\frac{\delta}{1-\delta} p' e^{\frac{\alpha}{2}\sigma_\nu^2} e^{\frac{\alpha^2}{2}\sigma_1^2}\left[1 - e^{\frac{\alpha^2}{2}\sigma_1^2\left(\frac{1+\lambda_1}{2^2}-1\right)}\right] < e^{-(2\log 2+\alpha\underline{\varepsilon})-\alpha\underline{\nu}}, \tag{47}$$

$$\frac{\delta}{1-\delta} p' e^{\frac{\alpha}{2}\sigma_\nu^2} e^{\frac{\alpha^2}{2}\sigma_2^2}\left[1 - e^{\frac{\alpha^2}{2}\sigma_2^2\left(\frac{1+\lambda_2+\lambda_2\lambda_3}{3^2}-1\right)}\right] > e^{-(3\log 3/2+\alpha\underline{\varepsilon})-\alpha\underline{\nu}} \tag{48}$$

$$\lambda_2 < 3 \tag{49}$$

$$\lambda_2\lambda_3 < (\lambda_2 + 1)\,\frac{5}{4} \tag{50}$$

and

$$1 < \frac{\lambda_1 + \lambda_1\lambda_2 + \lambda_1\lambda_2\lambda_3}{6} \tag{51}$$

Consider that, for any σ_1^2, λ_1 we can find parameters $\{\delta, p, \alpha, \sigma_\nu^2, \underline{\varepsilon}, \underline{\nu}\}$ such that (42) is satisfied. Let us set $\lambda_1 = 2, \lambda_2 = 2.9$ and $\lambda_3 = 1.1$, it is then easy to check that (43), (44), (45), (49), (50), (51) are satisfied. It is also easy to check that for any $\sigma_1^2 > 0$ and given the values of λ_1, λ_2 and λ_3 the LHS of (46) is higher than the LHS of (42). Hence, (46) is also satisfied. Since both RHS of (42) and (47) are the same and the LHS of (47) is lower than the RHS of (42) because $p' < p$, then we can always set p' so that (47) is satisfied. Further, it can be easily checked that the RHS of (48) is lower than the LHS of (47). Hence, a sufficient condition for (48) to be satisfied would be to set p' so that the LHS of (47) is lower but close from the RHS of (47) and to have that the LHS of (48) is higher than the LHS of (47) that is

$$e^{\frac{\alpha^2}{2}\sigma_2^2}\left[1 - e^{\frac{\alpha^2}{2}\sigma_2^2\left(\frac{1+\lambda_2+\lambda_2\lambda_3}{3^2}-1\right)}\right] > e^{\frac{\alpha^2}{2}\sigma_1^2}\left[1 - e^{\frac{\alpha^2}{2}\sigma_1^2\left(\frac{1+\lambda_1}{2^2}-1\right)}\right]$$

which is equivalent to

$$e^{\frac{\alpha^2}{2}\lambda_1\sigma_1^2} - e^{\frac{\alpha^2}{2}\lambda_1\sigma_1^2\left(\frac{1+\lambda_2+\lambda_2\lambda_3}{3^2}\right)} > e^{\frac{\alpha^2}{2}\sigma_1^2} - e^{\frac{\alpha^2}{2}\sigma_1^2\left(\frac{1+\lambda_1}{2^2}\right)}.$$

Given $\lambda_1 = 2, \lambda_2 = 2.9$ and $\lambda_3 = 1.1$, it can be checked that for any positive $\alpha^2\sigma_1^2$ the above inequality is satisfied . This completes the proof.

A.5 Computation of consumption correlations $r^2_{c_i,c_{i'}\in v}$ within and outside coalitions

Within a village v, $r^2_{c_i,c_{i'}\in v}$, the correlation between the consumptions of two households i and i', denoted by c_i and $c_{i'}$, depends on whether i and i' belong to the same coalition or not.

If i and i' belong to the same coalition S_j and since $c_z = \omega_z + \nu_t + \frac{\sum\limits_{k\in S_{j,t}(z)} \varepsilon_{k,t}}{n_{j,t}}$ for $z = i, i'$, we have

$$r^2_{(c_i,c_{i'})} = \frac{(cov\,(c_i, c_{i'}))^2}{var\,(c_i)\cdot var\,(c_{i'})}$$

with

$$cov\,(c_i, c_{i'}) = \mathbb{E}\left[(c_i \text{ - } \mathbb{E}\,(c_i)\ )\,(c_i'\text{- } \mathbb{E}\,(c_{i'})\ )\right]$$

leading to

$$cov\,(c_i, c_{i'}) = \mathbb{E}\left[\left(\nu_t + \frac{\sum\limits_{k\in S_j,t} \varepsilon_k, t}{n_j, t}\right)^2\right]$$

and for $z = i, i'$

$$var\,(c_z) = \mathbb{E}\,[c_z - \mathbb{E}\,(c_z)]^2$$

leading to

$$var\left(c_z\right) = \mathbb{E}\left[\left(\nu_t + \frac{\sum\limits_{k\in S_j,t} \varepsilon_k, t}{n_j, t}\right)\right]^2.$$

We thus have

$$r^2_{(c_i, c_{i'})} = \frac{\left(\sigma^2_\nu + \frac{\sum_{k\in S_j}\sigma^2_k}{(n_j)^2}\right)^2}{\left(\sigma^2_\nu + \frac{\sum_{k\in S_j}\sigma^2_k}{(n_j)^2}\right)\left(\sigma^2_\nu + \frac{\sum_{k\in S_j}\sigma^2_k}{(n_j)^2}\right)}$$

which is obviously equal to 1 when i and i' belong to the same coalition.

If i and i' do not belong to the same coalition S_j, the same reasoning leads to

$$r^2_{(c_i, c_{i'})} = \frac{\left(\sigma^2_\nu\right)^2}{\left(\sigma^2_\nu + \frac{\sum_{k\in S_j}\sigma^2_k}{(n_j)^2}\right)\left(\sigma^2_\nu + \frac{\sum_{k\in S_j}\sigma^2_k}{(n_j)^2}\right)}$$

which is obviously lower than 1, and even nil when $\sigma^2_\nu = 0$, that is to say when there is no aggregate income risk at the village level.

B Empirical results

B.1 Econometric Estimates

B.1.1 Tables

Table 1: Cross Village Results
Without Social Control

	Dependent variable: Correlation of consumption $\mathbf{r^2_{c_i,c_{i'}\in v}}$				
	(1)	(2)	(3)	(4)	(5)
Distance in income risk $\lvert\sigma^2_{\mathbf{i}} - \sigma^2_{\mathbf{i'}}\rvert$	-0.0240***	-0.0214***	-0.0221***	-0.0221***	-0.0192***
	(0.0052)	(0.0055)	(0.0054)	(0.0053)	(0.0051)
Difference in control variables $\lvert\mathbf{z_i} - \mathbf{z_{i'}}\rvert$	No	Yes	Yes	Yes	Yes
Sum of idiosyncratic risks $\sigma^2_{\mathbf{i}} + \sigma^2_{\mathbf{i'}}$	No	No	Yes	Yes	Yes
Sum of control variables $\mathbf{z_i} + \mathbf{z_{i'}}$	No	No	Yes	Yes	Yes
Individual level of idiosyncratic income risk $\sigma^2_{\mathbf{i}}$	No	No	No	Yes	Yes
Village FE $\alpha_{\mathbf{v}}$	No	No	No	No	Yes
Total number of observations	30362	26230	26230	26230	26230

This table presents the results of estimations of Equation (13) (see section 5.2.3).
An observation is a dyad of households in a given village. For a detailed description of the variables used in the estimations, see Section 5.2.2. Control variables have been selected by a lasso procedure from all household-level variables available from the baseline survey of the panel dataset, and eventually include: numbers of household members (total and adults), average age and education and maximum education among household members.
The standard errors reported in parentheses are dyadic-robust, following the methodology of Fafchamps & Gubert (2007). Significance levels: * 0.1; ** 0.05; *** 0.01.

Table 2: Cross Village Results
With Social Control

	Dependent variable: Correlation of consumption $\mathbf{r^2_{c_i,c_{i'}\in v}}$				
	(1)	(2)	(3)	(4)	(5)
Distance in income risk $\lvert\sigma^2_{\mathbf{i}} - \sigma^2_{\mathbf{i'}}\rvert$	-0.1345**	-0.1384**	-0.1255**	-0.1254**	-0.1422***
	(0.0529)	(0.0554)	(0.0530)	(0.0531)	(0.0488)
Interaction: Risk distance × Social control $\lvert\sigma^2_{\mathbf{i}} - \sigma^2_{\mathbf{i'}}\rvert \times \mathbf{SC_v}$	0.1256**	0.1323**	0.1170**	0.1168**	0.1391***
	(0.0564)	(0.0587)	(0.0561)	(0.0562)	(0.0518)
Social Control: Probability of sanction in case of default $\mathbf{SC_v}$	Yes	Yes	Yes	Yes	No
Difference in control variables $\lvert\mathbf{z_i} - \mathbf{z_{i'}}\rvert$	No	Yes	Yes	Yes	Yes
Sum of idiosyncratic risks $\sigma^2_{\mathbf{i}} + \sigma^2_{\mathbf{i'}}$	No	No	Yes	Yes	Yes
Sum of control variables $\mathbf{z_i} + \mathbf{z_{i'}}$	No	No	Yes	Yes	Yes
Individual level of idiosyncratic income risk $\sigma^2_{\mathbf{i}}$	No	No	No	Yes	Yes
Village FE $\alpha_{\mathbf{v}}$	No	No	No	No	Yes
Total number of observations	30362	26230	26230	26230	26230

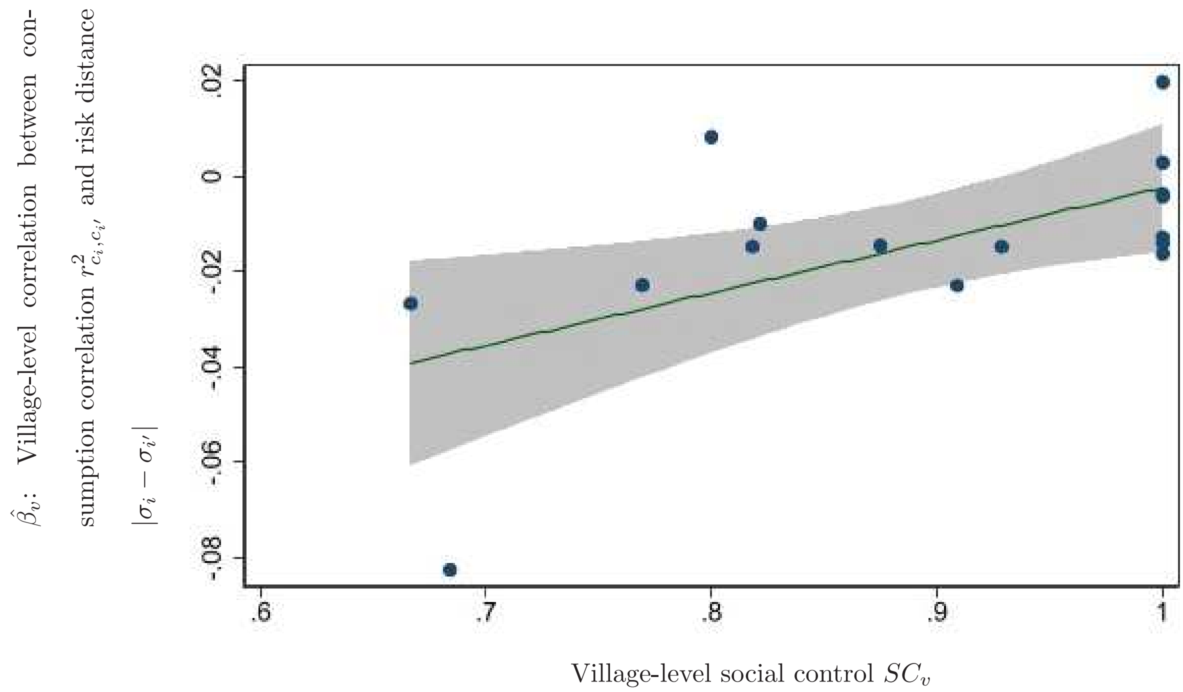
This table presents the results of estimations of Equation (14) (see section 5.2.3).

An observation is a dyad of households in a given village. For a detailed description of the variables used in the estimations, see Section 5.2.2. Control variables have been selected by a lasso procedure from all household-level variables available from the baseline survey of the panel dataset, and eventually include: numbers of household members (total and adults), average age and education and maximum education among household members.

The standard errors reported in parentheses are dyadic-robust, following the methodology of Fafchamps & Gubert (2007). Significance levels: * 0.1; ** 0.05; *** 0.01.

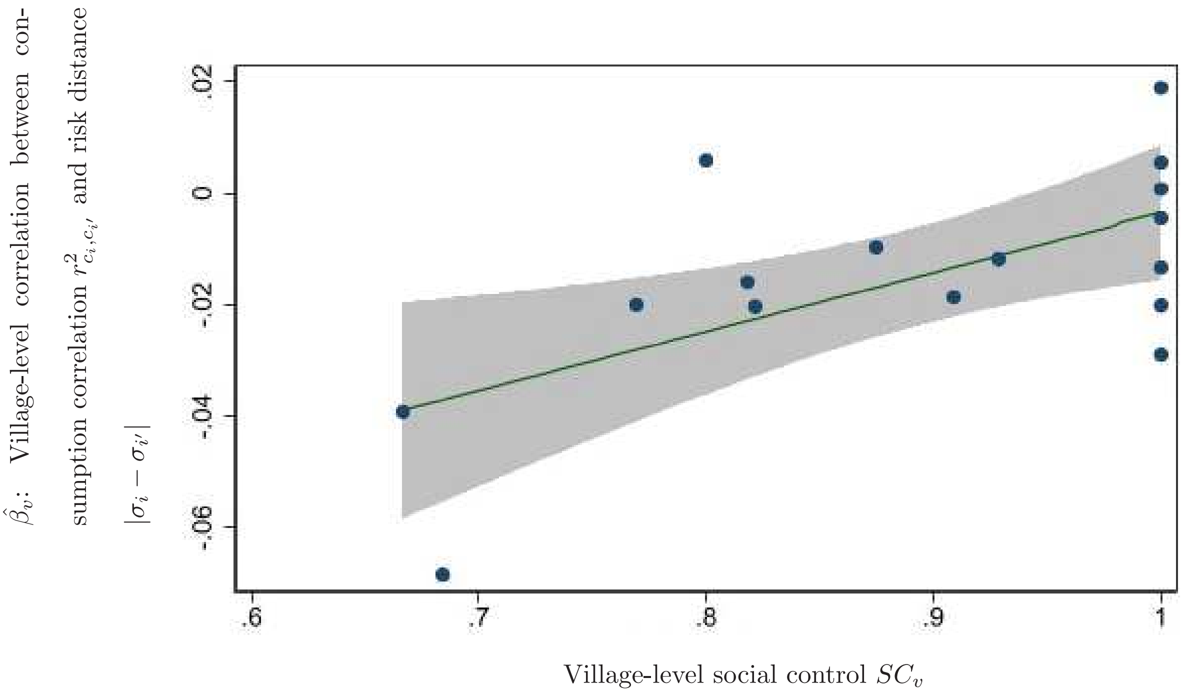
B.1.2 Figures

Figure 1: Risk-Sharing and Social Control - Village-Level Evidence - Without Controls



Each dot represents, on the y-axis, a $\hat{\beta}_v$, that is to say, for each of the sixteen villages v in our sample, the estimated coefficient for the effect of distance in terms of idiosyncratic risk $|\sigma_i - \sigma_{i'}|$ on consumption correlation $r^2_{c_i,c_{i'}}$, for two households i and i' in the same village v, ie the coefficient β of equation (13) estimated at the village level, without any control variables. The shaded area shows the 95% confidence interval. On the x-axis is reported our measure of social control at the village-level, SC_v.

Figure 2: Risk-Sharing and Social Control - Village-Level Evidence - With Controls



Each dot represents, on the y-axis, a $\hat{\beta}_v$, that is to say, for each of the sixteen villages v in our sample, the estimated coefficient for the effect of distance in terms of idiosyncratic risk $|\sigma_i - \sigma_{i'}|$ on consumption correlation $r^2_{c_i,c_{i'}}$, for two households i and i' in the same village v, ie the coefficient β of equation (13) estimated at the village level, with all control variables included. The shaded area shows the 95% confidence interval. On the x-axis is reported our measure of social control at the village-level, SC_v.

B.2 Simulation Results

Table 3: Simulation Results - With and Without Commitment

Villages	Number of coalitions		Aggregate risk premium		Zeta	
	with commitment	without commitment	with commitment	without commitment	with commitment	without commitment
761	10	2020	0.0140	0.0393	0.0030	0.6730
762	10	1879	0.1307	0.0701	0.0030	0.6260
763	9	2236	0.0294	0.0371	0.0027	0.7450
765	10	1878	0.0539	0.0644	0.0030	0.6257
2751	8	1974	0.0176	0.0609	0.0023	0.6577
2753	9	1576	0.0376	0.0558	0.0027	0.5250
2756	8	1081	0.0309	0.0451	0.0023	0.3600
2765	8	1422	0.0329	0.0371	0.0023	0.4737
4957	8	2071	0.0103	0.0534	0.0023	0.6900
4959	8	2035	0.0079	0.0380	0.0023	0.6780
4960	8	1464	0.0101	0.0353	0.0023	0.4877
4961	8	1675	0.0126	0.0358	0.0023	0.5580
5353	7	2077	0.0402	0.0510	0.0020	0.6920
5354	7	1807	0.0262	0.0476	0.0020	0.6020
5359	8	575	0.0447	0.0474	0.0023	0.1913
5360	5	494	0.0041	0.0182	0.0013	0.1643

Figure 3: Simulated distribution of idiosyncratic income risks σ_i in the 16 villages

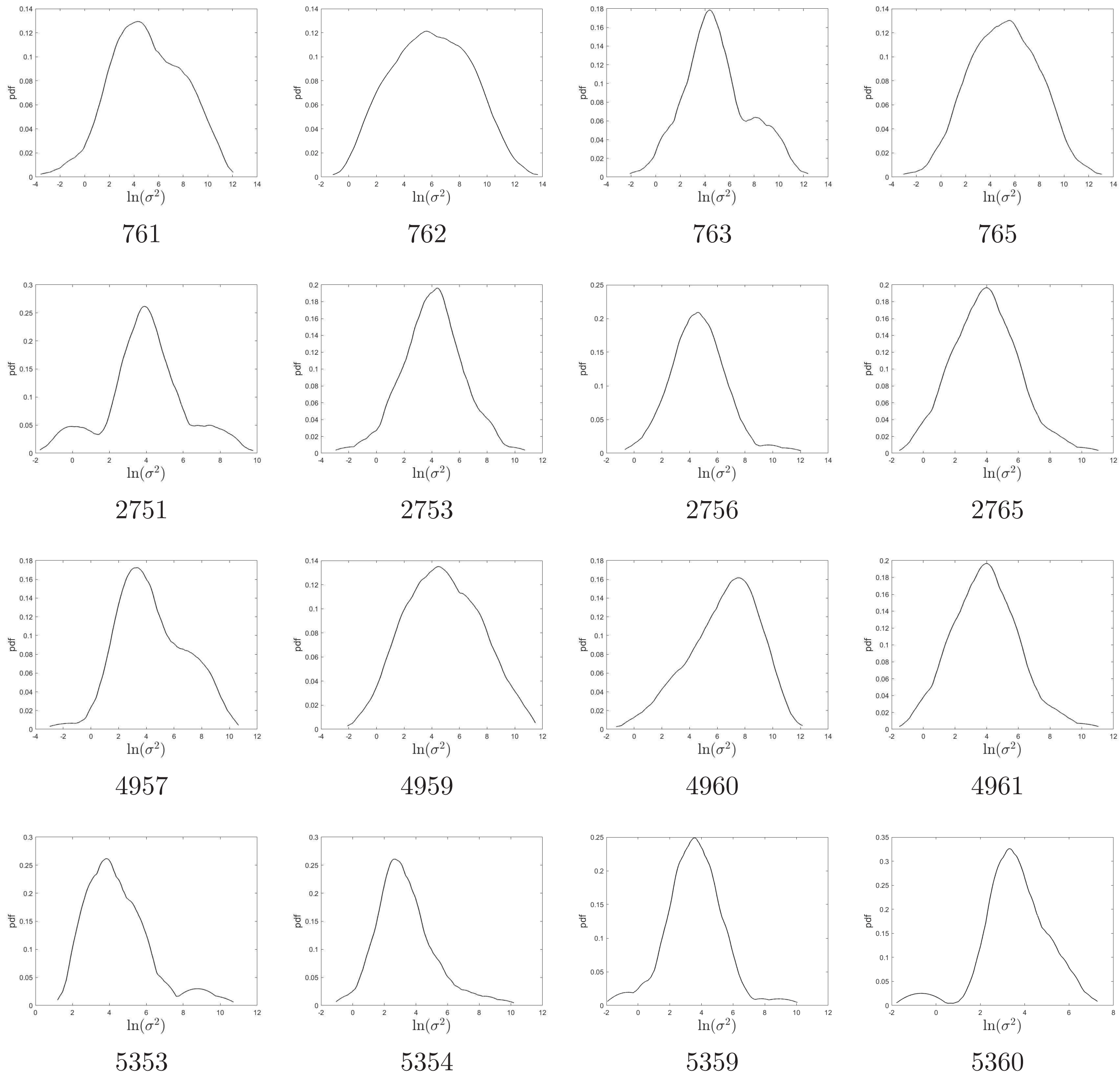


Figure 4: Risk Schedule for Village #5359

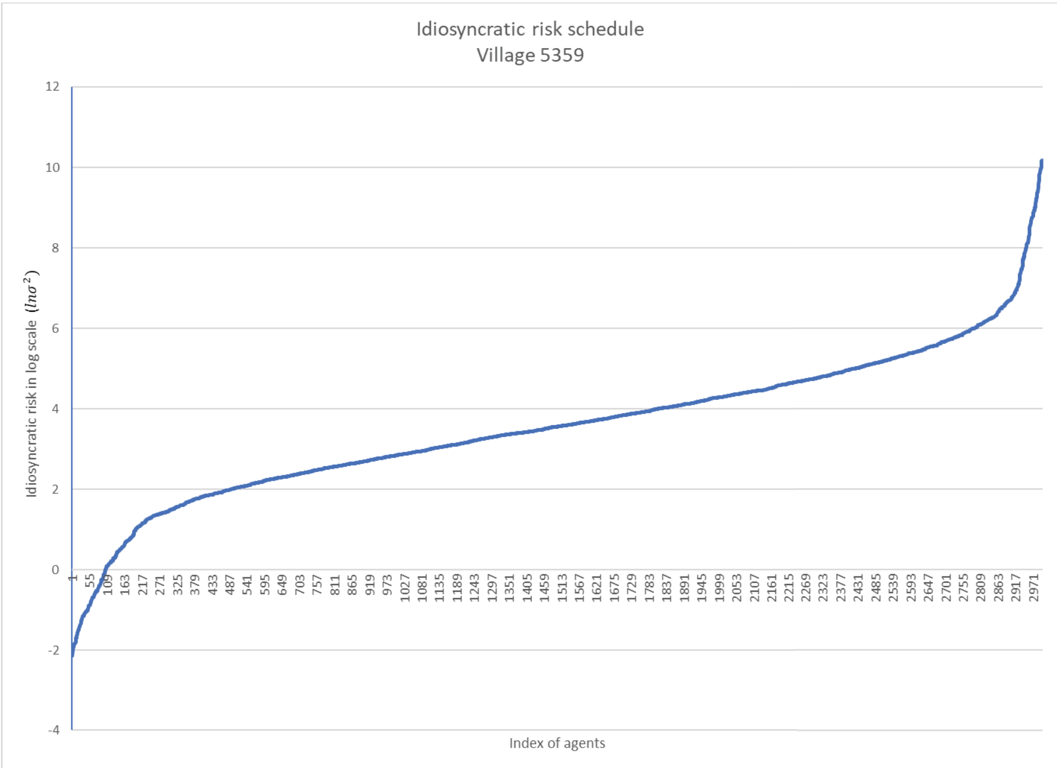
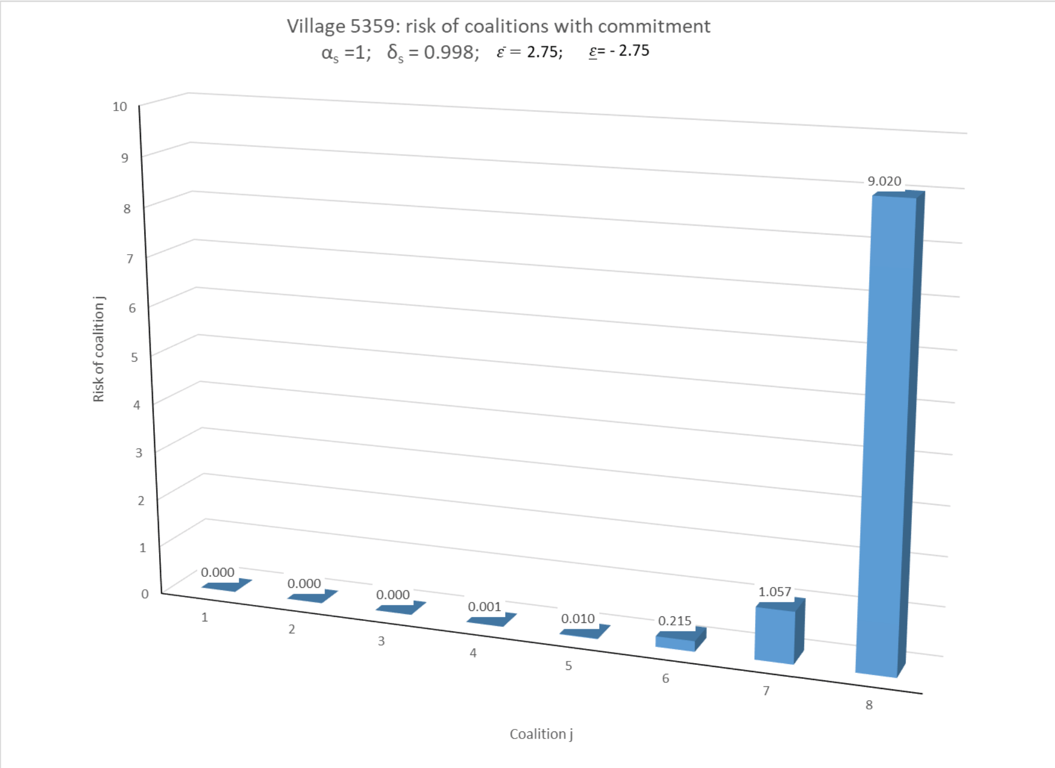


Figure 5: Simulated Risk Premia for Village #5359



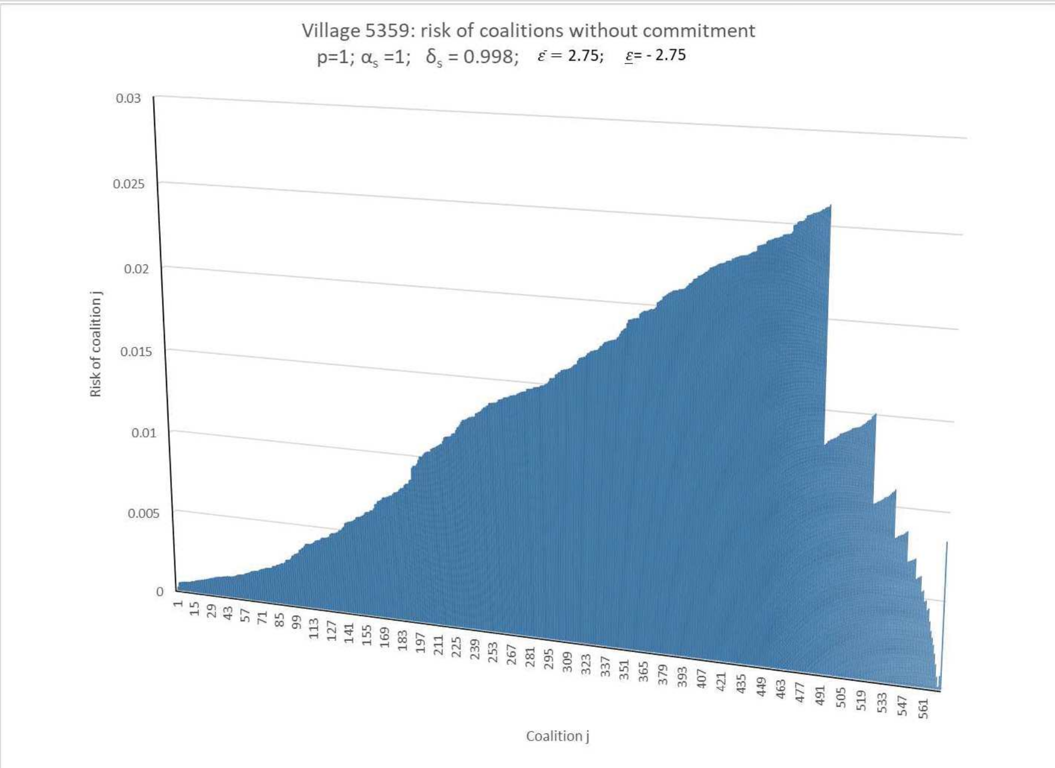


Figure 6: Simulated Number of Coalitions for Village #5359

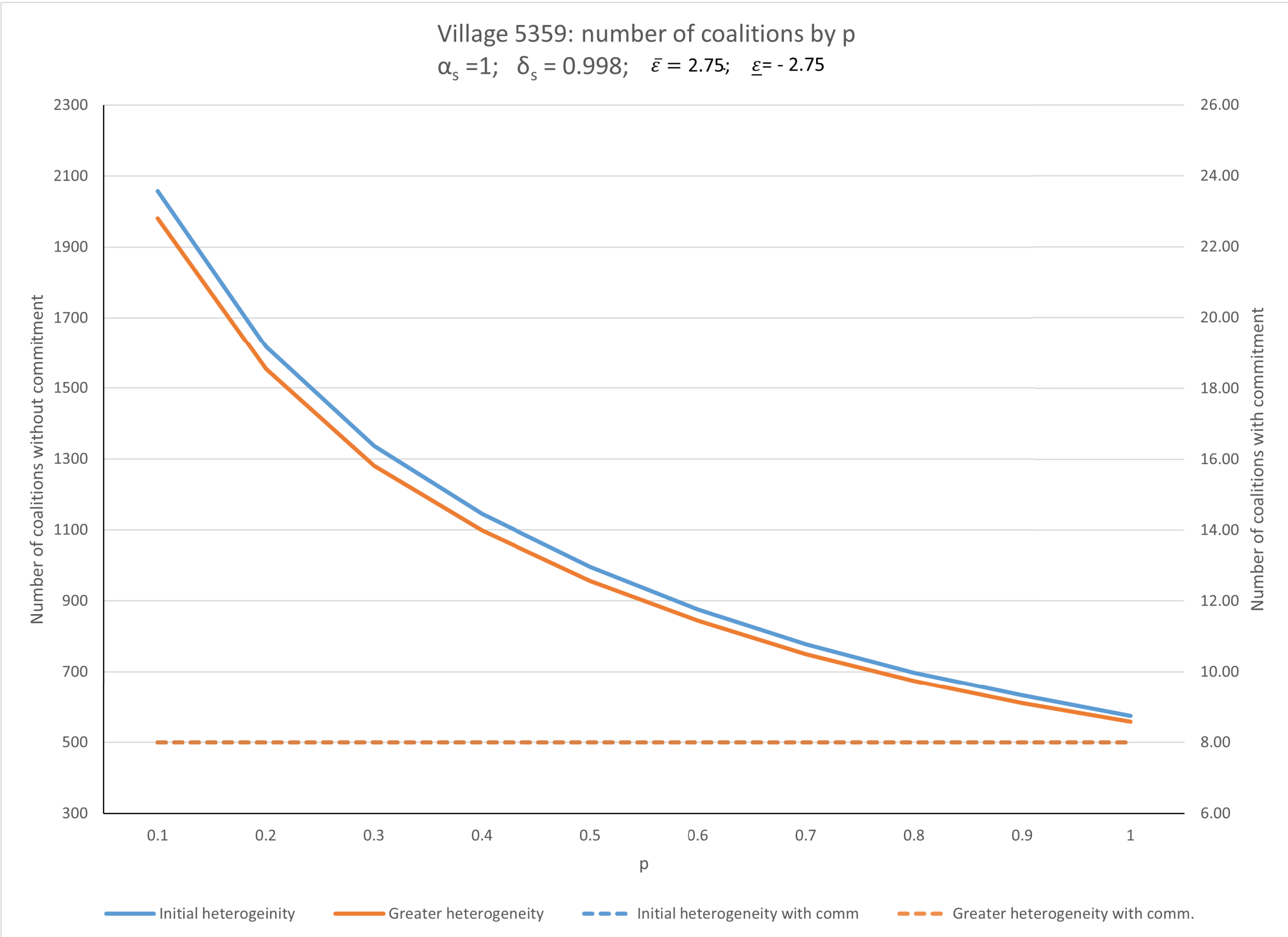
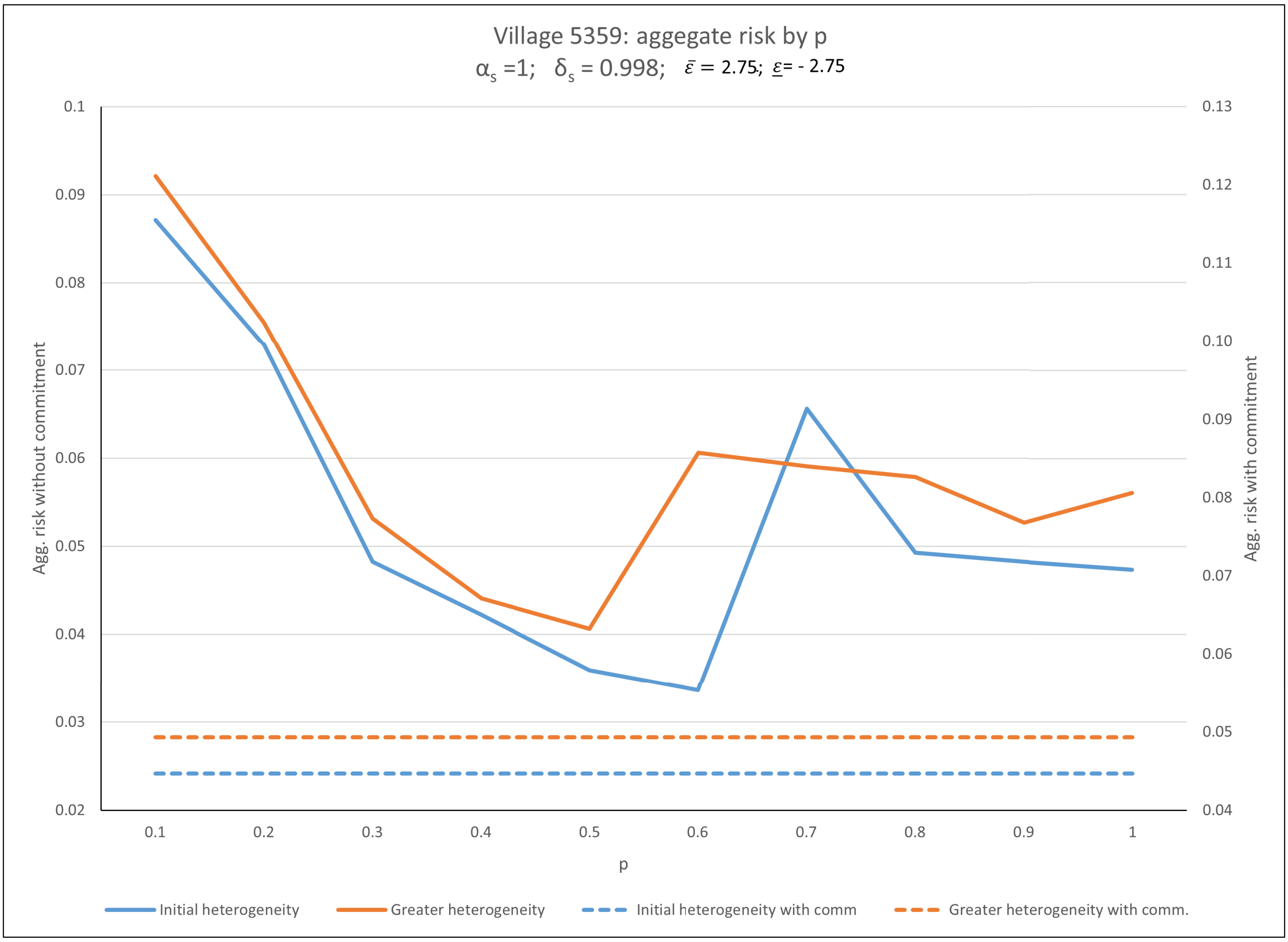


Figure 7: Simulated Aggregate Risk Premium for Village #5359



References

Árpád Ábrahám and Laczó, S. (2018). Efficient Risk Sharing with Limited Commitment and Storage. *Review of Economic Studies*, **85** (3), 1389–1424.

Attanasio, O., Barr, A., Cardenas, J. C., Genicot, G. and Meghir, C. (2012). Risk Pooling, Risk Preferences, and Social Networks. *American Economic Journal: Applied Economics*, **4** (2), 134–167.

Banerjee, S., Konishi, H. and Sönmez, T. (2001). Core in a Simple Coalition Formation Game. *Social Choice and Welfare*, **18** (1), 135–153.

Barro, R. J. and Sala-i-Martin, X. (2004). *Economic Growth (2nd ed.)*. MIT Press.

Bellemare, M. F. and Wichman, C. J. (2020). Elasticities and the inverse hyperbolic sine transformation. *Oxford Bulletin of Economics and Statistics*, **82** (1), 50–61.

Bloch, F., Genicot, G. and Ray, D. (2008). Informal Insurance in Social Networks. *Journal of Economic Theory*, **143** (1), 36–58.

Bold, T. (2009). Implications of Endogenous Group Formation for Efficient Risk-Sharing. *Economic Journal*, **119** (536), 562–591.

— and Broer, T. (2021). Risk Sharing in Village Economies Revisited. *Journal of the European Economic Association*, **19** (6), 3207–3248.

— and Dercon, S. (2014). *Insurance Companies of the Poor*. CEPR Discussion Papers 10278.

Bourlès, R., Bramoullé, Y. and Perez-Richet, E. (2021). Altruism and Risk Sharing in Networks. *Journal of the European Economic Association*, **19** (3), 1488–1521.

Bowles, S. and Gintis, H. (2000). Walrasian Economics in Retrospect. *The Quarterly Journal of Economics*, **115** (4), 1411–1439.

Bramoullé, Y. and Kranton, R. (2007). Public Goods in Networks. *Journal of Economic Theory*, **135** (1), 478–494.

Chiappori, P.-A., Samphantharak, K., Schulhofer-Wohl, S. and Townsend, R. M. (2014). Heterogeneity and risk sharing in village economies. *Quantitative economics*, **5** (1), 1–27.

Chriss, J. J. (2022). *Social Control: An Introduction*. John Wiley & Sons.

Coate, S. and Ravallion, M. (1993). Reciprocity Without Commitment : Characterization and Performance of Informal Insurance Arrangements. *Journal of Development Economics*, **40** (1), 1–24.

Cochrane, J. H. (1991). A Simple Test of Consumption Insurance. *Journal of Political Economy*, **99** (5), 957–976.

Cole, H., Krueger, D., Mailath, G. J. and Park, Y. (2020). *Coalition-Proof Risk Sharing Under Frictions*. CEPR Discussion Papers 14333.

Comola, M. and Fafchamps, M. (2014). Testing Unilateral and Bilateral Link Formation. *The Economic Journal*, **124** (579), 954–976.

Conley, T. G. (1999). Gmm estimation with cross sectional dependence. *Journal of Econometrics*, **92** (1), 1–45.

Croux, C. and Dehon, C. (2010). Influence functions of the spearman and kendall correlation measures. *Statistical Methods & Applications*, **19** (4), 497–515.

De Weerdt, J. and Dercon, S. (2006). Risk-Sharing Networks and Insurance Against Illness. *Journal of Development Economics*, **81** (2), 337–356.

— and Fafchamps, M. (2011). Social identity and the formation of health insurance networks. *The Journal of Development Studies*, **47** (8), 1152–1177.

— and Hirvonen, K. (2016). Risk Sharing and Internal Migration. *Economic Development and Cultural Change*, **65** (1), 63–86.

Dubois, P. (2006). *Heterogeneity of Preferences, Limited Commitment and Coalitions: Empirical Evidence on the Limits to Risk Sharing in Rural Pakistan*. CEPR Discussion Papers 6004.

—, Jullien, B. and Magnac, T. (2008). Formal and Informal Risk Sharing in LDCs: Theory and Empirical Evidence. *Econometrica*, **76** (4), 679–725.

Fafchamps, M. and Gubert, F. (2007). The formation of risk sharing networks. *Journal of Development Economics*, **83** (2), 326 – 350.

— and Lund, S. (2003). Risk-Sharing Networks in Rural Philippines. *Journal of Development Economics*, **71** (2), 261–287.

Farrell, J. and Scotchmer, S. (1988). Partnerships. *The Quarterly Journal of Economics*, **103** (2), 279–297.

Fitzsimons, E., Malde, B. and Vera-Hernández, M. (2018). Group Size and the

Efficiency of Informal Risk Sharing. *Economic Journal*, **128** (612), 575–608.

Foster, A. D. and Rosenzweig, M. R. (2001). Imperfect Commitment, Altruism, And The Family: Evidence From Transfer Behavior In Low-Income Rural Areas. *The Review of Economics and Statistics*, **83** (3), 389–407.

Genicot, G. and Ray, D. (2003). Group formation in risk-sharing arrangements. *The Review of Economic Studies*, **70** (1), 87–113.

Henderson, H. and Alam, A. (2021). The structure of risk-sharing networks. *Empirical Economics*, pp. 1–34.

Hoang, Q., Pasquier-Doumer, L. and Saint-Macary, C. (2021). Ethnicity and risk sharing network formation: Evidence from rural vietnam. *The Journal of Development Studies*, **0** (0), 1–18.

Jaramillo, F., Kempf, H. and Moizeau, F. (2003). Inequality and club formation. *Journal of Public Economics*, **87** (5-6), 931–955.

—, — and — (2015). Heterogeneity and the Formation of Risk-Sharing Coalitions. *Journal of Development Economics*, **114** (C), 79–96.

Karaivanov, A. and Townsend, R. M. (2014). Dynamic financial constraints: Distinguishing mechanism design from exogenously incomplete regimes. *Econometrica*, **82** (3), 887–959.

Kinnan, C. (2022). Distinguishing Barriers to Insurance in Thai Villages. *Journal of Human Resources*, **57** (1), 44–78.

Kocherlakota, N. R. (1996). Implications of Efficient Risk Sharing without Commitment. *Review of Economic Studies*, **63** (4), 595–609.

Laczó, S. (2015). Risk Sharing With Limited Commitment And Preference Heterogeneity: Structural Estimation And Testing. *Journal of the European Economic Association*, **13** (2), 265–292.

Ligon, E. (1998). Risk Sharing and Information in Village Economies. *Review of Economic Studies*, **65** (4), 847–864.

—, Thomas, J. P. and Worrall, T. (2000). Mutual Insurance, Individual Savings and Limited Commitment. *Review of Economic Dynamics*, **3** (2), 216–246.

—, — and — (2002). Informal Insurance Arrangements with Limited Commitment: Theory and Evidence from Village Economies. *Review of Economic Studies*, **69** (1), 209–244.

Meghir, C., Mobarak, A. M., Mommaerts, C. D. and Morten, M. (2019). *Migration and Informal Insurance: Evidence from a Randomized Controlled Trial and a Structural Model*. NBER Working Papers 26082.

Morten, M. (2019). Temporary Migration and Endogenous Risk Sharing in Village India. *Journal of Political Economy*, **127** (1), 1–46.

Platteau, J.-P. (2006). Solidarity norms and institutions in village societies: Static and dynamic considerations. In S.-C. Kolm and J. M. Ythier (eds.), *Foundations*, *Handbook of the Economics of Giving, Altruism and Reciprocity*, vol. 1, *12*, Elsevier, pp. 819–886.

— and Abraham, A. (2002). Participatory Development in the Presence of Endogenous Community Imperfections. *Journal of Development Studies*, **39** (2), 104–136.

Samphantharak, K. and Townsend, R. M. (2009). *Households as Corporate Firms: An Analysis of Household Finance Using Integrated Household Surveys and Corporate Financial Accounting*. Cambridge University Press.

Spearman, C. (1904). General intelligence objectively determined and measured. *American Journal of Psychology*, **15**, 201–293.

Townsend, R. M. (1994). Risk and Insurance in Village India. *Econometrica*, **62** (3), 539–591.

— (2016). Village and larger economies: The theory and measurement of the townsend thai project. *Journal of Economic Perspectives*, **30** (4), 199–220.

— (2017). Financial account manual instruction. In *Townsend Thai Monthly Survey Household Financial Accounting (Month 0-172)*, Harvard Dataverse.

—, Sakunthasathien, S. and Jordan, R. (2013). *Chronicles from the Field: The Townsend Thai Project*. The MIT Press, MIT Press.