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Abstract

This paper reexamines the Rosenstein-Rodan idea of the Big Push but from the supply side view. We incorporate education in a model of the Big Push where industrialization requires skilled labor. We show that initially unequal distribution of wealth can prevent the economy from taking a sustainable industrialization path. Moreover, financial imperfections, by affecting the elasticity of skill supply, imposes a constraint on the set of technologies that can be adopted. In particular, credit imperfections can support selffulfiling beliefs that can either put the economy on a high productivity equilibrium or a low productivity equilibrium. Finally, in this context, it is shown that the new organizational innovations in the North may widen the productivity gap between North and South.

Keywords: Income Distribution, Credit Constraints, Human Capital, Productivity Differences

JEL Classification: D31, I22, O14

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Introduction

The original idea of Rosenstein-Rodan (1943) was concerned manly with income effects associated to the use of increasing returns to scale technologies. The first authors who modeled this idea are Murphy, Shleifer and Vishny (1989). They built a simple static model with demand spillovers. They then show the existence of two equilibria, namely an underdevelopment trap with no industrialization and an equilibrium with industrialization. This argument of "market size" for successfull industrialization has been shown to be important by Ades and Glaeser (1999). They found that openess to international trade is a substitute for a large domestic market. However needless to say that histories of LDCs show that openess is not a sufficient condition for successfull development.

Our paper builds a Supply-Big-Push model. In the context of increasing globalization, it can be thought that "market size" is becoming a less binding constraint on development. We rather argue that the supply side is more relevant to understand the nowadays problem of industrialization.

We start from the empirical and historical evidence that industrialization of LDCs takes the form of the adoption of technologies developed in the North, and the implementation of these technologies necessitates human capital[1]. We then concentrate on knowledge spillovers. In our model, as in MSV, it is not profitable for a single firm to use the modern technology. But as more firm enter, a knowldge of how to use efficiently the technology is generated and thus the cost of entry decreases. Thus the effect of the entry of a firm[2] is not to generate demand for other firms but rather generate knowledge about the modern tehnology which reduces the cost of entry of further entrants.

But as mentioned, human capital is necessary to implement these technologies[3]. Skilled workers are needed to produce, assimilate the new technology and efficiently organize production. We assume that there are fixed costs of entry in the modern sector, which are complementary to unskilled labor[4]. This hypothesis that the fixed cost is complementary to unskilled

[1] This is the assimilationist view of development

[2] We do not totally reject the "market size" effect. In our model, one could argue that the introduction of modern technologies which use skilled workers creates a pool of "wealthy" consumers and thus creates the demand for other firms. But ofen trade liberalization, in countries which lack a strong industrial structure, leads only to more "importations".

[3] It is all the more important as technological progress in the North appears to be skill-biased, see Bernan, Bound and Griliches (94).

[4] Although we don't have capital, one can think about this costs as machines and

labor is the crude assumption which traduces the fact that technical progress is skill-biased. As it wil be clear in the model, skilled workers are needed to “absorb” the cost of organization. Thus this fixed cost can be viewed as a loss of output which comes from the fact that the economy lacks the knowledge to efficiently use the technology.

If the agents could easily finance their education, then the model would exibit one unique equilibrium with full industrialization, namely an equilibrium where all the sectors in the economy adopt the modern technology. However credit markets are imperfect, and because of the indivisibility of the cost of education some agents won't be able to get educated. Because of the knowledge externality and free entry in the modern technology, the wage of skilled workers is increasing with the measure of skilled workers. Consequently, as the decision to go to school depends on the expected return to education, the size of the educated workforce is determined by wealth distribution. This realtion gives rise to multiple equilibria. Therefore we are able to generate a link between wealth distribution and the “size” of the Big Push. However we obtain that the link between wealth distribution and development size is not univoque. When a wealth distribution G dominates in the first order stochastic sens another distribution H, then the high equilibrium under the G-distribution is higher than the high equilibrium under H, but the opposite occurs for the low equilibria.

Weath distribution appears also to be importnat in determining total factor productivity. Indeed, contrary to MSV where technological opportunities are fixed, we allow the country to get access to a menu of modern technologies. Then the novel feature is that the technology adopted is constrained by wealth distribution. Specifically there are multiple equilibria for the productivity and the more equal is initial wealth distribution, the higher the high-equilibrium-productivity of the adopted technology.

In this respect our work is also related to the litterature on inequality and development. However in this litterature, the links between inequality and growth, can be ranged into two categories. The first, surveyed in Benabou (96), is a political economy link. The median voter in an inegalitarian economy is poor and votes for too much redistribution, then reducing the incentives for factor accumulation and thus the growth rate. The second, more related to our paper, underlines that credit constraints prevent the poor from taking economically profitable investments. Thus inequality has bad impacts on growth since it reduces investment opportunities (Galor and Zeira (93), Banerjee and Newman (93) for example).

Our paper shows that, to the extent that there is some irreversibility in

organizational capital complementary to unskilled labor.

technology adoption[5], then initial wealth distribution can have a persistent impact on total factor productivity.

This paper is also related to Acemoglu and Zilliboti (1999) who explain the productivity differential between the North and the South by the fact that machines produced in the North are developed for the north which has more skilled people than Southern countries. Consequently, although using the same machines, LDCs are less productive. However our focus is rather on internal organization of firms. Indeed, we consider that importing a machine is also importing an organization of work. If we consider that there is a complementarily between the quality-productivity of a machine and the quality of the organization of the firm, then on top of not being endowed with the skill level necessary to operate the machine at its best level, developing countries do not have also the right internal organizations of firms to operate the machine at its most efficient level.

In section 2.1 we look at the possible impact of these new "skill consuming" organizational innovations in the North on the development of LDCs. We show that rising organizational costs can have different consequences on the productivities of the technologies adopted by different countries. With multiple equilibria, those countries in the low equilibrium are forced to switch to lower organisational costs technologies (and thus less productive), but those countries on the high equilibrium level can use these more productive technologies.

The main reason is that the supply of skill, under credit imperfection, can not respond enough to rising organizational costs in the low equilibrium level.

The paper is organized as follows. Section 1 presents the model. Section 2 deals with the interaction between income distribution and technology choice. Section 3 presents an extension allowing rent seeking. All the proofs are relegated to an Appendix.

1 The model

1.1 Firms

We consider a small open economy producing a continuum of goods. The world is endowed with a storage technology which yields r units of any good per unit stored[6].

[5] At least in the medium run.

[6] So we don't have capital in our model. Since capital is not central to our discussion, we can abstract from it. Although capital constraint can be an other part of our story.

There is a continuum of sectors each producing a good indexed by $q \in [0,1]$. A good can be produced either with a backstage technology with the production function $y = l$, where l is physical labor, or with an industrial technology[7] which necessitates when implemented at t an entry cost $\Upsilon(\phi_t, X_t)$ in terms of unskilled labor, where ϕ_t is the measure of skilled workers available at t and X_t is the number of industrialized sectors. After the cost of entry is incurred, the production necessitates $\frac{1}{\alpha}$, with $\alpha > 1$, units of skilled labor per unit of output[8].

The expression of the fixed cost deserves some comments. For a mater of tractability we choose a simple form specified by $\Upsilon(\phi_t, X_t) = F(1-\phi_t)/X_t$, where F is a constant, X_t is an aggregate spillover effect which means that the greater the number of firms (or sectors) which have adopted the new technology, the lower the cost of entry of a marginal firm (or sector)[9]. So the effect of human capital is embedded in the term $1/X_t$, which is taken to be external to the firm. Thus it can be interpreted as a "learning by doing" externality *à la* Arrow (1962). As more sectors use the technology there is a public knowledge generated that increases the efficiency use of the modern technology (better understanding of the production process, more efficient organization of labor..)[10]. Here we take the number of industrialized sectors as a proxy of the knowledge generated. Finally, when $X_t = 0$, the cost of entry is infinite, but this is only a simplification, and we could just have supposed that this cost of adoption is too high for a single firm when no other firm adopts the technology[11].

---

[7]Only one firm (a monopolist) can implement the modern technology. We take the assumption that firms live one period. So the fixed cost is incurred each period by the new monopolist, but it is decreasing in time because of the knowledge spillover.

[8]So the fixed cost permits to save on the variable cost of producing one unit of output but the labor required is skilled labor. Allowing a production function in the modern technology which uses unskilled and skilled workers (for example $A(L_s)^{\beta}(L_u)^{1-\beta}$), rather than the $1/\alpha$ units of skilled labor per unit output, complicates seriously the calculus but does not change the analysis.

[9]The economy as a whole preforms better the new technology. But note also that the fact that F is divided by X_t means that there is a kind of "sharing" of the fixed cost between all sectors. Indeed without the term $(1-\phi_t)$, we see that the aggregate fixed cost is constant through the industrialization process and equal to F, but the effect of industrialization is to reduce the cost paid by and individual firm equal to $\frac{F}{X_t}$.

[10]Note that as we choose the fixed cost in terms of unskilled labor, it means that the cost of adoption is a cost of organization complementary to unskilled labor. As X_t will be determined by the mass of skilled workers, the knowledge externality is consequently generated by human capital which allows a better understanding of the technology and allows the reduction of organizational costs.

[11]Or we could take an other specification for the externality, $f(X_t)$ with $f(0) > 0$ and $f' > 0$.

The term $(1-\phi_t)$ is just a way of getting vanishing fixed costs in order to permit the transition towards an economy producing with the constant returns to scale technology using skilled labor $y = \alpha l$ in all sectors[12].

When a firm decides to implement the new technology it replaces the competitive fringe which produced the good with the backstage technology. As in Murphy and *alii.* (1989), because demand is inelastic the price of any good will be 1, and the wage paid to unskilled workers is 1 too (marginal productivity)[13].

At period t each firm takes as given aggregate demand d_t, the wage paid to skilled workers w_t^s, the measure ϕ_t of skilled workers available at t, and the fraction of industrialized sectors X_t. Then the profit of a firm producing an industrialized good q is

$$\pi_t (q, \phi_t, X_t) = a_t d_t - \frac{F(1-\phi_t)}{X_t} \ , \tag{1}$$

where $a_t = 1 - \frac{w_t^s}{\alpha}$. Thus aggregate profits are[14]

$$\Pi_t (\phi_t, X_t) = \int_0^{X_t} \pi_t (q, \phi_t, X_t) \, dq = a_t X_t d_t - F(1-\phi_t) \ , \tag{2}$$

The equilibrium condition on the labor market for skilled people is $X_t d_t / \alpha = \phi_t$. Using the expression of the profit of a firm producing an industrialized good q (equation (1)) and aggregate profits (equation (2)), we get

$$\Pi_t (\phi_t, X_t) = a_t \alpha \phi_t - F(1-\phi_t) = \left(1 - \frac{w_t^s}{\alpha}\right) \alpha \phi_t - F(1-\phi_t) \ , \tag{3}$$

which we will write as $\Pi(w_t^s, \phi_t)$.

1.2 Households

The economy is composed of overlapping generations, and each generation contains a continuum of size 1 of individuals. An agent lives two periods and

[12] All the results derived in the paper are still holding if we take an entry cost of the form F/X_t, that is to say without the term $(1-\phi_t)$. The difference will simply be that there will still remain a cost F of entry at the new stationary equilibrium (we will come back to this case later on), and F will determine the fraction of unskilled people in the long run (the details are available upon request). Just for a matter of analytical convenience, we keep the form $F(1-\phi_t)/X_t$.

[13] Alternatively as we deal with a small open economy, it could be more convenient to assume that prices are given by the rest of the world.

[14] As the identity of the good does not matter, we assume that those sectors which first industrialize are those with lower indexes.

is endowed each period with one unit of labor which he supplies inelastically. The utility of an agent born at $t-1$ is

$$(1-\beta)\int_0^1 \ln x_t(q)dq + \beta \ln e_t \ , \tag{4}$$

where $x_t(q)$ is the consumption of good q and e_t is the bequest he gives to his offspring. Then when income[15] is y_t, utility maximization simply yields[16] $x_t(q) = (1-\beta)y_t$ and $e_t = \beta y_t$. The indirect utility is therefore $U(y_t) = v + 2\ln y_t$, where $v = (1-\beta)\ln(1-\beta) + \beta \ln \beta$.

During the first period of his life he decides wether to acquire education (to become skilled) or not. If he decides not to become skilled he works as an unskilled worker (n) during both periods of life. Becoming skilled requires a fixed indivisible investment h. When the young decides to become skilled, he has to spend h, devote all his "youth-time" to learn and he works as a skilled worker (s) when adult[17][18]. During the second period of his life the agent has an offspring and allocates his income between consumption and a bequest to his child. For simplicity we assume that agents only consume in the second period. When the bequest is sufficient to pay h, the young will go to school (this will be the case in the model given that income is sufficiently higher for skilled people), when it is not the case the young will have to borrow. But financial imperfections will prevent poor people to borrow, and they will stay poor as their parents.

Noting $y_t^n(e_{t-1})$ and $y_t^s(e_{t-1})$, the expected incomes in the second period of life respectively when unskilled and skilled and when the bequest received is e_{t-1}, then a young will decide to become skilled if $U[y_t^s(e_{t-1})] > U[y_t^n(e_{t-1})]$, which is equivalent to $y_t^s(e_{t-1}) > y_t^n(e_{t-1})$.

The world interest rate is r (independent of time), and we assume that because of financial imperfections the borrowing interest rate[19] is $i > r$.

1.3 Education decision

A young individual will face the choice in the first period of his life between going to school or working as a laborer. Given that the indirect utility

[15] Recall that agents consume only in second period of life, thus what we call income is first plus second period earnings

[16] As in Murphy et al (1989), an individual spends an equal fraction of income on each good. Thus demand is the same for all sectors.

[17] The amount h is redistributed in the economy by, say, wages to teachers.

[18] h is in terms of all the goods, that is to say $\int_0^1 h = h$

[19] Refer to Galor and Zeira (1993) to see how to get simply an $i > r$

function he gets depends only on his lifetime income, he will choose the occupation that gives him the higher expected lifetime income.

A young agent born at date $t-1$ who received a bequest $e_{t-1} \geq h$, an who expects future wage of skilled workers w_t^s, will compare the incomes he will get as a skilled and unskilled

$$
\begin{aligned}
y_t^s(e) &= w_t^s + (1+r)(e-h) \ , &(5)\\
y_t^n(e) &= 1 + (1+r)(e+1) \ . &(6)
\end{aligned}
$$

When he decides to acquire education, he pays the cost h, saves the remain amount of his bequest $e-h$ at the rate r and receives the expected wage w_t^s. When he chooses to become a laborer, he saves his bequest and earns the wage equal to 1 each period of his life. Thus he will choose education iff $y_t^s(e) \geq y_t^n(e)$, which is equivalent to

$$
w_t^s - \{(2+r) + (1+r)h\} \geq 0 \ , \tag{7}
$$

We will note $\widehat{w} = (2+r) + (1+r)h$.

We examine now the decision of someone who received $e < h$. The incomes he will get if he is respectively unskilled and skilled are

$$
y_t^n(e) = 1 + (1+r)(e+1) \ , \tag{8}
$$

$$
y_t^s(e) = w_t^s + (1+i)(e-h) \ . \tag{9}
$$

When he chooses to become a laborer, he saves his bequest and earns the wage equal to 1 each period of his life. When he decides to acquire education, he has to borrow the difference between his inherited wealth e and the cost of education h, but at a higher interest rate than r, and receives the expected wage w_t^s. Thus he will choose education iff

$$
w_t^s - \widehat{w} + (i-r)(e-h) \geq 0 \ , \tag{10}
$$

Which is equivalent to

$$
e \geq \tilde{e}(h) = h - \frac{w_t^s - \widehat{w}}{i-r} \ , \tag{11}
$$

Equation (11) traduces the fact that the investment of h on education must give him a reward greater than the extra cost of the funds he gets to finance education.

Lemma 0 Profits are null and the wage of skilled workers at t is given by

$$
w_t^s = w(\phi_t) = \alpha - \frac{F(1-\phi_t)}{\phi_t} \ , \tag{12}
$$

Lemma 0 says that entry in the modern sectors will drive profits to zero. One has to note that profits are null because of the competition between firms who want to attract skilled workers in order to use the modern technology. Profits will be absorbed by wages of skilled workers and the fixed costs. First, because an extra entry of firms into the modern sectors requires skilled workers. But as individuals are unequally endowed with wealth, attracting an extra fraction of young into the education sector necessitates higher wages. So wages will increase until profits are exhausted. Second, as the fixed cost is decreasing, skilled workers' wages will indeed absorb a higher fraction of profits until all individuals have acquired education, and thus the fixed cost has disappeared (at the new stationary equilibrium if full industrialization is achievable)[20].

Definition of equilibrium *A perfect foresight equilibrium is a sequence* $(G_{t-1}, \phi_t, X_t, w(\phi_t))_{t=1}^{\infty}$*, where* G_{t-1} *is the distribution of wealth at* $t-1$*, such that*
i) Given G_{t-1}*, and future expected wage* $w(\phi_t)$ *there are* ϕ_t *young agents at t-1 who decide to become skilled*
ii) Given ϕ_t *,* X_t *sectors industrialize*
iv) ϕ_t *is the fixed point of the mapping* $f_{t-1} : x \longrightarrow 1 - G_{t-1}\left[h - \frac{w(x)-\widehat{w}}{i-r}\right]$*, and* $w(\phi_t)$ *by (12)*
(v) Markets clear

We now state *Proposition 1* which sumarizes the discussion above, and proves the existence of an equlibrium as defined above, using equations (7), and (11).

Assumption 1 : there exists $\theta \in]0,1[$ *such that* $1 - G_0\left(h - \frac{w(\theta)-\widehat{w}}{i-r}\right) > \theta$

Proposition 1 *Under initial wealth distributions satisfying assumption 1 we have*

(i) Given G_{t-1} *the distribution of wealth at date* $t-1$ *and the arbitrage conditions* (7) *and* (11)*, the measure of young agents who will choose education at t exists and is the fixed point of the mapping* $f_{t-1} : x \longrightarrow 1 - G_{t-1}\left[h - \frac{w(x)-\widehat{w}}{i-r}\right]$*. (ii) there exist three equilibrium values of* ϕ_t*, which are* 0*,* $\phi_{t,l} < \phi_{t,h}$

Proof : See Appendix B.1

[20] Remark : in the case we take rather $\frac{F}{X_t}$, the rise of the wage of skilled workers will be limited by the remaining fixed cost F.

The intuition behind the existence of multiple equilibria is the intersection between the first $45°$ line and the concave function f_{t-1}. This result can explain different paterns of indsutrialization speeds between similar countries.

One can take an example to have a first look at the impact of wealth distribution on the equilibrium value of ϕ.

Assume $G(x) = (\frac{x}{\bar{h}})^{\gamma}$ where $\bar{h}$ represents the maximum wealth level in the economy. Define $f(x) = 1 - G\left(h - \frac{w(x)-\widehat{w}}{i-r}\right)$ and $g(x) = h - \frac{w(x)-\widehat{w}}{i-r}$. Then we easily get the following relation $-x\frac{f^{''}}{f^{'}} = 2\left[1 + (1-\gamma)\frac{x}{2}\frac{g^{'}}{g}\right]$.

So take $G(x) = (\frac{x}{\bar{h}})^{\gamma}$ and $H(x) = (\frac{x}{\bar{h}})^{\gamma^{'}}$ with[21] $1 > \gamma > \gamma^{'}$, consequently G is less concave than H. Define $f_{\gamma}(x) = 1 - G\left(h - \frac{w(x)-\widehat{w}}{i-r}\right)$ and $f_{\gamma^{'}}(x) = 1 - H\left(h - \frac{w(x)-\widehat{w}}{i-r}\right)$, then we get $-x\frac{f_{\gamma}^{''}}{f_{\gamma}^{'}} > -x\frac{f_{\gamma^{'}}^{''}}{f_{\gamma^{'}}^{'}}$. Assuming[22] a range of parameters such that $f_{\gamma^{'}}$ is concave (so it will be for f_{γ}), then $f_{\gamma^{'}}$ is less concave[23] than f_{γ}. As f_{γ} is more concave than $f_{\gamma^{'}}$, one would expect to get a lower value of ϕ under f_{γ} than under f_{γ}. But, as will be more extensively discussed, f_{γ} is above $f_{\gamma^{'}}$ and consequently the high equilibrium value $\phi_{G,h}$ under the G distribution is above its value under the H distribution. But for the low equilibrium values we have $\phi_{G,l} < \phi_{H,l}$. However these results come from the first order stochastic dominance of G, i.e. $\forall x\ G(x) \leq H(x)$.

In Appendix B.1 we extent the analysis to distribution functions which do not not dominate each other in the first order stochastic dominance sens.

We turn now to the dynamics of wealth and industrialization.

1.4 Dynamics

To study the dynamics of industrialization, that is to say the dynamics of ϕ_t, using *Proposition 1*, we see that we have to derive the dynamics of wealth distribution function G_t.

At period $t-1$ the distribution of wealth G_{t-1} is predetermined. In order to determine the period t distribution G_t, we must characterize the dynamics of $(e_t)_{t \in N}$. Let us define $v_t = e_t - h$, $R = 1 + r$, $I = 1 + i$, $\theta(\phi_t) = \beta w(\phi_t) - h$,

[21] When $\gamma \geq 1$, f_{γ} is always concave, and in this case the results are more obvious, and the higher γ the higher the level of ϕ.

[22] See Appendix for a more detailed discussion.

[23] Note that this reversion of concavity is intuitive. Indeed, since $g(x)$ is a decreasing function and $\lim\limits_{0} g(x) = +\infty$, one gets that the density function of G decreases less rapidly, as the level of wealth increases, than the density of H.

$\widehat{\theta} = \beta\widehat{w} - h$ and note $v_t = v_t^i$ if wealth belongs to category (i) defined by : (1) is the child of a wealthy parent who worked as a skilled, (2) is the child of a parent who borrowed to become skilled (an indebted parent), (3) is the child of an unskilled. The term $\theta(\phi_t)$ is the difference between the bequest an educated old individual gives to his child and the cost of education h, and $\widehat{\theta}$ is the difference between the amount an educated old laborer bequeath to his child and h.

Then using the fact that $e_t = \beta y_t$ and using expressions (5), (8) and (9) we get

$$v_t^1 = \beta R v_{t-1}^1 + \theta(\phi_t) \qquad \Longleftrightarrow \qquad v_{t-1} > 0 \ , \tag{13}$$

$$v_t^2 = \beta I v_{t-1}^2 + \theta(\phi_t) \qquad \Longleftrightarrow \qquad v_{t-1} \in \left[-\frac{w(\phi_t) - \widehat{w}}{i-r}, 0\right] \ , \tag{14}$$

$$v_t^3 = \beta R v_{t-1}^3 + \widehat{\theta} \qquad \Longleftrightarrow \qquad v_{t-1} \in \left]-\infty, -\frac{w(\phi_t) - \widehat{w}}{i-r}\right] \ , \tag{15}$$

We then make the following assumption.

Assumption 2 : $\beta\alpha - h > 0$ *and* $\beta\widehat{w} - h < 0$

The first inequality ensures that the equilibrium where every body has acquired the level h of education is a stationary equilibrium (it ensures that the transmission of wealth between generations is enough to finance h). The second inequality guaranties that a dynasty of unskilled is trapped in a "low wealth" point (otherwise the model is irrelevant!).

Assumption 3 : $h - \frac{\alpha - \widehat{w}}{I-R} > 0$

This inequality is necessary in order not to get the trivial case where $\phi = 1$ whatever wealth distribution.

Further, with some abuse of language, we will talk about "moving attraction point" for a sequence of real numbers with reference to another sequence of points towards which the first sequence is attracted. For example in equation (17), if ϕ_t was constant then the sequence $(v_t^2)_t$ would converge towards $\theta(\phi_t)/1 - \beta R$.

The three sequences defined by equations (17), (18) and (19) have 3 moving attraction points which are respectively

$$\frac{\widehat{\theta}}{1-\beta R} < \frac{\theta(\phi_t)}{1-\beta R} < \frac{\theta(\phi_t)}{1-\beta I} \ , \tag{16}$$

The first one is the moving attraction point of $(v_t^3)_{t\in IN}$, the second is the moving attraction point of $(v_t^1)_{t\in IN}$ and the third one is the moving attraction point of $(v_t^2)_{t\in IN}$.

Figure 2 in appendix draws the evolution[24] of $(v_t)_{t\in IN}$.

Please insert FIGURE 2

Looking directly at the figure one sees that when v_{t-1} is above $\theta(\phi_t)/(1-\beta R)$, then the bequest is decreasing but remains above h. When v_{t-1} is under $\theta(\phi_t)/(1-\beta R)$, then the bequest is increasing. When $v_{t-1} \geq \theta(\phi_t)/(1-\beta I)$ then $v_t \leq v_{t-1}$, and if $v_{t-1} \leq \theta(\phi_t)/(1-\beta I)$ then $v_t \geq v_{t-1}$. Finally if $v_{t-1} \geq \widehat{\theta}/(1-\beta R)$ then $v_t \leq v_{t-1}$, and if $v_{t-1} \leq \widehat{\theta}/(1-\beta R)$ then $v_t \geq v_{t-1}$.

This will be the starting point of the study of wealth dynamics. We will prove two lemmas and a proposition that will enable us to describe entirely the pace of industrialization without explicitly determining point by point the sequence of distribution functions. Before we can intuitively describe what happens. At the beginning of the first period, the economy begins with a continuum of old unskilled workers among whom wealth is distributed according to the concave distribution function G_0, the upper indexed designing old. Then these old individuals bequeath to their child a fraction β of their wealth, and the distribution function of the bequest is G_0 (concave).

Using *Proposition 1*, G_0 will determine ϕ_1, and using Figure 2 one also gets that the old of period 1 (those who were young at the beginning of the world) who inherited more than $\bar{e}_1 = h - \frac{w(\phi_1)-\widehat{w}}{i-r}$ will bequest more than they received, and those who received less than $\bar{e}_1$ will bequest more if their wealth was under the poverty attraction point and less if above the poverty attraction point. Thus, intuitively, we need to know how is $\bar{e}_1$ relatively to the poverty attraction point $h + \frac{\widehat{\theta}}{1-\beta R}$ (a dynasty of poor will converge towards this wealth level). Fortunately, given the assumptions we made, $\bar{e}_1$ will be under $h+\frac{\widehat{\theta}}{1-\beta R}$ when industrialization is sustainable, and consequently wealth will be globally increasing (to be defined soon).

We will then use a convenient and intuitive result that states that if one distribution dominates in the first stochastic order sense an other distribution, then it will be associated with a higher high-equilibrium level of

[24] That is to say : the difference between the bequest an old laborer received when he was young and the bequest he gives to his child. The difference between the bequeath an old in debt skilled received when he was young and the bequest he gives to his child. And finally the difference between the bequest an old wealthy skilled received when he was young and the bequest he gives to his child

industrialization. This will enable us to show that, when the economy enters a "good beliefs" path, the sequence $(\phi_t)_t$ is increasing, and then using recursively the lemmas, one proves that it converges towards one. But all this discussion holds only, as stated by *Proposition 2*, when industrialization can begin and is sustainable.

We will say further that wealth is "globally increasing" if for the relevant range of wealth levels, *i.e.* around h, each date more people cross the line defined by h. Lemma 1 below shows that wealth is globally increasing if $\theta(\phi_t) \geq 0$ and $-\left[w(\phi_t) - \widehat{w}\right] / (i-r) \leq \widehat{\theta} / (1 - \beta R)$. This last inequality states that the threshold level for education i.e. $h - \left[w(\phi_t) - \widehat{w}\right] / (i-r)$ is under the fixed point of the wealth of "unskilled dynasties" i.e. $h + \widehat{\theta} / (1 - \beta R)$. In appendix it is shown that this last inequality is always holding for taking-off economies. The first inequality $\theta(\phi_t) \geq 0$ is required for industrialization to be sustainable, because if $\theta(\phi_t) < 0$ then wealth is decreasing and *lemma 2* below shows that it means "de-industrialization".

Lemma 1 *When $\theta(\phi_t) \geq 0$ wealth is globally increasing and $-\left[w(\phi_t) - \widehat{w}\right] / (i-r) \leq \widehat{\theta} / (1 - \beta R)$*

The first part of the lemma says that each individual receives from his parent more than his parent received from his grand parents. The second part says that the threshold level of education is under the poverty trap.

Lemma 2 *Let $H(x)$ be a real valued function decreasing and convex, and G and T two distribution functions. If $T(x) \leq G(x)\ \forall x$, then the positive solutions $(\phi_{G,i})_{i=l,h}$ and $(\phi_{G,i})_{i=l,h}$ of respectively the equations $1-\phi = G(H(\phi))$ and $1 - \phi = T(H(\phi))$ verify $\phi_{T,h} > \phi_{G,h}$ and $\phi_{T,l} < \phi_{G,l}$.*

Lemma 2 demonstrates an intuitive result, which is that when one distribution function T dominates an other distribution function G in the sense of first order stochastic dominance, then high-industrialization equilibrium is higher when wealth is distributed according to H. But it also shows that the low-industrialization equilibrium is lower under T. Thus it is only when people can coordinate on the good high-equilibrium that stochastic dominance leads to a higher level of development. One other consequence of lemma 2 is that the difference $\phi_{T,h} - \phi_{T,l}$ is greater the more equal is wealth distribution. Thus we can hope that the more equal is wealth distribution the higher the probability of coordination on a high-level equilibrium.

Dynamics along a "good beliefs path"

To understand what we mean by good beliefs, let us recall that lemma 2 says that the low equilibrium under T is lower than the low equilibrium under G. When $\theta(\phi_t) \geq 0$, it is shown that we have $G_1 \leq G_0$. Then we know by lemma 2 again that $\phi_{G_0,h} < \phi_{G_1,h}$. Then if have $\phi_1 = \phi_{G_0,h}$ and $\phi_2 = \phi_{G_1,h}$ and $\phi_t = \phi_{G_t,h}$ for $t \geq 3$, then we say that the economy is on "good beliefs path". So we first consider this case.

Then Lemma 1 and lemma 2 enable us to study the dynamics of industrialization without deriving explicitly G_t.

The fact that $-\left[w(\phi_t) - \widehat{w}\right] / (i - r) \leq \widehat{\theta} / (1 - \beta R)$ is sufficient to show that ϕ_t converges towards 1 because this inequality and the fact that $(\phi_t)_t$ is increasing guaranty that there will not subsist any "dynasty of unskilled". This is simply because the poor are attracted by the point $\widehat{\theta}/(1-\beta R)$ and the threshold level for education is under this point, then as $(\phi_t)_t$ is increasing the threshold level will decrease and as wealth levels are attracted by the moving points $h + \frac{\theta(\phi_t)}{1-\beta R}$ and $h + \frac{\theta(\phi_t)}{1-\beta I}$, every dynasty of poor would cross one day a threshold level and become a dynasty of wealthy and educated individuals[25].

But this will hold only if certain conditions are satisfied, and this is the object of the following proposition.

Proposition 2 *For industrialization in the "good beliefs" path to begin and to be sustainable it is necessary and sufficient that (i)* $w(\phi_1) \geq \widehat{w} \Leftrightarrow \phi_1 \geq \phi^* = \frac{F}{\alpha - \widehat{w} + F}$ *and (ii)* $\theta(\phi_1) > 0 \Leftrightarrow \phi_1 > \phi^{**} = \frac{\beta F}{\beta\alpha - h + \beta F}$

So it is equivalent to $\phi_1 > \Omega(\alpha, F) = \max(\phi^{**}, \phi^*) = \phi^{**}$

The intuition behind this proposition is the following. The inequality (i) is essential for anyone to want to become skilled. The second inequality (ii) is essential for a virtuous dynamics of industrialization, because it states that, recalling that $\theta(\phi_1)$ is the difference bewteen $\beta w(\phi_1)$ and the cost of education, aggregate wealth is increasing. If this inequality was not satisfied, industrialization would be impoverishing, since the transmitted wealth from generation to generation would be decreasing (this can be easily understood by looking at equations (13)-(15)). If (i) is satisfied but not (ii), then industrialization would start but will return to its initial null level after some time.

[25] The speed of industrialization will depend on the initial degree of inequality in wealth distribution. When wealth is initially too unequally distributed, industrialization will not at all begin or be sustainable (see proposition 3). When wealth is not too unequally distributed, then $\bar{e_1}$ will be near the poverty trap, and if there remain a lot of people under this level, industrialization will be slow.

So when wealth is very unequally distributed, the economy cannot industrialize. Note also that, if the economy as a whole is not constrained, in our model all wealth above h could be efficiently redistributed to the poor and make industrialization possible. This is very important, because if aggregate wealth is very under h (remember that we have a continuum of mass 1 of individuals), then the economy cannot aford the cost of education for everybody. But even if aggregate wealth is under h, industrialization may be possible if wealth is not held by a small group of individuals. One can therefore suspect that wealth distribution is more a constraint for poor countries.

One can also argue that, given the equation that gives the equilibrium value of ϕ, we do not necessarily need to get an important class of "rich", what we rather need is a sufficient concentration of people at any "non too low level of wealth". Indeed, looking at the most favorable case where at the beginning of the world every body has a bequest greater or equal to $h - (\alpha - \widehat{w})/(I - R)$, then industrialization will be complete as soon as the first period.

Concretely, for any $\tilde{\phi} \in [0, 1]$, we can find a distribution $G_{\tilde{\phi}}$ such that $\tilde{\phi}$ is solution of the equation[26] $x = 1 - G_{\tilde{\phi}}\left(h - (w(x) - \widehat{w})/(I - R)\right)$. One can argue that the higher the level ϕ targeted, the less unequal has to be the initial distribution of wealth.

When the economy takes off, it converges towards one with the more productive technology $y = \alpha l$, but with l representing an educated work force. So what we modeled is the transition from an economy producing with a constant returns to scale technology $y = l$ using unskilled labor to an economy using a more productive constant returns to scale technology $y = \alpha l$ using a better educated work force. This transition necessitates the payment of fixed costs which are progressively reduced as the fraction of skilled workers grows (because of the know-how accumulation effect). During this transition the wage of skilled workers progressively attains the marginal product of skilled workers i.e. α. But as it was discussed in subsection 1.1, the fact that the fixed cost is null at the full industrialization equilibrium is not essential for the results derived in this paper. The fixed cost could still be present with an other specification, for example $\frac{F}{X_t}$ and the economy would converge to an other stationary equilibrium described by $(X_\infty = 1, \phi_\infty = 1 - F)$.

[26] The proof is intuitive : suppose this is not the case, then $\forall G$ a distribution function on $[0, \bar{h}]$, either $\tilde{\phi} > 1 - G(g(\tilde{\phi}))$ or $\tilde{\phi} < 1 - G(g(\tilde{\phi}))$. Then take two distributions G_1 and G_2 such that $\tilde{\phi} < 1 - G_1(g(\tilde{\phi}))$ and $\tilde{\phi} > 1 - G_2(g(\tilde{\phi}))$. Then $\exists \lambda \in]0, 1[$ such that $\tilde{\phi} = \lambda(1 - G_1(g(\tilde{\phi}))) + (1 - \lambda)(1 - G_2(g(\tilde{\phi}))) =$
$1 - \lambda G_1(g(\tilde{\phi})) + (1 - \lambda)G_2(g(\tilde{\phi}))$. Then Take $G_{\tilde{\phi}} = \lambda G_1 + (1 - \lambda)G_2$, it is a distribution function (concave).

“Bad beliefs” paths

Now we can consider the other cases that can happen.

first case : note first that the case $\Omega(\alpha, F) > \phi_{G_0,h}$ is the most simple case since then we have $G_0 > G_1$ and using lemmas 1 and 2, we have $\phi_{G_0,h} > \phi_{G_1,h}$ and industrialization is not sustainable.

second case : suppose $\phi_{G_0,h} \geq \Omega(\alpha, F) > \phi_{G_0,l}$, then if the economy at date 1 is on the low equilibrium level, we know again that $G_0 < G_1$. But as $\phi_{G_1,l} > \phi_{G_0,l}$ it is not obvious that the economy doesn't jump to a high equilibrium if $\phi_{G_2,h} \geq \Omega(\alpha, F)$ (otherwise we are in the first case above). Thus it is possible that a “shock” to beliefs pushes the economy on an “industrialization road”. However the initial date zero “bad belief” equilibrium may be a trap.

third case : suppose $\phi_{G_0,h} > \phi_{G_0,l} \geq \Omega(\alpha, F)$, then even if the economy is on the low-equilibrium level, conditions of proposition 3 are satysfied. However, we know that $\phi_{G_{0,l}} > \phi_{G_1,l}$, then if equilibrium value of ϕ_2 is $\phi_{G_1,l}$ the economy may enter a “bad beliefs” path. It seems difficult to imagine that, although the first period satisfies the conditions for a sustainable development, industrialization doesn't grow. However if beliefs are not so "good" (as the initial equilibrium is $\phi_{G_0,l}$), any small “shock” to beliefs can push the economy at second period on the low equilibrium $\phi_{G_0,l}$ and as this value is under the first period equilibrium, then beliefs are selfullfiling and the economy can get "traped" in a bad beliefs path and industrialization will decline!

2 Wealth inequality and technology choice

In this section we use the previous analysis to show the perverse effect of inequality on technology adoption.

2.1 Endogenous organizational costs

In this section, we consider a situation where the cost of adoption, F, increases with the level of productivity α. What we have in mind, is to consider the impact of new organizational innovations in the North on technology adoption by the South. To be clear, we consider that importing a machine is also importing an organization of work. If we consider that there is a complementarily between the quality-productivity of a machine and the quality of the organization of the firm, then on top of not being endowed with the skill level necessary to operate the machine at its best level, developing countries do not have also the right internal organizations of firms to operate the machine at its most efficient level. So what we argue is that the concern is not only a problem of a lack of the right people who have the necessary skills to operate a machine. We rather think that the organization "around the machine" is as much important than the workers who directly operate the machine. Think of machine operation as one task among many others that are complementary. Although you have someone who knows how works the machine, if the workers who are performing the other tasks do not have the appropriate skills, then the productivity will be low. Note also that production is no longer, in developed countries, the main activity in the firm. If work evolves towards more interaction between workers, more communication, more involvement (see Gant, Ichinovski and Shaw (1999)), then LDCs will have some trouble to be as productive as the North. There is some evidence that technical progress in the north is associated to new organizational innovations that are more "skill consuming". These facts are documented in Bernan, Bound and Griliches (1994), Caroli and Van Reenen (1999), Greenan and Mairesse (1999).

In our modelling strategy, as the costs of organization increase, more skilled workers are needed to reduce them, and the knowledge generated by the utilization of the technology contributes to the reduction of the costs of adoption. To this purpose, the knowledge externality, modeled here by X_t, has to seen as a social-organizational knowledge.

The threshold level of ϕ defined in *Proposition 3* by $\Omega(\alpha, F)$ is decreasing in α and increasing in F. As ϕ_1 depends on the initial distribution of income, we can suspect that inegalitarian economies will be forced to adopt technologies with a low F.

Proposition 3 *Suppose a menu of technologies $\{(\alpha, F(\alpha)) / \alpha \in R^+\}$ with $F'(\alpha) > 0$ and $F''(\alpha) > 0$, is available to a country. Then α^* the optimal α chosen by a social planner is determined by initial wealth distribution G_0. When initial wealth distribution G_0 is too unequal, the economy is constrained to adopt a low α technology. There exist inital distributions that give rise to multiple equilibrium values of α^*, a low productivity equilibrium α_L and a high productivity equilibrium α_H.*

The social planner wants to choose α that maximizes the size of the initial group who chooses education in the high equilibrium, namely $\phi_{G_0,h}(\alpha)$ which we note $\phi(\alpha)$, subject to the constraint that it gives rise to a full industrialization path. The program is

$$\underset{\alpha}{Max}\left[1 - G_0\left(h - \frac{\alpha - \frac{F(\alpha)(1-\phi(\alpha))}{\phi(\alpha)} - \hat{w}}{I - R}\right)\right]$$

$$st\ \phi(\alpha) \geq \frac{\beta F(\alpha)}{\beta\alpha - h + \beta F(\alpha)} \tag{(FIC)}$$

We know that $\phi(\alpha)$ is implicitly defined by

$$\phi(\alpha) = 1 - G_0 \left[h - \frac{\alpha - \frac{F(\alpha)(1-\phi(\alpha))}{\phi(\alpha)} - \widehat{w}}{I - R} \right] , \tag{17}$$

and from Appendix $B.5.3$, we know that $\phi(\alpha)$ admits a maximum[27]. Then differentiating[28] the equation above and taking $\phi^{'}(\alpha) = 0$ we get that

$$\phi^{'}(\alpha) = 0 \Longleftrightarrow \phi(\alpha) = \frac{F^{'}(\alpha)}{1 + F^{'}(\alpha)} ,$$

and putting this in equation (17), one gets that the optimal α noted α^* is defined by

$$\frac{F^{'}(\alpha^*)}{1 + F^{'}(\alpha^*)} = 1 - G_0 \left(h - \frac{\frac{1}{2}\alpha^* - \widehat{w}}{I - R} \right) \tag{18}$$

The left hand side of equation (18) is a concave function of α if $F^{'''} < 0$, and the right hand side is convexe. Thus there exist functions G_0 sucht that (18) admits two equilibrium values, $\alpha_L^* < \alpha_H^*$. In Appendix $B.6.2$, we show that what maters is the difference between the speed at which $G_0^{'}$ (that is to say the density) decreases and the size of $F^{'}(\alpha)$, that is to say the marginal cost of adoption. For high values of α, the cost of adoption rises to much, and consequently more skilled workers are needed to absorb these more productive technologies.

Suppose first that $\alpha_L^* < \alpha_{FIC} < \alpha_H^*$, where α_{FIC} is defined by

$$\frac{F^{'}(\alpha_{FIC})}{1 + F^{'}(\alpha_{FIC})} = \frac{\beta F(\alpha_{FIC})}{\beta\alpha - h + \beta F(\alpha_{FIC})}.$$

Then in the decentralized equilibrium, the economy may either adopt the high productivity technology and fully industrialize, or stay at its backstage level.

Now, if $\alpha_{FIC} < \alpha_L^* < \alpha_H^*$, then the economy can either be at a high productivity long run equilibrium with full industrialization or a low productivity long run equilibrium with full industrialization. So selffulfiling propheties can either put the economy on a high equilibrium or a low equilibrium. The

[27]We show that $\phi(\alpha)$ is $S-$shaped

[28]See appendix B.5.1 for more details

intuition behind selffulfiling beliefs is as follows. When entreprenenurs (those who create the firms in the model) believe that the supply of skill will not be high enough to support the adoption costs of a high-productivity technology, they implement a technology with a low α. Anticipating that the return to education will not be high, there will be less young people who will be willing to finance education. As a result tomorow the supply of skill will be low, and the technology adopted will be indeed a low α technology. Note that for lower values of α, the marginal individual in the equation that gives the equilibrium value of ϕ for the α chosen is richer than the marginal one corresponding to a high α. Consequently, as G_0 is concave, indeed the supply of skill, in the case where a low α is chosen, is not very responsive.

So the model predicts that initially identical countries (with the same distribution of wealth) can be at different long run total factors productivity levels.

Effect of a shift of the organizational costs curve

Let us suppose that the costs function F shifts upward, for example F becomes $(1 + v)F$, with $v > 0$. Then α_L^* decreases and α_H^* increases, thus productivity differences between countries at different equilibrium points increases. The intuition can be captured with beliefs. In the model, a country trapped in the low equilibrium is a country where people believe that the supply of skill is not enough to reduce the costs of adoption, and consequently the lower costs technology is chosen. As the productivity associated is lower, indeed the supply of skill will be lower, and determined in the region of wealthy people. This region, as argued in Appendix, is a region where the density of G_0 is lower, and thus the elasticity of the skill supply is lower too. Consequently an increase in the costs of adoption will direct beliefs towards a lower equilibrium level, which indeed will be the case. But the mechanism is reversed for a country in the high equilibrium point. Here the elasticity of the skill supply is higher, thus an increase in the costs of adoption, will not provoque bad beliefs. This is because, an increase in the costs will necessitate a higher skill supply, thus people need to be attracted towards the education sector, thus α has to rise, and as the initial equilibrium was in a "good" beliefs point, a higher α will be taken, and as the skill supply is in a more elastic region, more people will engage in school, and beliefs will be selffulfiling.

Example :

Let us take $F(\alpha) = A.\alpha^2/2$. Then the optimal α noted α^* is defined by

$$\frac{A.\alpha^*}{1 + A.\alpha^*} = 1 - G\left(h - \frac{\frac{1}{2}\alpha^* - \widehat{w}}{I - R}\right) . \tag{19}$$

The function $\alpha \longrightarrow \frac{A\alpha}{1+A\alpha}$ is concave and one verifies that the function $\zeta : \alpha \longrightarrow 1 - G\left(h - \frac{\frac{1}{2}\alpha - \widehat{w}}{I-R}\right)$ is convex. If we restrict ourself to situations where α^* is defined[29][30], it is immediate that the higher γ the higher will be α^*, because of first order stochastic dominance. But one has to note that first order dominance is not necessary, as it is suggested in Appendix B.1. In this example, what is important is that the elasticity of the density function be not too low at wealth levels above h (by analogy with $figure$ b in Appendix B.1).

One could argue that as wealth increases (this is the case in both models) sectors could choose an other technology $(\alpha', F(\alpha'))$ with a higher α'. But if there are high social costs to changing the technology, for example the "learning by doing" effect can be technology-specific, then switching to an other technology may be very costly for one sector. This effect can be even higher with an input-output structure, that is to say if each technology is associated with different sets of intermediate inputs, then the degree of irreversibility of technology choice is increased since it entails even a greater need for coordination between input suppliers and final goods producers (an also between input suppliers) etc. So what we advance is that with a kind of irreversibility in the technology choice (or at least high cost to changing the technology), wealth inequality may force the economy to adopt a low-productivity technology. Note also that when the economy is constrained to choose low-α technologies, it implies that it has less wealth to redistribute to foster industrialization, since the redistributed wealth comes from the pool of educated people extra wealth i.e. $\beta\alpha - h$ (which is increasing with α). Another consequence is also a reduced supply of skilled labor.

2.2 Endogenous human capital

Let us suppose that the productivity parameter α is a function of h, and that there are decreasing returns to human capital in the production function. So

[29] To the extent that the intersection is in the range authorized for the productivity parameter α. Indeed, productivity is bounded above and below. First we have from assumption 2, $\alpha \geq \frac{h}{\beta}$, and second from assumption 3, we have $\alpha < (I - R)h + \hat{w}$.

Note that Assumption 3 garanties $\alpha < 2\hat{w}$ (otherwise it is immediate, looking at (17) that $\phi = 1$ is a trivial solution).

[30] See appendix B.7

let us take $\alpha(h) = Bh^{\upsilon}$ with $\upsilon < 1$. First we look at the program solved by a social planner. So we first assume that only one level of human capital is chosen.

Invariable costs

For a given G, the level h that maximizes[31] ϕ will be given by taking ϕ as a function of h and differentiating the equation that determines ϕ. We obtain $\phi(h) = 1 - G\left[h - \frac{\alpha(h) - \frac{F(1-\phi(h))}{\phi(h)} - \widehat{w(h)}}{I-R}\right]$ and consequently

$$\phi^{'}(h) = \left[1 - \frac{1}{I-R}\left(\phi^{'}(h)\frac{\partial w}{\partial \phi} + \alpha^{'}(h)\frac{\partial w}{\partial \alpha}\right)\right] G^{'}\left(h - \frac{\alpha(h) - \frac{F(1-\phi(h))}{\phi(h)} - \widehat{w(h)}}{I-R}\right),$$

and taking $\phi^{'}(h) = 0$, one obtains $1 - \frac{1}{I-R}(\alpha^{'}(h) - R) = 0$. The level of human capital chosen is thus given by

$$h^* = \left(\frac{\upsilon B}{1+i}\right)^{1/(1-\upsilon)} = \left(\frac{\upsilon B}{1+r+(i-r)}\right)^{1/(1-\upsilon)}, \tag{20}$$

Note that it is the level which minimizes[32] $\tilde{e}(h)$ the threshold level of wealth that permits education. This is not surprising to obtain a negative relation between h^* and the "level" of financial imperfections measured here by $(i-r)$. The higher the financial imperfection the lower the level h of education and the slower the path of industrialization, and also the steady state total factor productivity $\alpha(h^*)$. One can note that in this particular context, wealth distribution does not at all influence the level of education h^*, and only the extent of credit imperfection determines h^*. But now we can use *Proposition 2* to argue that in the case where the credit imperfection is severe (so h^* is low) and the entry cost F is too high, then wealth distribution can prevent the economy from taking a sustainable industrialization path.

[31] One would argue that this not obvious that the objective of the social planner is to maximize ϕ. Taking $\exp((1-\beta)\int_0^1 \ln x_i di + \beta \ln e)$ rather than $(1-\beta)\int_0^1 \ln x_i di + \beta \ln e$, all the calculus would remain unchanged, and the utility of the agents would be linear in lifetime income. So the social planner can choose to maximise the discounted aggregate welfare from $t = 0$ to infinity. This compatible with maximizing ϕ, first because the productivity of labor is higher in moden sectors, second because of the trickle -down growth effect of an increase in ϕ, the mass of individuals who will have access to education will increase and third proposition 1 has to be satisfied.

[32] This is indeed a minimum, since $\frac{d^2\tilde{e}(h)}{dh^2} = -\frac{1}{i-r}\alpha^{''}(h) > 0$

Variable costs

For a given G, the level h that maximizes ϕ will be given by taking ϕ as a function of α which is a function of h and differentiating the equation that determines ϕ. We obtain $\phi(h) = 1 - G\left[h - \frac{\alpha(h) - \frac{F(\alpha(h))(1-\phi(h))}{\phi(h)} - \widehat{w(h)}}{I-R}\right]$. In Appendix B.7, we show that the optimal h_R^*, taking again $F(\alpha) = A\alpha^2/2$ is given by

$$\frac{A\alpha(h_R^*)}{1 - \frac{I}{A\alpha'(h_R^*)} + A\alpha(h_R^*)} = 1 - G\left(h_R^* - \frac{\frac{1}{2}\alpha(h_R^*) - \widehat{w(h_R^*)}}{I-R} - \frac{1}{2}\frac{\alpha(h_R^*)}{\alpha'(h_R^*)}\right). \quad (21)$$

In the general case, it is hard to study the properties of equation (21). However we can take particular cases that are easier to deal with.

First example

Let us take $\alpha(h) = \sqrt{h}$, $G(x) = \left(\frac{x}{h}\right)^{\gamma}$. Then in Appendix B.7 it is shown that the equilibrium value of h_R is an increasing function of γ.

Second example : multiple equiulibria for the education level

Here we take $\alpha(h) = Bh$ and no particular form for G. The optimal h_R^* is the solution of

$$\frac{Bh}{1 - \frac{I}{B} + Bh} = 1 - G\left(\left(\frac{1}{2} - \frac{\frac{1}{2}B - R}{I-R}\right)h + \frac{1+R}{I-R}\right) \quad (22)$$

The left hand side of (22) is a concave function of h, and for $\frac{1}{2}B - R < 0$, the right hand side is convexe. Thus in this example, there may exist multiple equilibria for h_R^*.

This last example may help us assert that there has to be a strategic choice of education policy and technology adoption policy. Adopting a technology that requires a high degree of education is not a good policy in a inegalitarian country since the supply of skill will not respond. Choosing a medium technology may be better, although it is not the most efficient, as it would permit education of more people and a rapid increase of productivity[33].

We think that this is a relevant aspect of the link between inequality and development. Moreover, the positive interaction between technology adoption and income redistribution can be used to explain part of the rapid growth

[33] Remember that the speed of development depends on the interaction of wealth distribution and the cost of education.

of Asian countries like Korea. Keller (1996) argues, for example, that trade liberalization has to be accompanied by more human capital accumulation in order to absorb the higher arrival rate of new technologies. One can then argue that the argument is not only a problem of level of human capital. Although more complex technologies necessitate higher skilled workers, we think that it also necessitates more skilled workers[34]. We then claim that income redistribution is necessary to increase the proportion of skilled workers in the economy and thus to be able to absorb more productive technologies.

3 Extension

Rent seeking

In this section we look at the impact of the existence of rent seekers in the modern sector. We add to the previous model a mass m of individuals at the bottom of the wealth distribution, who have the political power and extract rents from the modern sector. We choose[35] a very simple way of introducing a rent seeking activity in our economy. The point is just made to show that on top of diverting productive resources, rent seeking distorts the reward of other more socially productive activities and consequently slows the pace of economic development by constraining technology adoption.

We just assume that a fraction τ of the output in modern sectors is extracted by the rent seekers and consumed. Rent seekers don't work, they spend time controlling and extracting rents from the modern firms.

It is then immediate to establish the equivalent of *lemma 0*.

Lemme 4 *Profits are null and the wage of skilled workers is given by*

$$w_t^s = w(\phi_t) = (1-\tau)\alpha - \frac{F(1-\phi_t)}{\phi_t} \tag{23}$$

The income of a rent seeker at t is then $\tau\alpha\phi_t/m$. It remains to see wether rent seeking is more profitable than working as a skilled worker in the modern sector. The condition is that

$$\tau\alpha\phi_t/m \geq (1-\tau)\alpha - \frac{F(1-\phi_t)}{\phi_t} \tag{24}$$

[34]This may be a more important aspect if we consider the impact of new organizational change in the north, which is skill-intensive.

[35]A rather more rigorous treatment of the effect of the political regime on technology adoption and human capital accumulation is under examination in an other paper.

For small values of ϕ_t this inequality is likely to be satisfied. As ϕ_t is inversely related to τ, one can argue that rent seeking and low industrialization are positively related.

Looking at *(21)* for $\phi = 1$, we see that it is satisfied if $\tau \geq m/(1+m)$. This means that when the rent extracting group is able to "tax" a fraction of output higher than its relative size in the population, then rent seeking will not disappear with full industrialization. But if $\tau < m/(1+m)$, then rent seeking will vanish at some degree of industrialization.

The results of section 2 can be applied here. Equation (20) shows that rent seeking reduces the reward of education, and consequently the supply of skill. With endogenous choice of technology, then rent seeking will increase the burden of wealth distribution on technology adoption.

Conclusion

This paper has tried to model the consequences of wealth inequality on industrialization. Industrialization was assimilated to the use of increasing returns to scale technologies. We assumed that the industrial technologies required a fixed cost in terms of unskilled labor and then each unit of output required $1/\alpha$ units of skilled labor. We showed that when industrial profit are exhausted by entry, there is a positive correlation between the size of the skilled work force and returns to education. This positive correlation permits, as the size of the last generation educated work force increases, to the poor to progressively get access to education. When initial distribution of wealth is not too unequally distributed, the economy can take-off an *converge* to an economy with a constant returns to scale technology using skilled labor and more productive than the initial backstage technology. When the distribution is inegalitarian, then without redistribution, the economy cannot industrialize. More over as wealth above h is *socially unproductive,* redistribution is valuable and has a kind of "trickle-down" effect.

We finally emphasized something we think to be a relevant question for developing countries, which is the perverse effect of wealth inequality on technology adoption. In the paper the threshold level of ϕ which permits the take-off is positively correlated to the size of the fixed cost F. Consequently, in a world where more productive technologies necessitates a higher fixed cost, a too inegalitarian distribution of wealth may force the economy to adopt low productivity technologies.

We think that the combination of a non well functioning labor market with financial imperfections are of great importance for studying economic development and is an interesting direction for further research. For education to be stimulated, the returns to skill have to reflect the true social value of education. Indeed, growing education in the absence of a well functioning modern sector has no effect. We thus think that a further inquiry into the link between corporate governance and the labor market conditions could be an interesting direction for further research. For example, the internal organization of firms is of great importance to study human capital accumulation. Another important aspect is the determinant of surplus sharing between owners and workers in the context of financial imperfections. When financial imperfections are strong, human capital is more tied to the physical assets because workers cannot quit their employer to build their own firm, and this may create a higher distortion of the returns to education.

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B Appendix

B.1 Comparing two economies

We want here to show that we can compare two economies starting with two different wealth distributions, without the need to assume a strong condition of first order stochastic dominance.

In fact until now the first stochastic dominance criteria was used mainly to show that under a successful take off, we have $G_t(x) \leq G_{t-1}(x)\ \forall x$ and $\forall t \geq 1$.

It is a trivial result that starting with two different distribution functions G and H, if $G(x) \leq H(x)\ \forall x$, then "sustainable" industrialization will be higher and faster under G than under H.

However, the following lemma states that if G is less concave than H in the upper levels of wealth, then industrialization may be higher under the G distribution.

Lemma 3 *Let $g(x)$ be a real valued function decreasing and convex, and G and H any two distribution functions. If $-x\frac{G''}{G'}$ is sufficiently lower than $-x\frac{H''}{H'}$ for $x \in [\tilde{x}, \infty[$, where $\tilde{x}$ verifies $H'(g(\tilde{x})) << G'(g(\tilde{x}))$, then it is possible that the solutions ϕ_G and ϕ_H of the following equations $1 - \phi_G = G(g(\phi_G))$ and $1 - \phi_H = H(g(\phi_T))$ verify $\phi_G > \phi_H$*

The intuitive explanation of Lemma 3 is illustrated by the following figures.

Figure a *Figure b*

Figure a depicts the case where the concavity[36] of G is not low enough relatively to the concavity of H for $1 - G(g(\phi))$ to cross the $45°$ line after $1 - H(g(\phi))$. Figure b represents the favorable case where the concavity of G is sufficiently lower than the concavity of H for $1 - G(g(\phi))$ to cross the $45°$ line after $1 - H(g(\phi))$. The condition $H'(g(\tilde{x})) << G'(g(\tilde{x}))$ guaranties that the derivative of $\phi \longrightarrow 1 - G(g(x))$ at $\tilde{x}$ is greater than the derivative at $\tilde{x}$ of $\phi \longrightarrow 1 - H(g(x))$. This condition is necessary since otherwise $\phi \longrightarrow 1 - G(g(x))$ would be always under $\phi \longrightarrow 1 - H(g(x))$. Adding the condition that $-xG''/G'$ is sufficiently lower than $-xH''/H'$ for $x \in [\tilde{x}, \infty[$, guaranties that $H'(g(x)) < G'(g(x))$ for a sufficiently large interval $[\hat{x}, \tilde{x}]$ in order that despite its greater concavity, the function $\phi \longrightarrow 1 - G(g(x))$ remains above[37] $\phi \longrightarrow 1 - H(g(x))$.

[36] We talk about $1 - \gamma$

[37] That is to say despite the fact that the density decreases more rapidly, the distribution remains above.

B.2 Proof of proposition 1

B.2.1 Existence and multiple equilibria

Define the function g of ϕ as $g(\phi) = h - \frac{w(\phi)-\widehat{w}}{i-r}$. Then ϕ is the intersection of the line $y = x$ and the function $y = 1 - G(g(x))$. First to see when ϕ_1 exists, we have to show that the function $u(x) = x - 1 + G(g(x))$ crosses the horizontal axe in other points than $x = 0$ or $x = 1$. Note first that first $u(0) = 0$ is always holding.

Assumption 1 garanties theexistence of multiple equilibria. Note first that, given the expression of the skilled workers' wage as a function of ϕ, we have that the function $x \to 1 - G(g(x))$ is constant equal to 0 for $x \in [0, \bar{x}]$ (this is simply because $\underset{x\to 0}{lim}\ w(x) = -\infty$). The value of $\bar{x}$ depends on the wealth of the reachest individual in the economy. Let M be the coniunuous extension of $x \to 1 - G(g(x))$ over $[0, 1]$, defined by $M(x) = 0$ for $xx \in [0, \bar{x}]$ and $M(x) = 1 - G(g(x))$ for $x \geq \bar{x}$. Then it is immediate that M is a continuous function and given assumptions 1 and 3, we have that M crosses the $45°$ line two times between in $]\bar{x}, 1[$.

For the existence of ϕ_t for $t \geq 2$, note simply that wealth at $t + 1$ is the transformation by the continuous function piecewise linear f_t defined by

$$f_t(x) = \begin{cases} \beta R(x-h) + \Theta(\phi_t) + h & \text{if } x \geq h \\ \beta I(x-h) + \Theta(\phi_t) + h & \text{if } x \in \left[-\frac{w(\phi_t)-\widehat{w}}{i-r} + h, h\right] \\ \beta R(x-h) + \overset{\wedge}{\Theta} + h & \text{if } x \in \left]\underline{h}, -\frac{w(\phi_t)-\widehat{w}}{i-r} + h\right] \end{cases} \tag{25}$$

The continuity is clear except for the point $-\frac{w(\phi_t)-\widehat{w}}{i-r} + h$, but one easily verifies the following equality $\beta I\left(-\frac{w(\phi_t)-\widehat{w}}{i-r}\right) + \Theta(\phi_t) = \beta R(-\frac{w(\phi_t)-\widehat{w}}{i-r}) + \overset{\wedge}{\Theta}$. We have to add that G_t verifies also $1 - G_t(g(\theta)) > \theta$ because of lemma 1.

B.2.2 Markets clearing

The condition of market clearing for skilled labor is

$$\frac{X_t d_t}{\alpha} = \phi_t \tag{(*)}$$

The condition of market clearing for unskilled labor is

$$1 - \phi_t = (1 - X_t)d_t + X_t\left(F\frac{(1-\phi_t)}{X_t}\right) \tag{(**)}$$

Then eliminating d_t from $(*)$ and putting it in $(**)$ we get the equilibrium value of X_t

$$X_t = \frac{\alpha\phi_t}{(1-\phi_t)(1-F)+\alpha\phi_t}$$

And one verifies that when $\phi_t = 0$ then $X_t = 0$, and when $\phi_t = 1$ then $X_t = 1$.

B.3 Proof of proposition 2

The first inequality (i) is necessary because otherwise, using *proposition 1*, we see that nobody would want to acquire education. The second (ii) follows from lemma 1 and lemma 2 and the discussion above. First when (ii) is verified the second period wealth distribution verifies the conditions of lemma 2. Consequently when lemma 1 is verified, $(\phi_t)_t$ is a strictly increasing sequence. Finally as the threshold level for education (which is a decreasing function of ϕ) is under the poverty attraction, one can state that $\lim \phi_t = 1$.

B.4 Proof of Lemma 1

B.4.1 First part : wealth is "globally increasing"

Recall the function f_t defined above, we then just have to show that it is increasing, and this point is clear, and that $\forall x \geq \frac{\Theta(\phi_t)}{1-\beta R}$, $f_t(x) \geq x$. This last point comes from lemma 1 (the proof of which is obvious).

B.4.2 Second part : $-\left[w(\phi_t)-\widehat{w}\right]/(i-r) \leq \widehat{\theta}/(1-\beta R)$ for taking off economies

Recall that $\widehat{\theta} = \beta\widehat{w} - h$. Then $\frac{-[w(\phi_t)-\widehat{w}]}{I-R} - \frac{\widehat{\theta}}{1-\beta R} = -\frac{(w(\phi_t)-\widehat{w})(1-\beta R)+(I-R)(\beta\widehat{w}-h)}{(I-R)(1-GbR)}$. The numerator of this fraction is equal to $-\left[(1-\beta R)w(\phi_t)-(1-\beta I)\widehat{w}-(I-R)\right]$, and it remains to determine the sign of the expression in brackets. For this recall that $-\frac{w(\phi_t)-\widehat{w}}{i-r} \leq 0$ and by *proposition 3* $-\frac{\beta w(\phi_t)-h}{1-\beta I} \leq 0$, so $-\left[\frac{\beta w(\phi_t)-h}{1-\beta I}+\frac{w(\phi_t)-\widehat{w}}{i-r}\right] \leq 0$ so $\frac{\beta w(\phi_t)-h}{1-\beta I}+\frac{w(\phi_t)-\widehat{w}}{i-r} \geq 0$, which is equivalent to $(1-\beta R)w(\phi_t)-(1-\beta I)\widehat{w}-(I-R) \geq 0$. This completes the proof.

B.5 Proof of Lemma 2

As $T(x) \leq G(x)\ \forall x$, then $1-T(H(x)) \geq 1-G(H(x))$, consequently as the functions $x \longrightarrow 1-G(H(x))$ and $x \longrightarrow 1-T(H(x))$ are concave and above the $45°$ line around $x = 0$, one can state the result. Note that if we define $\bar{x}_T$ and $\bar{x}_G$ the correponding values for T and G of the $\bar{x}$ defined in $B.2.1$, then we have $\bar{x}_T \leq \bar{x}_G$.

B.6 Proof of proposition 3

For the problem to exist, we suppose that α evolves in $]1\ ,\ \bar{\alpha}]$ where $\bar{\alpha} < (I - R)h + \widehat{w}$.

Sign of $\phi^{'}(\alpha)$

As ϕ is likely to be an increasing function of α, then it is possible that as α increases, although the threshold level of ϕ for a successful industrialization rises, ϕ may increase.

Let us then pose that ϕ is a function of α, $\phi(\alpha)$, and study the sign of its derivative.

We know that $\phi(\alpha)$ is implicitly defined by the equation

$$\phi(\alpha) = 1 - G\left[h - \frac{\alpha - \frac{F(\alpha)(1-\phi(\alpha))}{\phi(\alpha)} - \widehat{w}}{I - R}\right] \ , \tag{26}$$

So taking the derivative on both sides one gets

$$\phi^{'}(\alpha) =$$

$$\frac{1}{I-R}\left[1 - \frac{F^{'}(\alpha)\,(1-\phi(\alpha))}{\phi(\alpha)} + \phi^{'}(\alpha)\frac{F}{\phi^2(\alpha)}\right] G^{'}\left(h - \frac{\alpha - \frac{F(1-\phi(\alpha))}{\phi(\alpha)} - \widehat{w}}{I - R}\right)$$

Rearranging, one gets

$$\phi^{'}(\alpha) = \frac{1}{I-R}\frac{\left(1 - \frac{F^{'}(\alpha)(1-\phi(\alpha))}{\phi(\alpha)}\right) G^{\prime}\left(h - \frac{\alpha - \frac{F(1-\phi(\alpha))}{\phi(\alpha)} - \widehat{w}}{I-R}\right)}{1 - \frac{1}{I-R}\frac{F}{\phi^2(\alpha)}G^{\prime}\left(h - \frac{\alpha - \frac{F(1-\phi(\alpha))}{\phi(\alpha)} - \widehat{w}}{I-R}\right)}. \tag{27}$$

Consequently the sign of $\phi^{'}(\alpha)$ is equal to

$$\frac{sign\left(1 - \frac{F^{'}(\alpha)(1-\phi(\alpha))}{\phi(\alpha)}\right)}{sign\left(1 - \frac{1}{I-R}\frac{F}{\phi^2(\alpha)}G^{\prime}\left(h - \frac{\alpha - \frac{F(1-\phi(\alpha)}{\phi(\alpha)} - \widehat{w}}{I-R}\right)\right)}.$$

First $1 - \frac{F^{'}(\alpha)(1-\phi(\alpha))}{\phi(\alpha)} \geq 0 \iff \phi(\alpha) \geq \frac{F^{'}(\alpha)}{1+F^{'}(\alpha)}$.

It remains to evaluate the sign of $\frac{1}{I-R}\frac{F}{\phi^2(\alpha)}G^{'}\left(h-\frac{\alpha-\frac{F(\alpha)(1-\phi(\alpha))}{\phi(\alpha)}-\widehat{w}}{I-R}\right)$. To this purpose let us assume the following specification for G, $G(x)=\left(\frac{x}{h}\right)^{\gamma}$. Then one gets

$$\begin{aligned}
& \frac{1}{I-R}\frac{F}{\phi^2(\alpha)}G^{'}\left(h-\frac{\alpha-\frac{F(\alpha)(1-\phi(\alpha))}{\phi(\alpha)}-\widehat{w}}{I-R}\right) \\
= \ & \gamma\frac{1}{\bar{h}^{\gamma}}\frac{1}{I-R}\frac{F(\alpha)}{\phi(\alpha)^{1+\gamma}}\left(\frac{1}{\left(\phi(\alpha)(h-\frac{\alpha-\hat{w}}{I-R})+\frac{F(1-\phi(\alpha))}{I-R}\right)^{1-\gamma}}\right)
\end{aligned}$$

The term $\left(\phi(\alpha)(h-\frac{\alpha-\hat{w}}{I-R})+\frac{F(1-\phi(\alpha))}{I-R}\right)$ is positive and is bounded below; $\exists Z_1, Z_2 \in R^+$, and $Z_1, Z_2 > 0$, such that $\forall\alpha$, $Z_2 > \phi(\alpha)(h-\frac{\alpha-\hat{w}}{I-R})+\frac{F(1-\phi(\alpha))}{I-R} > Z_1$. Note $\Delta=\gamma\frac{1}{h^{\gamma}}\frac{1}{I-R}$.

For low values of α, $\phi(\alpha)$ is very low and $\Delta\frac{F(\alpha)}{\phi(\alpha)^{1+\gamma}}Z_2 > 1$, so unambiguously $sign(\phi^{'}(\alpha)) = +$. For other values, we need to make an assumption that permits to conclude. For this let us take $F(\alpha)=A\alpha^2/2$.

Assumption 3 : A is such that $\forall\alpha>1$, $\Delta F(\alpha)Z_1<1$.

Then $sign(\phi^{'}(\alpha)) = +$ iff $\phi(\alpha)\leq\frac{F^{'}(\alpha)}{1+F^{'}(\alpha)}$.

Concavity of $\phi(\alpha)$

Recall the function $f(x)=1-G(g(x))$, we know that it is concave. We know that $\phi(\alpha)$ verifies

$$\phi(\alpha)=f(\phi(\alpha))\ .$$

Let us take the derivative of this relation at the second order. One obtains $\phi^{''}(\alpha)=\phi^{''}(\alpha)f^{'}(\phi(\alpha))+\phi^{'2}(\alpha)f^{''}(\phi(\alpha))$ and then we get $\phi^{''}(\alpha)=\frac{\phi^{'2}(\alpha)f^{''}(\phi(\alpha))}{1-f^{'}(\phi(\alpha))}$. Under assumption 3 or for γ not too high we have $f^{''}\leq 0$ always, and $f^{'}<1$.

Multiple equilibria

Let us take again $F(\alpha)=A\alpha^2/2$. Then $\Psi:\alpha\rightarrow\frac{F^{'}(\alpha)}{1+F^{'}(\alpha)}$ is a concave function. One easily verifies that $\chi:\alpha\rightarrow 1-G_0(h-\frac{\frac{\alpha}{2}-\widehat{w}}{I-R})$ is convexe. Then there are three possible situations.

1) Ψ and χ cross each other at a a unique point, which may satisfy or not condition (FIC)

2) Ψ and χ cross each other at two points, one statisfying (FIC) and the other (tus the lower) not.

3) Ψ and χ cross each other at two points, both statisfying (FIC)

B.7 Appendix of Section 2.2.2

Result : We are going to show here that the equilibrium value of h_R is greater for high values of γ than for lower values.

We have $\phi(h) = 1 - G\left[h - \frac{\alpha(h) - \frac{F(\alpha(h))(1-\phi(h))}{\phi(h)} - \widehat{w(h)}}{I-R}\right]$ and consequently

$$\begin{aligned}\phi^{'}(h) &= \left[1 - \frac{1}{I-R}\left(\phi^{'}(h)\frac{\partial w}{\partial \phi} + \alpha^{'}(h)\frac{\partial w}{\partial \alpha} - R\right) + \frac{1}{I-R}\alpha^{'}(h)F^{'}(\alpha(h))\frac{(1-\phi(h))}{\phi(h)}\right] \\ &\quad \times G^{'}\left(h - \frac{\alpha(h) - \frac{F(1-\phi(h))}{\phi(h)} - \widehat{w(h)}}{I-R}\right) ,\end{aligned}$$

and taking $\phi^{'}(h) = 0$, one obtains $1 - \frac{1}{I-R}(\alpha^{'}(h) - R) + \frac{1}{I-R}\alpha^{'}(h)F^{'}(\alpha(h))\frac{(1-\phi(h))}{\phi(h)} = 0$, and rearranging one finally gets that the optimal level of education h_R^* verifies

$$1 + i = \alpha^{'}(h_R^*)\left[1 - F^{'}(\alpha(h_R^*))\frac{(1-\phi(h_R^*))}{\phi(h_R^*)}\right] . \tag{28}$$

Putting (21) into the equation which defines implicitly $\phi(h)$ one gets

$$\frac{F^{'}(\alpha(h_R^*))}{1 - \frac{I}{\alpha^{'}(h_R^*)} + F^{'}(\alpha(h_R^*))} = 1 - G\left(h_R^* - \frac{\alpha(h_R^*) - \frac{F(\alpha(h_R^*))}{F^{'}(\alpha(h_R^*))}\left(1 - \frac{I}{\alpha^{'}(h_R^*)}\right) - \widehat{w(h_R^*)}}{I-R}\right) ,$$

and taking $F(\alpha) = \alpha^2/2$ one gets

$$\frac{\alpha(h_R^*)}{1 - \frac{I}{\alpha^{'}(h_R^*)} + \alpha(h_R^*)} = 1 - G\left(h_R^* - \frac{\frac{1}{2}\alpha(h_R^*) - \widehat{w(h_R^*)}}{I-R} - \frac{1}{2}\frac{\alpha(h_R^*)}{\alpha^{'}(h_R^*)}\right) . \tag{29}$$

Let us use again the following specification for $\alpha(h) = Bh^v$. Then equation (22) becomes

$$\frac{B(h_R^*)^v}{1 - \frac{I}{vB(h_R^*)^{v-1}} + B(h_R^*)^v} = 1 - G\left(\left(1 - \frac{1}{2v}\right)h_R^* - \frac{\frac{1}{2}B(h_R^*)^v - \widehat{w(h_R^*)}}{I-R}\right) . \tag{30}$$

Proof for example 1

Let us take an example : take $v = 1/2$, $B = 1$, and again $G(y) = \left(\frac{y}{h}\right)^\gamma$then (29) becomes

$$\frac{\sqrt{h}}{1 - \frac{I\sqrt{h}}{2} + \sqrt{h}} = 1 - G\left(-\frac{\frac{1\sqrt{h}}{2} - (1+R) - Rh}{I-R}\right) \tag{31}$$

Let us pose $x=\sqrt{h}$, then one verifies that the function $x \longrightarrow \frac{1}{2}x-(1+R)-Rx^2$ is always negative. More over the function $x \longrightarrow \frac{x}{1-\frac{Ix}{2}+x}$ is concave. It remains to evaluate the concavity of $\vartheta : x \longrightarrow 1-G\left(-\frac{\frac{1x}{2}-(1+R)-Rx^2}{I-R}\right)$. By differentiating two times and putting $G^{'}$ in factor one gets that $sign\left(\vartheta^{''}\right) = sign\left(-2R+(1-\gamma)\frac{\left(\frac{1}{2}-2Rx\right)^2}{Rx^2+1+R-\frac{x}{2}}\right)$.

After some manipulations, we get

$$sign\left(-2R+(1-\gamma)\frac{\left(\frac{1}{2}-2Rx\right)^2}{Rx^2+1+R-\frac{x}{2}}\right)$$
$$=$$
$$sign\left(Rx\left(2(1-\gamma)-1\right)\left(2Rx-1\right)+\frac{1}{4}(1-\gamma)-2R(1+R)\right)$$

Then remember that $\alpha(h) > \hat{w} > R$ by assumptions in the texte. Then $Rx\left(2(1-\gamma)-1\right)\left(2Rx-1\right)+\frac{1}{4}(1-\gamma)-2R(1+R) > R(1+R)\left(2(1-\gamma)-1\right)\left(2R(1+R)-1\right)+\frac{1}{4}(1-\gamma)-2R(1+R)$

Then one verifies easily that for $\gamma = 0.25$ for example, then for any $R \geq 1$ $R(1+R)\left(2(1-\gamma)-1\right)\left(2R(1+R)-1\right)+\frac{1}{4}(1-\gamma)-2R(1+R) > 0$. Consequently for $\gamma = 0.25$, $\text{sign}\left(\vartheta^{''}\right)$ is positif. And one also shows that $sign\left(\vartheta^{'}\right)$ is negative (this sign is always neagtive independently of the value of γ). Consequently for small values of γ, ϑ is decreasing and convexe.

For high values of γ , for example $\gamma > 1/2$, it is easy to show that $Rx\left(2(1-\gamma)-1\right)\left(2Rx-1\right)+\frac{1}{4}(1-\gamma)-2R(1+R) < 0$, and thus $sign\left(\vartheta^{''}\right) < 0$, and as $sign\left(\vartheta^{'}\right)$ is negative, then ϑ is decreasing and convexe.

Finally, as the function $x \longrightarrow \frac{x}{1-\frac{Ix}{2}+x}$ is independent of γ, we have that the equilibrium value of h_R is greater for high values of γ than for lower values.

B.8 Appendix of the example in section 2.1

We have to show that the equation

$$\frac{A.\alpha^*}{1+A.\alpha^*} = 1-G\left(h-\frac{\frac{1}{2}\alpha^*-\widehat{w}}{I-R}\right) . \tag{32}$$

admits a solution for well chosen parameters. First remember that the function $\zeta_1 : \alpha \longrightarrow \frac{A\alpha}{1+A\alpha}$ is concave and the function $\zeta_2 : \alpha \longrightarrow 1-G\left(h-\frac{\frac{1}{2}\alpha-\widehat{w}}{I-R}\right)$ is convex.

Although not in the range of values allowed for α, the derivative of $\alpha \longrightarrow \frac{A\alpha}{1+A\alpha}$ at

$\alpha = 0$ is equal to A. The derivative of ζ_2 at $\alpha = 0$ is equal to $\frac{\gamma}{2}\frac{1}{((I-R)h+\widehat{w})}\left(\frac{h+\frac{\widehat{w}}{I-R}}{\bar{h}}\right)^{\gamma}$ which can be made as small as desired. We can choose A and $\bar{h}$ and γ such that $\frac{\gamma}{2}\frac{1}{((I-R)h+\widehat{w})}\left(\frac{h+\frac{\widehat{w}}{I-R}}{\bar{h}}\right)^{\gamma} < A$. Then if $1 - G\left(h + \frac{\widehat{w}}{I-R}\right) = 0$ then we know that ζ_1 and ζ_2 will cross each other at a unique other point $\alpha^* > 0$. If $1 - G\left(h + \frac{\widehat{w}}{I-R}\right) > 0$, then if ζ_2 is too convex then there may be no intersection. When we evaluate the convexity of ζ_2, we get that the higher γ the less convex is ζ_2.

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