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**Growth and Inflation
in a Monetary "Selling-Cost" Model**

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Abstract

This paper presents a simple monetary "selling-cost" model in the spirit of Dornbusch and Frenkel [1973] who assume that there are real costs in terms of resources to "delivering" output to consumers and that producers may reduce these transaction costs by accepting money in exchange for products. Within an exogenous growth framework, this assumption easily justifies the existence of a negative relationship between *i)* inflation and transition growth in the short run and *ii)* inflation and output in the long run. Within an endogenous growth framework, this kind of transaction costs allows the existence of multiple steady states when monetary authorities control the rate of money creation. Under a mild assumption concerning the specification of the transaction costs, two stationary growth rates arise and the higher one is indeterminate. When indeterminacy occurs the monetary growth rate displays a positive impact on the balanced growth rate. Moreover the higher growth rate is recognized to be superior in terms of welfare. Eventually we investigate the role of monetary and fiscal policy to coordinate agents and drive the economy towards virtuous growth rates.

JEL classification: E63; E52; E31

Keywords: Transaction costs; Indeterminacy; Monetary policy; Fiscal theory of the price level

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1 Introduction

How to best introduce money into intertemporal growth models is still an open question. Great attention has been payed to the analysis of consumers' direct costs and benefits of money holding. This class of models includes Money-in-Utility-Function (MIUF), Cash-in-Advance (CIA) and Shopping-Time (or other shopping cost) models.

In an exogenous growth framework, the effect of these formalizations of money holding on the long-run output-per-capita ratio crucially depends on the presence of leisure in the utility function. If labor is exogenously supplied, the well-known (long-run) superneutrality result due to Sidrauski [1967] generally applies in representative agent's models. However, in heterogenous agents' models, a Tobin effect[1] could arise if, for instance, new generations or dynasties appear at each period (see Orphanides and Solow [1990]). Conversely, when the supply of labor is endogenous, there exists a negative relationship between money growth and output per capita, *i.e.* a reverse Tobin effect. This result has been shown by Brock [1975] in a MIUF model and by Cooley and Hansen [1989] in a stochastic model with a CIA constraint on consumption. In these cases, inflation acts as a tax on market activities and induces households to switch from market to non-market activity (leisure).

The same kind of results hold in endogenous growth models but the output-per-capita ratio has to be replaced by the capital growth rate in order to evaluate the effect of the money growth rate[2]. However, as noted by Chari, Jones, and Manuelli [1995], those models are unable to replicate the strong negative (non-linear) relationship between inflation and growth that empirical cross-country studies generally find. Facts from data are highlighted in Fischer [1993], Barro [1995], Bruno and Easterly [1995], Judson and Orphanides [1996], Sarel [1996] and Ghosh and Phillips [1998]. These studies conclude to a negative relationship between inflation and growth which robustly holds at all but the lowest inflation rate[3].

Chari, Jones, and Manuelli [1995] show that the models' inability to reproduce these facts may be due to their standard narrow assumption that all money is held by public (the consumers) for making transactions.

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[1] In Tobin [1965] money competes with capital for a place in the households' portfolio. One prediction from Tobin's model is that the inflationary money growth and capital stock are positively correlated.

[2] See Chari, Jones, and Manuelli [1995] for a review and Van der Ploeg and Alogoskoufis [1994] for a model with new dynasties.

[3] The evidence varies from one study to another. However the empirical works agree about the sign of long run inflation impact on growth. 10 percentage point increase in the average inflation rate is associated with a decrease in the average growth rate of between about 0.2 and 0.7 percentage points (Fischer [1993], Chari, Jones and Manuelli [1995]). In the short run there is a less agreement about the effects of moderate inflation and a positive link is detected for very low inflation rate (Ahmed and Rogers [2000]).

A way investigated by Stockman [1981] consists in examining the implications of a CIA constraint applied to investment. Stockman's insight is prompted by the fact that firms frequently put up some cash in financing their investment project. Money is then a complement to capital, accounting for a negative relationship between the steady state level of output-per-capita and the inflation rate in a neoclassical environment.

Unfortunately, Chari, Jones, and Manuelli [1995] find this assumption powerless to explain a strong negative relationship between inflation and growth in an endogenous growth framework. The authors[4] then suggest to incorporate banks in the models in order to match the empirical outcomes. Their analysis is based upon the effect of a reserve requirement constraint on the credit cost.

Exploring an other side of banking activity, Aiyagari, Braun, and Eckstein [1998] incorporate into a monetary growth model a credit sector providing services that allow firms to economize on money. The paper is motivated by empirical observations on the comovements of output-currency ratio, inflation and the relative size of the credit sector.

In an earlier work, Dornbusch and Frenkel [1973] stressed the importance of money holding by firms in a neoclassical growth model. We use a modified version of the Dornbusch-Frenkel selling-cost technology and extend the analysis to a simple endogenous growth framework. More precisely there are real costs in terms of resources to 'delivering' output to consumers. Producers may reduce these transaction costs by accepting money in exchange for products[5]. However the producers implicitly bear an opportunity cost of holding money, given as usual by the nominal interest rate. The effect of inflation on output-per-capita depends on the relative complementariness (versus substitutability) between money and capital. In the case of complementariness, inflation and output-per-capita are negatively correlated. Money is now viewed as a substitute to costly financial services.

Within an exogenous growth framework, we emphasize the existence of a negative relationship between inflation and transition growth rate in the short run and between inflation and output in the long run. Within an endogenous growth framework, the transaction costs allow the existence of multiple steady states when monetary authorities control the rate of money creation. Under a mild assumption concerning the specification of the transaction costs, two stationary growth rates arise and the higher one is indeterminate. Moreover when indeterminacy occurs the monetary growth rate displays a positive impact on the balanced growth rate and the higher growth rate is recognized to be superior in terms of welfare.

In our context the alternative monetary policy of interest pegging rules out

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[4]See Chari, Jones, and Manuelli [1996] for an extension.

[5]Empirically, the importance of the money holding by firms has been recently documented by Mulligan [1997].

the real indeterminacy. However the nominal indeterminacy (i.e. the equilibrium multiplicity for prices and nominal variables) persists if the monetary policy is not accompanied by a specific fiscal policy. More precisely in a large class of fiscal policies the government's solvability condition plays the role of an additional equilibrium condition which completes the usual market clearing conditions and fixes prices (Leeper [1991], Sims [1994] and Woodford [1995]). We then investigate the role of such a mix of monetary and fiscal policy in our model.

The rest of the paper proceeds as follows. In section 2 we define the representative agent's program, the policy rule and the market clearing conditions. In section 3 the neoclassical framework is characterized. Our results are very close to those of Dornbusch and Frenkel [1973] but without ambiguity about the sign of the relationship between inflation (and the monetary growth rate) and growth. The Fisher relation does hold despite the presence of a reverse Tobin effect. Section 4 is devoted to endogenous growth. The Fisher relation no longer holds. This kind of transaction costs allows the existence of multiple steady states when monetary authorities control the rate of money creation. Under a mild assumption concerning the specification of the transaction costs, two stationary growth rates arise and the higher one is indeterminate. When indeterminacy occurs the monetary growth rate displays a positive impact on the balanced growth rate. Moreover the higher growth rate is found to be Pareto-superior. In Section 5, we investigate the role and the interplay of monetary and fiscal policy to coordinate agents and drive the economy towards better growth rates. A policy mix constituted by an interest rate peg and an appropriate fiscal policy prevents from real and nominal indeterminacy. The last section proposes a conclusion.

2 The model

Following the literature on the "shopping costs", we suppose that real resources are necessary for the task of facilitating transactions. Real money holdings provide "shopping services" in the sense that the more money is held, the more real resources are freed from the transaction task. In the spirit of Dornbusch and Frenkel [1973] we suppose more precisely that there are real resources costs to delivering output to consumers ("selling-costs") and that producers can reduce those by accepting money in exchange for their products. Although we deal with a single-good economy, our setup is intended to provide a simplified representation in which the household-producer does not consume his own production and sells his specialized output to a large number of other households-producers.

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2.1 The representative agent's program

Let $s\left(f\left(k_t\right)/m_t\right)$ denote the unitary transaction cost of selling one unit of product, where $f\left(k_t\right)$ is the total production and m_t the level of real balances. We suppose that the function $s\left(\cdot\right)$ satisfies :

Assumption 1:
- $s'\left(v_t\right) > 0$,   if $v_t > \underline{v}$,
- $2s'\left(v_t\right) + v_t s''\left(v_t\right) > 0$,   if $v_t > \underline{v}$,
- $s'\left(v_t\right) = s'\left(\underline{v}\right) = 0$,   if $v_t \leq \underline{v}$,

where $v_t = f\left(k_t\right)/m_t$ denotes the output-real balances ratio. One can easily verify than these conditions are always met with the function:

$$s\left(v\right) = \begin{cases} \beta\left[\underline{v}^\delta + \left(v - \underline{v}\right)^\delta\right]^{1/\delta}, & \text{if } v > \underline{v}, \\ \beta\underline{v}, & \text{if } v \leq \underline{v}, \end{cases} \tag{1}$$

with $\beta > 0$ and $\delta > 1$.

The unitary selling-cost is then an increasing function of the total production relative to real balances $f\left(k_t\right)/m_t$. This specification is symmetric to the more traditional[6] formalization of transaction costs where the shopping costs are increasing in consumption relative to real balances[7]. The point of satiation in real balances $m_t = f\left(k_t\right)/\underline{v}$ is defined as satisfying: $s'\left(\underline{v}\right) = 0$. It is not worthwhile to increase m_t beyond this point since, by doing it, it is not possible to save additional resources. The hypothesis $2s'\left(v\right) + vs''\left(v\right) > 0$ is equivalent to condition $\partial^2 s/\partial m^2 > 0$, so that an increase in the real quantity of money decreases the transaction costs at a decreasing rate. This is a sufficient condition jointly with the other traditional hypotheses, to assure the convexity of the representative agent's program.

The representative consumer maximizes an intertemporal utility functional

$$\int_0^\infty e^{-\rho t} u\left(c_t\right) dt, \tag{2}$$

where c_t denotes consumption at time t and ρ represents the time preference rate, subject to a portfolio decision constraint,

$$a_t = k_t + m_t + b_t, \tag{3}$$

the equation for real asset accumulation,

$$\dot{a}_t = \left[1 - s\left(f\left(k_t\right)/m_t\right)\right] f\left(k_t\right) - \eta k_t + r_t b_t - \pi_t m_t - \tau_t - c_t \tag{4}$$

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[6]See for instance Kimbrough [1986], McCallum and Goodfriend [1987] and Correia and Teles [1996].

[7]An other and interesting case is captured by the following limit form : $s\left(v\right) = +\infty$ if $v > \underline{v}$, and $s\left(v\right) = 0$ if $v \leq \underline{v}$, which is to the selling cost technology what the Cash-in-Advance constraint is to the shopping cost technology.

and the limit condition,

$$\lim_{t\to\infty} a_t \cdot e^{-\int_0^t r_s ds} \geq 0. \tag{5}$$

As usual k_t is the stock of capital and η the depreciation rate. b_t is the real amount of bonds held by the household and r_t is the real interest rate. The inflation rate is denoted by π_t, while $-\pi_t m_t$ represents the inflationary tax. Finally, real lump-sum taxes are denoted by τ_t.

Utility is assumed to be isoelastic, with elasticity of intertemporal substitution equal to σ :

$$u(c_t) = \begin{cases} \frac{c_t^{1-1/\sigma}-1}{1-1/\sigma}, & \text{if } \sigma \neq 1, \\ \ln c_t, & \text{if } \sigma = 1. \end{cases}$$

Setting the Hamiltonian and arranging the first order conditions, one obtains:

$$\dot{c}_t/c_t = \sigma\{r_t - \rho\}, \tag{6}$$
$$s'(v_t)v_t^2 = r_t + \pi_t, \tag{7}$$

$$[1 - s(v_t) - s'(v_t)v_t]f'(k_t) - \eta = r_t \tag{8}$$

and

$$\lim_{t\to\infty} u'(c_t) a_t \cdot e^{-\rho t} = 0. \tag{9}$$

Equation (6) is a traditional Euler equation and (7) implicitly determines the desired amount of real balances: the left hand side is the marginal rate of return ("productivity") on real balances and the right hand side is the marginal cost of money holding, that is the nominal interest rate, $i_t = r_t + \pi_t$. Equation (8) equalizes the real rate of return of capital to the real interest rate on bonds and finally (9) is the transversality condition.

Rewriting equation (7) as

$$s'(v_t)v_t^2 = i_t, \tag{10}$$

one can express implicitly the output-currency ratio as a function of the nominal interest rate, $v_t = \tilde{v}(i_t)$, with,

$$\tilde{v}'(i_t) = \frac{1}{\tilde{v}(i_t)}\left[2s'(\tilde{v}(i_t)) + \tilde{v}(i_t)s''(\tilde{v}(i_t))\right]^{-1}, \tag{11}$$

which is always positive under assumption 1.

2.2 The government budget constraint

The nominal government budget constraint is given by:

$$\dot{M}_t + \dot{B}_t = i_t B_t - p_t \tau_t. \tag{12}$$

M_t is the nominal stock of money and B_t the amount of nominal bonds. Defining $d_t = (M_t + B_t)/p_t$ and rewriting the preceding expression in real terms, one obtains :

$$\dot{d}_t = r_t d_t - (i_t m_t^s + \tau_t), \tag{13}$$

where $m_t^s = M_t/p_t$ is the real money supply.

Integrating the government budget constraint at time 0, we can construct the intertemporal real budget constraint:

$$\frac{M_0 + B_0}{p_0} = \int_0^\infty (i_t m_t^s + \tau_t) \cdot e^{-\int_0^t r_s ds} dt + \Delta_0, \tag{14}$$

where Δ_0 is given by:

$$\Delta_0 = \lim_{t \to \infty} d_t \cdot e^{-\int_0^t r_s ds}. \tag{15}$$

2.3   Policy regime and market equilibrium

In section 3 and 4, we shall suppose that the monetary authority follows a simple monetary rule : $\dot{M}_t/M_t = \mu$, and that the government adopts for instance a fiscal policy neutral for prices which respects the no-Ponzi game condition $\Delta_0 = 0$. At the money market equilibrium, we have $m_t = M_t/p_t$ and the real balance law of motion is given by :

$$\dot{m}_t = (\mu - \pi_t) m_t. \tag{16}$$

The good market equilibrium is obtained by aggregating the capital asset accumulation constraint of household (3) and (4) with the government real budget constraint (13), that is,

$$\dot{k}_t = [1 - s(v_t)] f(k_t) - \eta k_t - c_t, \tag{17}$$

which corresponds to the law of motion of capital stock.

3   The neoclassical framework

The results we obtain within an exogenous growth setup under the above transaction technology are very close to those found by Dornbusch and Frenkel [1973]. In particular the complementariness between money and capital entails a negative relation between inflation and output-per-capita.

Assume the production function to be concave $(f''(\cdot) < 0)$. Integrating equation (8) in equation (6) one obtains a modified Euler condition.

$$\dot{c}_t = \sigma [(1 - s(v_t) - s'(v_t) v_t) f'(k_t) - \eta - \rho] c_t. \tag{18}$$

Combining (7), (8) and (16), we obtain the following law of motion for real balances :

$$\dot{m}_t = \left\{ \mu + \left[1 - s\left(v_t\right) - s'\left(v_t\right) v_t \right] f'\left(k_t\right) - \eta - s'\left(v_t\right) v_t^2 \right\} m_t. \tag{19}$$

Equations (17), (18) and (19) constitute a dynamic system

$$\left(\dot{m}_t, \dot{k}_t, \dot{c}_t \right) \equiv \varphi\left(m_t, k_t, c_t \right). \tag{20}$$

3.1 Steady state

At steady state, $\dot{k}_t = \dot{m}_t = \dot{c}_t = 0$. By imposing these conditions in (6) and (16), we obtain $\pi^* = \mu$ and $r^* = \rho$ and the output-currency ratio v^* is implicitly given by equation (7) :

$$s'\left(v^*\right) v^{*2} = \rho + \mu. \tag{21}$$

By totally differentiating (21), we notice the positive impact of the parameters μ and ρ on the steady state v^*. The steady state value of the capital stock, k^*, is then derived from (8) with $r_t = \rho$:

$$k^* = f'^{-1}\left(\frac{\rho + \eta}{1 - s\left(v^*\right) - s'\left(v^*\right) v^*} \right). \tag{22}$$

3.2 Dynamic analysis

Initial conditions of the reduced system (20) concern only nominal money, which is denoted by M_0, and capital, k_0. As consumption and real balances are independently non-predetermined, neither is the output-money ratio. Dynamics described by (20) are three-dimensional and the local dynamics around the steady state are linearized by the following Jacobian matrix :

$$J = \begin{bmatrix} \left(f' + v\right)\left(2s' + vs''\right)v & \left(1 - s - vs'\right)f''m - \left(f' + v\right)\left(2s' + vs''\right)f' & 0 \\ v^2 s' & \rho & -1 \\ \sigma c\left(2s' + vs''\right)f'v/m & \sigma c\left[\left(1 - s - vs'\right)f'' - \left(2s' + vs''\right)\left(f'\right)^2/m\right] & 0 \end{bmatrix}.$$

Under assumption 1 the trace, $tr\left(J\right) = \left(f' + v\right)\left(2s' + s''v\right)v + \rho$, is always positive and the determinant

$$\det\left(J\right) = \sigma c\left(2s' + vs''\right)\left(1 - s - vs'\right)vf''$$

is always negative.

In this case one eigenvalue is negative and the other two have positive real parts. As there are one predetermined variable (capital) and two non-predetermined variables (real balances and consumption), the equilibrium turns out to be determinate. The result is summarized in the following proposition :

Proposition 1 *In a neoclassical version of the "selling-cost" model equilibrium determinacy always prevails under assumption* 1.

This result rules out the possibility of endogenous fluctuations and sunspot equilibria.

3.3 Monetary policy and the Friedman rule

By totally differentiating (21) and (22), we obtain:

$$\frac{\partial k^*}{\partial \mu} = \frac{f'(k^*)^2}{(\rho + \eta) v^* f''(k^*)} < 0.$$

The effect of a rise in the money growth rate is a drop in the steady state capital level. By the quantitative equation, at the steady state, inflation as well as the nominal interest rate rise linearly with μ. Consequently, agents reduce their real balances $(dv^*/d\mu > 0)$. This in turn raises the cost to "delivering" output to other consumers. Because at the steady state the net marginal productivity, $[1 - s(v) - s'(v)v] f'(k) - \eta$, is equal to ρ, the rise of selling costs, *i.e.* the decline of $[1 - s(v) - s'(v)v]$, has to be compensated by a rise of the gross marginal productivity $f'(k)$ which is obtained by a drop of capital stock level. During the transition the growth rate becomes negative and converges to zero.

We then highlight without any ambiguity a direct negative effect of a rise in the money growth rate on the stationary capital level. In contrast, in the seminal paper of Dornbusch and Frenkel [1973][8], this direct effect is challenged by a positive cross-effect. This secondary effect appears because the (inverse of) real money enters alone the transaction cost function[9]. Following the more recent literature on transaction costs-based monetary model, we have supposed that the (inverse of) real balances-output ratio enters in the transaction cost function removing by this way the cross-effect. Nevertheless, Dornbusch and Frenkel [1973] introduce a convexity argument which permits them to conclude that a direct negative effect dominates the cross effect. Actually, this kind of argument is equivalent to assumption 1 in the present paper.

The optimal monetary policy is solution of the following simple program:

$$\max \quad \int_0^\infty e^{-\rho t} u(c_t)\, dt$$
$$s.t. \quad \dot{k}_t = [1 - s(v_t)] f(k_t) - \eta k_t - c_t.$$

The first order conditions are given by equations (8) and (17) and by:

$$s'(v_t) v_t^2 = 0 \quad (= r_t + \pi_t) \quad \forall t, \tag{23}$$

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[8]Part IV, p. 151.

[9]Within our definition of the transaction cost function.

that is, $v_t = \underline{v} \; \forall t$ and $\pi_t = -r_t$. The real balances are at satiation level and the nominal interest rate is equal to zero. The Friedman rule applies even during transition and the relevant path is solution of a simple dynamical system composed by the following couple of differential equations:

$$\dot{k}_t = [1 - s(\underline{v})] f(k_t) - \eta k_t - c_t,$$
$$\dot{c}_t = \sigma \{[1 - s(\underline{v})] f'(k_t) - \eta - \rho\} c_t.$$

This optimal path can be implemented by choosing the money growth rate μ_t verifying (16), $\pi_t = -r_t$ and $\underline{v} m_t = f(k_t)$, that is :

$$
\begin{aligned}
\mu_t &= \frac{\dot{m}_t}{m_t} + \pi_t \\
&= \left(\frac{f'(k_t)}{f(k_t)}\right) \dot{k}_t - r_t \\
&= \eta - (\eta k_t + c_t)\left(\frac{f'(k_t)}{f(k_t)}\right).
\end{aligned}
$$

The optimal steady state is then defined by equation (22) with $v^* = \underline{v}$:

$$\hat{k} = f'^{-1}\left(\frac{\rho + \eta}{1 - s(\underline{v})}\right) \tag{24}$$

and the steady state value for the rate of monetary expansion is simply $\hat{\mu} = -\rho$. By comparison of (22) and (24), the optimal steady state is clearly associated to a higher value of capital stock.

4   Endogenous growth

The neoclassical growth is characterized by a negative trade-off between inflation and output-per-capita. However the exogenous growth model by definition is powerless to take in account the link between inflation and the long run growth. Therefore we are naturally brought to focus on the endogenous causes of growth.

The impact of inflation on growth is clarified within the endogenous growth framework. Furthermore quite richer dynamics arise accounting for the complexity of observed phenomena.

The main outcomes of the paper are presented in this section.

As we are chiefly interested in the monetary dynamics entailed by the "selling-cost" transaction technology, we avoid weighing down the production side with richer specifications and we straight assume a basic linear form:

$$f(k_t) \equiv A k_t.$$

By repeating the steps performed in the previous section, i.e. combining equations (6), (7), (8), (16) and (17), we obtain a reduced dynamic system :

$$\dot{v}_t = [(A + v_t) s' (v_t) v_t - \mu - x_t] v_t, \tag{25}$$

$$\dot{x}_t = \sigma \{(1 - s (v_t) - s' (v_t) v_t) A - \eta - \rho\} x_t$$
$$- \{[1 - s (v_t)] A - \eta - x_t\} x_t. \tag{26}$$

where $x_t = c_t/k_t$ is the consumption-capital ratio. The stationary growth rates for real balances, capital, production and consumption are equal, i.e. the growth, as usual in this class of models, is balanced.

4.1 Balanced growth: multiplicity

By definition the derivatives in the right hand sides of (25) and (26) are equal to zero at the steady state. Thereby the two differential equations become simple algebraic expressions and the stationary output-real balances ratio v^* solves the reduced equation:

$$s' (v^*) v^{*2} + (\sigma - 1) \{[1 - s (v^*) - s' (v^*) v^*] A - \eta\} = \mu + \sigma\rho. \tag{27}$$

The balanced growth rate $\gamma^* \equiv (\dot{c}_t/c_t)^* = (\dot{m}_t/m_t)^* = (\dot{k}_t/k_t)^*$ is given by (6) :

$$\gamma^* = \sigma \{r^* - \rho\}, \tag{28}$$

where the equilibrium real interest rate r^* is easily obtained by putting $f' (k_t) = A$ and $v_t = v^*$ (from (27)) in (8) :

$$r^* = [1 - s (v^*) - s' (v^*) v^*] A - \eta. \tag{29}$$

Using (10), equation (27) becomes:

$$i^* = \mu + \sigma\rho - (\sigma - 1) \{[1 - s (\tilde{v}(i^*)) - s' (\tilde{v}(i^*)) \tilde{v}(i^*)] A - \eta\}. \tag{30}$$

Differentiating this expression in i^* and μ, we get the derivative of the implicit function $\mu = \tilde{\mu} (i^*)$:

$$\tilde{\mu}' (i^*) = 1 - (\sigma - 1) \frac{A}{\tilde{v}(i^*)}. \tag{31}$$

We then make the following assumption:

Assumption 2:
- $\underline{v} < (\sigma - 1) A$,
- $\tilde{\mu}(0) = (\sigma - 1) \{[1 - s (\underline{v})] A - \eta\} - \sigma\rho > 0$.

Under this assumption, the function $\mu = \tilde{\mu} (i^*)$ is qualitatively represented by the following convex curve :

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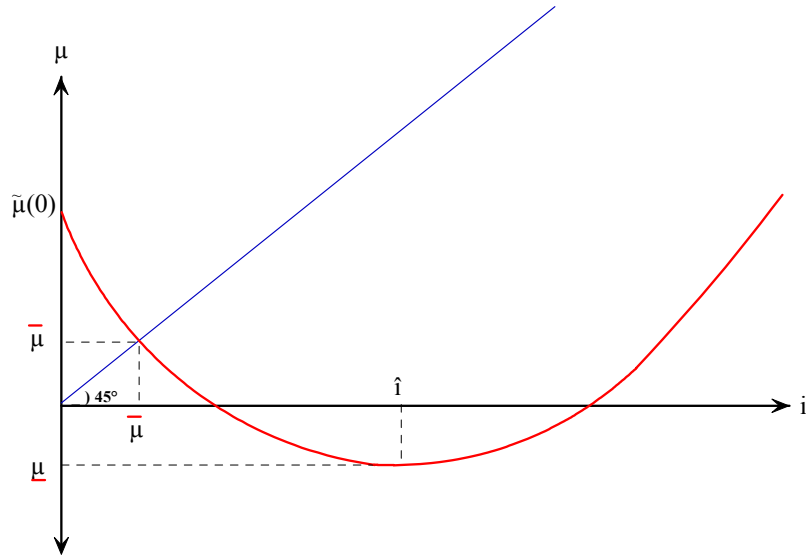
Figure 1

As we will show, the 45° line permits us to exclude from the solution space some stationary equilibria which violates the transversality condition. At the steady state, the transversality condition (9) becomes:

$$\lim_{t\to\infty} c_0^{1-1/\sigma} a_0 \cdot e^{-[\rho-(1-1/\sigma)\gamma^*]t} = 0$$

or equivalently

$$\rho > (1 - 1/\sigma)\gamma^*. \tag{32}$$

By inspecting (28), this condition coincides with the more traditional no-Ponzi game condition:

$$r^* > \gamma^* \tag{33}$$

Using (28) and (29), one can rewrite equation (30) in the following form :

$$i^* = \mu + r^* - \gamma^*, \tag{34}$$

which is simply, by noting that $i^* - r^* = \pi^*$, the dynamic version of the quantitative equation. Condition (33) is equivalent to $i^* > \mu$ and constitutes a restriction in the solution space: one can exclude the area above the 45° line in the figure 1. This implies the following proposition :

Proposition 2 *Under Assumptions 1 and 2, the Friedman rule, i.e. $i^* = 0$, violates the transversality condition $r^* > \gamma^*$ and is not achievable.*

The monetary growth rate μ affects negatively the balanced growth rate γ^* if and only if the following condition holds:

$$\tilde{v}(i^*) > A_\sigma = (1 - \sigma) A, \tag{35}$$

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where $\widetilde{v}$ is defined by (11) (see appendix).

This result is simply interpreted in a model where real balances increase the (net of selling-costs) productivity of capital. When inflation rises, agents reduce their real balances which in turn raise the cost to "delivering" output to other consumers. In contrast the impact is positive if condition (35) is violated.

In figure 1 $\underline{\mu}$ denotes the minimum of the convex curve $\mu = \tilde{\mu}\left(i^*\right)$, while $\bar{\mu}$ corresponds to the intersection between this curve and the 45° line. The analytical expressions for $\underline{\mu}$ and $\bar{\mu}$ are given in appendix.

Observing figure 1, we easily obtain the following results:

- for $\mu < \underline{\mu}$, there exist no stationary equilibria at all;

- for $\mu \in \left]\underline{\mu}, \bar{\mu}\right[$, there are two stationary equilibria: the first verifies, $\tilde{v}\left(i^-\right) < A_\sigma$ and the second, $\tilde{v}\left(i^+\right) > A_\sigma$;

- for $\mu > \bar{\mu}$, there exists only one stationary equilibrium which satisfies the transversality condition; it verifies $\tilde{v}\left(i^+\right) > A_\sigma$.

More details are provided in appendix.

Using (27) to (10) and $\pi^* = i^* - r^*$, it is easy to check that the equilibrium growth rate is always negatively correlated to the inflation rate. We get $\gamma^* = \gamma\left(\pi^*\right)$, with :

$$\frac{d\gamma^*}{d\pi^*} = \frac{-\sigma A}{A + \tilde{v}\left(i^*\right)} < 0.$$

Consequently, when (35) is not verified, the positive impact of a rise of monetary growth rate on the growth rate is associated with a negative relation between the monetary growth and the inflation rate. Totally differentiating the dynamical version of the quantitative equation,

$$\mu = \pi^* + \gamma\left(\pi^*\right),$$

we find :

$$\frac{d\pi^*}{d\mu} = \frac{A + \tilde{v}\left(i^*\right)}{\tilde{v}\left(i^*\right) - A_\sigma},$$

which is positive if and only if condition (35) holds. If not, the (rather surprising) negative relation between money growth and inflation is clearly due to a (negative) over-reaction of the growth rate to a modification of the inflation rate.

4.2 Dynamic analysis: determinacy and local indeterminacy

We carry out the local analysis in the neighborhood of a steady state. As usual we linearize the dynamic system of (25) and (26) around the steady state. The

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Jacobian matrix evaluated at the steady state is:

$$J \equiv \begin{bmatrix} v^* \left\{ s'(v^*) v^* + (A + v^*) \left[s'(v^*) + s''(v^*) v^* \right] \right\} & -v^* \\ A \left\{ s'(v^*) - \sigma \left[2s'(v^*) + s''(v^*) v^* \right] \right\} x^* & x^* \end{bmatrix}. \tag{36}$$

Under Assumption 1 the trace is always positive. The determinant is positive if and only if (35) hold. Sink configurations are ruled out by a positive trace. The steady state is a source if and only if (35) holds. Otherwise it is a saddle.

As real balances and consumption are non-predetermined variables, the output-real balances ratio v_t ($= Ak_t/m_t$) and the consumption-capital ratio x_t ($= c_t/k_t$) are non-predetermined as well. More precisely even if either the real balances or consumption are fixed, the other variable remains non-predetermined. Thereby the output-money ratio v and the consumption-capital ratio x are independently non-predetermined.

Local indeterminacy is that particular multiplicity of equilibrium trajectories arising whenever the dimension of the stable manifold is greater than the number of predetermined variables. As now two variables are non-predetermined and the occurrence of sinks is definitely ruled out, local indeterminacy requires a saddle configuration.

Under Assumption 1 and 2 for every monetary growth $\mu \in \left(\underline{\mu}, \overline{\mu} \right)$, there are two steady states according to figure 1. The lower growth rate is associated to the higher interest rate and satisfies condition (35). According to the previous discussion, this equilibrium is determinate. To the converse the higher growth rate is indeterminate. The following proposition sums up the results.

Proposition 3 *Under Assumption 1 and 2, if $\underline{\mu} < \mu < \overline{\mu}$, there are two steady states of endogenous growth, the lower balanced growth rate is determinate while the higher one is indeterminate. The impact of monetary growth is negative on the lower growth rate and positive on the higher.*

Under indeterminacy there is room for stochastic sunspot equilibria, i.e. endogenous fluctuations. The sense of this statement is illustrated among others by Woodford [1986] and Chiappori and Guesnerie [1989].

A geometric representation
Now we shape the phase diagram in the (v, x)-space. We plot the two curves $\dot{v}_t = 0$ and $\dot{x}_t = 0$ in the (v, x)-space. From (25) and (26) we obtain respectively

$$\begin{aligned} x &= (A + v) s'(v) v - \mu, \\ x &= [1 - s(v)] A - \eta - \sigma \left\{ [1 - s(v) - s'(v) v] A - \eta - \rho \right\}. \end{aligned}$$

If $\underline{\mu} < \mu < \overline{\mu}$, the two curves intersect twice at the stationary velocity v^- and $v^+ > v^-$. Thereby inside the interval (v^-, v^+) the locus $\dot{v}_t = 0$ lies below $\dot{x}_t = 0$,

and above outside. Moreover if the elasticity $s''(v) v/s'(v)$ is greater than -1, both the curves $\dot{v}_t = 0$, $\dot{x}_t = 0$ are positive-sloped.
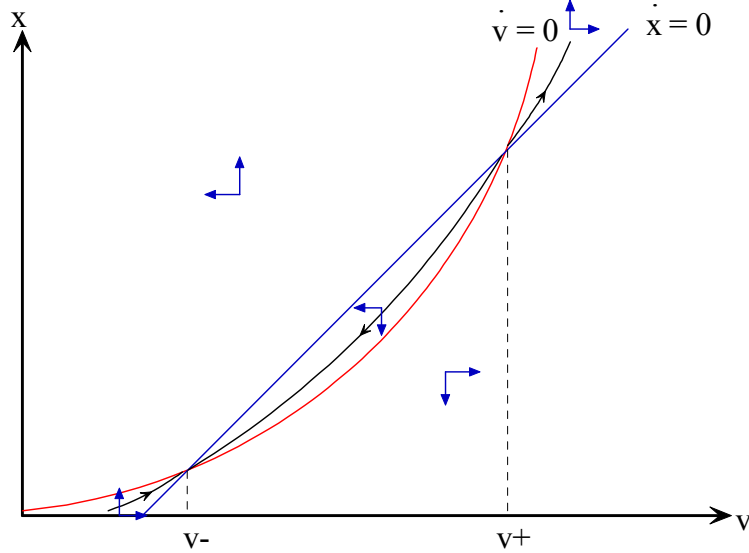
Figure 2

As seen above (v^-, x^-) is a saddle and (v^+, x^+) is a source.

An example

To perform a numerical simulation, we specify a functional form of transaction costs and the parameter values. First we take a limit version of the function (1) when $\delta \to 1^+$ such that $s(v)$ becomes linear for $v > \underline{v}$, i.e. $s(v) = \beta v$. The coefficient β simply captures the sensitivity of transaction costs to the output-real balances ratio. The two curves $\dot{v}_t = 0$ and $\dot{x}_t = 0$ are positive-sloped and the phase diagram looks like that of figure 2.

Second we fix the measurable parameters as follows $A = 0.15$, $\eta = 0.1$, $\mu = 0$, $\rho = 0.02$. As the measure of the intertemporal substitution σ and sensitivity β of transaction costs is problematic, we locate the admissible parameter region in the (σ, β)-space. Conditions for reality, positivity and determinacy of the steady states define two curve in the (σ, β)-space and divide it in three regions (see figure 3). In the region I, on the left of the vertical line, there is a unique steady state which is determinate. Two stationary equilibria arise in region II: the lower is determinate, while the higher one, is indeterminate. Below the non-linear curve,

inside the shaded area, there are no steady states at all.
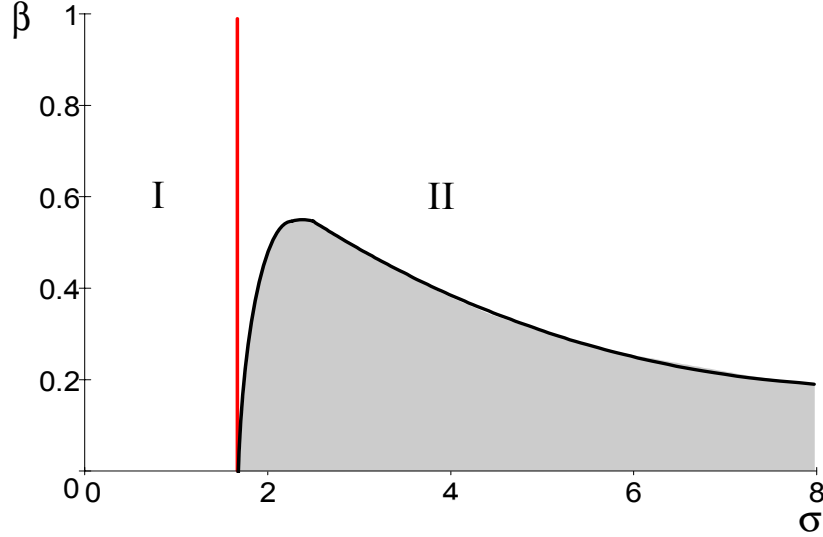
Figure 3

To complete the characterization of the steady states, their Pareto ranking is now provided.

4.3 Welfare analysis

We are interested in evaluating the welfare associated to each steady state. As we consider a representative agent, the measure of welfare is simply given by the value of the intertemporal utility functional evaluated along the optimal consumption path. The optimal trajectory associated to a steady state in endogenous growth is characterized by a constant balanced growth rate. Thereby growth is exponential and we can compute the intertemporal utility. In our paper felicity $u(c_t)$ takes a CES form. The optimal consumption path is $c_t = c_0 e^{\gamma t}$, where $\gamma = \sigma \left[(1 - s - vs') A - \eta - \rho\right]$ is the balanced growth rate and $s = s(v)$. The initial consumption of steady state $c_0 = k_0 \left[(1 - s) A - \eta - \gamma\right]$ is determined by the law of capital accumulation (17). Integrating all these informations and computing the intertemporal utility along the exponential consumption path, we obtain the optimal value function, which is a welfare index.

$$U(v) = \frac{\sigma}{\sigma - 1} \left\{ k_0^{1 - 1/\sigma} \frac{\left\{(1 - s) A - \eta - \sigma\left[(1 - s - vs') A - \eta - \rho\right]\right\}^{1 - 1/\sigma}}{\rho - (\sigma - 1)\left[(1 - s - vs') A - \eta - \rho\right]} - \frac{1}{\rho} \right\}.$$

Note that this computation is possible if and only if the transversality condition (32) holds. Otherwise the intertemporal utility (2) diverges under the law of

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motion (4) and maximization fails. The value $U(v)$ depends on the stationary production-real balance ratio v we consider. Such a welfare measure allows to compare and rank the steady states. Under the transversality condition the welfare measure is always strictly decreasing, i.e.

$$U'(v) < 0.$$

This inequality provides a straightforward and robust conclusion: the higher the growth rate, the lower the output-money ratio, the higher the welfare. The higher stationary growth is implemented by a lower initial consumption. Nevertheless from the welfare point of view the higher stationary growth rate γ^+ is always preferred to γ^-, irrespective to the initial consumption. Welfare analysis is the convenient introduction to policy analysis we develop in the next section.

5 Fiscal and monetary policy

As seen above under a constant money growth there is room for real indeterminacy. The equilibrium multiplicity can be removed by adopting a simple alternative monetary rule, *i.e.* an interest peg. However the nominal indeterminacy viewed as price indetermination, may persist and we need to complete the monetary policy with a specific fiscal policy.

5.1 Monetary policy: interest rate peg

Consider the maximization of a usual intertemporal utility under constraints (3) and (4). Moreover assume the following interest rate rule

$$i_t = i > \bar{\mu} \tag{37}$$

even during transition[10]. The new constraint enters the first order condition (10) defining the output-currency ratio $v_t = \tilde{v}(i)$, and the consumption growth is given by (6) with (8):

$$\dot{c}_t/c_t = \sigma\left\{[1 - s(\tilde{v}(i)) - s'(\tilde{v}(i))\tilde{v}(i)]A - \eta - \rho\right\} = \tilde{\gamma}(i). \tag{38}$$

The law of motion (17) determines a unique value for initial consumption, which is compatible with the equilibrium

$$c_0 = k_0\left\{[1 - s(\tilde{v}(i))]A - \tilde{\gamma}(i) - \eta\right\}.$$

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[10]Notice that $i = \mu + \varepsilon$, where ε is very small, is a good candidate for such a policy because it maximizes welfare.

Hence the economy jumps on its balanced growth (38) and transition is ruled out as in Rebelo's Ak model. Finally notice that the interest rate rule (37) determines the initial real balances $m_0 = Ak_0/v(i)$ but that the general level of prices is not yet determined because M_0 is now endogenous. Actually, under an interest rate peg, the relevant state variable is $M_0 + B_0$, the total government liability, but M_0 (as well as B_0) jumps in response to a modification of the nominal interest rate.

5.2 Fiscal policy and the price level determination

The monetary policy plays a key role for equilibrium selection in real terms. However it is powerless to stabilize the prices. Conversely fiscal policy matters for price determination. More precisely the question is whether the government can use some other policy tools, such as taxes or debt policy, in conjunction with monetary policy to fix the initial price level and thereby the price path. According to Leeper [1991], Sims [1994] and Woodford [1995] the government's choice of how to finance its debt is not neutral. Following Woodford [1995], a fiscal regime is said to be neutral for prices, if the present discounted value of government liabilities converges to zero irrespective of the price path[11]. More precisely the limiting condition (a no-Ponzi game condition) holds for the government intertemporal budget constraint regardless of the evolution of the other endogenous variables. It is actually a special regime; the fiscal policy plays no role at all in price-level determination, while the path of the money supply clearly does. This is the domain of validity for the quantity theory. In contrast, in a regime where the government solvability condition matters, the interest rate pegging delivers a unique price level, while a constant money supply growth yields a contradictory over-determination.

At equilibrium, the no-Ponzi game condition (15) has to be satisfied for a constant real interest rate, *i.e.*

$$\Delta_0 = \lim_{t \to \infty} d_t \cdot e^{-rt} = 0. \tag{39}$$

5.2.1 A general class of fiscal policy

Let us consider the following class of fiscal policy:

$$\tau_t = \alpha_{bt} \frac{iB_t}{p_t} - \alpha_{mt} \frac{iM_t}{p_t} + \delta_t, \tag{40}$$

where α_{bt}, α_{mt}, δ_t are exogenous time functions with $\alpha_{bt}, \alpha_{mt} \in [0, 1]$ $\forall t$.

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[11]Barro's Ricardian equivalence simply means the irrelevance of government debt for real quantities (Barro [1974]).

The nominal and the real government budgetary constraints, respectively (12) and (13), then become:

$$\dot{M}_t + \dot{B}_t = (1 - \alpha_{bt}) iB_t + \alpha_{mt} iM_t - \delta_t \tag{41}$$

and

$$\dot{d}_t = (r - \alpha_{bt}i) d_t - [(1 - \alpha_{bt} - \alpha_{mt}) im_t + \delta_t]. \tag{42}$$

Integrating the last equation between 0 an $+\infty$, we get:

$$d_0 = \int_0^\infty [(1 - \alpha_{bt} - \alpha_{mt}) im_t + \delta_t] \cdot e^{-\int_0^t (r - \alpha_{b\tau}i)d\tau} dt + \tilde{\Delta}_0, \tag{43}$$

where $\tilde{\Delta}_0$ is defined by:

$$\tilde{\Delta}_0 \equiv \lim_{t \to \infty} d_t e^{-\int_0^t (r - \alpha_{b\tau}i)d\tau}. \tag{44}$$

Noting that the real balance level grows at the constant rate γ, i.e. $m_t = m_0 e^{\gamma t}$, we can easily verify than equation (43) holds under two assumptions on the exogenous time functions[12] $\alpha_{bt} + \alpha_{mt}$ and δ_t, i.e.:

$$\left| \int_0^\infty [1 - (\alpha_{bt} + \alpha_{mt})] \cdot e^{\int_0^t -(r - \alpha_{b\tau}i - \gamma)d\tau} dt \right| < +\infty$$

and

$$\left| \int_0^\infty \delta_t e^{-\int_0^t (r - \alpha_{b\tau}i)d\tau} dt \right| < +\infty.$$

5.2.2   Neutral fiscal policy

Consider now the following restriction :

$$(1 - \alpha_{bt} - \alpha_{mt}) im_t + \delta_t = 0 \quad \forall t.$$

(41), (42) and (43) become respectively:

$$\dot{M}_t + \dot{B}_t = (1 - \alpha_{bt}) i (B_t + M_t),$$

$$\dot{d}_t = (r - \alpha_{bt}i) d_t \tag{45}$$

and

$$d_0 = \tilde{\Delta}_0. \tag{46}$$

[12]For instance such assumptions are satisfied, if the function $\alpha_{bt} = \alpha_b$ is constant over time and the inequality $r - \alpha_b i > \gamma$ holds. The latter condition is equivalent to $\mu(i) < (1 - \alpha_b) i$, which constitutes a new restriction on the choice of i in figure 1.

Equation (45) indicates that the real government liability grows at a rate, $(r - \alpha_{bt}i)$, that allows us to write:

$$d_t = d_0 e^{\int_0^t (r - \alpha_{b\tau}i)d\tau}.$$

Then, by taking the limit of d_t the equation (44) give immediately $\tilde{\Delta}_0 = d_0$. In consequence, equation (46) is uninformative about the equilibrium value of $d_0 = (M_0 + B_0)/p_0$ which is indeterminate as well as the initial price level p_0. We can easily verify that the no-Ponzi game condition (39) is always satisfied[13] for $\alpha_{bt} > 0$.

5.2.3 Price anchoring fiscal policy

Generically the exogenous time paths α_{bt}, α_{mt}, δ_t satisfy the condition

$$\int_0^\infty \left[(1 - \alpha_{bt} - \alpha_{mt}) \, im_t + \delta_t \right] \cdot e^{-\int_0^t (r - \alpha_{b\tau}i)d\tau} dt \neq 0.$$

Under some feasibility condition $\left| \tilde{\Delta}_0 \right| < \infty$, the initial value of the government liability d_0 is uniquely determinate as well as the price level p_0. These results are summarized in the following proposition :

Proposition 4 *Under an interest rate rule $(i_t = i)$ the real equilibrium is always determinate. If the interest pegging is accompanied by a fiscal policy where the government solvability is an additional market clearing condition, then the nominal equilibrium is determinate as well.*

Summing up, a monetary policy which pegs the interest rate, determines the real equilibrium and in particular the inflation. A fiscal policy characterized by a non-redundant government solvability condition, fixes the initial price p_0. Therefore this policy mix jointly determines the whole price path.

6 Conclusion

In this paper, we have investigated the role of firm's money holding which is supposed to reduce selling costs.

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[13]Our example of neutral fiscal policy slightly generalizes the case studied by Schmitt-Grohé and Uribe [2000]. In this article a secondary surplus policy is considered, i.e. taxes are used for paying the debt interests: $\tau_t = ib_t$. This policy is captured in our context by simply setting $\alpha_{bt} = 1$, $\alpha_{mt} = \delta_t = 0$ for every t.

Within a neoclassical setup standard results arise. In particular we highlight the equilibrium determinacy and a long run negative relationship between inflation and output.

In endogenous growth dynamics are richer and less obvious. If the monetary authority controls the money growth, under mild assumptions there are two stationary growth rates. The higher growth rate is indeterminate, superior in terms of welfare and increasing in money growth. A robust negative relation between inflation and growth holds.

If the monetary authority pegs the interest rate, there is no longer room for real indeterminacy. Moreover the residual nominal indeterminacy is ruled out by fiscal policies where the government solvability plays the role of an effective equilibrium condition.

Among the model extensions we envisage, two research lines matter more. On the one side usual consumer's transaction costs can interact with the selling costs. On the other side the framework can be augmented by theoretical blocks such as an overlapping dynasties hypothesis or a New Keynesian Phillips Curve in order to account for the non-linearity of the growth-inflation relationship, i.e. a positive and negative link for respectively low and high inflation rates.

A   Appendix

Let us define $\bar{\mu}$ and $\underline{\mu}$ such that $\bar{\mu} = \tilde{\mu}(\bar{\mu})$, and $\underline{\mu} = \tilde{\mu}(\hat{\imath})$ where $\hat{\imath}$ verifies $\tilde{\mu}'(\hat{\imath}) = 1 - A_\sigma/\tilde{v}(\hat{\imath}) = 0$ with $A_\sigma \equiv (\sigma - 1)A$, or equivalently

$$0 = [1 - s(\tilde{v}(\bar{\mu})) - s'(\tilde{v}(\bar{\mu}))\tilde{v}(\bar{\mu})] A_\sigma - [\sigma\rho + (\sigma - 1)\eta]$$

and, using (30),

$$\underline{\mu} = \tilde{v}^{-1}(A_\sigma) + [1 - s(A_\sigma) - s'(A_\sigma)A_\sigma] A_\sigma - [\sigma\rho + (\sigma - 1)\eta].$$

We are now able to provide useful comparative statics. Using (28), (29) and (11), one obtain:

$$\frac{d\gamma^*}{di^*} = \frac{-\sigma A}{\tilde{v}(i^*)}$$

and combining this result with the inverse of (31), we get:

$$\frac{d\gamma^*}{d\mu} = \frac{\sigma A}{A_\sigma - \tilde{v}(i^*)}.$$

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