EPEE
UNIVERSITE D'EVRY
VAL D'ESSONNE

DOCUMENT DE RECHERCHE

EPEE

CENTRE D'ETUDE DES POLITIQUES ECONOMIQUES DE L'UNIVERSITÉ D'EVRY

Financial Effects of Privatizing the Production of Investment Goods

Stefano BOSI & Carine NOURRY

01 – 10

UNIVERSITE D'EVRY – VAL D'ESSONNE, 4 BD. FRANCOIS MITTERRAND, 91025 EVRY CEDEX

Financial Effects of Privatizing the Production of Investment Goods [1]

Stefano BOSI

EPEE, Université d'Evry - Val d'Essonne
E-mail: Stefano.Bosi@eco.univ-evry.fr

and

Carine NOURRY

EPEE, Université d'Evry - Val d'Essonne
E-mail: Carine.Nourry@eco.univ-evry.fr

September 2001

Abstract: *This paper focuses on a two-sector overlapping generations model with productive capital, where the investment good is jointly provided by government and private firms. Keeping, for simplicity, the production technologies identical for both sectors, we look at the issue of existence and stability of steady states and we derive some necessary and sufficient conditions to observe a unique rational expectations equilibrium. In particular we study the real and financial effects of privatization. In our very specific context, we highlight, by means of numerical simulations, a negative privatization impact on the utility level and the speed of absorbing exogenous shocks*

[1] We would like to thank Jean-Paul Barinci, Guillaume Girmens, Roger Guesnerie, Michel Guillard, Frédéric Jouneau-Sion, Alain Venditti and the participants to the conference "New Perspective on (In)Stability : the Role of Heterogeneity" (7-9 juin 2001, GREQAM, Marseille) and to the workshop "Journées générations imbriquées" (11-12 juin 2001, Université Charles de Gaulle, Lille) for helpful comments and suggestions. Any remaining errors are solely our own.

1 Introduction

This paper deals with a standard two-sector overlapping generations model with productive capital. The Diamond [6] formulation is adopted, but augmented to include a second sector, producing an investment good. We consider the OLG counterpart to the two-sector growth model (Uzawa [17], [18]) developed by Galor [10]. But, unlike in the Galor formulation where private firms produce goods in both sectors, we assume that there are two kinds of firms producing the investment good. A proportion π of firms is private, whereas a proportion $1-\pi$ is public. Schmitz [16] also considers a two-sector model where government produces investment goods, but he assumes the government technology to be different from the private technology. In our setup both use the same technology and also the same technology is used in both sectors.

The situation where the government produces the vast majority of investment good purchased in a country is not unrealistic since in a number of countries the government restricts imports of investment goods and then produces domestic investment goods. Egypt, but also India and Turkey are characteristic of such a policy. Nevertheless, there is not much research examining the aggregate consequences of government production of goods. The World Bank's (1995) report about government enterprise tells us that the share of public output on total output is 10 to 15% in poor as well as in rich market economies. As it is the same whatever country is considered, we could conclude that there are no aggregate consequences. But Schmitz [15] proves that the types of goods produced by government are different and correlated with income. The share of manufacturing output is large in many poor countries and very small in all rich countries. Thus, despite the fact that there is some overlap between the manufacturing and investment sectors, it seems that government production may have important aggregate consequences and thus generate different types of dynamical behavior of variables like the interest rate on the financial market.

Schmitz [16] considers a growth model where productivities are different when the firm is public or private. That is because he wants to explain that the policy which consists in making the investment good produced by government reduces drastically the productivity in both sectors. We are interested in another economic feature which is privatization. As we know public firms have to show that they are competitive before being sold to private agents, we assume that they act competitively and have reached the same technology than private firms. The other difference with the Schmitz' assumptions, is that we consider that, initially, public firms own their capital. In Schmitz, the only difference between a public firm and a private one is that their technology is different. Actually, both firms are owned by private agents since they buy shares of the firm in each period and receive the return of their last period investment.

Our aim is to study the short and long-term effects of privatization. As a first step, in this paper, we consider the very special case where the government owns all, or a part, of the investment sector, and wants to improve the capital stock of the economy. We will say that the capital stock of the economy is improved if the level goes closer the golden rule value. To implement such a policy, the State employes the dividends from investment firms to offer first-period transfers to agents. When dividends are positive, then young agents receive positive transfers. But when they are negative, agents are then taxed. As we will show, this last case corresponds to an overaccumulation: government decreases the first-period income of agents and thus their saving. When the capital stock is lower than the golden rule value, then young agents receive additional lump-sum transfers allowing them to consume and save more. We will then analyze the consequences of the privatization of the investment sector during and after the transition in terms of price stability and welfare level.

To reach this goal, we first have to study the existence and stability of steady state equilibria for all proportions of private firms in the investment good sector, *i.e.* for every degree of privatization. We analyze the determinacy properties of equilibrium paths for capital and labor supply allocation between the two sectors. A given configuration will be referred to as locally indeterminate as soon as there exists a continuum of distinct equilibrium paths starting from the same initial value for the capital stock. It is now well-known that indeterminacy of perfect foresight equilibrium is a sufficient condition for the existence of sunspot equilibria and stochastic fluctuations based upon extrinsic uncertainty[1]. In the past the occurrence of indeterminacy has been considered to be a theoretical weakness, while a recent literature admits that the theory of indeterminacy has

[1]See Cass and Shell [5], and Woodford [19].

a role to play, to explain a large part of economic fluctuations. The traditional approach highlights the existence of shocks on the fundamentals such as technology and tastes, and simulates the propagation mechanism of these shocks. Indeterminacy may constitute a rich source of propagation dynamics to an equilibrium model and sunspot may provide an alternative shock to technology or taste changes.

Such shocks would be inconsistent if the rational expectations equilibrium were unique. Under indeterminacy this is no longer the case. As long as the sunspot process is consistent with the expectations of agents, equilibrium conditions are satisfied, and sunspots will affect the real dynamics. The sunspot theory is not a mere intellectual trick to perform an equilibrium selection.

We shall focus on both the classes of shocks: on the beliefs to generate endogenous real business cycles, on fundamentals to obtain exogenous business cycles. For the sake of plausibility and simplicity we shall take in account a temporary shock on the beliefs about employment, and a permanent shock on the proportion of public firms viewed as the main fundamental among the others. Actually, we note that in a period of privatization, it is not unrealistic to assume that agents can be influenced by extrinsic signals about the risk of unemployment like just some warning messages. Moreover, a real shock on the proportion of public firms producing the investment good, for example a decrease of this proportion, just means a partial privatization of the investment sector.

With imperfect competition, local indeterminacy easily arises in Ramsey as well as in OLG models[2]. In the case of perfect competition, Kehoe and Levine [12] consider a pure exchange economy, and prove that there exist some robust examples of overlapping generations models with a continuum of equilibria. This indeterminacy result has been extended to production economies by Muller and Woodford [13]. These contributions are concerned with multi-dimensional OLG models with many consumption and capital goods. The coexistence of state and forward variables allows therefore to consider local indeterminacy of perfect foresight equilibrium near steady state equilibria. On the contrary, in the standard Diamond model, perfect foresight equilibria are locally unique. Nevertheless, as soon as labor supply is endogenous, Nourry [14] shows that for large range of fundamentals parameters, a steady state equilibrium is locally indeterminate. In

[2] See Benhabib and Nishimura [4] and the recent survey by Benhabib and Farmer [3].

a one-sector growth model, Benhabib and Farmer [1] provide a necessary and sufficient condition for indeterminacy in a relative simple way. However this condition requires large externalities. In a two-sector model, Benhabib and Farmer [2] introduce increasing returns to scale that occur as a consequence of sector specific externalities. Keeping the production technology identical for both sectors, they show indeterminacy occurs for realistic parameter values. In a two-sector OLG model, Galor [10] proves that, when the technology is different in the consumption good sector and the investment good sector, indeterminacy easily arises. But as we will check in our paper, if we consider the Galor's model with identical technology in both sectors, local indeterminacy of steady state equilibria is no longer possible.

It suffices to write the dynamical system corresponding to the evolution of the capital stock in a two-sector model without government, to see that it is exactly the same than the system used by Farmer [8] and corresponding to a one-sector model with a government debt. In this last model, Farmer proves the existence of deterministic fluctuations. Stochastic ones are not possible since a steady state equilibrium cannot be indeterminate. We thus have the same characteristics for our model in the extreme case where all the investment sector in private and government does not play any role (of course, there is no debt).

Adding the assumption that public firms supply a part of the investment good, we completely characterize the existence of steady state equilibria and the dynamical behavior near these stationary states. We establish, thereby, the necessary and sufficient conditions for the existence of indeterminate equilibria. We prove that, when there exist several steady state equilibria, some of them can be locally indeterminate. If the steady state equilibrium is unique, then, for a proportion of public firms in the investment good sector which is not "too large", perfect foresight equilibria are determinate. The proportion of public production that allows the existence of endogenous fluctuation depends on the saving behavior of private agents. The existence of government firms in a two-sector model introduces then a drastic modification of the dynamical behavior involved since we obtain indeterminacy even when the technology is the same in both sector.

The lack of economic stability is interpreted in two different ways. On the one hand the equilibrium multiplicity allows the expectation-driven stochastic fluctuations. On the other hand the slowness of convergence to a stationary state after a shock, even in the case of unique equilibrium path, can be viewed as a form of economic instability.

The first non-intuitive result we obtain is thus that, when the steady state is unique, the economic system can be destabilized when the government owns a large part of the sector, because there is room for a multiplicity of transition equilibria and stochastic fluctuations.

However, according to our second interpretation of economic instability, we are able to show within a plausible example, that privatization has a destabilizing impact on the economic system because as the proportion of private ownership increases, the speed of convergence after a shock on the degree of privatization or on other fundamental parameters, decreases. It is thus more and more costly, in terms of instability, to absorb real shocks when the government is leaving the sector.

In a long-run perspective, the welfare impact of the privatization is not surprising. We show on a standard example that the utility of agents decreases with the degree of privatization of the sector. In our example, where the stationary level is below the golden rule, this is just the consequence of the government policy who improves the steady state level of capital.

But, when the level of private ownership of the sector is increased, the utility level jumps to a level above the initial one, because of the expectations of agents. Only afterwards it begins to decrease continuously towards a level below the initial one.

The paper is organized as follows. In the next section we shall present the model. In section 3, we focus on the steady state and establish an existence result. In section 4, we analyze the local dynamics and derive our main theoretical results. Section 5 shows the financial aspects, while section 6 studies the occurrence of endogenous fluctuations generated by shocks on the beliefs as well as the transmission of exogenous shocks on the fundamentals. Numerical computations are performed in section 7 to simulate the shock impact on capital and labor dynamics. Section 8 gives some concluding comments. All the proofs are gathered in section 9.

2 The model

Consider a perfectly competitive world where economic activity is performed over infinite discrete time in which there are identical selfish agents. In every period, two goods, a perishable consumption good and an investment good, are produced using two factors, labor and capital, in the production process.

Each agent lives for two periods: he works during the first, supplying inelastically one unit of labor. He has preferences for his consumption of private good (c, when he is young, and d, when he is old), which are summarized by the utility functions $U(c,d)$.

Assumption 1. *$U(c,d)$ is strictly increasing with respect to each argument ($U_1(c,d) > 0$, and $U_2(c,d) > 0$), C^2 with negative definite Hessian matrix over the interior of the set R_+^2. Moreover, for all consumption levels c, $d > 0$, $U_1(0,d) = U_2(c,0) = \infty$.*

Each agent is assumed to have $1+n$ children, with $n \geq 0$, and the number of individuals born in period t is denoted N_t. During his first period of life, he receives a lump-sum amount T_t from the government which can be seen as a tax if it is negative. He maximizes his utility function over his life-cycle as follows:

$$\begin{aligned} \max_{c_t, d_{t+1}, z_t^c, z_t^I} \quad & U(c_t, d_{t+1}) \\ \text{s.t.} \quad & w_t + T_t = c_t + q_t^c z_t^c + q_t^I z_t^I, \\ & \left(q_{t+1}^c + \delta_{t+1}^c\right)^e z_t^c + \left(q_{t+1}^I + \delta_{t+1}^I\right)^e z_t^I = d_{t+1}, \end{aligned} \tag{1}$$

where q_t^c and q_t^I are respectively the asset prices of the two representative firms in the consumption and the investment sector. δ_t^c and δ_t^I denote the dividends of these two firms per asset share. z_t^c and z_t^I are the demands of shares addressed respectively to the two representative firms.

The consumer is price-taker as usual and maximizes the utility function with respect to the consumption and asset demands c_t, d_{t+1}, z_t^c, z_t^I. The first order conditions with respect to z_t^c, z_t^I are resumed by a no-arbitrage condition, which can be viewed as an equilibrium condition:

$$\frac{\left(q_{t+1}^c + \delta_{t+1}^c\right)^e}{q_t^c} = \frac{\left(q_{t+1}^I + \delta_{t+1}^I\right)^e}{q_t^I} \equiv R_{t+1}^e, \tag{2}$$

where the common return factor is denoted by R^e_{t+1} and is simply interpreted as an interest factor. The assets value constitutes the consumer's saving and we are allowed to set

$$s_t \equiv q^c_t z^c_t + q^I_t z^I_t.$$

After resetting, the program gets a more convenient form:

$$\begin{aligned} \max_{c_t, d_{t+1}} \quad & U(c_t, d_{t+1}) \\ \text{s.t.} \quad & w_t + T_t = c_t + s_t, \\ & R^e_{t+1} s_t = d_{t+1}. \end{aligned}$$

Agents are assumed to expect perfectly the interest factor $R^e_{t+1} = R_{t+1}$. Assumption 1 implies the existence and uniqueness of an interior optimal saving level s_t. The first order condition

$$U_1(w_t + T_t - s_t, s_t R_{t+1}) = R_{t+1} U_2(w_t + T_t - s_t, s_t R_{t+1}) \tag{3}$$

determines the saving supply of each agent as a function

$$s_t = s(w_t + T_t, R_{t+1}).$$

Let us denote the initial disposable income as follows: $\omega \equiv w + T$. Under Assumption 1, the saving function $s(.,.)$ is differentiable for $(\omega, R) \in \mathbf{R}^2_{++}$, with value in $\mathbf{R}_+$. We will make use of the following restriction throughout the paper.

Assumption 2. *Consumptions c and d are normal goods.*

Under Assumption 2, it is well known that the saving function is increasing with respect to the first period income, *i.e.* $s_\omega(\omega, R) \geq 0$.

We assume the technology to produce the consumption and the investment good to be identical. The production function of a representative firm, denoted $F(K^j, L^j)$, is thus the same within the two sectors. It depends on the stock of capital K^j and labor L^j used in the sector j, and is assumed to be homogeneous of degree one. Assuming also that capital depreciation is complete in each period (the active life of a generation), and denoting $k^j = K^j/L^j$ the capital stock per labor unit in the sector j, we may define the production function in intensive form as $f(k^j) = F(k^j, 1)$.

Assumption 3. *$f(k)$ is positive valued, C^2, strictly increasing, strictly concave over $\mathbf{R}_{++}$, and satisfies* $\lim_{k\to 0} f'(k) = \infty$, $\lim_{k\to\infty} f'(k) = 0$ *and* $f(0) = 0$.

Let the consumption good be the numeraire, and let p_t denote the price of the investment good, K_t^c and L_t^c the stock of capital and labor used in the consumption good sector. The firm maximizes a sum of discounted future profits:

$$\max_{K_t^c, L_t^c} F(K_t^c, L_t^c) - w_t L_t^c - p_t K_{t+1}^c + \sum_{\tau=t+1}^{\infty} \frac{1}{\prod_{i=t+1}^{\tau} R_i} \left[F(K_\tau^c, L_\tau^c) - w_\tau L_\tau^c - p_\tau K_{\tau+1}^c\right].$$

In the investment good sector, there exists a proportion $1-\pi$ of the representative firm owned by the government and π owned by private agents. We denote by K_t^I and L_t^I the stock of capital and labor employed in the investment good sector. The only difference with a model, where the whole firm is private, is that a share $1-\pi$ of the dividends goes to government. We assume that the dividends received by the government in each period are used to finance a lump-sum additional income for the young (if the dividends are negative, then the young pay a tax), whereas the private firms use it to pay interests on the capital borrowed to consumers. Actually, in our model, a share $1-\pi$ of the capital used in the investment sector is owned by the public sector, and a share π is borrowed to agents. Thereby we can define π as a relevant measure of the privatization extent. The representative firm of the investment sector maximizes a sum of discounted future profits:

$$\max_{K_t^I, L_t^I} p_t F\left(K_t^I, L_t^I\right) - w_t L_t^I - p_t K_{t+1}^I + \sum_{\tau=t+1}^{\infty} \frac{1}{\prod_{i=t+1}^{\tau} R_i} \left[p_\tau F\left(K_\tau^I, L_\tau^I\right) - w_\tau L_\tau^I - p_\tau K_{\tau+1}^I\right].$$

Thereby the government transfers are given by

$$N_t T_t = (1-\pi)\left[p_t F\left(K_t^I, L_t^I\right) - w_t L_t^I - p_t K_{t+1}^I\right]. \tag{4}$$

If labor and private capital are perfectly mobile across sectors and if both goods are produced, then the competitive equilibrium conditions imply that the interest factor R_t and the wage rate w_t satisfy:

$$R_t = \frac{f'(k_t^c)}{p_{t-1}} = \frac{p_t f'\left(k_t^I\right)}{p_{t-1}}, \tag{5}$$

$$w_t = f(k_t^c) - k_t^c f'(k_t^c) = p_t\left(f\left(k_t^I\right) - k_t^I f'\left(k_t^I\right)\right). \tag{6}$$

We thus obtain the capital intensity in each sector is the same[3]: $k_t^c = k_t^I \equiv k_t$ for every t, and the investment good price turns out to be equal to unity[4].

Since each young agent supplies 1 unit of labor, the total labor in the economy in period t is: $L_t = N_t$. As $l_t^j \in [0,1]$ is the proportion of the labor force employed in sector j at time t, we have $l_t^c + l_t^I = 1$.

From (4) we derive the transfer to a young consumer.

$$T_t = (1-\pi)\left[(1-l_t)\, k_t f'(k_t) - (1+n)(1-l_{t+1})\, k_{t+1}\right].$$

The only way to save for agents in this model is the private capital. Thus, the capital accumulation equation simply states

$$K_{t+1}^c + \pi K_{t+1}^I = L_t s(\omega_t, R_{t+1}).$$

Moreover, the capital of the economy in period $t+1$ is the investment good produced in period t by the investment sector:

$$K_{t+1}^c + K_{t+1}^I = F\left(K_t^I, L_t^I\right).$$

We denote $l_t \equiv l_t^c$.

In this economy, the dynamics of capital and labor allocation are thus given by the Euler-Lagrange equation (3), and the following conditions:

$$(1+n)\, k_{t+1} = (1-l_t)\, f(k_t), \tag{7}$$

$$\frac{s(\omega_t, R_{t+1})}{1+n} = l_{t+1}k_{t+1} + \pi(1-l_{t+1})\, k_{t+1}. \tag{8}$$

3 Steady state equilibria

We denote $w(k_t) \equiv f(k_t) - k_t f'(k_t)$. Restricting our attention to the steady state equilibria, and using competitive equilibrium prices, we may define the following equilibrium

[3]Equations (5) and (6) imply

$$\frac{f(k_t^c) - k_t^c f'(k_t^c)}{f(k_t^I) - k_t^I f'(k_t^I)} = \frac{f'(k_t^c)}{f'(k_t^I)},$$

then $G(k_t^c) = G\left(k_t^I\right)$ with $G(k) \equiv [f(k) + kf'(k)]/f'(k)$, a monotonous function.

[4]From equation (6).

conditions which corresponds to the conditions (7) and (8) with $k_{t+1} = k_t = \bar{k}$ and $l_{t+1} = l_t = \bar{l}$:

$$\begin{aligned}
(1+n)\,k &= (1-l)\,f(k)\,, & (9)\\
\frac{s\left(w(k)+T, f'(k)\right)}{1+n} &= lk + \pi(1-l)\,k, & (10)\\
T &= (1-\pi)(1-l)\,k\left(f'(k)-(1+n)\right).
\end{aligned}$$

From this last equation, we know that the amount of $\bar{T}$ is negative only in the dynamically inefficient case of over-accumulation $(f'(k) < 1+n)$ in which the stationary state is greater than that of golden rule.

As in the one-sector OLG model, the existence of an interior steady state equilibrium is not guaranteed even with strengthened Inada conditions[5]. Restrictions on the nature of the interaction between preferences and technology are required as well. Let us denote

$$\varphi(k) \equiv (1+n)\,k\left[1-(1-\pi)(1+n)\,k/f(k)\right] - s\left(\omega(k), f'(k)\right). \qquad (11)$$

Proposition 1 . *Under Assumptions* $1-3$*, there exists a trivial steady state* $\hat{k}=0$*.*

Proposition 2 . *Under Assumptions* $1-3$*, there exists a* $\bar{\pi} \in (0,1)$ *such that,*

(i) if $\left[\lim_{k\to 0}\varphi'\left(\bar{k}\right)\right][\bar{\pi}-\pi] \geq 0^+$*, there exists at least one interior steady state. Generically, the number of steady states is odd;*

(ii) if $\left[\lim_{k\to 0}\varphi'\left(\bar{k}\right)\right][\bar{\pi}-\pi] \leq 0^+$*, then the number of interior steady states is generically even, and can be zero.*

4 Analysis of the local dynamics

Let us now precise the notion of indeterminacy that will be used in the following.

Definition 3 . *Let* $\{k_t\}_{t=0}^{\infty}$ *denote an equilibrium for an economy with initial condition* k_0*. It is said to be locally indeterminate if for every* $\varepsilon > 0$ *there exists another sequence* $\{k'_t\}_{t=0}^{\infty}$*, with* $0 < |k'_1 - k_1| < \varepsilon$ *and* $k'_0 = k_0$*, which is also an equilibrium.*

[5] See Galor and Ryder [11].

If an equilibrium is not indeterminate, then it is said to be determinate. We will discuss the dynamic properties of the equilibrium according to the proportion of private firms in the investment good sector, and we will study first the specific case where all the investment good is produced by private firms.

4.1 The case $\pi = 1$

We establish the relation between our two-sector model with no public production and no public debt, and the one-sector model with debt due to an initial deficit and zero deficits in the other periods. We obtain a first result:

Proposition 4 . *The evolution of capital stock in a two-sector overlapping generations model without debt is represented exactly by the same dynamical equation of a one-sector model with initial debt. The involved dynamical behavior is thus the same in both of the models.*

We deduce from the results obtained in Farmer [8], that deterministic fluctuations can actually be observed in our model when the investment good is produced with private capital.

The following equations describe the equilibrium paths in a neighborhood of the steady state $(\bar{k}, \bar{l})$:

$$k_{t+1} = \frac{(1 - l_t) f(k_t)}{1+n}, \tag{12}$$

$$k_{t+1} = \frac{s(w(k_t), f'(k_{t+1}))}{1+n}. \tag{13}$$

As the equation (13) does not depend on the labor supply used in the consumption good sector, it determines the dynamic behavior of k_t. Equation (12) gives then the behavior of the labor supply[6]. We remark that the dynamic behavior of k_t, in each sector, is exactly the same than in a one-sector model *à la* Diamond. If we denote $s_\omega \equiv \partial s(\bar{\omega}, \bar{R}) / \partial \omega$ and $s_R \equiv \partial s(\bar{w}, \bar{R}) / \partial R$ the derivatives of the saving function evaluated at $\bar{k}$, we thus obtain the usual stability condition:

[6] A stationary solution $\bar{k}$ of the second equation gives the relevant value for labor supply in the first one. As $\bar{s}/(1+n) = \bar{k} = (1-\bar{l})\bar{f}/(1+n)$, then $1 - \bar{l} = \bar{s}/\bar{f} < \bar{w}/\bar{f} < 1$.

Proposition 5 . *Let* $\Delta \equiv \bar{k} f''\left(\bar{k}\right) s_w + \left|1 + n - f''\left(\bar{k}\right) s_R\right|$. *Under Assumptions* $1-3$, *the following cases hold.*

(*i*) *If* $\Delta > 0$, *then* $\bar{k}$ *is stable.*

(*ii*) *If* $\Delta < 0$, *then* $\bar{k}$ *is unstable.*

Let n be the number of steady state equilibria. These states are ranked as follows: $k_1 > k_2 > \ldots > k_n$.

Proposition 6 . *Under Assumptions* $1-3$, *all steady states with an odd number are locally stable, whereas a sufficient condition for the steady states with an even index to be unstable is* $1 + n - f''\left(\bar{k}\right) s_R > 0$.

Corollary 7 . *Under Assumptions* $1-3$, *if there exists a unique steady state equilibrium, then it is stable.*

4.2 The case $\pi \in [0, 1)$

We obtain the dynamical system which describes the equilibrium paths in a neighborhood of the steady state $\left(\bar{k}, \bar{l}\right)$:

$$\begin{aligned} (1+n) k_{t+1} &= (1 - l_t) f(k_t), & (14) \\ s_t &= (1+n)\left[l_{t+1} k_{t+1} + \pi (1 - l_{t+1}) k_{t+1}\right], & (15) \end{aligned}$$

where

$$s_t \equiv s\left(f(k_t) - k_t f'(k_t) + (1-\pi)\left[(1-l_t) k_t f'(k_t) - (1+n)(1-l_{t+1}) k_{t+1}\right], f'(k_{t+1})\right).$$

This is a two-dimensional dynamical system with one predetermined variable, k_t, and one forward variable, l_t. The dimension of indeterminacy can not be greater than one. Actually, the steady state $\left(\bar{k}, \bar{l}\right)$ is indeterminate if and only if the local stable manifold is two-dimensional. System (14-15) is equivalently restated:

$$\varphi = 0,$$

where

$$\begin{aligned} \varphi &\equiv (1+n) k_{t+1} - (1-\pi)(1+n)^2 \frac{k_{t+1}}{f(k_{t+1})} k_{t+2} - s_t, \\ s_t &\equiv s\left(f(k_t) - k_t f'(k_t) + (1-\pi)(1+n)\left[\frac{k_t f'(k_t)}{f(k_t)} k_{t+1} - (1+n)\frac{k_{t+1}}{f(k_{t+1})} k_{t+2}\right], f'(k_{t+1})\right). \end{aligned}$$

A definition of saddle point is required.

Definition 8 . *A steady state* $\bar{k}$ *of the second order difference system (14-15) is saddle-point stable if and only if the dimension of the local stable manifold is equal to 1.*

We can now state the following result.

Proposition 9 . *Under Assumptions* $1-3$*,*

(1) *if* $\varphi'\left(\bar{k}\right) > 0$ *or* $\varphi'\left(\bar{k}\right) - 2\partial\varphi/\partial k_{t+1} > 0$*, the steady state* $\bar{k}$ *is saddle-point stable;*

(2) *if* $\varphi'\left(\bar{k}\right) < 0$ *and* $\varphi'\left(\bar{k}\right) - 2\partial\varphi/\partial k_{t+1} < 0$*, the steady state* $\bar{k}$ *is*

(2.1) *unstable if and only if* $\partial\varphi/\partial k_t + \partial\varphi/\partial k_{t+2} > 0$*,*

(2.2) *locally indeterminate if and only if* $\partial\varphi/\partial k_t + \partial\varphi/\partial k_{t+2} < 0$*.*

Proposition 10 . *Under Assumptions* $1-3$*,*

(i) *if* $\pi > \bar{\pi}$*, all steady states with an odd index are saddle-point stable.*

(ii) *if* $\pi < \bar{\pi}$*, all steady states with an even index are saddle-point stable.*

We observe from the last proposition that indeterminacy may occur even if there is a unique steady state equilibrium. We know that indeterminacy generates endogenous fluctuations and can be seen as a source of financial markets instability. Let us assume that the State owns a given part of the investment sector and that the only non-trivial steady state is indeterminate. The economy can be stabilized by selling shares of the investment firms to private agents and thereby ruling out the occurrence of indeterminacy. When the level of privatization is higher enough, the system is determinate. But the destabilizing real effects of this privatization in terms of speed of convergence after a shock have to be taken into account too. Summing up, the privatization rules out the multiplicity of equilibrium paths, but it can decrease the speed of convergence to a long-run equilibrium. This counter-effect will be highlighted in the numerical example of section 7.

5 Financial aspects

(i) First, we focus on the equilibrium quantities in the asset markets. Assets demand adjusts to satisfy the no-arbitrage condition (2). On the supply side we assume an inelastic asset provision: the number of shares per firm at each period is equal to Z. In other terms the young representative agent will buy Z/N_t shares of the first firm (consumption sector) and a fraction $\pi Z/N_t$ of the shares of the second firm (investment sector), because a proportion $(1-\pi)$ of the second firm is owned by the government. Therefore at the equilibrium:

$$\begin{aligned} N_t z_t^c &= Z, \\ N_t z_t^I &= \pi Z. \end{aligned}$$

The global amount of dividends distributed to the old, born at $t-1$, by the two firms as returns on financial investment are respectively

$$\begin{aligned} Z\delta_t^c &= L_t \left[l_t k_t f'(k_t) - (1+n)\, l_{t+1} k_{t+1} \right], \\ \pi Z \delta_t^I &= \pi L_t \left[(1-l_t)\, k_t f'(k_t) - (1+n)(1-l_{t+1})\, k_{t+1} \right]. \end{aligned}$$

We notice that an exogenous fraction $1-\pi$ of the dividends in the investment sector are transferred to the young, born at t, as consumption subsides:

$$N_t T_t = (1-\pi) L_t \left[(1-l_t)\, k_t f'(k_t) - (1+n)(1-l_{t+1})\, k_{t+1} \right].$$

(ii) Second, we compute the equilibrium prices. The dividends per share are respectively given by δ_τ^c and δ_τ^I for the first and the second firm. The equilibrium prices q_t^c and q_t^I are simply obtained by computing the discounted sum of future dividends under a usual no-bubble condition.

$$\begin{aligned} q_t^c &= \sum_{\tau=t+1}^{\infty} \frac{\delta_\tau^c}{\prod_{i=t+1}^{\tau} R_i} = \frac{1}{Z} \sum_{\tau=t+1}^{\infty} \frac{N_\tau \left[l_\tau k_\tau f'(k_\tau) - (1+n)\, l_{\tau+1} k_{\tau+1} \right]}{\prod_{i=t+1}^{\tau} f'(k_i)} \\ &= p_t K_t^c = K_t^c, \\ q_t^I &= \sum_{\tau=t+1}^{\infty} \frac{\delta_\tau^I}{\prod_{i=t+1}^{\tau} R_i} = \frac{1}{Z} \sum_{\tau=t+1}^{\infty} \frac{N_\tau \left[(1-l_\tau)\, k_\tau f'(k_\tau) - (1+n)(1-l_{\tau+1})\, k_{\tau+1} \right]}{\prod_{i=t+1}^{\tau} f'(k_i)} \\ &= p_t K_t^I = K_t^I. \end{aligned}$$

6 Endogenous and exogenous fluctuations

Indeterminacy means the existence of an infinite number of equilibria, all very close to each other. In the past the occurrence of indeterminacy has been considered to be a theoretical weakness, while a recent literature admits that the theory of indeterminacy has a role to play to explain a large part of economic fluctuations. The traditional approach highlights the existence of shocks on the fundamentals such as technology and tastes, and simulates the propagation mechanism of these shocks. Indeterminacy may provide a rich source of propagation dynamics to an equilibrium model and sunspot may provide an alternative shock to technology or taste changes. More precisely the equilibrium multiplicity we call indeterminacy, allows the introduction of exogenous shocks that are not based on fundamentals. Such shocks would be inconsistent with equilibrium if the rational expectations equilibrium were unique. Under indeterminacy this is no longer the case. As long as the sunspot process as a chain of extrinsic stochastic signals is consistent with the expectations of agents, equilibrium conditions can be satisfied, and sunspots will affect the evolution of real economic variables. The prevailing equilibrium is *de facto* selected by this exotic process.

In the following we will focus on both the classes of shocks from a theoretical viewpoint: on the beliefs, to generate endogenous real business cycles; on fundamentals, to obtain exogenous business cycles.

As we are interested in the impact of such shocks on the financial market through the interest rate, we will study directly the capital dynamics and thus the evolution of the interest rate by the production function.

The Jacobian matrix evaluated at the steady state is detailed in appendix in the proof of proposition 9. For notational simplicity we call the Jacobian components as follows:

$$\mathcal{J} \equiv \begin{bmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{bmatrix}. \tag{16}$$

Let (k_0, l_0) be an initial condition. We assume that the initial condition lies in a neighborhood of the steady state and we approximate the non-linear dynamics (14-15) with the following linearized ones:

$$\begin{bmatrix} k_{t+1} - \bar{k} \\ l_{t+1} - \bar{l} \end{bmatrix} = \mathcal{J} \begin{bmatrix} k_t - \bar{k} \\ l_t - \bar{l} \end{bmatrix}. \tag{17}$$

We provide the explicit analytical form of these linearized dynamics to study the shock propagation and to perform numerical simulations.

Proposition 11 . *If the eigenvalues of the Jacobian matrix are complex (say $\lambda_1 \equiv a+bi$ and $\lambda_2 \equiv a-bi$), then the approximated dynamics become*

$$
\begin{aligned}
k_t &= \bar{k} + \left(\sqrt{a^2+b^2}\right)^t \left\{ \left[\cos(t\theta) - \frac{a-j_{11}}{b}\sin(t\theta)\right](k_0-\bar{k}) \right. \\
&\quad \left. + \frac{j_{12}}{b}\sin(t\theta)\left(l_0-\bar{l}\right)\right\}, \qquad (18) \\
l_t &= \bar{l} + \left(\sqrt{a^2+b^2}\right)^t \left\{ -\frac{b^2+(a-j_{11})^2}{bj_{12}}\sin(t\theta)\left(k_0-\bar{k}\right) \right. \\
&\quad \left. + \left[\cos(t\theta) + \frac{a-j_{11}}{b}\sin(t\theta)\right]\left(l_0-\bar{l}\right)\right\}, \qquad (19)
\end{aligned}
$$

where $\theta \equiv \arccos\left(a/\sqrt{a^2+b^2}\right)$.

If the eigenvalues λ_1 and λ_2 are real, then the linearized dynamics are:

$$
\begin{aligned}
k_t &= \bar{k} - \left(\frac{\lambda_2-j_{11}}{\lambda_1-\lambda_2}\lambda_1^t - \frac{\lambda_1-j_{11}}{\lambda_1-\lambda_2}\lambda_2^t\right)\left(k_0-\bar{k}\right) + \frac{j_{12}}{\lambda_1-\lambda_2}\left(\lambda_1^t-\lambda_2^t\right)\left(l_0-\bar{l}\right), \qquad (20) \\
l_t &= \bar{l} - \frac{(\lambda_1-j_{11})(\lambda_2-j_{11})}{j_{12}(\lambda_1-\lambda_2)}\left(\lambda_1^t-\lambda_2^t\right)\left(k_0-\bar{k}\right) \\
&\quad + \left(\frac{\lambda_1-j_{11}}{\lambda_1-\lambda_2}\lambda_1^t - \frac{\lambda_2-j_{11}}{\lambda_1-\lambda_2}\lambda_2^t\right)\left(l_0-\bar{l}\right). \qquad (21)
\end{aligned}
$$

We apply the theory to our specific model and distinguish the following cases.

(1) The steady state is a sink. We observe indeterminacy. There is room for shocks on the fundamentals and also on the beliefs. For the sake of simplicity we consider only the latter ones. There are two sub-cases: (1.1) the eigenvalues are complex and conjugated, and convergence displays deterministic fluctuations; (1.2) the eigenvalues are real.

(2) The steady state is a saddle. The equilibrium is always determinate and there is no room for shocks on the beliefs. We focus only on the shocks on the fundamentals.

6.1 Shocks on the beliefs

We follow Farmer and Guo [9] and Benhabib and Farmer [3]. We refer to linearized dynamics and we assume that the realization of the vector at time τ depends on the expectation formed at time τ of the vector at time $\tau+1$:

$$
\begin{bmatrix} k_\tau - \bar{k} \\ l_\tau - \bar{l} \end{bmatrix} = ME_\tau \begin{bmatrix} k_{\tau+1} - \bar{k} \\ l_{\tau+1} - \bar{l} \end{bmatrix}, \qquad (22)
$$

where M is the matrix of the forward looking dynamics and E is the usual expectation operator. Equation (22) is reset as follows.

$$\begin{aligned} M\begin{bmatrix} k_{\tau+1}-\bar{k} \\ l_{\tau+1}-\bar{l} \end{bmatrix} &= \begin{bmatrix} k_{\tau}-\bar{k} \\ l_{\tau}-\bar{l} \end{bmatrix} + M\left(\begin{bmatrix} k_{\tau+1}-\bar{k} \\ l_{\tau+1}-\bar{l} \end{bmatrix} - E_{\tau}\begin{bmatrix} k_{\tau+1}-\bar{k} \\ l_{\tau+1}-\bar{l} \end{bmatrix}\right) \\ &= \begin{bmatrix} k_{\tau}-\bar{k} \\ l_{\tau}-\bar{l} \end{bmatrix} + M\begin{bmatrix} k_{\tau+1}-E_{\tau}k_{\tau+1} \\ l_{\tau+1}-E_{\tau}l_{\tau+1} \end{bmatrix}. \end{aligned}$$

If M is invertible we get

$$\begin{aligned} \begin{bmatrix} k_{\tau+1}-\bar{k} \\ l_{\tau+1}-\bar{l} \end{bmatrix} &= M^{-1}\begin{bmatrix} k_{\tau}-\bar{k} \\ l_{\tau}-\bar{l} \end{bmatrix} + \begin{bmatrix} k_{\tau+1}-E_{\tau}k_{\tau+1} \\ l_{\tau+1}-E_{\tau}l_{\tau+1} \end{bmatrix} \\ &= \mathcal{J}\begin{bmatrix} k_{\tau}-\bar{k} \\ l_{\tau}-\bar{l} \end{bmatrix} + \begin{bmatrix} k_{\tau+1}-E_{\tau}k_{\tau+1} \\ l_{\tau+1}-E_{\tau}l_{\tau+1} \end{bmatrix}, \end{aligned}$$

where $\mathcal{J} = M^{-1}$ is the Jacobian matrix (16). We observe that the deterministic system (17) has been augmented by taking into account the vector of forecasting errors at time $\tau + 1$:

$$\begin{bmatrix} k_{\tau+1}-E_{\tau}k_{\tau+1} \\ l_{\tau+1}-E_{\tau}l_{\tau+1} \end{bmatrix}.$$

In our model the capital is a predetermined variable known at time τ after the investment decision: $E_{\tau}k_{\tau+1} = k_{\tau+1}$. For simplicity we assume that $E_{\tau}l_{\tau+1} = \bar{l}$, *i.e.* before the shock on the beliefs the agents expect the labor at time $\tau + 1$ to get a long-run value. The temporary shock on the non-predetermined variable $l_{\tau+1}$ occurs between τ and $\tau + 1$ according to a sunspot signal. Agents coordinate on the new value $l_{\tau+1}$. This freedom of choice is compatible with a rational behavior because there is indeterminacy. Summing up, the vector of forecasting errors becomes $(0, \varepsilon_{\tau+1})$, where $\varepsilon_{\tau+1} \equiv l_{\tau+1} - \bar{l}$.

To simplify the notation we set $\tau + 1 = 0$. The shock is performed in 0 on the non-predetermined variable l_0. Before the shock the economy is at the steady state $(\bar{k}, \bar{l})$. The predetermined variable is still $k_0 = \bar{k}$. The shock is temporary. The steady state is not affected by the shock because it depends only on the fundamentals. We still assume the initial condition $(\bar{k}, l_0)$ to lie in a sufficiently small neighborhood of $(\bar{k}, \bar{l})$, *i.e.* the vertical shock $\varepsilon_0 = l_0 - \bar{l}$ to be not too large.

As seen above there are two cases.

(1.1) The eigenvalues are complex and conjugated and deterministic oscillations arise. Under the condition $k_0 = \bar{k}$ system (18-19) becomes:

$$\begin{aligned} k_t &= \bar{k} + \left(\sqrt{a^2+b^2}\right)^t \frac{j_{12}}{b} \sin(t\theta)\, \varepsilon_0, \\ l_t &= \bar{l} + \left(\sqrt{a^2+b^2}\right)^t \left[\cos(t\theta) + \frac{a - j_{11}}{b} \sin(t\theta)\right] \varepsilon_0. \end{aligned}$$

(1.2) The eigenvalues are real. As above we set $k_0 = \bar{k}$. System (20-21) simplifies:

$$\begin{aligned} k_t &= \bar{k} + \frac{j_{12}}{\lambda_1 - \lambda_2} \left(\lambda_1^t - \lambda_2^t\right) \varepsilon_0, \\ l_t &= \bar{l} + \left(\frac{\lambda_1 - j_{11}}{\lambda_1 - \lambda_2}\lambda_1^t - \frac{\lambda_2 - j_{11}}{\lambda_1 - \lambda_2}\lambda_2^t\right) \varepsilon_0, \end{aligned}$$

where $\varepsilon_0 \equiv l_0 - \bar{l}$ still measures the shock on the beliefs.

6.2 Shocks on the fundamentals

The shock is performed in $t = 0$ on the privatization degree π. The degrees before and after the shock are respectively π_0 and π. The steady state of π_0 is (k_0, l_0), the steady state of π is $\left(\bar{k}, \bar{l}\right)$. We assume that (k_0, l_0) is in a sufficiently small neighborhood of $\left(\bar{k}, \bar{l}\right)$. k_0 is predetermined. Agents have rational expectations and respect the transversality condition. The unique initial condition compatible with a long run equilibrium is $\left(k_0, \widehat{l_0}\right)$, where $\widehat{l_0}$ is such that the starting point lies on the saddle path converging to the new steady state $\left(\bar{k}, \bar{l}\right)$. We consider an approximated $\widehat{l_0}$ because we compute a linearized saddle path around the new steady state.

Both the eigenvalues are real. Let λ_1 be the stable eigenvalue and λ_2 be the explosive one. The linearized saddle path equation is satisfied in $\left(k_0, \widehat{l_0}\right)$:

$$\widehat{l_0} - \bar{l} = \frac{\lambda_1 - j_{11}}{j_{12}} \left(k_0 - \bar{k}\right).$$

We notice that the slope is negative if and only if $(\lambda_1 - j_{11})/j_{12} < 0$.

The linearized dynamics (20-21) simplify:

$$k_t = \bar{k} + \lambda_1^t \left(k_0 - \bar{k}\right), \tag{23}$$

$$l_t = \bar{l} + \frac{\lambda_1 - j_{11}}{j_{12}} \lambda_1^t \left(k_0 - \bar{k}\right). \tag{24}$$

The shock on the capital level $\bar{k}-k_0$ depends on the shock on the privatization degree $\pi-\pi_0$ according to the linear approximation: $\bar{k}-k_0 \approx (\pi-\pi_0)\, d\bar{k}/d\pi$, where the privatization impact on the stationary capital $d\bar{k}/d\pi$ is obtained by totally differentiating the steady state equation (11). More precisely the system (23-24) becomes

$$\begin{aligned} k_t &= \bar{k} - \lambda_1^t \frac{d\bar{k}}{d\pi} (\pi - \pi_0), \\ l_t &= \bar{l} - \frac{\lambda_1 - j_{11}}{j_{12}} \lambda_1^t \frac{d\bar{k}}{d\pi} (\pi - \pi_0), \end{aligned}$$

where

$$\frac{d\bar{k}}{d\pi} = \frac{(1+n)^2 \bar{k}^2/f}{s_k - (1+n)\left[1 + (1-\pi)(1+n)\left(\bar{k}/f\right)\left(\bar{k}f'/f - 2\right)\right]}$$

and s_k is the derivative of the stationary saving with respect to the steady capital.

7 Numerical simulation

For simplicity the utility function is assumed to be a separable CES:

$$U(c_t, d_{t+1}) \equiv \frac{c_t^{1-1/\sigma} - 1}{1 - 1/\sigma} + \beta \frac{d_{t+1}^{1-1/\sigma} - 1}{1 - 1/\sigma},$$

where σ denotes the elasticity of intertemporal substitution, while the production function is assumed to be a Cobb-Douglas: $F(K, L) = K^\alpha L^{1-\alpha}$, *i.e.* in an intensive form $f(k) = k^\alpha$, where $f \equiv F/L$ and $k \equiv K/L$.

Under this fundamentals specification the steady state is given by the system

$$\begin{aligned} 0 &= (1+n)\left[k - (1-\pi)(1+n)k^{2-\alpha}\right] - \frac{(1-\alpha)k^\alpha + (1-\pi)(1+n)\left[\alpha k - (1+n)k^{2-\alpha}\right]}{1 + \beta^{-\sigma}\left(\alpha k^{\alpha-1}\right)^{1-\sigma}}, \\ l &= 1 - (1+n)k^{1-\alpha}. \end{aligned}$$

To perform the numerical simulations we choose the parameter values according to those commonly adopted in the business cycle literature. We set $\alpha = 1/3$, $\sigma = 2$. As there are two overlapping generations, we assume the length of a period (the active life) equal to 35 years. Thereby $\beta = 0.5$ corresponds to a per year subjective discount factor of 0.98039, while $n = 0.5$ corresponds to an annual population growth rate of 1.17%.

The following table describes the evolution of the steady states in terms of capital and labor $(\bar{k}, \bar{l})$, consumption $(\bar{c}, \bar{d})$ and utility $(\bar{U})$ as long as the privatization degree increases. The progressive eigenvalues values are provided too.

π	$\bar{k}_1$	$\bar{l}_1$	$\bar{c}$	$\bar{d}$	$\bar{U}$	λ_1	λ_2
0.0	0.084	0.71230	0.20754	0.15671	-1.6930	0.22208	5.2521
0.1	0.081	0.71919	0.20357	0.16135	-1.6959	0.22236	6.2681
0.3	0.075	0.73323	0.19501	0.17127	-1.7030	0.22508	9.2869
0.5	0.071	0.74280	0.18850	0.17810	-1.7097	0.22531	14.493
0.7	0.066	0.75503	0.18129	0.18880	-1.7139	0.23002	27.381
0.9	0.063	0.76325	0.17631	0.19659	-1.7168	0.23158	90.388

There is always a trivial steady state given by $(\bar{k}, \bar{l}) = (0, 1)$. Under our parameter choice the meaningful steady state is characterized by under-accumulation for every privatization degree $\pi \in [0, 1)$. To see the point compare the column of the stationary capital $\bar{k}$ with the golden rule value: $k^* = [\alpha/(1+n)]^{1/(1-\alpha)} = 0.10476$.

Moreover we notice that the steady state is always a saddle point. As there is only one non-predetermined variable the equilibrium is always determinate (unique). There is no indeterminacy and no room for endogenous fluctuations. Thereby we consider only shocks on the fundamentals and in particular the relevant shock on the privatization degree π.

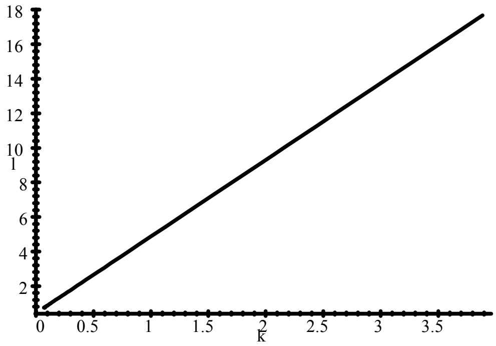
7.1 An example of shock on the fundamentals

We fix the privatization degrees before and after the shock respectively equal to $\pi_0 = 40\%$ and $\pi = 50\%$. In other words the shock is set equal to 0.1. The non-trivial stationary capital is given by $\bar{k} = 0.071$, while the labor share employed in the consumption good sector is equal to $\bar{l} = 74,28\%$. We observe a negative privatization impact on the stationary capital:

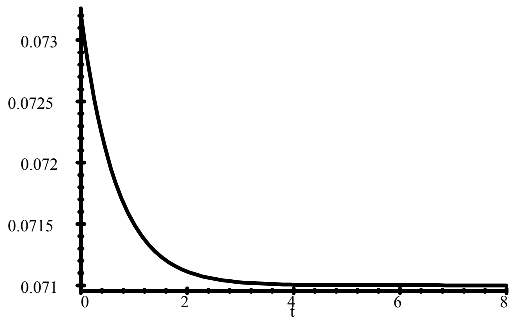
$$\bar{k} - k_0 \approx \frac{d\bar{k}}{d\pi}(\pi - \pi_0) = -0.0022141. \tag{25}$$

The privatization effect is positive on the fraction of workers employed in the production of consumption good and thereby negative in the investment sector. The equilibrium intensive capital is the same in both the sectors $(k^c = k^I)$, but it decreases with respect to the saddle point. Therefore the aggregate capital employed in the investment sector drops off.

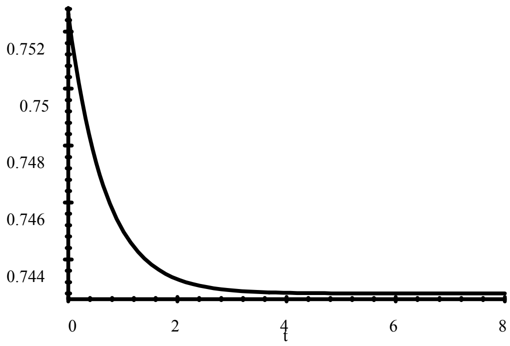
The stable and the unstable eigenvalues are respectively $\lambda_1 = 0.22531$, $\lambda_2 = 14.493$. Dynamics are described by the following trajectories:



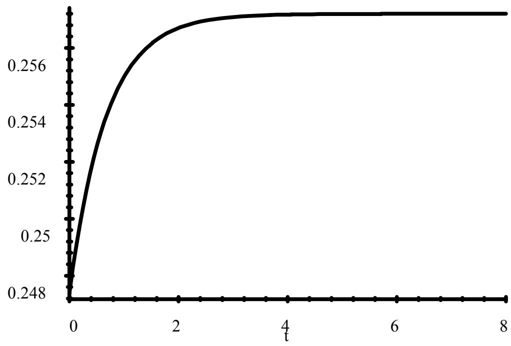
Saddle path.



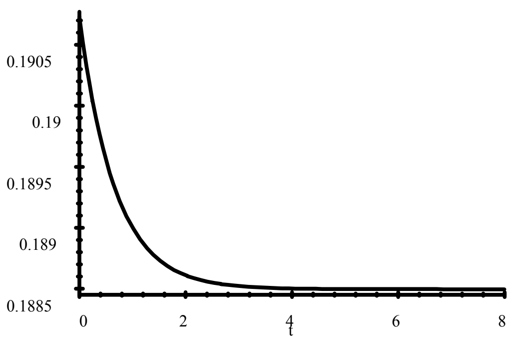
Transition for k.



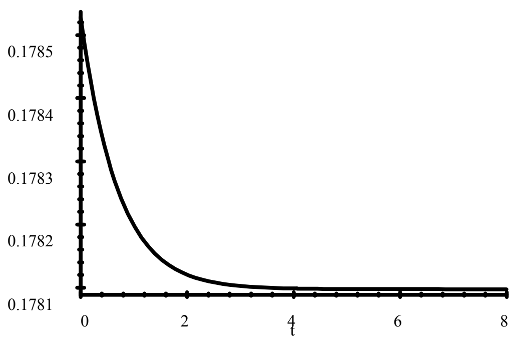
Transition for l^c.



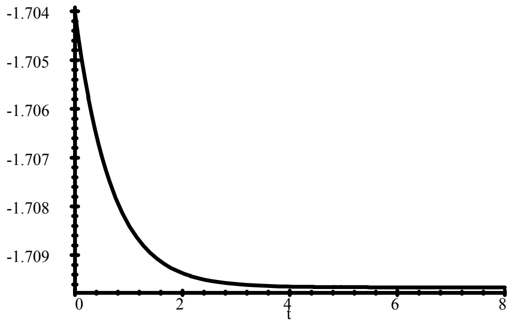
Transition for l^I.



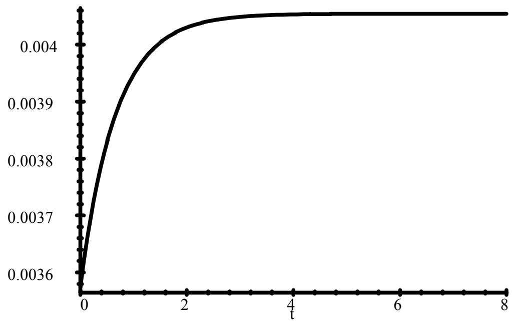
Transition for c.



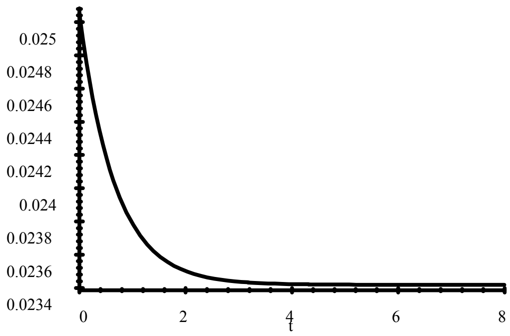
Transition for d.



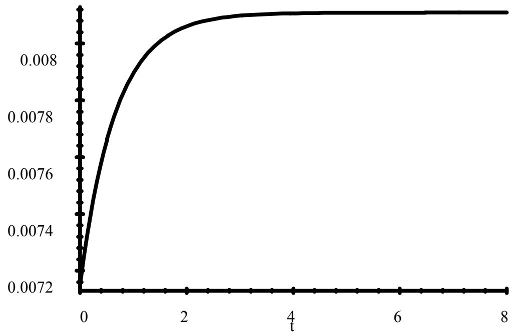
Transition for U.



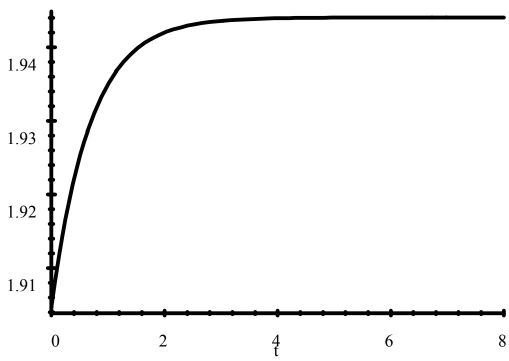
Transition for T.



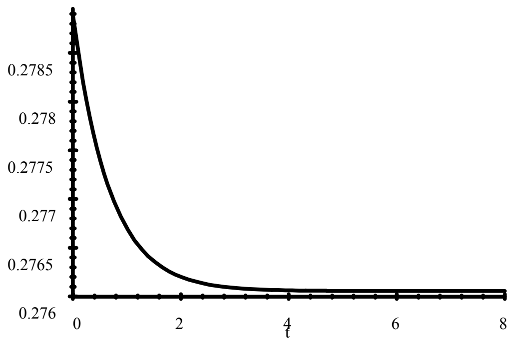
Transition for δ^c.



Transition for δ^I.



Transition for R.



Transition for w.

The impact of the shock on the asset prices is depicted in the next figure.

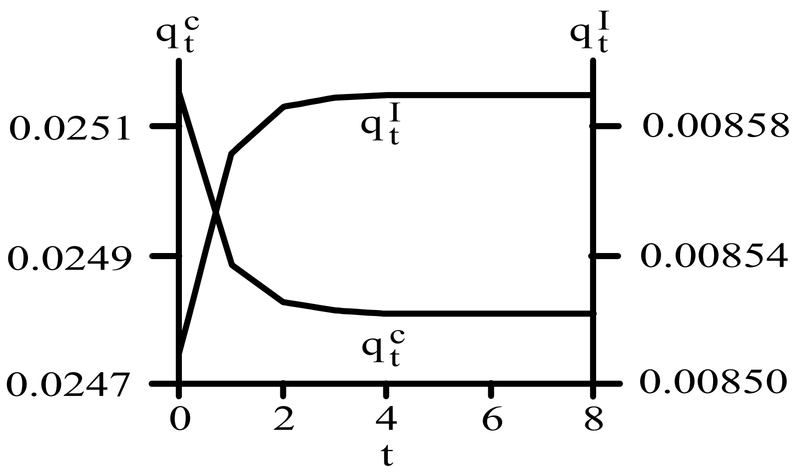
We first note that the level of the stationary stock of capital decreases as the proportion of investment firms owned by private agents increases (see the table). This is due to the particular government way of allocating the dividends. Actually, for the private proportion of the sector, dividends are received by agents when they are old, corresponding to the dividends from the shares sold when they were young. Whereas the dividends obtained by the government, if they are positive, are given to young agents as an additional income.

When private agents own a larger part of the investment firms, they receive a lower proportion of their income, when they are young, and a greater one, when they are old. Then they decrease their saving. As the capital accumulation is the counterpart of private saving, the capital stock decreases. The interest rate increases, generating an indirect substitution effect straightening the direct one.

Nevertheless, we note that this saving decrease is not enough in the long-run to compensate the income transfers from young to old since when π is raising, the first period consumption decreases, and the second period consumption increases. The resulting effect is that the utility level of agents decreases (see the table). We can remark that, just after the privatization, the utility is first above the former level, then decreases during the transition to a lower level (see the figure with the transition for U). The result is not surprising since, as the stationary level of capital is less than the golden rule value, when this capital stock decreases, it just goes away from the golden rule. The agents' utility would increase, if the transfers were destined to old agents and the stationary capital was above the golden rule.

Another effect of the decrease of capital stock is that the investment good has to be produced in reduced quantity. The investment sector uses lower quantities of inputs, namely the stationary employed capital and labor are lower. For a constant labor supply, a higher quantity of labor is then used in the consumption sector.

We eventually observe that, when the shock occurs, in order to reach the saddle path by opportunely setting the non-predetermined variable, agents' expectations focus on a too high second period consumption with respect to the steady state value, increasing temporarily the utility level above its initial value. The different levels converge then slowly to the new steady state, where the utility level is decreased. The stationary values of the labor supply in both sector, as well as the share prices, are also overshooted in the short term. As the increase of π means that the number of shares of the investment sector firms supplied to private agents increases, in the long-run, their price then decreases. In the same time, the number of shares of firms in the consumption good sector becomes relatively lower with respect to the total number of shares, and then their price becomes higher (see the figure with the transition share prices).



Privatization and financial adjustment (to be checked).

7.2 Destabilizing power of privatization

The speed of convergence is informative about the stability of the economic system under exogenous shocks. A usual indicator of this speed is the stable eigenvalue λ_1. More precisely the time to absorb a given fraction of the shocks is an increasing function of the modulus[7] of λ_1. In our example the saddle case is characterized by a stable eigenvalue which is increasing with π. Therefore the privatization of public firms has a destabilizing effect over a longer period in more privatized economies. In countries where the state owns a larger part of the investment sector, the privatization is thus less troubling.

The same conclusion holds for all the shocks on the other fundamentals such as the technological parameter A in the production function $f(k_t) = Ak_t^\alpha$ (notice that in our model we have normalized A to one for simplicity). Hence in general the stabilization is faster in less privatized economies.

8 Concluding comments

We have studied a two-sector overlapping generations model with production and fully characterized the local dynamics generated by the model.

[7] The absorption period of a part d of the shock is given by $T = \ln d / \ln \lambda_1$.

In an economy where the state owns a large part of the firms producing the investment goods, business cycles may arise even if the fundamentals remain unchanged, even in a model with a unique steady state equilibrium. Actually, agents' beliefs can be different from the perfect foresight. Thus the stationary state is not robust and the interest rate can fluctuate endogenously. To avoid endogenous fluctuations, the solution is to have no government or to have a government who owns a weak part of the investment sector. In this sense, the privatization can be stabilizing.

But in the standard example studied in this paper, we have not found any endogenous fluctuation because the unique steady state turns out to be a saddle point. Moreover, the stationary level of capital is below the golden rule value. The only source of economic instability are the exogenous real shocks on the fundamentals of the economy. A privatization can be seen as a shock, since that just means, in our model, that the proportion of the shares of the investment sector sold to private agents increases. We thus analyze a permanent shock on the privatization degree and the convergence along the saddle path has been also simulated. We observe that such a shock destabilizes the economy for a longer period when the privatization degree increases. In this terms, the privatization can be seen as destabilizing, as it slows down the processus driving prices and quantities back to the new stationary values. When, to the contrary, the government owns a large part of the investment sector, exogenous fluctuations are limited.

This example shows us that, in a standard representation of the economy used in the theory of real business cycles, privatization could contribute to destabilize the economy as it represents a real shock on the fundamentals. And the destabilizing effect is worth and worth when the proportion of the sector owns by private agents increases.

The welfare effect of privatization is negative in the long run and positive just after the shock. If the firms partly owned by government are as efficient as private firms, and the government really cares about the agents' utility, it can improves the life-cycle level of agents' consumption. We thus show that the general idea about the improvement of the agent's consumption after privatization is linked to the fact that firms where government is a partial owner are less efficient, or to the goal aimed by the State.

Of course, if the transfers to agents from the government are received during the second period of life of individuals, the result would be that the welfare of agents increases after privatization.

It is of interest to study the effects of privatization for different dividend redistribution policies the government could choose. Among various research lines, we would like to investigate whether a debt financing policy would give different results, and to compare our analyses to the case where government owns shares of the sector producing the consumption good.

9 Appendix

Proof of Proposition 1. A steady state $\bar{k}$ is a solution of the equation

$$(1+n)\,k\,[1-(1-\pi)\,(1+n)\,k/f\,(k)] = s\,(\omega\,(k)\,,f'\,(k))\,.$$

As agents can not save more than their initial income $\omega\,(k)$, then for all $k \geq 0$,we have $0 \leq s\,(\omega\,(k)\,,R\,(k)) \leq \omega\,(k)$, with $\omega\,(k) \equiv w\,(k) + T\,(k)$. Under Assumption 3, $f\,(0) = 0$ and $w\,(k)$ is non negative, then $\lim_{k\to 0} T\,(k) = 0$, $w\,(0) = 0$ (see de la Croix and Michel [7] that $f\,(0) = w\,(0)$) and $\lim_{k\to 0}\,[k/f\,(k)] = 0$, we deduce that $\lim_{k\to 0} s\,(\omega\,(k)\,,R\,(k)) = 0$. The result therefore follows from Assumption 3. ■

Proof of Proposition 2. The system of equations (9) and (10) can be written

$$\begin{aligned} l &= 1-(1+n)\,k/f\,(k)\,, \\ s\,(\omega\,(k)\,,f'\,(k))\,/\,(1+n) &= k-(1-\pi)\,(1+n)\,k^2/f\,(k)\,. \end{aligned}$$

Thus a stationary $\bar{k}$ is characterized by $\varphi\left(\bar{k}\right) = 0$, where φ is defined by (11) and, as $l \in (0,1)$, the first equation of the system tells us that this $\bar{k}$ must be such that $\bar{k}/f\left(\bar{k}\right) < 1/\,(1+n)$. Under Assumption 3, as $k/f\,(k)$ is increasing, $\lim_{k\to 0} k/f\,(k) = 0$ and $\lim_{k\to\infty} k/f\,(k) = +\infty$ then there exists a k_{max} such that $(1+n)\,k_{max}/f\,(k_{max}) = 1$. We state now three important Lemmas.

Lemma 12 . *There exists a value $\bar{\pi} \in (0,1)$ such that for every $\pi \in [0,\bar{\pi})$, $\varphi\,(k_{max}) < 0$, and for every $\pi \in [\bar{\pi},1)$, $\varphi\,(k_{max}) > 0$. If $\pi = \bar{\pi}$, then k_{max} is a steady state equilibrium.*

Proof. We have $\varphi(k_{max}) = \pi(1+n)k_{max} - s(\omega(k_{max}), f'(k_{max}))$. Thus when $\pi = 0$, $\varphi(k_{max}) = -s(\omega(k_{max}), f'(k_{max}))$ and is strictly negative under Assumption 1. Whereas when $\pi = 1$, $\varphi(k_{max}) = (1+n)k_{max} - s(\omega(k_{max}), f'(k_{max}))$. As k_{max} is greater than the golden rule, we know that $T < 0$, and, therefore, $\omega < w$ and $s(\omega(k_{max}), f'(k_{max})) < w(k_{max}) < f(k_{max})$ by definition. We deduce that $\varphi(k_{max}) > (1+n)k_{max} - f(k_{max})$ which is zero. As $\varphi(k_{max})$ is increasing with π, we finally obtain the result.

Lemma 13 . *If* $\lim_{k\to 0}\varphi'(k) > 0$*, then, when* $\pi \in [0, \bar{\pi})$*, there exists at least one steady state equilibrium. Generically, the number of steady state is odd. And when* $\pi \in (\bar{\pi}, 1)$*, the number of steady state equilibria is generically even, and can be zero.*

Proof. As $\lim_{k\to 0}\varphi'(k) > 0$ and φ is a continuous function of k, thus if the value of this function is positive when k tends to zero, and negative when k is at his maximum value (as $\pi < \bar{\pi}$), there exists a $\bar{k} \in (0, k_{max})$ such that $\varphi(\bar{k}) = 0$. When $\varphi(k) = 0$ for several values of $\bar{k}$, the number of these values is even only if there exists a value $\bar{k}_1$ such that $\varphi(\bar{k}_1) = 0$ and $\varphi'(\bar{k}_1) = 0$. This situation is not robust to any small change in the parameters, we will thus refer to it as 'non-generic'. The result is obtained similarly when $\varphi(k_{max})$ is now positive (*i.e.* when $\pi < \bar{\pi}$).

Lemma 14 . *If* $\lim_{k\to 0}\varphi'(k) < 0$*, then when* $\pi \in (\bar{\pi}, 1)$*, there exists at least one steady state equilibrium. Generically, the number of steady state is odd. And when* $\pi \in [0, \bar{\pi})$*, the number of steady state equilibria is generically even, and can be zero.*

Proof. The proof is the symmetric of the preceding one. ■

Proof of Proposition 4. In the two-sector model considered in this paper, when there is no government production, the equilibrium in the consumption good market is: $c_t + d_t/(1+n) = l_t f(k_t)$. Using the consumer's budget constraints, his optimal solution for the consumer, the equilibrium factor prices and equation (7) we finally obtain:

$$w(k_t) - s(w(k_t), R(k_{t+1})) + R(k_t)\frac{s(w(k_{t-1}), R(k_t))}{1+n} = f(k_t) - (1+n)k_{t+1}. \quad (26)$$

If we now consider the one-sector model with debt used by Farmer [8], and we denote b_t the per young debt amount at date t, the debt evolution equation is: $b_t = R(k_t)b_{t-1}/(1+n)$, and the agents' saving finances the actual debt and the capital of the next period: $b_t = s(w(k_t), R(k_{t+1})) - (1+n)k_{t+1}$. The dynamical behavior of the capital stock is thus given by the equation:

$$s(w(k_t), R(k_{t+1})) - (1+n)k_{t+1} = R(k_t)\left[\frac{s(w(k_{t-1}), R(k_t))}{1+n} - k_t\right]. \tag{27}$$

As $w(k_t) = f(k_t) - k_t R(k_t)$, then equation 26 and 27 are identical. ■

Proof of Proposition 5. We remark that the capital evolution equation allows to compute dk_{t+1}/dk_t evaluated at the steady state $\bar{k}$ which is

$$\frac{-\bar{k}f''(\bar{k})\, s_w(w(\bar{k}), f'(\bar{k}))}{1+n-f''(\bar{k})\, s_R(w(\bar{k}), f'(\bar{k}))}.$$

As the steady state is stable if and only if $|dk_{t+1}/dk_t| < 1$, we obtain the proposition. ■

Proof of Proposition 6. As for $\pi = 1$, $T = 0$, and then $\varphi(k) = \varphi_1(k) \equiv (1+n)k - s(w(k), f'(k)) > (1+n)k - w(k) > (1+n)k - f(k)$ by definition of saving and wage, and Assumption 3 implies $\lim_{k\to+\infty}(1+n)k - f(k) > 0$, we know that $\lim_{k\to+\infty}\varphi_1(k) > 0$. Then the steady state with the higher index, and generically all the steady states with an odd index, are such that $\varphi_1'(\bar{k}) > 0$. As $\varphi_1'(k) = 1+n+kf''(k)\, s_w(w(k), f'(k)) - f''(k)\, s_R(w(k), f'(k))$, the positivity of this expression means $1+n-f''(k)\, s_R(w(k), f'(k)) > 0$ and thus $\Delta > 0$. The steady state equilibria with an even index correspond generically to $\varphi'(\bar{k}) < 0$. Under the assumption that $1+n-f''(k)\, s_R(w(k), f'(k)) > 0$, this property implies that $\Delta < 0$. ■

Proof of Proposition 9. The characteristic polynomial is obtained from the Jacobian matrix of the two-dimensional dynamical system evaluated at the steady state. Some tedious computations allows to obtain: $P(\lambda) = \lambda^2 - \mathcal{T}\lambda + \mathcal{D} = 0$, with

$$\begin{aligned}\mathcal{T} &= -(1-\alpha) - \frac{f''(\bar{k})\, s_R - (1+n)\left[1-(1-\pi)\alpha s_\omega\right]}{(1-\pi)(1+n)^2(1-s_\omega)}\frac{f(\bar{k})}{\bar{k}},\\ \mathcal{D} &= \frac{1}{1+n}\frac{s_\omega}{1-s_\omega}\left[(1-\alpha+\varepsilon_R)f'(\bar{k}) - \frac{f(\bar{k})f''(\bar{k})}{(1-\pi)(1+n)}\right],\end{aligned}$$

where $\alpha = \alpha(\bar{k}) \equiv \bar{k}f'(\bar{k})/f(\bar{k})$ is the capital share on total income and $\varepsilon_R \equiv \bar{k}f''(\bar{k})/f'(\bar{k})$ is the elasticity of the interest factor. Using the positivity of the wage rate and the fact that every stationary value of $\bar{k}$ has to be greater than $k_{\max}$, we can deduce the numerator of $\mathcal{D}$ is negative, and thus $\mathcal{D}$ is positive. Moreover, it is easy to show that:

$$\begin{aligned} P(1) &= -\frac{\varphi'\left(\bar{k}\right)}{(1-\pi)(1+n)^2(1-s_\omega)}\frac{f\left(\bar{k}\right)}{\bar{k}}, \\ P(-1) &= \frac{2\partial\varphi/\partial k_{t+1} - \varphi'\left(\bar{k}\right)}{(1-\pi)(1+n)^2(1-s_\omega)}\frac{f\left(\bar{k}\right)}{\bar{k}}. \end{aligned}$$

Since under Assumptions $1-3$, as $P(0) = \mathcal{D}$ is positive, the eigenvalues, if they are real, are both positive or negative. We note that if $\varphi'\left(\bar{k}\right) > 0$, then $P(1) < 0$ and if $\varphi'\left(\bar{k}\right) - 2\partial\varphi/\partial k_{t+1} > 0$, $P(-1) < 0$. The condition $P(1) < 0$ or $P(-1) < 0$, is a necessary and sufficient condition for the steady state to be saddle-point stable. As soon as $P(1) > 0$ and $P(-1) > 0$, the eigenvalues are real or complex. In this case, the steady state is locally indeterminate if and only if the product of eigenvalues, *i.e.* $\mathcal{D}$ is less than one, and locally unstable if and only if the determinant is greater than one. ■

Proof of Proposition 10. As if $\pi > \bar{\pi}$, $\varphi(k_{max}) > 0$, then, $\varphi'\left(\bar{k}\right) > 0$ and $P(1) < 0$ for steady states with an odd index; and $\varphi'\left(\bar{k}\right) < 0$ and $P(1) > 0$ for those with an even index.

And if $\pi > \bar{\pi}$, $\varphi(k_{max}) < 0$, then, $\varphi'\left(\bar{k}\right) < 0$ $(P(1) > 0)$ for steady states with an odd index and $\varphi'\left(\bar{k}\right) > 0$ $(P(1) < 0)$ for those with an even index. The rest of the proof is obvious. ■

Proof of Proposition 11. The Jacobian matrix evaluated at the steady state is provided by equations (16). System (14-15) is linearized to obtain the following approximated trajectories:

$$\begin{bmatrix} k_{t+1} - \bar{k} \\ l_{t+1} - \bar{l} \end{bmatrix} = \mathcal{J}^t \begin{bmatrix} k_0 - \bar{k} \\ l_0 - \bar{l} \end{bmatrix}.$$

We provide the explicit analytical form of the linearized dynamics to make numerical simulations. Let

$$\Lambda \equiv \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix},$$

where λ_1 and λ_2 are the eigenvalues of $\mathcal{J}$ and let

$$V = \begin{bmatrix} j_{12} & j_{12} \\ \lambda_1 - j_{11} & \lambda_2 - j_{11} \end{bmatrix}$$

be an associated transformation matrix such that $V\Lambda V^{-1} = \mathcal{J}$. We observe that

$$v_i = \begin{bmatrix} j_{12} \\ \lambda_i - j_{11} \end{bmatrix}$$

is an eigenvector associated to the eigenvalue λ_i, $i = 1, 2$. Two cases matter. The eigenvalues are (i) complex $(\lambda = a \pm bi)$ or (ii) real.

(i) We write the complex eigenvalues in polar coordinates $\lambda = a + bi = r(\cos\theta + i\sin\theta)$, where $r^2 = a^2 + b^2$, $\cos\theta = a$. Therefore $\lambda^t = r^t(\cos\theta + i\sin\theta)^t = r^t\left(e^{\theta i}\right)^t = r^t e^{(t\theta)i} = r^t(\cos t\theta + i\sin t\theta)$. We obtain the linearized dynamics:

$$\begin{aligned}
\begin{bmatrix} k_t - \bar{k} \\ l_t - \bar{l} \end{bmatrix} &= \mathcal{J}^t \begin{bmatrix} k_0 - \bar{k} \\ l_0 - \bar{l} \end{bmatrix} = V\Lambda^t V^{-1} \begin{bmatrix} k_0 - \bar{k} \\ l_0 - \bar{l} \end{bmatrix} \\
&= V \begin{bmatrix} \lambda_1^t & 0 \\ 0 & \lambda_2^t \end{bmatrix} V^{-1} \begin{bmatrix} k_0 - \bar{k} \\ l_0 - \bar{l} \end{bmatrix} \\
&= \begin{bmatrix} j_{12} & j_{12} \\ a + bi - j_{11} & a - bi - j_{11} \end{bmatrix} \\
&\quad \begin{bmatrix} r^t(\cos t\theta + i\sin t\theta) & 0 \\ 0 & r^t(\cos t\theta - i\sin t\theta) \end{bmatrix} \\
&\quad \begin{bmatrix} j_{12} & j_{12} \\ a + bi - j_{11} & a - bi - j_{11} \end{bmatrix}^{-1} \begin{bmatrix} k_0 - \bar{k} \\ l_0 - \bar{l} \end{bmatrix} \\
&= r^t \begin{bmatrix} \cos t\theta - [(a - j_{11})/b]\sin t\theta & (j_{12}/b)\sin t\theta \\ -\left[b^2 + (a - j_{11})^2/(bj_{12})\right]\sin t\theta & \cos t\theta + [(a - j_{11})/b]\sin t\theta \end{bmatrix} \begin{bmatrix} k_0 - \bar{k} \\ l_0 - \bar{l} \end{bmatrix}.
\end{aligned}$$

More explicitly we get the system (18-19).

(ii) Let λ_1 and λ_2 be real. The linearized dynamics become:

$$\begin{aligned}
\begin{bmatrix} k_t - \bar{k} \\ l_t - \bar{l} \end{bmatrix} &= \mathcal{J}^t \begin{bmatrix} k_0 - \bar{k} \\ l_0 - \bar{l} \end{bmatrix} = V\Lambda^t V^{-1} \begin{bmatrix} k_0 - \bar{k} \\ l_0 - \bar{l} \end{bmatrix} \\
&= \begin{bmatrix} j_{12} & j_{12} \\ \lambda_1 - j_{11} & \lambda_2 - j_{11} \end{bmatrix} \begin{bmatrix} \lambda_1^t & 0 \\ 0 & \lambda_2^t \end{bmatrix} \\
&\quad \begin{bmatrix} j_{12} & j_{12} \\ \lambda_1 - j_{11} & \lambda_2 - j_{11} \end{bmatrix}^{-1} \begin{bmatrix} k_0 - \bar{k} \\ l_0 - \bar{l} \end{bmatrix}.
\end{aligned}$$

We obtain the system (20-21). ■

References

[1] Benhabib J. and R.E.A. Farmer (1994), Indeterminacy and Growth, *Journal of Economic Theory,* **63,** 19-41.

[2] Benhabib J. and R.E.A. Farmer (1996), Indeterminacy and Sector-Specific Externalities, *Journal of Economic Theory,* **37,** 421-43.

[3] Benhabib J. and R.E.A. Farmer (1999), Indeterminacy and Sunspots in Macroeconomics, *Handbook of Macroeconomics,* Taylor and Woodford eds., Elsevier.

[4] Benhabib J. and K. Nishimura (1998), Indeterminacy and Sunspots with Constant Returns, *Journal of Economic Theory,* **81,** 58-96.

[5] Cass D. and K. Shell (1983), Do Sunspots Matter ?, *Journal of Political Economy,* **91,** 193-227.

[6] Diamond P.A. (1965), National Debt in a Neoclassical Growth Model, *American Economic Review,* **55,** 1126-50.

[7] De la Croix D. and P. Michel (1998), The Overlapping Generations Approach to Capital Accumulation, Chapter 1, *GREQAM Mimeo,* **98-01.**

[8] Farmer R.E.A. (1986), Deficits and Cycles, *Journal of Economic Theory,* **40**, 77-88.

[9] Farmer R.E.A. and J.T. Guo (1994), Real Business Cycles and the Animal Spirit Hypothesis, *Journal of Economic Theory,* **63,** 42-73.

[10] Galor O. (1992), A Two-Sector Overlapping-Generations Model: a Global Characterization of the Dynamical System, *Econometrica,* **60,** 1351-86.

[11] Galor O. and H.E. Ryder (1989), Existence, Uniqueness, and Stability of Equilibrium in an Overlapping-Generations Model with Productive Capital, *Journal of Economic Theory,* **49,** 360-75.

[12] Kehoe T. and D. Levine (1985), Comparative Statics and Perfect Foresight in Infinite Horizon Economies, *Econometrica,* **53,** 433-53.

[13] Muller W.J. and M. Woodford (1988), Determinacy of Equilibria in Stationary Economies with Both Finite and Infinite Lived Consumers, *Journal of Economic Theory,* **46,** 255-90.

[14] Nourry C. (2001), Stability of Equilibria in the Overlapping Generations Model with Endogenous Labor Supply, *Journal of Economic Dynamics and Control,* forthcoming.

[15] Schmitz J.A. Jr. (1996), The Role Played by Public Enterprises: How Much Does it Differ across Countries?, *Federal Reserve Bank of Minneapolis Quarterly Review,* **20,** 2-15.

[16] Schmitz J.A. Jr. (2001), Government Production of Investment Goods and Aggregate Labor Productivity, *Journal of Monetary Economics,* **47,** 163-87.

[17] Uzawa H. (1961), On a Two-Sector Model of Economic Growth, *Review of Economic Studies,* **29,** 40-7.

[18] Uzawa H. (1964), Optimal Growth in a Two-Sectors Model of Capital Accumulation, *Review of Economic Studies,* **31,** 1-24.

[19] Woodford M. (1986), Stationary Sunspot Equilibria: the Case of Small Fluctuations around a Deterministic Steady State, *Chicago University Mimeo.*

Documents de recherche EPEE

2002

02 - 01 **Inflation, salaires et SMIC: quelles relations?**
Yannick L'HORTY & Christophe RAULT

02 - 02 **Le paradoxe de la productivité**
Nathalie GREENAN & Yannick L'HORTY

02 - 03 **35 heures et inégalités**
Fabrice GILLES & Yannick L'HORTY

02 - 04 **Droits connexes, transferts sociaux locaux et retour à l'emploi**
Denis ANNE & Yannick L'HORTY

02 - 05 **Animal Spirits with Arbitrarily Small Market Imperfection**
Stefano BOSI, Frédéric DUFOURT & Francesco MAGRIS

02 - 06 **Actualité du protectionnisme :**
l'exemple des importations américaines d'acier
Anne HANAUT

2001

01 - 01 **Optimal Privatisation Design and Financial Markets**
Stefano BOSI, Guillaume GIRMENS & Michel GUILLARD

01 - 02 **Valeurs extrêmes et series temporelles :**
application à la finance
Sanvi AVOUYI-DOVI & Dominique GUEGAN

01 - 03 **La convergence structurelle européenne :**
rattrapage technologique et commerce intra-branche
Anne HANAUT & El Mouhoub MOUHOUD

01 - 04 **Incitations et transitions sur le marché du travail :**
une analyse des stratégies d'acceptation et des refus d'emploi
Thierry LAURENT, Yannick L'HORTY, Patrick MAILLE & Jean-François OUVRARD

01 - 05 **La nouvelle economie et le paradoxe de la productivité :**
une comparaison France - Etats-Unis
Fabrice GILLES & Yannick L'HORTY

01 - 06 **Time Consistency and Dynamic Democracy**
Toke AIDT & Francesco MAGRIS

01 - 07 **Macroeconomic Dynamics**
Stefano BOSI

01 - 08 **Règles de politique monétaire en présence d'incertitude :**
une synthèse
Hervé LE BIHAN & Jean-Guillaume SAHUC

01 - 09 **Indeterminacy and Endogenous Fluctuations**
with Arbitrarily Small Liquidity Constraint
Stefano BOSI & Francesco MAGRIS

01 - 10 **Financial Effects of Privatizing the Production of Investment Goods**
Stefano BOSI & Carine NOURRY

01 - 11 **On the Woodford Reinterpretation of the Reichlin OLG Model : a Reconsideration**
Guido CAZZAVILLAN & Francesco MAGRIS

01 - 12 **Mathematics for Economics**
Stefano BOSI

01 - 13 **Real Business Cycles and the Animal Spirits Hypothesis in a Cash-in-Advance Economy**
Jean-Paul BARINCI & Arnaud CHERON

01 - 14 **Privatization, International Asset Trade and Financial Markets**
Guillaume GIRMENS

01 - 15 **Externalités liées dans leur réduction et recyclage**
Carole CHEVALLIER & Jean DE BEIR

01 - 16 **Attitude towards Information and Non-Expected Utility Preferences : a Characterization by Choice Functions**
Marc-Arthur DIAYE & Jean-Max KOSKIEVIC

01 - 17 **Fiscalité de l'épargne en Europe : une comparaison multi-produits**
Thierry LAURENT & Yannick L'HORTY

01 - 18 **Why is French Equilibrium Unemployment so High : an Estimation of the WS-PS Model**
Yannick L'HORTY & Christophe RAULT

01 - 19 **La critique du « système agricole » par Smith**
Daniel DIATKINE

01 - 20 **Modèle à Anticipations Rationnelles de la COnjoncture Simulée : MARCOS**
Pascal JACQUINOT & Ferhat MIHOUBI

01 - 21 **Qu'a-t-on appris sur le lien salaire-emploi ? De l'équilibre de sous emploi au chômage d'équilibre : la recherche des fondements microéconomiques de la rigidité des salaires**
Thierry LAURENT & Hélène ZAJDELA

01 - 22 **Formation des salaires, ajustements de l'emploi et politique économique**
Thierry LAURENT

2000

00 - 01 **Wealth Distribution and the Big Push**
Zoubir BENHAMOUCHE

00 - 02 **Conspicuous Consumption**
Stefano BOSI

00 - 03 **Cible d'inflation ou de niveau de prix : quelle option retenir pour la banque centrale dans un environnement « nouveau keynésien » ?**
Ludovic AUBERT

00 - 04 **Soutien aux bas revenus, réforme du RMI et incitations à l'emploi : une mise en perspective**
Thierry LAURENT & Yannick L'HORTY

00 - 05 **Growth and Inflation in a Monetary « Selling-Cost » Model**

Stefano BOSI & Michel GUILLARD

00 - 06 **Monetary Union : a Welfare Based Approach**
Martine CARRE & Fabrice COLLARD

00 - 07 **Nouvelle synthèse et politique monétaire**
Michel GUILLARD

00 - 08 **Neoclassical Convergence versus Technological Catch-Up : a Contribution for Reaching a Consensus**
Alain DESDOIGTS

00 - 09 **L'impact des signaux de politique monétaire sur la volatilité intrajournalière du taux de change deutschemark - dollar**
Aurélie BOUBEL, Sébastien LAURENT & Christelle LECOURT

00 - 10 **A Note on Growth Cycles**
Stefano BOSI, Matthieu CAILLAT & Matthieu LEPELLEY

00 - 11 **Growth Cycles**
Stefano BOSI

00 - 12 **Règles monétaires et prévisions d'inflation en économie ouverte**
Michel BOUTILLIER, Michel GUILLARD & Auguste MPACKO PRISO

00 - 13 **Long-Run Volatility Dependencies in Intraday Data and Mixture of Normal Distributions**
Aurélie BOUBEL & Sébastien LAURENT

1999

99 - 01 **Liquidity Constraint, Increasing Returns and Endogenous Fluctuations**
Stefano BOSI & Francesco MAGRIS

99 - 02 **Le temps partiel dans la perspective des 35 heures**
Yannick L'HORTY & Bénédicte GALTIER

99 - 03 **Les causes du chômage en France : Une ré-estimation du modèle WS - PS**
Yannick L'HORTY & Christophe RAULT

99 - 04 **Transaction Costs and Fluctuations in Endogenous Growth**
Stefano BOSI

99 - 05 **La monnaie dans les modèles de choix intertemporels : quelques résultats d'équivalences fonctionnelles**
Michel GUILLARD

99 - 06 **Cash-in-Advance, Capital, and Indeterminacy**
Gaetano BLOISE, Stefano BOSI & Francesco MAGRIS

99 - 07 **Sunspots, Money and Capital**
Gaetano BLOISE, Stefano BOSI & Francesco MAGRIS

99 - 08 **Inter-Juridictional Tax Competition in a Federal System of Overlapping Revenue Maximizing Governments**
Laurent FLOCHEL & Thierry MADIES

99 - 09 **Economic Integration and Long-Run Persistence of the GNP Distribution**
Jérôme GLACHANT & Charles VELLUTINI

99 - 10 **Macroéconomie approfondie : croissance endogène**
Jérôme GLACHANT

99 - 11 **Growth, Inflation and Indeterminacy in a Monetary « Selling-Cost » Model**
Stefano BOSI & Michel GUILLARD

99 - 12 **Règles monétaires, « ciblage » des prévisions et (in)stabilité de l'équilibre macroéconomique**
Michel GUILLARD

99 - 13 **Educating Children : a Look at Household Behaviour in Côte d'Ivoire**
Philippe DE VREYER, Sylvie LAMBERT & Thierry MAGNAC

99 - 14 **The Permanent Effects of Labour Market Entry in Times of High Aggregate Unemployment**
Philippe DE VREYER, Richard LAYTE, Azhar HUSSAIN & Maarten WOLBERS

99 - 15 **Allocating and Funding Universal Service Obligations in a Competitive Network Market**
Philippe CHONE, Laurent FLOCHEL & Anne PERROT

99 - 16 **Intégration économique et convergence des revenus dans le modèle néo-classique**
Jérôme GLACHANT & Charles VELLUTINI

99 - 17 **Convergence des productivités européennes : réconcilier deux approches de la convergence**
Stéphane ADJEMIAN

99 - 18 **Endogenous Business Cycles : Capital-Labor Substitution and Liquidity Constraint**
Stefano BOSI & Francesco MAGRIS

99 - 19 **Structure productive et procyclicité de la productivité**
Zoubir BENHAMOUCHE

99 - 20 **Intraday Exchange Rate Dynamics and Monetary Policy**
Aurélie BOUBEL & Richard TOPOL

1998

98 - 01 **Croissance, inflation et bulles**
Michel GUILLARD

98 - 02 **Patterns of Economic Development and the Formation of Clubs**
Alain DESDOIGTS

98 - 03 **Is There Enough RD Spending ? A Reexamination of Romer's (1990) Model**
Jérôme GLACHANT

98 - 04 **Spécialisation internationale et intégration régionale. L'Argentine et le Mercosur**
Carlos WINOGRAD

98 - 05 **Emploi, salaire et coordination des activités**
Thierry LAURENT & Hélène ZAJDELA

98 - 06 **Interconnexion de réseaux et charge d'accès : une analyse stratégique**
Laurent FLOCHEL

98 - 07 **Coût unitaires et estimation d'un système de demande de travail : théorie et application au cas de Taiwan**
Philippe DE VREYER

98 - 08 **Private Information :**
an Argument for a Fixed Exchange Rate System
Ludovic AUBERT & Daniel LASKAR

98 - 09 **Le chômage d'équilibre. De quoi parlons nous ?**
Yannick L'HORTY & Florence THIBAULT

98 - 10 **Deux études sur le RMI**
Yannick L'HORTY & Antoine PARENT

98 - 11 **Substituabilité des hommes aux heures et ralentissement de la productivité ?**
Yannick L'HORTY & Chistophe RAULT

98 - 12 **De l'équilibre de sous emploi au chômage d'équilibre :**
la recherche des fondements microéconomiques de la rigidité des salaires
Thierry LAURENT & Hélène ZAJDELA