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# On the Woodford Reinterpretation of the Reichlin OLG Model : a Reconsideration

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# On the Woodford Reinterpretation of the Reichlin OLG Model: A Reconsideration

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## Abstract

In this paper we investigate whether competitive equilibrium dynamics similar to those generated in a non-monetary two-period overlapping generations model, such as that studied by Reichlin [4], can result in a monetary framework with infinite-lived workers and capitalists from the existence of a cash-in-advance constraint on workers' consumption purchases, as in Woodford [6]. In contrast with the reinterpretation of the OLG framework provided by the latter author, we show that the existence of finance constraints in the monetary infinite horizon model of that type can never result, generically, in equilibrium dynamics "similar" (or equal) to those obtained from a non-monetary two-period overlapping generations model where people work when young and consume when old. In particular, we prove that the parameter configurations leading to local indeterminacy and stochastic endogenous fluctuations in either model necessarily imply local determinacy and the absence of sunspot equilibria in the other. *Journal of Economic Literature* Classification Numbers: E32, C62.

## 1. INTRODUCTION

Dating from the early contributions of Reichlin [4] and Woodford [6], it as been pointed out by a number of authors (see, among the others, Boldrin and Woodford [1], Grandmont *et al*. [3], Woodford [6]) that equilibrium dynamics similar to those in non-monetary, two-period overlapping generations model with production can result in a model with infinitely-lived workers and entrepreneurs, from the existence of a cash-in-advance constraint on workers' purchases, together with the usual budget constraint. In this case, as stated in Boldrin and Woodford [1], *the competitive equilibrium dynamics turns out to be very close to those originated in the* Reichlin [4] *model, and, consequently, endogenous fluctuations are possible phenomena under circumstances very close to those discussed in* Reichlin [4]. As a result, it would not be entirely fair to criticize (see Sims [5]) the examples of endogenous fluctuations in economies with overlapping generations of two-period-lived agents as involving mechanisms that generate cyclic equilibrium paths only at very low frequencies. Although Reichlin's overlapping generations model yields endogenous fluctuations that are approximately three times the consumers' lifetime, or six periods in length, in the life-cycle interpretation of his model, it as been observed (see Boldrin and Woodford [1]) that in the Woodford's reinterpretation of that model, six periods is still too short for a business cycle, under the most reasonable interpretation of the period length in an infinite horizon model with cash-in-advance constraint.

More recently, Grandmont *et al*. [3], although focusing on the link between the local dynamics generated by *monetary* overlapping generations models and those resulting from the infinite horizon framework with liquidity constraints along the lines of Woodford [6], have still recognized that, in the case of a Leontief technology, i.e. when there is no capital-labor substitution, the equations governing the dynamics of Reichlin's non-monetary OLG model are practically identical to those that describe the local dynamics, in a neighborhood of the stationary state, of the Woodford's model.

In this paper we propose a reconsideration of the Woodford [6] reinterpretation of the Reichlin [4] *non-monetary* overlapping generations model. In particular, we shall demonstrate that local indeterminacy and stochastic endogenous fluctuations can never occur, for any given parameter configuration, at the same time in the two economies under study. We shall show, in fact, that the conditions leading to an asymptotically stable steady-state in either model necessarily imply local determinacy and the absence of sunspot equilibria in the other, a result generically true even when both models exhibit no capital-labor substitution, in contrast with Grandmont *et al*. [3]. Hence, the idea that local dynamics near the monetary steady-state in the Woodford's model are similar (or even identical) to what would come out from an OLG structure à-la-Reichlin turns out to be false.

The rest of the paper is organized as follows. In the next section, we shall briefly recall the basic features of the two models and discuss the conditions on the parameters leading to local indeterminacy and endogenous fluctuations. In Section 3 we shall compare the implications of the two settings and state the main result. The final remarks will be summarized in Section 4.

## 2. THE BENCHMARK MODELS

In this sections we shall shortly recall the two benchmark frameworks between which our subsequent comparison will be carried out, namely the Reichlin [4] OLG model with production, and the Woodford [6] infinitely horizon model with heterogeneous agents and cash-in-advance constraint. In both cases, we shall consider the extended versions to account for partial capital depreciation and capital-labor substitution, along the lines of Cazzavillan [2], and Grandmont *et al*. [3].

### 2.1. The Rechlin model

Reichlin [4] considers a non-monetary overlapping generations model involving a unique perishable good, that can be either consumed or saved as investment, and a large number of competitive firms endowed with an identical technology. Agents, who live two periods only, supply labor $\ell_t \geq 0$ when young in period *t*, and consume $c_{t+1} \geq 0$ units of output when old at time *t*+1. We assume that agents' behavior is described by a "representative" consumer with an additively separable utility function of the form $-V(\ell_t) + U(c_{t+1})$, in which $U(c)$ is the utility of consumption $c \geq 0$, whereas $V(\ell)$ is the disutility of labor $\bar{\ell} \geq \ell \geq 0$, where $\bar{\ell} > 0$ represents the labor endowment. On the above preferences structure, we shall assume the following.

***Assumption 2.1***. The functions $U(c)$ and $V(\ell)$ are defined and continuous for all $c \geq 0$ and all $0 \leq \ell \leq \bar{\ell}$ respectively, where the labor endowment $\bar{\ell} > 0$ may be finite or infinite. Moreover, they are $C^r$, for *r* large enough, with $U'(c) > 0$, $V'(\ell) > 0$, $U''(c) < 0$, $V''(\ell) > 0$, for all $c > 0$ and $\bar{\ell} > \ell > 0$, with $Lim_{\ell \to \bar{\ell}} V'(\ell) = +\infty$. In addition, consumption and leisure are gross substitutes, i.e. $R_U(c) \equiv -c\,U''(c)/U'(c) < 1$.

A representative agent born at time $t$ wishes to maximize his utility, subject to the budget constraints given by $z_t = \omega_t \ell_t$, and $c_{t+1} = R_{t+1} z_t$, where $z_t > 0$, $\omega_t > 0$, and $R_{t+1} > 0$ are saving, the real wage at time $t$, and the real gross interest rate at time $t$+1, respectively. When interior, the unique solution of an agent born at time $t$ is characterized by the budget constraints and the first order condition

$$u(c_{t+1}) = v(\ell_t), \quad c_{t+1} > 0, \ \ell_t > 0, \tag{2.1}$$

where $u(c) \equiv cU'(c)$, and $v(\ell) \equiv \ell V'(\ell)$.

Output $y \geq 0$ is produced by an aggregate production function $F(k, \ell)$ that combines current labor $\ell \geq 0$ and capital services $k \geq 0$. Technology is assumed to satisfy the following properties.

***Assumption 2.2.*** The function $F(k, \ell)$ is defined, continuous, strictly concave for $k \geq 0$, $\ell \geq 0$, and homogeneous of degree one. The intensive form production function $F(k, \ell) \equiv \ell f(a)$, with $a \equiv k / \ell$, satisfies, accordingly, $f'(a) > 0$ and $f''(a) < 0$ for $a > 0$, while the marginal product of capital $\rho(a) \equiv f'(a) > 0$ and the marginal product of labor $\omega(a) \equiv f(a) - af'(a) > 0$ are, respectively, decreasing and increasing in the capital-labor ratio $a$.

Letting $0 \leq \delta \leq 1$ and $q > 0$ be the capital depreciation rate and the real rental of capital, factor prices, given profit maximization, are $\omega_t = \omega(a_t)$ and $q_t = \rho(a_t)$. In equilibrium, $R_t = q_t + 1 - \delta$, for all $t \geq 0$. As a result, from the first order conditions for the firms and the arbitrage condition on the capital market, one has

$$\omega_t = \omega(a_t), \quad R_t = \rho(a_t) + 1 - \delta \equiv R(a_t). \tag{2.2}$$

Equilibrium in the good market requires that investment be equal to saving, i.e. $k_{t+1} = z_t$. The equilibrium dynamics are therefore obtained by putting together the consumer's budget constraints, Eqs. 2.1 and 2.2, as well as the condition $k_{t+1} = z_t$. *An intertemporal equilibrium with perfect foresight is then a sequence* $(k_t, a_t) > 0$, *with* $k_0 > 0$ *given, that satisfies, for all* $t \geq 1$, *the following equations*

$$k_{t+1} = \frac{k_t}{a_t}\omega(a_t) \tag{2.3}$$

$$u(k_{t+1}(\rho(a_t)+1-\delta)) = v(k_t / a_t)\,. \tag{2.4}$$

To study local dynamics one computes, as usual, the Jacobian of (2.3) and (2.4) evaluated at a steady-state $(a^*, k^*)_{OG} > 0$.[1] The trace and the determinant, i.e. the sum and the product of the characteristic roots, of this Jacobian are given by the expressions

$$T = T_1 + \frac{\varepsilon_\gamma - 1}{|\varepsilon_R|}\text{, with } T_1 = 1 + \frac{\varepsilon_\omega}{|\varepsilon_R|}\text{, and } D = D_1\varepsilon_\gamma\text{, with } D_1 = \frac{\varepsilon_\omega}{|\varepsilon_R|}, \tag{2.5}$$

where $\varepsilon_R$ and $\varepsilon_\omega$ are the elasticities of the functions $R$ and $\omega$ respectively, whereas $\varepsilon_\gamma$ represents the elasticity of the local inverse of the offer curve $\gamma(\ell) \equiv u^{-1}(v(\ell))$ derived from Eq. (2.1), all evaluated at the steady-state. Following the geometrical approach developed by Grandmont *et al.* [3], as shown in Cazzavillan [2], one can focus on two parameters of interest, namely the elasticity of input substitution and the elasticity of labor supply whose estimates are somewhat imprecise on empirical grounds. To that purpose, one can fix the technology, i.e. $\varepsilon_\omega$ and $\varepsilon_R$, and make $\varepsilon_\gamma$ vary continuously in the open interval (1, + ∞). From the expressions of *D* and *T* given (2.5), one sees that $(T(\varepsilon_\gamma), D(\varepsilon_\gamma))$ describes a half-line $\Delta$ which starts for $\varepsilon_\gamma = 1$ at the point $(T_1, D_1)$ that satisfies the equation $T_1 = 1 + D_1$ and belongs, therefore, to the line (AC) (see point *Q* in Fig. 1). In addition, the slope of $\Delta$ is equal to $\varepsilon_\omega$. One can then easily look at local stability and bifurcations simply by locating the half-line $\Delta$ when all the other parameters are kept fixed. For that purpose, it is convenient to express $\varepsilon_R$ and $\varepsilon_\omega$ as functions of the technological parameters, more specifically, as functions of the share of capital in total income $0 < s(a) \equiv a\rho(a) / f(a) < 1$, the depreciation rate $0 \le \delta \le 1$, and the elasticity of input substitution $\sigma(a) \ge 0$. Easy computations yield

$$|\varepsilon_R(a)| = \frac{s(a)}{\sigma(a)s^*(a)} \quad \text{and} \quad \varepsilon_\omega(a) = \frac{s(a)}{\sigma(a)}, \tag{2.6}$$

[1] For a complete discussion on existence of a stationary state we shall refer the reader to Cazzavillan [2].

where $s^*(a) \equiv (s(a)+(1-s(a))(1-\delta))/(1-s(a)).$ As a result, the coordinates of the origin of the half-line $\Delta(\sigma)$ are $T_1 = 1+s^*$ and $D_1 = s^*$, where $s^* = s^*(a^*)$ is evaluated at the steady-state under consideration. From Eqs. (2.5), one also sees that the slope of $\Delta(\sigma)$ is equal to $s/\sigma$, where $s = s(a^*)$ and $\sigma = \sigma(a^*)$ are evaluated at the steady-state. If $s = s(a^*)$ and $s^* = s^*(a^*)$ are kept fixed, while the elasticity of input substitution $\sigma = \sigma(a^*)$ is treated as an independent parameter, one sees that the slope of $\Delta(\sigma)$ decreases from $+\infty$ to 0 as $\sigma$ increases from 0 to $+\infty$. The qualitative behavior of the half-line $\Delta(\sigma)$ is then summarized in Fig. 1: it pivots rightward around the point $Q$ of coordinates $1+s^*$ and $s^*$ and it is vertical when $\sigma = 0$ and horizontal when $\sigma = +\infty$. *Therefore local indeterminacy and endogenous fluctuations obtain if and only if the point Q belongs to the interior of the segment* [AC], *i.e.* $D_1 < 1,$ that is when the constant capital depreciation rate exceeds the ratio between the private capital and labor shares evaluated at the steady-state, i.e. $\delta > s/(1-s).$ Given that the ratio between the private capital share and the labor share is close to ½ on the empirical ground, the latter condition requires a large depreciation rate. Such a feature should be expected in view of the long time-period implied by the OLG framework. When $\delta > s/(1-s)$, local indeterminacy and deterministic cycles arise when $\sigma$ is in $(0,\ s)$ as $\Delta(\sigma)$ always crosses the interior of [BC] in that open interval. On the other hand, if $\sigma$ is in $(s,\ +\infty),$ the half-line $\Delta(\sigma)$ always lies below the line (AC), so local determinacy occurs.

Insert Fig. 1 here

### 2.2. The Woodford model

We now turn to the infinite horizon model with heterogenous agents studied by Woodford [6]. The economy is populated with two types of agents endowed with perfect foresight who consume and trade over their infinite lifetime. The first class of agents, called workers, consume, supply labor, and save a fraction of income either as physical capital or as nominal fiat money. They buy a unique consumption good at the outset of each period while firms pay rental returns on capital and real wage at the end of each period. In addition, workers cannot consume out of their money balances and the returns earned on capital held at the outset of each period. The second class of agents, called capitalists, consume and save their income, made up of real money balances and returns on capital, while operating the production process. In addition, they are more patient than workers and, therefore, discount their future utility less than workers. As shown in Woodford [6], such an assumption implies that, at the stationary state and in a neighborhood of it, the real return on capital is strictly larger than that of money holdings which is close to zero, as long as there is positive discounting and/or capital depreciation. As a result, capitalists choose not to hold fiat money.

Given the structure of the economy shortly recalled above, we shall focus on the overlapping generations structure that results from the infinite horizon framework of the Woodford type near the monetary steady-state.[2] In this region, in fact, workers behave as if they come in a two-period overlapping generations economy: they save $m_t \geq 0$ units of outside money, and supply $\ell_t \geq 0$ units of labor when young in period *t*, whereas they consume $c_{t+1} \geq 0$ units of consumption when old in the next period. If we posit that preferences of a representative worker are represented by the separable utility function $-V(\ell_t) + U(c_{t+1})$, where $V(\ell)$ and $U(c)$ satisfy Assumption 2.1, and that he maximizes at time *t*, the objective $-V(\ell_t) + U(c_{t+1})$ subject to the budget constraint $w_t \ell_t = m_t = p_{t+1} c_{t+1}$, where $w_t > 0$ and $p_{t+1} > 0$ are, respectively, the current monetary wage and the next period monetary price of consumption, the first order condition of his maximization problem are identical to those derived from Reichlin [4] and studied in the previous subsection, i.e. $u(c_{t+1}) = v(\ell_t),\ \ c_{t+1} > 0,\ \ell_t > 0,$ where, again, $u(c) \equiv cU'(c),$ and $v(\ell) \equiv \ell V'(\ell)$.

Firms behavior and technology being exactly the same as those assumed in the previous sub-section (c.d. Assumption 2.2), the marginal product of capital and labor, given profit maximization, must equate the real rental price of capital $\rho_t = r_t / p_t > 0$ and the real wage $\omega_t = w_t / p_t > 0$, respectively. In equilibrium, therefore, where labor's demand equates workers' labor supply $\ell_t > 0$ and the demand for capital is equal to the available stock $k_{t-1} > 0$, one has $\omega_t = \omega(a_t)$, and $\rho_t = \rho(a_t)$, where $a_t \equiv k_{t-1} / \ell_t$. It follows that the real gross rate of return on capital in equilibrium is $R(a_t) = \rho(a_t) + 1 - \delta$. As in the original Woodford [7] specification it is assumed that the real rate of return on capital exceeds the real return on money at the steady-state and in a neighborhood of it, it follows that capitalists end up with holding physical capital only. The capitalists' behavior, in equilibrium, is then described by the following equation

$$k_t = \beta R(a_t) k_{t-1} \tag{2.7}$$

where $0 < \beta < 1$ represents the capitalists' discount factor (greater than the workers' discount factor).

[2] For a more detailed analysis, see Grandmont et *al*. [3].

Fiat money supply is assumed to be constant over time. Since it is held by workers only, competitive equilibrium in the money market at date *t* requires that real balances $M_t / p_t$ be equal to consumption $c_t$ and to the real wage income $\omega(a_t)\ell_t$. Therefore,

$$\omega(a_t)\ell_t = c_t = M_t / p_t. \tag{2.8}$$

Finally, Walras' law ensures equilibrium in the remaining good market. Using Eqs. (2.7) and (2.8), as well as the first order conditions for workers' and firms' maximization problems, one can then conclude that *an intertemporal equilibrium with perfect foresight is a sequence* $(k_{t-1}, a_t) > 0$, *with* $k_0 > 0$ *given, that satisfies, for all* $t \geq 1$, *the following equations*

$$k_t = \beta(\rho(a_t) + 1 - \delta)k_{t-1} \tag{2.9}$$

$$v(k_{t-1} / a_t) = u(k_t \omega(a_{t+1}) / a_{t+1}). \tag{2.10}$$

Exactly as we did in sub-section 2.1, we now provide the expressions of the trace and the determinant of the Jacobian matrix evaluated at the steady-state $(a^*, k^*)_{IH} > 0$.[3] One can verify that these expressions, for $\varepsilon_\omega \neq 1$, are given by

$$T = T_1 - \frac{\varepsilon_\gamma - 1}{\varepsilon_\omega - 1}, \text{ with } T_1 = 1 + \frac{|\varepsilon_R| - 1}{\varepsilon_\omega - 1}, \text{ and } D = D_1 \varepsilon_\gamma, \text{ with } D_1 = \frac{|\varepsilon_R| - 1}{\varepsilon_\omega - 1}, \tag{2.11}$$

where the notation is the same as that used in Eqs. (2.5).

Following the same strategy adopted while studying the OLG framework, one can immediately verify that if the technology, i.e. $\varepsilon_\omega$ and $\varepsilon_R$, is kept fixed at the steady-state while varying $\varepsilon_\gamma$ form one to $+\infty$, the generic point (*T*, *D*) still describes a half-line $\Delta$ that starts at the point Q of coordinates $(T_1, D_1)$ when $\varepsilon_\gamma = 1$, and satisfies the equation $T_1 = 1 + D_1$ and, therefore, lies on the line (AC) as shown in Fig. 2. As in sub-section 2.1, we can express $\varepsilon_\omega$ and $\varepsilon_R$ in terms of the structural parameters of the model. This yields: $\varepsilon_\omega = s(a^*) / \sigma(a^*)$, and $\varepsilon_R = \phi(1 - s(a^*)) / \sigma(a^*)$, where $\phi \equiv 1 - \beta(1 - \delta) > 0$. It follows that Eqs. (2.11) can be rewritten as

[3] For a careful discussion on the existence of a steady-state, see Grandmont et *al*. [3].

$$T = T_1 - (\varepsilon_\gamma - 1)\frac{\sigma}{s-\sigma}, \quad with \ \ T_1 = 1 + D_1 \ , \ \ D = D_1\varepsilon_\gamma \ , \ \ with \ \ D_1 = \frac{\phi(1-s)-\sigma}{s-\sigma}, \qquad (2.12)$$

where $s \equiv s(a^*)$, $\sigma \equiv \sigma(a^*)$, with $s \neq \sigma$. From direct inspection of Eqs. (2.12), one sees that the slope of the half-line $\Delta(\sigma)$ is $1-\phi(1-s)/\sigma$. As shown by Grandmont et *al*. [3], *local indeterminacy and endogenous fluctuations can be obtained for reasonable parameter values if and only if* $\phi(1-s)/s<1$. Under such an inequality, the half-line $\Delta(\sigma)$ lies entirely above (AC) (see Fig. 2) when $s>\sigma$, and below (AC) when $s<\sigma$, whereas the slope of $\Delta(\sigma)$ increases with $\sigma$, from $-\infty$ to 1. In addition, the ordinate $D_1$ of the origin of $\Delta(\sigma)$ decreases from $\phi(1-s)/s$ to $-\infty$ when $\sigma$ increases form 0 to *s*, and from $+\infty$ to 1 when $\sigma$ goes from *s* to $+\infty$. As a result, as shown in Fig. 2, the half-line $\Delta(\sigma)$ pivots anti-clockwise around point Q which decreases monotonically with $\sigma$ along (AC). Local indeterminacy and endogenous fluctuations, therefore, occur for all $\sigma \in [0, \ \sigma_I)$, where $\sigma_I = (\phi(1-s)+s)/2 < s$.

Insert Fig. 2 here

One can immediately verify that the necessary and sufficient condition to get local indeterminacy and endogenous cycle for some $\varepsilon_\gamma > 1$, i.e. $\phi(1-s)/s<1$, can be rewritten as $\beta\delta < s/(1-s)+\beta-1$. The latter, to be fulfilled, requires a sufficiently low depreciation rate whose upper bound depends upon the value of the capital share in total income *s*. Such a constraint, given the short time period (say a quarter) implied by the infinite horizon model under study, is perfectly acceptable. For a capital share equal to 1/3, and a discount factor close to one, the above inequality is satisfied when the depreciation rate $\delta$ is less than, approximately, 1/2.

## 3. COMPARING THE MODELS

The above analysis shows that local indeterminacy and endogenous fluctuations arise in the two models under quite different conditions. In the overlapping generations setting, as studied by Reichlin [4] and Cazzavillan [2], the steady-state can be locally indeterminate for all $\sigma$ in $[0, \ s)$ if and only if the depreciation rate $0 \le \delta \le 1$ is large enough relatively to the capital share *s*, i.e.

$$\delta > \frac{s}{1-s}. \qquad (3.1)$$

On the contrary, as shown in Woodford [6] and Grandmond et *al*. [3],[4] the steady-state can be locally indeterminate if and only if

$$\delta < \beta^{-1}\left(\frac{s}{1-s}+\beta-1\right), \tag{3.2}$$

that is if and only if the depreciation rate is sufficiently small.

These findings are not very surprising, given the different nature of the two models: the Reichlin's model is a genuine overlapping generations model and, therefore, it is associated with a quite long time period. Conversely, the Woodford's model, in spite of the cash-in-advance constraint which makes workers behave as if they come in an OLG structure, still remains an infinite horizon model in which the time period can be interpreted as short. The two conditions in (3.1) and (3.2), thus, are consistent and "plausible" within each respective framework. However, as far as the conditions leading to local indeterminacy and sunspot equilibria, the two models are totally different. As a matter of fact, we are now going to show *that local indeterminacy and stochastic endogenous cycles, for any given parameter configuration, can never occur under the same circumstances in the two economies under study*. *We shall in fact demonstrate that the conditions leading to an asymptotically stable steady-state in one model necessarily imply local determinacy in the other. As a result, the idea that local dynamics near the monetary steady-state in the Woodford's model are similar (or even identical) to what would come out from an OLG structure à-la-Reichlin turns out to be false*.

To understand why the local dynamics originated in the Reichlin's model are not compatible with those arising from the Woodford's model, it is sufficient to study simultaneously the two inequalities in (3.1) and (3.2) and check if there exist some values of $\delta$ in $[0,\ 1]$ that satisfy both at the same time. For that purpose, it is worthwhile to notice that (3.1) is satisfied for some $\delta$ in $[0,\ 1]$ if and only if $s < 1/2$, i.e. the share of capital in total income has not to be too large. For $s \geq 1/2$, in fact, the domain of inequality (3.1) would include only depreciation rates strictly greater than one, a feature economically meaningless. A simple comparison between inequalities (3.1) and (3.2) shows that the upper bound for $\delta$ in the Woodford's setting is always less than the lower bound for $\delta$ in the Reichlin's framework for all $s < 1/2$ and $0 < \beta < 1$. As a result, if (3.1) is fulfilled, and, therefore, local indeterminacy and endogenous cycles occur in the OLG model, (3.2) is never met, i.e. local determinacy and no stochastic endogenous cycles near the steady-state arise in the

[4] Grandmont et *al*. [3] consider the limit case where capitalists' discount factor is equal to one.

Woodford's model. Conversely, if (3.2) is satisfied, and thus local indeterminacy and endogenous fluctuations are possible in the Woodford's model, (3.1) is never met, i.e. local determinacy and no periodic solutions occur in the Reichlin's model. Formally, the two inequalities (3.1) and (3.2) would be simultaneously satisfied if and only if

$$\left(\frac{s}{1-s}-1\right)\frac{1-\beta}{\beta}>0. \tag{3.3}$$

But (3.3) has no solutions for any given $s<1/2$ and $0<\beta<1$. Also in the limit case $\beta=1$ analyzed by Grandmont et *al*. [3], (3.3) is never satisfied. These simple observations lead to the following result.

**Proposition 3.1.** *Suppose that Assumptions 2.1 and 2.2 hold and let* $(a^*, k^*)_{OG}>0$ *and* $(a^*, k^*)_{IH}>0$ *be the steady-states under study in the OLG model and in the infinite horizon framework with cash-in-advance constraint, respectively. Moreover, assume that* $s<1/2$, *and* $0<\beta<1$. *Then, if* $(a^*, k^*)_{OG}>0$ *is asymptotically stable, hence locally indeterminate,* $(a^*, k^*)_{IH}>0$ *is locally unstable, hence locally determinate. On the contrary, if* $(a^*, k^*)_{IH}>0$ *is locally indeterminate, then* $(a^*, k^*)_{OG}>0$ *is locally determinate.*

In view of the results stated in the above Proposition, it appears quite clearly that the parameter configurations leading to local indeterminacy in the two models are mutually exclusive. It follows that sunspot equilibria arising from the OLG model under study cannot be given the Woodford's reinterpretation.

An interesting, though not robust, case is obtained when there is not capital-labor substitution, i.e., $\sigma=0$, as in the original Reichlin [4] and Woodford [6] specifications. From the previous section we learnt that, for $\sigma=0$, $D_1=s/(1-s)+1-\delta$ in the OLG model, whereas $D_1=(1-\beta(1-\delta))(1-s)/s$ in the cash-in-advance model. Simple computations show that the two expressions are identical if and only if $\beta=1$ (as in Grandmont *et al*. [3]) and $\delta=s/(1-s)$. Under these circumstances, the local dynamics are the same. The origins merge at the point C (see Fig. 3) of coordinates (2, 1), while the half-lines $\Delta(\sigma)$ coincide. In both models, therefore, the stationary state is a source, hence locally determinate. A slight departure from the limiting case $\beta=1$, i.e. $\beta<1$, makes, as shown in Proposition 3.1, the local indeterminacy regimes incompatible. In particular, if the condition leading to an asymptotically stable stationary state in the Reichlin's model is met, i.e. if $s/(1-s)+1-\delta<1$, the point Q belongs to the interior of the segment (AC), whereas it lies above the point C along the line (AC) in the Woodford's model as the condition leading to local indeterminacy and endogenous cycles, i.e. $s/(1-s)+\beta(1-\delta)>1$, cannot be fulfilled (see Fig. 3). On the contrary, if $s/(1-s)+\beta(1-\delta)>1$, and hence local indeterminacy and endogenous fluctuations obtain in the Woodford's model, the condition $s/(1-s)+1-\delta<1$ cannot be met and the origin of the half-line $\Delta(\sigma)$ in the Reichlin's model lies above the point C along the line (AC).

## 4. CONCLUSIONS

We have investigated whether competitive equilibrium dynamics similar to those generated in a non-monetary overlapping generations model such as that studied by Reichlin [4] can result, generically, in a monetary framework with infinite-lived workers and capitalists from the existence of a cash-in-advance constraint on workers' consumption purchases on the line of Woodford [6], as stated in Boldrin and Woodford [1]. We have shown, on the contrary, that the existence of finance constraints in the monetary infinite horizon model can never result in equilibrium dynamics similar (or equal) to those obtained for a two-period non-monetary overlapping generations models where people work when young and consume when old. As a result, the Woodford's reinterpretation of the Reichlin's model does not hold.

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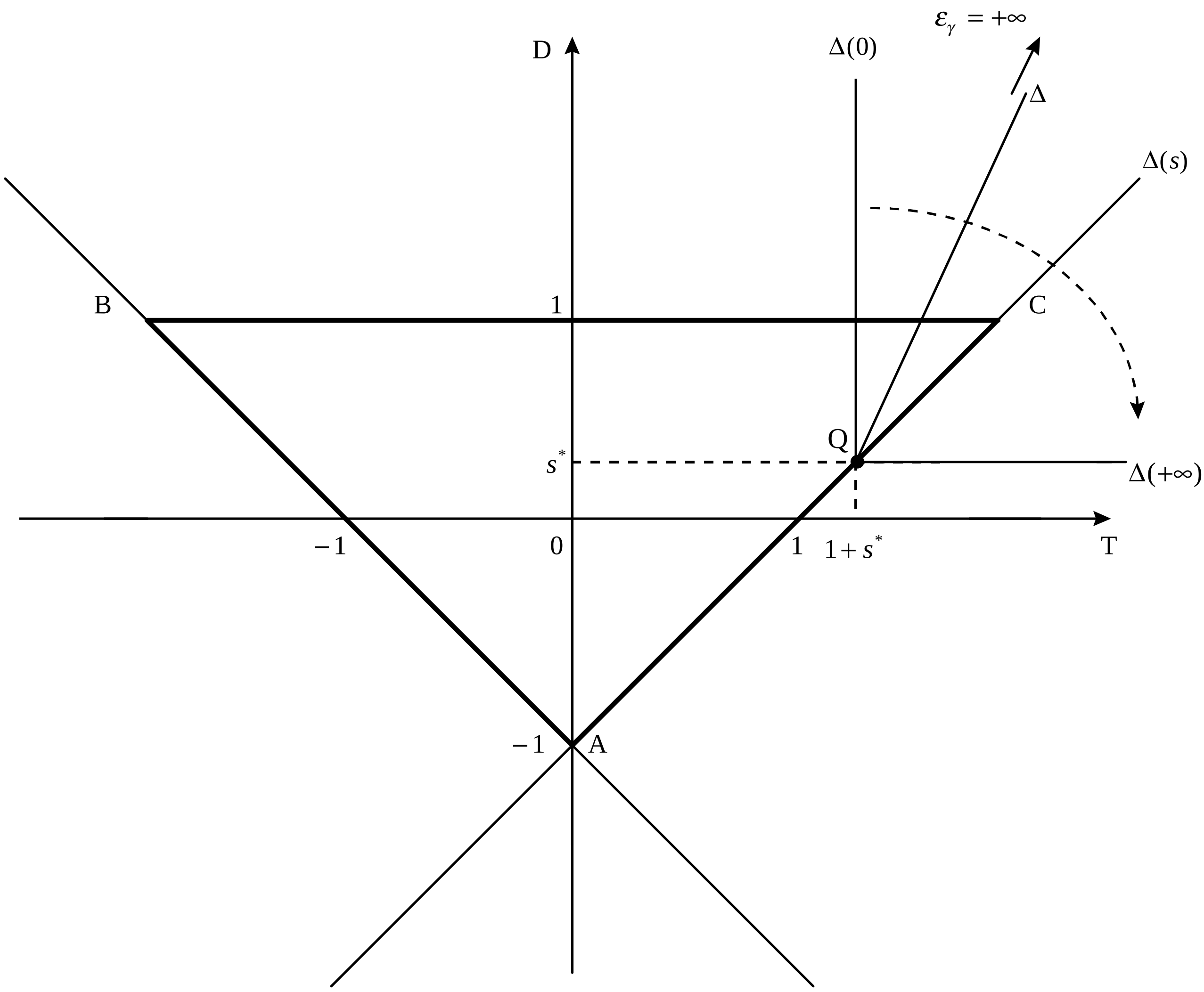
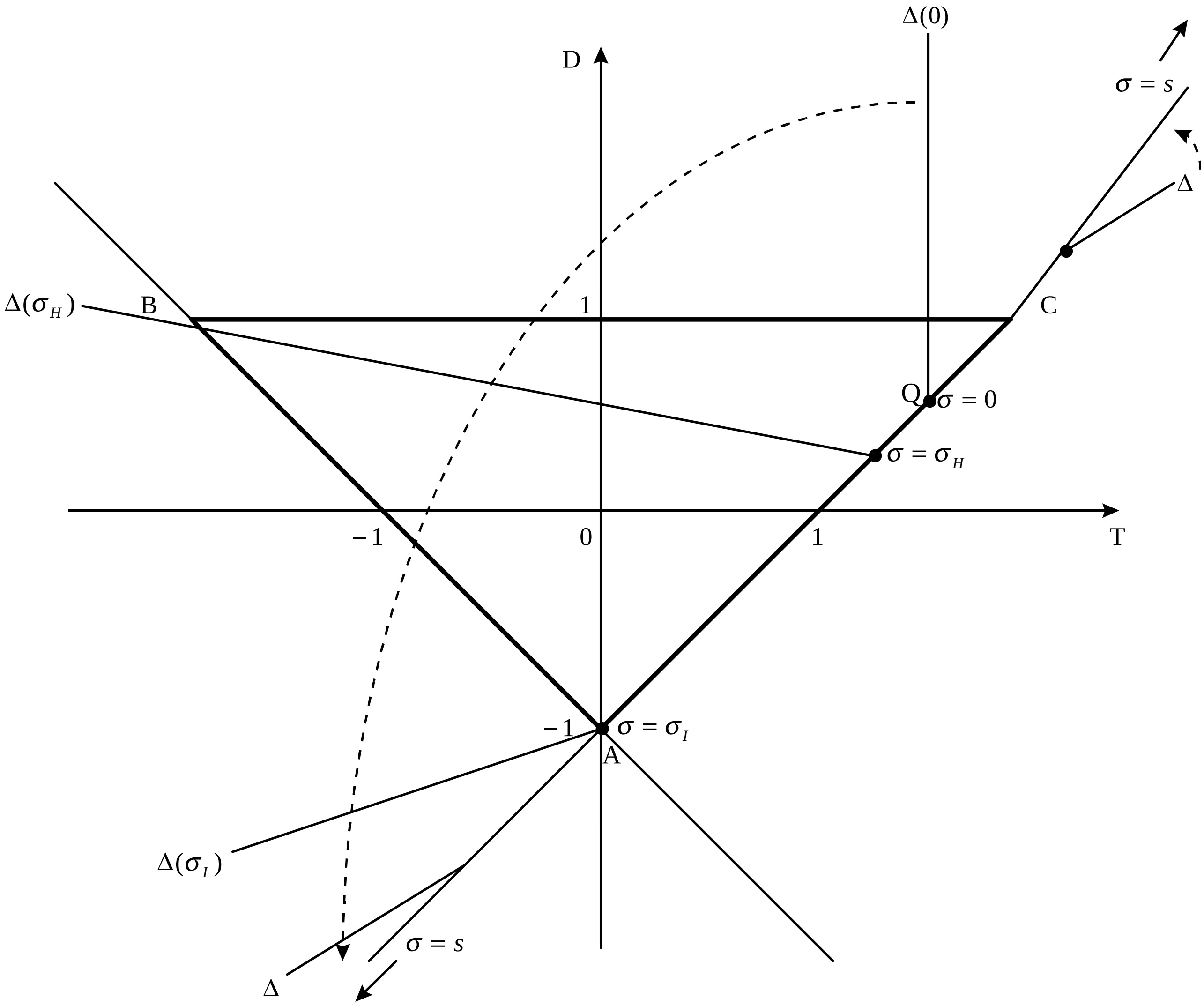


$\varepsilon_\gamma = +\infty$
D
$\Delta(0)$
$\Delta$
$\Delta(s)$
B
1
C
Q
$s^*$
$\Delta(+\infty)$
$-1$
0
1
$1+s^*$
T
$-1$
A


**Fig.1**



$\Delta(0)$
D
$\sigma = s$
$\Delta$
$\Delta(\sigma_H)$
B
1
C
Q
$\sigma = 0$
$\sigma = \sigma_H$
$-1$
0
1
T
$-1$
$\sigma = \sigma_I$
A
$\Delta(\sigma_I)$
$\sigma = s$
$\Delta$


**Fig.2**

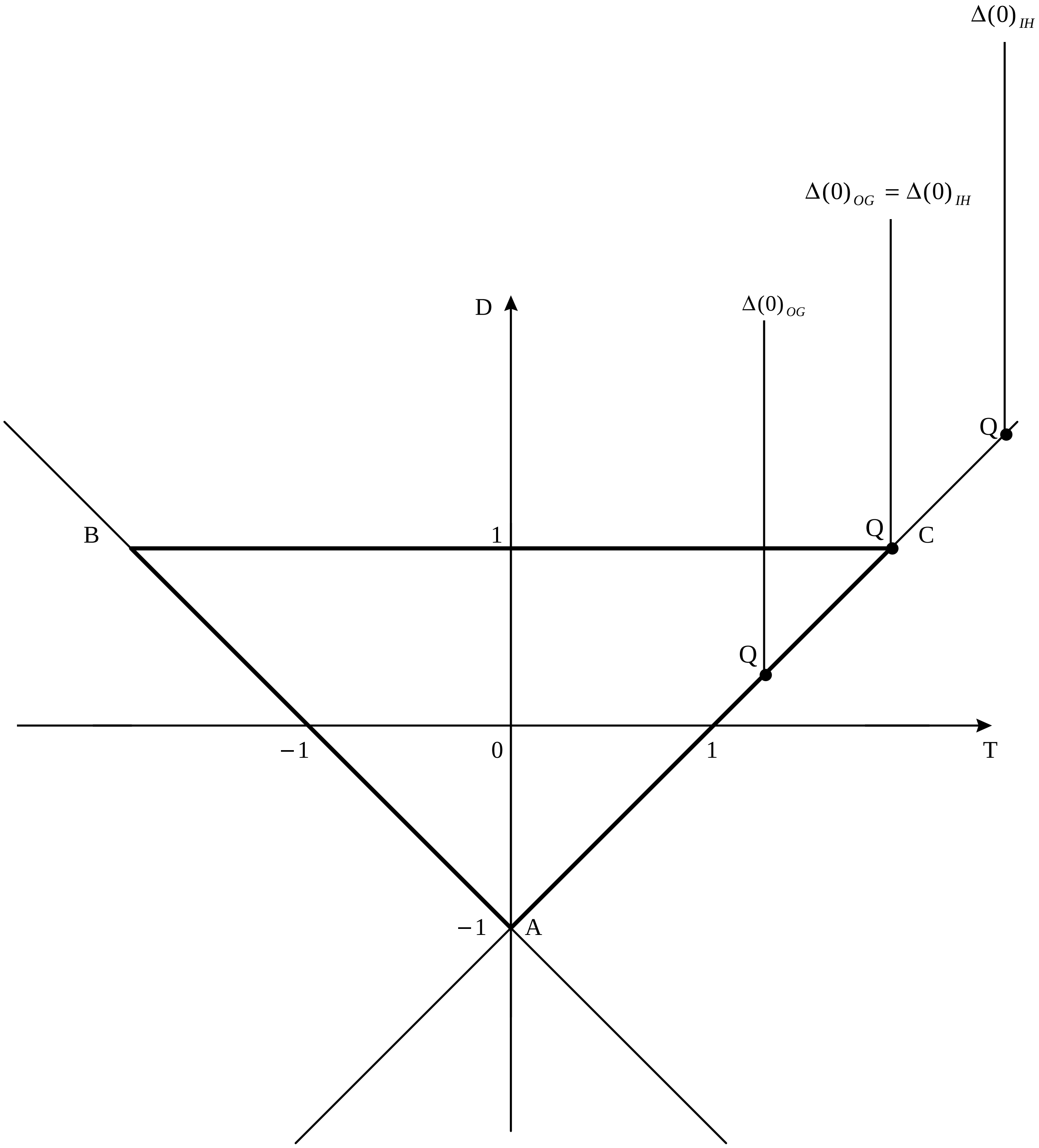



**Fig.3**

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