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THE FRAGILITY OF THE FISCAL THEORY OF PRICE DETERMINATION

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ABSTRACT. The fiscal theory of price determination asserts that the price level is determined by the ratio of nominal public debt to the present value of real primary surpluses. To show its fragility, we describe a cash-in-advance economy with infinitely lived real productive assets. The fiscal theory does not hold since speculative bubbles partly restore the classical indeterminacy result.

KEYWORDS. Money, bubbles, indeterminacy, monetary policy, fiscal policy.

JEL CLASSIFICATION NUMBERS. D50, E40, E50.

1. INTRODUCTION

The aim of this paper is to show that the fiscal theory of price determination does not hold when one introduces, in the description of a monetary economy, stocks of long term assets whose market values are to be determined at equilibrium.[1]

Roughly speaking, the fiscal theory affirms that the price level is determined by the ratio of nominal debt to the present value of real primary surpluses. To the purpose of simplification, one might assume that variables of real allocative relevance are given and only nominal values are to be determined. Equilibrium requires Walras Law, that is,

$$\frac{w_0}{p_0} = \sum_{t \geq 0} \left(\frac{1}{1+\rho} \right)^t g_t,$$

where w_0 is the given amount of nominal public debt at the initial date $t = 0$, p_0 is the price level, g_t is the primary public surplus at date t and ρ is the real rate of interest. If primary public surpluses adjust so as to satisfy Walras Law at any given price level, the latter is an identity or, equivalently, corresponds to the intertemporal public budget constraint. On the contrary, when primary public surpluses are preset, as it is assumed by the fiscal theory, Walras Law is to be regarded as an equation and the price level must serve to solve it. In this case, the government is not subject to any intertemporal budget constraint or, in the terminology borne out by the fiscal theory, fiscal policy is not Ricardian.

Suppose that one allows for a stock to be traded on the asset market, with price q_t and real dividend d_t at date t. At equilibrium, Walras Law now takes the form

$$\frac{w_0 + q_0}{p_0} = \sum_{t \geq 0} \left(\frac{1}{1+\rho} \right)^t (g_t + d_t) .$$

[1]References on the fiscal theory include, among others, Woodford [9, 10] and Cochrane [3, 4].

If one separates fundamental, $f_0 = p_0 \sum_{t \geq 0} (1+\rho)^{-t} d_t$, and bubble, b_0, component in the initial market value of the stock, q_0, Walras Law becomes

$$\frac{w_0 + b_0}{p_0} = \sum_{t \geq 0} \left(\frac{1}{1+\rho} \right)^t g_t.$$

Consistently, when primary public surpluses are preset, the price level is indeterminate, up to a lower bound, which takes into account the positivity of the bubble. A positive speculative bubble can be sustained at equilibrium since no wealth effect operates: initial 'outside' nominal claims held by the private sector, $w_0 + b_0$, are balanced by the nominal value of due tax payments, $\sum_{t \geq 0} p_0 (1+\rho)^{-t} g_t$.

One also verifies that sequential (or flow) public budget constraint is satisfied at every date. For a given constant nominal rate of interest, r, bubble and prices grow at a constant rate, $b_{t+1} = (1+r) b_t$ and $(1+\rho) p_{t+1} = (1+r) p_t$. One then defines

$$w_s = p_s \sum_{t \geq s} \left(\frac{1}{1+\rho} \right)^{t-s} g_t - b_s,$$

which implies the sequential (or flow) budget constraint of the government,

$$w_t = p_t g_t + \left(\frac{1}{1+r} \right) w_{t+1}.$$

However, when the speculative bubble is positive, public debt declines and, for large t, becomes negative.

We present our conclusion formally in a general sequential markets monetary economy with uncertainty. We modify the monetary economy described by Bloise, Drèze and Polemarchakis [2] so as to encompass trade in productive real assets.[2] These productive assets consist of firms whose property rights are traded on a stock market. To avoid issues related to the existence of an equilibrium with production over an infinite horizon, we assume that every production set is trivial (that is, it is a singleton). Thus, in fact, we consider a monetary economy with given stocks of, say, productive lands: each land is administrated by a firm whose ownership is sequentially traded by individuals.

2. A Monetary Economy with Production

Time and the resolution of uncertainty are described by an event-tree, a countable set, $\mathcal{S}$, endowed with a (partial) order, $\succeq$. For every date-event, σ, an element of $\mathcal{S}$, t_σ denotes its date. The unique initial date-event is ϕ, with $t_\phi = 0$. Finally, for a given date-event, σ, $\sigma_+ = \{\tau \succ \sigma : t_\tau = t_\sigma + 1\}$ denotes the set of its immediate successors, a finite set. The construction is standard: for details, see, for instance, Santos and Woodford [6].

At every date-event, markets are open for commodities, assets and balances, which are numéraire. The set of all tradable commodities is $\mathcal{S} \times \mathcal{N}$, with $\mathcal{N}$ being a finite set. Consistently, the commodity space coincides with the vector space of all bounded real valued maps on $\mathcal{S} \times \mathcal{N}$ and prices of commodities, p, are positive real valued maps on $\mathcal{S} \times \mathcal{N}$.[3]

[2] Bloise, Drèze and Polemarchakis [2], in turn, can be regarded as an extension of Woodford's [9] cash-in-advance representative-individual economy to many commodities and many individuals.

[3] Here, and in the following, the term 'positive' ('negative') is to be interpreted as 'greater than or equal to zero' ('less than or equal to zero').

There are sequentially complete markets for assets. Assets are priced using state prices, a, strictly positive real valued maps on $\mathcal{S}$ which satisfy, at the initial date-event, $a_\phi = 1$ and, at every date-event,

$$a_\sigma \geq \sum_{\tau \in \sigma_+} a_\tau = \left(\frac{1}{1+r_\sigma}\right) a_\sigma.$$

The above restriction implies positive nominal rates of interest, r, a positive real valued map on $\mathcal{S}$, and is needed to avoid arbitrage opportunities between short term bonds and balances. When nominal rates of interest are pegged, any reference to state prices is always to be understood as a reference to state prices consistent with nominal rates of interest.

Trading in assets is represented by transfers of revenues, v, real valued maps on $\mathcal{S}$. At a date-event, σ, the market value of a portfolio of assets, delivering revenues $(v_\tau : \tau \in \sigma_+)$ at the following date-events, is

$$a_\sigma^{-1} \sum_{\tau \in \sigma_+} a_\tau v_\tau.$$

The composition of portfolio needs no be made explicit, as does not the full specification of tradable nominal assets.

There is a finite set of long term assets, which are sequentially traded on the market subject to no short sale constraints. A long-term asset, j, is represented by its dividends, d^j, a positive real valued map on $\mathcal{S}$. By no arbitrage, its market price, q^j, a positive real valued map on $\mathcal{S}$, satisfies, at every date-event,

$$q_\sigma^j = a_\sigma^{-1} \sum_{\tau \in \sigma_+} a_\tau \left(q_\tau^j + d_\tau^j\right).$$

It follows that

$$q_\sigma^j \geq a_\sigma^{-1} \sum_{\tau \succ \sigma} a_\tau d_\tau^j,$$

which allows for speculative bubbles (see Santos and Woodford [6]).

There is a finite set of individuals. An individual, i, is described by preferences, $\succeq^i$, over the consumption space, the positive cone of the commodity space; an endowment of commodities, e^i, an element of the consumption space; initial nominal claims, δ^i, and initial shares into long term assets, $\left(\ldots, \zeta^{ij}, \ldots\right)$.

(U) Preferences. *For every individual, preferences are continuous (in the relative Mackey topology), convex and strictly monotone.*

(E) Endowments. *The endowment of every individual lies in the (norm) interior of the commodity space.*

These two assumptions are common in the analysis of competitive economies over an infinite horizon (*e.g.*, Bewley [1]).

A government conducts monetary, fiscal and debt policies. The supply of balances is a contingent plan, m, a positive real valued map on $\mathcal{S}$. A monetary policy consists of pegging nominal rates of interest, r, and supplying balances so as to accommodate demand.

(M) Monetary Policy. *Nominal rates of interest are bounded.*

A fiscal policy is represented by taxes, g, elements of the commodity space: given prices of commodities, p, at every date-event, the aggregate nominal tax on individuals is $p_\sigma \cdot g_\sigma$. Taxes across individuals, $(\dots, g^i, \dots)$, are subject to consistency, $g = \sum_i g^i$.

(F) Fiscal Policy. *For every individual, g^i belongs to the positive cone of the commodity space and, in the aggregate, g belongs to the (norm) interior of the positive cone of the commodity space. In addition, given nominal rates of interest, r, for every individual,*

$$\left(\frac{1}{1+r}\right) e^i - g^i$$

is in the (norm) interior of the consumption space.[4]

The second part of assumption (F) will be used to guarantee solvency of every individual. Its first part is unnecessarily restrictive and is maintained in order to facilitate presentation.

The government issues public debt. A debt policy is described by Θ, a strictly positive real valued map on $\mathcal{S}$. At every date-event, the portfolio of the government consists only of assets with returns $(\Theta_\tau : \tau \in \sigma_+)$, that is, of an element of $\langle(\Theta_\tau : \tau \in \sigma_+)\rangle$, the span of $(\Theta_\tau : \tau \in \sigma_+)$. For given monetary and fiscal policies, the actual evolution of public liabilities is determined by a sequential budget constraint.

Given prices of commodities, p, nominal rates of interest, r, the demand for balances, m, and a fiscal policy, g, debt policy determines the evolution of public liabilities using

$$\left(\frac{r_\sigma}{1+r_\sigma}\right) m_\sigma + a_\sigma^{-1} \sum_{\tau \in \sigma_+} a_\tau w_\tau = w_\sigma - p_\sigma \cdot g_\sigma,$$

jointly with

$$(w_\tau : \tau \in \sigma_+) \in \langle(\Theta_\tau : \tau \in \sigma_+)\rangle,$$

starting from an initial value for liability, $w_\phi = \delta$. At every date-event, one might interpret $(w_\tau - m_\sigma : \tau \in \sigma_+)$ as positions on elementary short-term Arrow securities taken by the government.

The initial liability of the public authority, δ, corresponds to initial claims of individuals, $(\dots, \delta^i, \dots)$, and firms, $(\dots, \delta^j, \dots)$, that is, $\delta = \sum_i \delta^i + \sum_j \delta^j$.

(L) Initial Public Liability. *The initial public liability, δ, is strictly positive.*

It is worth noticing that, in the spirit of the fiscal theory of price determination, no solvency constraints are imposed on public policies.

[4]For a real-valued map, x, on $\mathcal{S}$ and a real valued map, z, on $\mathcal{S} \times \mathcal{N}$, $xz = zx$ is the real valued map on $\mathcal{S} \times \mathcal{N}$ obtained by point-wise product. Moreover,

$$\left(\frac{1}{1+r}\right) = \left(\dots, \left(\frac{1}{1+r_\sigma}\right), \dots\right)$$

and

$$\left(\frac{r}{1+r}\right) = \left(\dots, \left(\frac{r_\sigma}{1+r_\sigma}\right), \dots\right)$$

are used for notational convenience.

The constraints that an individual faces, at every date-event, are a budget constraint,

$$\left(\frac{r_\sigma}{1+r_\sigma}\right) m^i_\sigma + a^{-1}_\sigma \sum_{\tau\in\sigma_+} a_\tau w^i_\tau + p_\sigma \cdot \left(x^i_\sigma - e^i_\sigma\right) + p_\sigma \cdot g^i_\sigma \leq w^i_\sigma,$$

a liquidity constraint,[5]

$$p_\sigma \cdot \left(x^i_\sigma - e^i_\sigma\right)^- - m^i_\sigma \leq 0,$$

and a solvency constraint,[6]

$$a^{-1}_\sigma \sum_{\tau\succeq\sigma} a_\tau \left(p_\tau \cdot g^i_\tau - \left(\frac{1}{1+r_\tau}\right) p_\tau \cdot e^i_\tau\right) \leq w^i_\sigma,$$

where initial nominal claims, w^i_ϕ, are determined according to

$$w^i_\phi = \delta^i + \sum_j \zeta^{ij}\left(q^j_\phi + d^j_\phi\right).$$

Individuals choose a consumption plan, x^i, balances, m^i, and asset holdings, w^i, so as to maximize their preferences subject to sequential constraints.

The sequence of budget constraints reduces to a unique intertemporal budget constraint,

$$\sum_{\sigma\in\mathcal{S}}\left(\frac{r_\sigma}{1+r_\sigma}\right) a_\sigma m_\sigma + \sum_{\sigma\in\mathcal{S}} a_\sigma p_\sigma \cdot \left(x^i_\sigma - e^i_\sigma\right) \leq w^i_\phi - \sum_{\sigma\in\mathcal{S}} a_\sigma p_\sigma \cdot g^i_\sigma,$$

which is well-defined since present value prices of commodities, ap, form a positive summable map on $\mathcal{S}\times\mathcal{N}$, for, otherwise, Ponzi schemes would be possible.

There is a finite set of firms. Every firm, j, has a unique production plan, y^j, a real valued map on $\mathcal{S}\times\mathcal{N}$.

(P) Production. *For every firm, the production plan belongs to the (norm) interior of the positive cone of the commodity space.*

A firm is a long-term institution whose ownership is sequentially traded on assets markets under no short sale constraints. Individuals are initially endowed with shares, $\left(\ldots,\zeta^{ij},\ldots\right) \geq 0$, such that $\sum_i \zeta^{ij} = 1$. A firm pays off dividends, d^j, a positive real valued map on $\mathcal{S}$, to share-holders.

At every date-event, a firm is subject to a budget constraint,

$$d_\sigma + \left(\frac{r_\sigma}{1+r_\sigma}\right) m^j_\sigma + a^{-1}_\sigma \sum_{\tau\in\sigma_+} a_\tau w^j_\tau \leq w^j_\sigma + p_\sigma \cdot y^j_\sigma,$$

a liquidity constraint,

$$p_\sigma \cdot y^j_\sigma - m^j_\sigma \leq 0,$$

and a solvency constraint,

$$-a^{-1}_\sigma \sum_{\tau\succeq\sigma}\left(\frac{1}{1+r_\tau}\right) a_\tau p_\tau \cdot y^j_\tau \leq w^j_\sigma.$$

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[5]For a real valued map, x, on $\mathcal{S}\times\mathcal{N}$, x^+ (x^-) denotes its positive (negative) part, so that $x = x^+ - x^-$.

[6]Solvency constraints on individuals, and on firms, are in the spirit of those considered by Woodford and Santos [6]: an individual, or a firm, can incur any amount of nominal debt which can be repayed in finite time.

The initial nominal claim, $w_\phi^j = \delta^j$, is taken as given.

For a given dividend policy, d^j, a firm is an infinitely lived asset priced by no-arbitrage conditions. Its price can be decomposed into a fundament value and a speculative bubble,

$$q_\sigma^j = f_\sigma^j + b_\sigma^j,$$

where

$$f_\sigma^j = a_\sigma^{-1} \sum_{\tau \succ \sigma} a_\tau d_\tau^j = a_\sigma^{-1} \sum_{\tau \succeq \sigma} a_\tau d_\tau^j - d_\sigma^j$$

and

$$b_\sigma^j = a_\sigma^{-1} \sum_{\tau \in \sigma_+} a_\tau b_\tau^j,$$

so that there are no arbitrage opportunities. The bubble component must be positive at all date-events, which reflects a primitive hypothesis of free-disposal.

The firm chooses its dividend policy, d^j, so as to maximize its market value *cum dividendo* subject to sequential constraints. This is equivalent to say that it distributes all the proceedings from the selling of production, plus the initial nominal claim, as dividends to share-holders. Dividends are in practice chosen, subject to positivity, so as to satisfy a unique solvency constraint,

$$\sum_{\sigma \succeq \phi} a_\sigma d_\sigma^j = \delta^j + \sum_{\sigma \succeq \phi} \left(\frac{1}{1 + r_\sigma} \right) a_\sigma p_\sigma \cdot y_\sigma^j \geq 0.$$

Nominal rates of interest reflect the effect of an operative cash-in-advance constraint since, at every date-event, production results in balances which can only be payed off as dividends at the following date-events.

Given monetary, fiscal and debt policies, an equilibrium consists of plans for consumption, balances, asset holdings and dividends, prices of commodities, state prices and prices of long term assets such that: (a) plans of individuals are optimal subject to sequential constraints, given initial nominal claims and market values of long term assets; (b) dividends of firms are set so as to maximize their market values *cum dividendo* subject to sequential constraints, given initial nominal claims; (d) market clearing is achieved for commodities,

$$\sum_i \left(x^i - e^i \right) = \sum_j y^j,$$

and assets,

$$\sum_i w^i + \sum_j w^j = w + \sum_j \left(q^j + d^j \right),$$

where public liabilities, w, are determined by sequential public budget constraints evaluated at balances demanded by individuals and firms,

$$m = \sum_i m^i + \sum_j m^j.$$

By the very definition, an equilibrium does not call for a Ricardian fiscal authority, according to the terminology borne out by the fiscal theory of price determination.

3. Equilibrium

For given monetary, fiscal and debt policies, an equilibrium exists under our assumptions. There is, however, an unavoidable multiplicity of equilibria if one takes into account the occurrence of speculative bubbles on long term assets. Apart from a distributive effect of initial claims across individuals, all degrees of indeterminacy are purely nominal and of no allocative relevance.

Existence and Indeterminacy. *(a) For given monetary, fiscal and debt policies, a multiplicity of equilibria exists, corresponding to arbitrarily chosen positive initial values of speculative bubbles, provided that there are no initial nominal debts of individuals and firms.*[7] *(b) For given monetary, fiscal and debt policies, every equilibrium with strictly positive speculative bubbles exhibits infinitely many degrees of nominal multiplicity, corresponding to consistent choices of state prices.*

In general, the existence of an equilibrium without speculative bubbles does not obtain when some individual, or some firm, holds an initial nominal debt. This is a matter of consistency: fiscal policy might require a low overall price level at which some individual, or some firm, might not be able to honor the initial debt. However, it can be shown that equilibria with speculative bubbles exist even when there are initial debt positions, since the overall price level is freed and can serve to avoid bankruptcies.

Assumptions (F) and (P) can be substantially weakened. It is clear that we exploit the strict positivity of taxes and production plans to sustain an equilibrium with a positive value of money. Dropping the hypothesis of strict positivity would require additional conditions for existence: as originally emphasized by Dubey and Geanakoplos [5], and further investigated in Bloise, Drèze and Polemarchakis [2], an equilibrium would exist provided that gains to trade offset the transaction costs related to nominal rates of interest.[8]

4. Conclusion

Under control of nominal rates of interest, exogenously set fiscal plans do not uniquely determine the overall price level in a monetary economy, as claimed by the fiscal theory of price determination. One degree of multiplicity remains, with allocative relevance through a distributive effect. Non-Ricardian fiscal plans only reduce the extent of the indeterminacy occurring under Ricardian fiscal plans (Sargent and Wallace [7, 8]).

Similarly, debt policy does not eliminate the indeterminacy of the variablity of inflation rates occurring under Ricardian fiscal plans, which is again only reduced. These are degrees of purely nominal multiplicity under sequentially complete markets, which would correspond to degrees of real multiplicity were markets sequentially incomplete and the asset structure partially nominal.

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[7] That is, $(\dots, \delta^i \dots) \geq 0$ and $(\dots, \delta^j \dots) \geq 0$, with $\sum_i \delta^i + \sum_j \delta^j = \delta > 0$.

[8] Since there is an initial public debt, if there were no taxes, then the government would have to rely on seignorage in order to honor its debt. For strictly positive nominal rates of interest, seignorage would be positive in our economy, since at least firms would be willing to hold positive amounts of balances. This seems, however, an artifact of our simple production sets and, in general, nominal rates of interest would have to be explicitly taken into account for the profitability of production plans, as well as of net trades of individuals, so that the demand for balances would not be necessarily positive at arbitrary nominal rates of interest.

Our result is due to the presence of long term assets. If one were allowed to restrict attention to equilibria without speculative bubbles on such assets, then a full determinacy result would obtain, consistently with the fiscal theory of price determination. However, speculative bubbles cannot be ruled out.

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Proof

Outline. The proof of the proposition is organized as follows. First, we show that, at equilibrium, present value prices of commodities can be determined independently of state prices, for arbitrarily set initial values of speculative bubbles. Second, we show that there is a multiplicity of state prices compatible with a given equilibrium allocation provided that speculative bubbles are positive.

Abstract Equilibrium. We consider the following abstract notion of equilibrium: it consists of present value prices of commodities, π, a summable positive real valued map on $\mathcal{S} \times \mathcal{N}$, an allocation, $\left(\ldots, x^i, \ldots\right)$, positive values for bubbles, $\left(\ldots, \beta^j, \ldots\right)$, and a strictly positive index for the overall price level, λ, such that: (a) market clearing is achieved,

$$\sum_i \left(x^i - e^i\right) = \sum_j y^j;$$

(b) for every individual,

$$z^i \succ^i x^i \text{ implies } \pi \cdot \left(z^i - x^i\right) + \left(\frac{r}{1+r}\right) \pi \cdot \left(\left(z^i - e^i\right)^- - \left(x^i - e^i\right)^-\right) > 0$$

and

$$\pi \cdot \left(x^i - e^i + g^i\right) + \left(\frac{r}{1+r}\right) \pi \cdot \left(x^i - e^i\right)^- = \lambda \alpha^i + \left(\frac{1}{1+r}\right) \pi \cdot y^i,$$

where $y^i = \sum_j \zeta^j y^{ij}$ and $\alpha^i = \delta^i + \sum_j \zeta^{ij} \left(\beta^j + \delta^j\right)$; (c) every firm satisfies its solvency requirement,

$$\lambda \delta^j + \left(\frac{1}{1+r}\right) \pi \cdot y^j \geq 0.$$

To offset the redundancy stemming from the choice of the unit of account, we add the normalization $\|\pi\|_1 = 1$. It is clear that, since no firm holds an initial nominal debt, $\left(\ldots, \delta^j, \ldots\right) \geq 0$, condition (c) is redundant, since it is always satisfied.

We shall prove existence of an abstract equilibrium for given $\left(\ldots, \beta^j, \ldots\right)$, so that λ is endogenous. This creates no problems under the assumption of no initial nominal debts of individuals (that is, $\left(\ldots, \delta^i, \ldots\right) \geq 0$) and the interiority assumption in (F), since nonemptyness of budget constraints of individuals can easily be shown at all positive λ.

Existence: Truncation. Suppose that all vector spaces are finite-dimensional, which corresponds to a truncated economy. We show that an abstract equilibrium exists. (Details are in Bloise, Drèze and Polemarchakis [2].)

Consider the space of all

$$f = \left(\left(\ldots, x^i, \ldots\right), \pi, \lambda\right) \in \cdots \times X^i \times \cdots \times \Pi \times \Lambda = F,$$

where X^i is consumption space of individual i, Π is the space of normalized present value prices and Λ is the set of positive real numbers. For given $\left(\ldots, \beta^j, \ldots\right)$, consider the correspondence $\hat{f} \twoheadrightarrow \bar{f}$ defined by:

- $\bar{x}^i$ is an optimal choice subject to

$$\hat{\pi} \cdot \left(x^i - e^i + g^i\right) + \left(\frac{r}{1+r}\right) \hat{\pi} \cdot \left(x^i - e^i\right)^- \leq \hat{\lambda} \alpha^i + \left(\frac{1}{1+r}\right) \hat{\pi} \cdot y^i;$$

- $\bar{\lambda}$ solves

$$\left(\frac{r}{1+r}\right) \hat{\pi} \cdot \sum_j y^j + \left(\frac{r}{1+r}\right) \hat{\pi} \cdot \sum_i \left(\hat{x}^i - e^i\right)^- + \hat{\pi} \cdot g = \lambda \sum_i \alpha^i;$$

- $\bar{\pi}$ maximizes

$$\pi \cdot \left(\sum_i \left(\hat{x}^i - e^i\right) - \sum_j y^j\right).$$

A fixed point exists and it can be shown to be an abstract equilibrium of the truncated economy. Moreover, it is clear that, since $\pi \cdot g > 0$, $\lambda > 0$.

Existence: Limit. For given values of speculative bubbles, $\left(\ldots, \beta^j, \ldots\right)$, consider a sequence of abstract equilibria of the t-truncated economies. One can show, using arguments similar to those of Bewley [1], that the limit is an abstract equilibrium of the original economy. (For details, see Bloise, Drèze and Polemarchakis [2].) It is clear that, in the limit, since $\pi \cdot g > 0$, $\lambda > 0$.

Indeterminacy. For preset values of speculative bubbles, $\left(\ldots, \beta^j, \ldots\right)$, there is an abstract equilibrium and, possibly rescaling present value prices, one can assume that $\lambda = 1$. State prices allow one to transform this abstract equilibrium into an equilibrium of the monetary economy. To simplify, we consider the case of a common value of speculative bubbles across long term assets, that is, $\left(\ldots, \beta^j, \ldots\right) = \left(\ldots, J^{-1}\beta \ldots\right)$, where J is the number of firms.

Arbitrarily set state prices, a, consistent with nominal rates on interest, r, determine prices of commodities through

$$\left(\ldots, a_\sigma p_\sigma, \ldots\right) = \left(\ldots, \pi_\sigma, \ldots\right).$$

Given such state prices and the corresponding prices of commodities, at every date-event, σ, for an individual, i, optimal holdings of balances and assets are, respectively, $m_\sigma^i = p_\sigma \cdot \left(x_\sigma^i - e_\sigma^i\right)^-$ and

$$a_\sigma w_\sigma^i = \sum_{\tau \succeq \sigma} \left(\frac{r_\tau}{1+r_\tau}\right) a_\tau m_\tau^i + \sum_{\tau \succeq \sigma} a_\tau p_\tau \cdot \left(x_\tau^i - e_\tau^i\right) + \sum_{\tau \succeq \sigma} a_\tau p_\tau \cdot g_\tau^i.$$

For a firm, j, they are, respectively, $m_\sigma^j = p_\sigma \cdot y_\sigma^j$ and

$$a_\sigma w_\sigma^j = a_\sigma^j f_\sigma^j + a_\sigma d_\sigma^j + \sum_{\tau \succeq \sigma} \left(\frac{r_\tau}{1+r_\tau}\right) a_\tau m_\tau^j - \sum_{\tau \succeq \sigma} a_\tau p_\tau \cdot y_\tau^j.$$

One then makes explicit equilibrium restrictions as follows.

(a) Given state prices, a, consistent with nominal rates of interest, r, using the above demands for assets and balances by individuals and firms, market clearing on the asset market, at every date-event, σ, takes the form

$$a_\sigma \left(w_\sigma + b_\sigma\right) = a_\sigma \left(\sum_i w_\sigma^i + \sum_j w_\sigma^j - \sum_j f_\sigma^j - \sum_j d_\sigma^j\right) = u_\sigma,$$

where

$$\begin{aligned} u_\sigma \quad = \quad & \sum_{\tau \succeq \sigma} \left(\frac{r_\tau}{1+r_\tau}\right) \pi_\tau \cdot \sum_j y_\tau^j \\ & + \sum_{\tau \succeq \sigma} \left(\frac{r_\tau}{1+r_\tau}\right) \pi_\tau \cdot \sum_i \left(x_\tau^i - e_\tau^i\right)^- + \sum_{\tau \succeq \sigma} \pi_\tau \cdot g_\tau > 0, \end{aligned}$$

from which it follows that

$$\sum_{\tau \in \sigma_+} u_\tau = u_\sigma - \left(\frac{r_\sigma}{1+r_\sigma}\right) a_\sigma m_\sigma - a_\sigma p_\sigma \cdot g_\sigma.$$

(b) Given state prices, a, consistent with nominal rates of interest, r, public liabilities, w, are subject, at every date-event, σ, to the sequential public budget constraint,

$$w_\tau = \left(\frac{\Theta_\tau}{\sum_{\tau \in \sigma_+} a_\tau \Theta_\tau}\right) \left(a_\sigma w_\sigma - a_\sigma p_\sigma \cdot g_\sigma - \left(\frac{r_\sigma}{1+r_\sigma}\right) a_\sigma m_\sigma\right).$$

(c) Given state prices, a, consistent with nominal rates of interest, r, values of speculative bubbles, b, are subject to positivity and, at every date-event, σ, satisfy the no arbitrage requirement,

$$\sum_{\tau \in \sigma_+} a_\tau b_\tau = a_\sigma b_\sigma.$$

One shows that there are state prices, a, consistent with nominal rates of interest, r, public liabilities, w, and positive values of speculative bubbles, b, which satisfy restrictions (a)-(c) with initial conditions $w_\phi = \delta$ and $b_\phi = \beta$. Moreover, if $\beta > 0$, there are infinitely many degrees of multiplicity in choosing such variables.

The proof is by induction. At the initial date-event, $a_\phi = 1$ and $\delta + \beta = u_\phi$. Suppose that state prices, public liabilities and values of speculative bubbles have been consistently determined up to a date-event, σ, under restrictions (a)-(c). If one chooses strictly positive $(a_\tau : \tau \in \sigma_+)$, subject to $(1 + r_\sigma) \sum_{\tau \in \sigma_+} a_\tau = a_\sigma$, and uses public sequential budget constraint, at all immediate successors, τ, market clearing is satisfied if

$$\begin{aligned} a_\tau b_\tau &= u_\tau - \left(\frac{a_\tau \Theta_\tau}{\sum_{\xi\in\sigma_+} a_\xi \Theta_\xi}\right)\left(a_\sigma w_\sigma - a_\sigma p_\sigma \cdot g_\sigma - \left(\frac{r_\sigma}{1+r_\sigma}\right) a_\sigma m_\sigma\right) \\ &= u_\tau - \left(\frac{a_\tau \Theta_\tau}{\sum_{\xi\in\sigma_+} a_\xi \Theta_\xi}\right)\left(\sum_{\xi\in\sigma_+} u_\xi - a_\sigma b_\sigma\right), \end{aligned}$$

where the last equality uses the fact that market clearing is satisfied at σ. In addition, if $(b_\tau : \tau \in \sigma_+)$ are determined by the above conditions, then they satisfy the no arbitrage requirement (c). The only remaining constraint is the positivity of bubble values. At the unique state prices, $(a_\tau : \tau \in \sigma_+)$, set so as to satisfy $(1+r_\sigma)\sum_{\tau\in\sigma_+} a_\tau = a_\sigma$ and

$$\frac{u_\tau}{\sum_{\xi\in\sigma_+} u_\xi} = \frac{a_\tau \Theta_\tau}{\sum_{\xi\in\sigma_+} a_\xi \Theta_\xi},$$

the bubble is determined by

$$b_\tau = \left(\frac{a_\sigma \Theta_\tau}{\sum_{\xi\in\sigma_+} a_\xi \Theta_\xi}\right) b_\sigma.$$

Therefore, if $b_\sigma > 0$ ($b_\sigma = 0$), $b_\tau > 0$ ($b_\tau = 0$). When the bubble is strictly positive at σ, perturbing state prices, around these set values, will still guarantee strict positivity of the bubble at every immediately following date-event, τ. If, at the initial date-event, $b_\phi = \beta > 0$, there are in fact infinitely many choices of state prices which satisfy equilibrium restrictions (a)-(c).

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