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Pervasiveness of Sunspot Equilibria

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Pervasiveness of Sunspot Equilibria*

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Abstract

In this paper we consider a monetary pure exchange infinite horizon economy in which a given share of consumption purchases must be paid cash in the hand of the representative consumer. We establish a result that challenges the viewpoint according to which indeterminacy and sunspot equilibria would rest upon empirically implausible features, as strong increasing returns to scale or ad hoc and unconventional fundamentals specifications. In our model, in fact, such phenomena are more and more likely to occur as soon as the amplitude of the liquidity constraint is set smaller, and finally, when it becomes low enough, they prevail for whatever fundamentals specification.

We also perform a complete bifurcation analysis of the change of stability occurring when the amplitude of the liquidity constraint is relaxed continuously.

Keywords: cash-in-advance, indeterminacy, sunspot, flip bifurcation.

JEL Classification: C62, E32, E41

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1 Introduction

It is well known that the main criticism addressed towards indeterminate models, rests mainly on the weak support such models receive from empirical studies estimating the degree of market imperfection exploited, as for example the amount of returns to scale. Since the early contributions on the subject by Benhabib and Farmer (1994) and Farmer and Guo (1994), the amount of increasing returns required have been judged to be too high to fall within standard errors of the empirical estimates of Basu and Ferland (1997) in which returns to scale seems to be essentially constant. More recent theoretical works have proposed alternative models in which indeterminacy can be obtained at more plausible degrees of increasing returns (see, e.g., Wen's 1998 one-sector model with variable capacity utilization), even though they remain strictly larger than unity. To drive returns arbitrarily close to one, one must shift from one-sector models to multi-sector models (e.g. Benhabib and Farmer, 1996; Benhabib and Nishimura, 1998; Venditti, 1998), at the costs of being unable to replicate stylized facts of the US business cycles, at least when solely driven by beliefs shocks.

Another class of indeterminate models exploits the frictions in the markets due to the introduction of money as a medium of exchange. Yet, their success appears to rely upon ad hoc if not even unconventional fundamental specifications. Subject to such a criticism is, for example, the Barinci and Chèron (2001) cash-in-advance economy, in which indeterminacy requires very strong income effects, the Farmer (1997) MIUF model with non-standard utility function and Benhabib and Farmer (2000) environment with money is included as an argument in the production function. Also Grandmont *et al.* (1998) model with heterogenous agents can be questioned on the basis that indeterminacy depends dramatically upon technologies exhibiting very low factors substitutability, ruling therefore out the widespread Cobb-Douglas case.

In this paper we challenge the viewpoint according to which indeterminacy would rest upon degrees of market imperfection too high to fit empirical evidence and prove that, on the contrary, it nature may be very pervasive. We do it by considering a simple monetary pure exchange infinite horizon economy *à la* Lucas and Stokey (1987), with the difference that only a given share of consumption purchases must be paid cash in the hand of the representative consumer. We find the surprising result that indeterminacy and sunspot equilibria are more and more likely to occur as soon as the market imperfection - in the present context the amount of consumption good to be paid cash - is decreased, and not as intuition would suggest, increased. What is even more surprising, is the finding that when the amplitude of the liquidity constraint is small enough, such phenomena arise in correspondence to whatever fundamental specification.

Bosi and Magris (2002) and Bosi, Dufourt and Magris (2002) find analogous results in an economy with capital accumulation. However, the three-dimensional system describing the law of motion of the economy they obtain, make it more difficult to provide some reliable economic intuition of the main driving force leading to an apparently counter-intuitive result. On the contrary, equilibrium in the present model is described by a relatively simple one-dimensional first order equation, which allows an easier interpretation for the conditions for indeterminacy, without altering in any relevant way the mechanisms at work (see Section 5). At the same time, this permits us, by assuming specific functional forms, to perform a complete bifurcation analysis of the change of stability occurring when the amplitude of the liquidity constraint is relaxed continuously.

2 The environment

We consider an infinite horizon economy with many identical infinite lived households whose size is normalized to unity. Preferences of the representative agent are given by the expected discounted sum of instantaneous utilities:

$$\sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right) - v\left(l_t\right) \right]. \tag{1}$$

The variables c and l stand, respectively, for consumption and labor and $\beta \in (0,1)$ for the discount factor. The per-period utility functions u and v possess the standard following features.

Assumption 1. *$u : R_+ \to R$ is smooth, strictly increasing and strictly concave. Moreover $\lim_{c \to 0^+} u'(c) = +\infty$. $v : R_+ \to R$ is smooth, strictly increasing and strictly convex. Moreover $\lim_{l \to l^{*+}} v'(l) = +\infty$, where l^* is the (maybe infinite) endowment of time.*

When maximizing (1), agents must take into account the dynamic budget constraint

$$M_{t+1} + B_{t+1} + p_t c_t \leq M_t + (1 + i_t) B_t + p_t w_t l_t, \tag{2}$$

where M denotes money balances, B nominal bonds, p the price of consumption good, i the nominal interest rate on bonds holding, and w the real unit wage. The initial endowment of bond holding is zero, i.e. $B_0 = 0$. Still, financial markets are assumed to be imperfect: Agents must in fact pay cash at least a share $q \in (0, 1]$ of their consumption purchases, i.e. they are subject to the additional financial constraint

$$qp_t c_t \leq M_t. \tag{3}$$

The program of the representative agent consists thus in maximizing (1) subject to (2) and (3). Denoting λ and μ the Lagrangian multipliers associated to budget constraint and financial constraint, respectively, and assuming that constraint (3) binds, which is true when the nominal interest rate i (roughly, the opportunity cost of holding money) is positive, the FOC's write:

$$-\lambda_{t-1} + \lambda_t + \mu_t = 0, \tag{4}$$

$$-\lambda_{t-1} + \lambda_t (1 + i_t) = 0, \tag{5}$$

$$\beta^t u' (c_t) - \lambda_t p_t - \mu_t q p_t = 0, \tag{6}$$

$$-\beta^t v' (l_t) + \lambda_t p_t w_t = 0. \tag{7}$$

By manipulating conditions (4)-(7) and defining $\pi_t \equiv p_t/p_{t-1}$ the inflation factor between period $t-1$ and period t, we obtain the following expressions which will turn out to be quite useful for the sequel of our analysis:

$$\beta^t u' (c_{t+1}) = (1-q) \beta^t v' (l_{t+1}) + q\beta^{t-1} v' (l_t) \frac{c_t}{c_{t+1}}, \tag{8}$$

$$u'(c_t) = \beta u'(c_{t+1}) \frac{q (1 + i_t) + (1-q)}{q\pi_{t+1} + (1-q) (1 + i_{t+1})^{-1} \pi_{t+1}}, \tag{9}$$

$$u' (c_t) = (1 + q i_t) v' (l_t). \tag{10}$$

For what concerns the production side of the economy, we assume that one unit of labor can be used to produce one unit of output y according to the aggregate constant returns to scale production function

$$y_t = L_t, \tag{11}$$

where L stands for aggregate labor. Finally, we suppose that an exogenously given amount $\bar{M}$ of *fiat* money is available in the economy in each period[1].

[1] Assuming government to peg the money growth at a constant rate would not change the results of our analysis from a qualitative point of view.

3 Intertemporal equilibrium

Equilibrium in factors market implies $L_t = l_t$ for all $t \geq 0$. Money market equilibrium requires $M_t = \bar{M}$ for every $t \geq 0$: It follows that, when constraint (3) binds, $p_t/p_{t-1} = c_{t-1}/c_t$ in each period. Firm's profit maximization problem entails an unitary real wage, i.e. $w_t = 1$ for all $t \geq 0$. Good market equilibrium requires all output to be consumed, i.e., in the light of (11), $c_t = l_t = y_t$. Finally, by Walras law, the real interest rate adjusts in order to ensure equilibrium in bonds market, that means, since agents are all equals, $B_t = 0$ for $t = 1, 2...$

Combining equation (8) with equilibrium conditions, and setting $U(y) \equiv u'(y)\, y$ and $V(y) \equiv v'(y)\, y$, we can fully describe a perfect foresight equilibrium as follows.

Definition 1 *An intertemporal interior equilibrium with perfect foresight is a sequence $\{y_t\}_{t=0}^{\infty}$, $y_t > 0$ for all $t \geq 0$, satisfying the one-dimensional first order difference equation*

$$U(y_{t+1}) - (1-q)\, V(y_{t+1}) = \frac{q}{\beta} V(y_t) \tag{12}$$

and the transversality condition.

Notice that in the special case $q = 1$, the law of motion for the economy boils down to $\beta U(y_{t+1}) = V(y_t)$ which presents very close analogies with that obtained by Lucas and Stokey (1987) in their economy with cash good and credit good[2] and with that derived in Azariadis (1981) and Grandmont (1985) in a standard *OLG* framework.

One immediately verifies that a stationary solution of system (8) is a level of output $\bar{y}$ solving equation

$$u'(y)/v'(y) = 1 - q + q/\beta. \tag{13}$$

By observing, in the light of Assumption 1, that $u'(y)/v'(y)$ is decreasing in y, one deduces that there exists at most one steady state. In particular, the

[2] In the present context, consumption plays the role of cash good (although imperfectly, in view of the partial liquidity constraint), whilst leisure can be interpreted as credit good, since its purchases do not require money balances. Therefore, in spite of the presence of a production function, our model can be completely assimilated to a pure exchange economy with (partial) cash good and credit good.

boundary conditions

$$\lim_{y \to 0+} u'(y)/v'(y) > 1 - q + q/\beta, \tag{14}$$

$$\lim_{y \to +\infty} u'(y)/v'(y) < 1 - q + q/\beta \tag{15}$$

ensure the existence of a unique non-trivial steady state $\bar{y}$. Moreover, from (10) and taking into account (13), one obtains that the stationary nominal interest rate is strictly positive:

$$\bar{\imath} = \frac{1}{q}\left[\frac{u'(\bar{y})}{v'(\bar{y})} - 1\right] = \frac{1}{q}\left(1 - q + \frac{q}{\beta} - 1\right) = \frac{1-\beta}{\beta} > 0.$$

It follows that constraint (3) binds at the steady state, and thus in a small neighborhood of it. Therefore system (12) is consistent, in the sense that it describes equilibria near the steady state. Finally, notice that at the steady state the price level is constant and equal to $\bar{M}/q\bar{y}$.

4 Linearized dynamics: Stability analysis

Recall that output y in each period is a non-predetermined variable. Therefore the steady state of system (12) will be indeterminate if the unique eigenvalue of the Jacobian matrix J belongs to the interior of the unit circle, i.e. will belong to the interval $(-1, 1)$. Straightforward computations yield the following expression for J :

$$J = \left.\frac{dy_{t+1}}{dy_t}\right|_{\bar{y}} = \frac{1+\varepsilon}{1 - \frac{1}{\sigma} - \beta\frac{1-q}{q}\left(\varepsilon + \frac{1}{\sigma}\right)}, \tag{16}$$

where $\sigma \equiv u'/u''y$ defines the elasticity of intertemporal substitution in consumption and $\varepsilon \equiv v''y/v'$ the inverse of the elasticity of labor supply.

By a direct inspection of (16), one easily verifies that $J = -1$ when

$$q = q^* \equiv \frac{1}{1 + \frac{1}{\beta}\frac{2(1+\varepsilon)-(\varepsilon+1/\sigma)}{\varepsilon+1/\sigma}}, \tag{17}$$

that J belongs to $(-1, 0)$ for $q < q^*$ and $|J| > 1$ otherwise. Moreover, fixing all the parameters (in particular ε) others than σ, it is easy to verify that q^* is decreasing in σ and that

$$q^{**}(\varepsilon) = \lim_{\sigma \to +\infty} q^* = \frac{1}{1 + \frac{1}{\beta}\frac{2+\varepsilon}{\varepsilon}} \in (0, 1). \tag{18}$$

Therefore

$$q < q^{**}(\varepsilon)$$

implies indeterminacy whatever σ is. At the same time, simple computations show that $q^* = 1$ for $\sigma < \sigma^* \equiv 1/(2+\varepsilon) < 1$. This means that under the full cash-in-advance constraint $q = 1$, indeterminacy requires income effect to dominate substitution one, result in line with Azariadis (1981) and Grandmont (1985). Conversely, as soon as q is progressively relaxed, the range of σ inducing indeterminacy improves, including eventually, for $q < q^{**}(\varepsilon)$, its whole domain of definition. Finally, since when q goes through q^* one has $J = -1$, a flip bifurcation occurs. Accordingly, a stable or unstable two-period cycle will appear for all $q's$ belonging to a sufficiently small neighborhood of q^*. Nevertheless, in order to detect the stability of the cycle, one must specify fundamentals, namely the functions u and v, exercise that we actually carry out in Section 5 of the paper.

The above considerations prove the following proposition.

Proposition 2 *The steady state of system (12) is stable, thus locally indeterminate, when* $0 < q < q^*$, *otherwise it is unstable, thus locally determinate. In addition, when* q *goes through* q^*, *it undergoes a flip bifurcation. Moreover,* q^* *is decreasing in* σ : *In particular, for a given* ε, *when* $q < q^{**}(\varepsilon)$, *the steady state is locally indeterminate whatever* σ *is.*

5 Indeterminacy: An interpretation

Proposition 2 claims that indeterminacy becomes more and more likely to occur as soon as the amplitude of the liquidity constraint, q, decreases. This result could appear counter-intuitive - if not even troublesome - if compared to those found in most of the literature focusing on indeterminacy, in which the likelihood of the latter seems very often to be positively correlated - and not negatively as in our case - with the magnitude of the market imperfection accounted for, be it due to the presence of external effects in production or to possible frictions in the credit market induced by the introduction of money. Nevertheless, if we analyze with care the arbitrage equations (9) and (10), we can show that this is not the case, and that the mechanisms leading to indeterminacy is rather simply interpretable.

In order to better illustrate this, let us focus, for sake of simplicity but without any relevant loss in generality, on an utility logarithmic in consumption and linear in labor with unitary coefficient. Under these specifications, equations (9) and (10) can be written, respectively,

$$p_{t+1}c_{t+1}/p_t c_t = \beta \frac{q\,(1+i_t) + (1-q)}{q + (1-q)\,(1+i_{t+1})^{-1}} \tag{19}$$

and

$$1 = (1 + q i_t)\, c_t. \tag{20}$$

To carry out our exercise, let us suppose that the system is initially at the steady state and let us analyze under which conditions it is possible to construct an alternative equilibrium in which agents anticipate, say, a fall in next period price level. Under this conjecture, if the liquidity constraint is still binding, it is easy to see that tomorrow consumption will be driven up meanwhile, according to the static equation (20), tomorrow nominal interest rate i_{t+1} will fall down. Now, since $p_{t+1}c_{t+1} = p_t c_t = M/q$, the Euler equation (19) in order to be still satisfied requires its right-hand side to remain constant. Suppose first that q is rather close to one; In such a case, for a given diminution of i_{t+1}, the re-establishment of (19) will require only a slight increase in today nominal interest rate i_t, so, according again to (20), only a slight decline of c_t below its steady state level. But this will induce an explosive dynamics and violate the transversality condition. Consider now the case in which q is conversely small: Under this hypothesis, in order to compensate the fall of i_{t+1}, i_t must now know a sharp increase, its weight q being relatively small to that of i_{t+1}, $1-q$. Therefore, again according to (20), today consumption will sharply contract with respect to its initial steady state level and such a contraction will be actually greater than the corresponding expansion of c_{t+1}. Therefore, the system will move back towards its steady state, although following an oscillatory path.

6 Non-linear dynamics in the CES economy

In this section we specify the form of the functions u and v, in order to improve our qualitative analysis and in particular characterize the direction of the flip bifurcation occurring when q goes through q^*. To this end, consider the isoelastic utility functions:

$$u\,(c) = \frac{c^{1-1/\sigma}}{1-1/\sigma} \tag{21}$$

and

$$v(l) = \frac{l^{1+\varepsilon}}{1+\varepsilon} \tag{22}$$

with $\sigma > 0$ and $\varepsilon \geq 0$. Under these specifications, it is possible to explicitly derive the backward dynamics of system (12):

$$y_t = \left\{ \frac{\beta}{q} \left[y_{t+1}^{1-1/\sigma} - (1-q)\, y_{t+1}^{1+\varepsilon} \right] \right\}^{1/(1+\varepsilon)}. \tag{23}$$

Let us fix the fundamental parameters. According to consensual estimated values when the period is set equal to a year one has $\beta = 0.99$. In addition let us set $\varepsilon = 1$ and $\sigma = 2$, values falling within standard errors of empirical estimates. Under this calibration, we finds that the bifurcation value for the degree of market imperfection defined in (17) is $q^* = 0.37265$.

In Figures 1-3 we have depicted the graph of (23) corresponding to three different values of $q : q_3 > q_2 > q^* > q_1$. It is there possible to appreciate the modifications undergone by the graph as soon as q is relaxed and in particular how the steady state from stable (unstable in forward dynamics) becomes unstable (stable in forward dynamics).

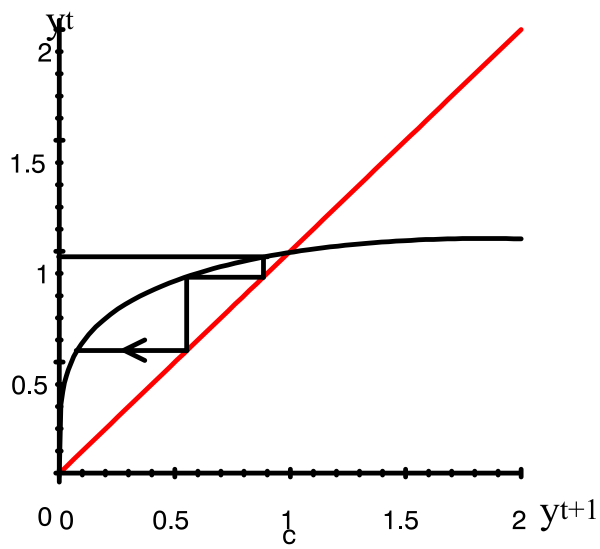

Figure 1: $q = 0.9$

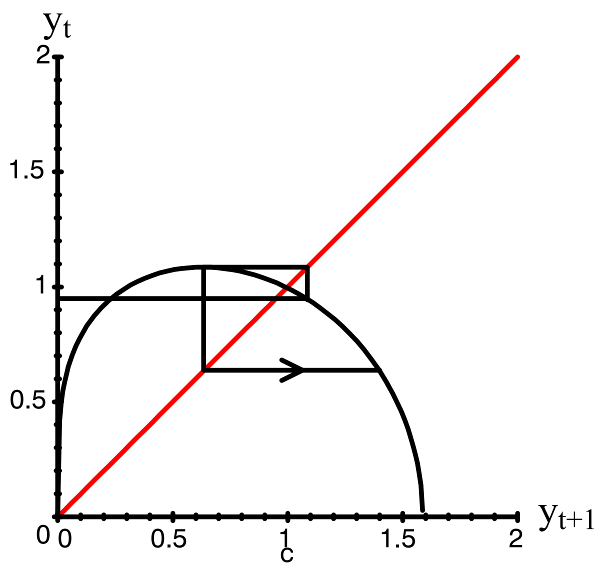

Figure 2: $q = 0.5$

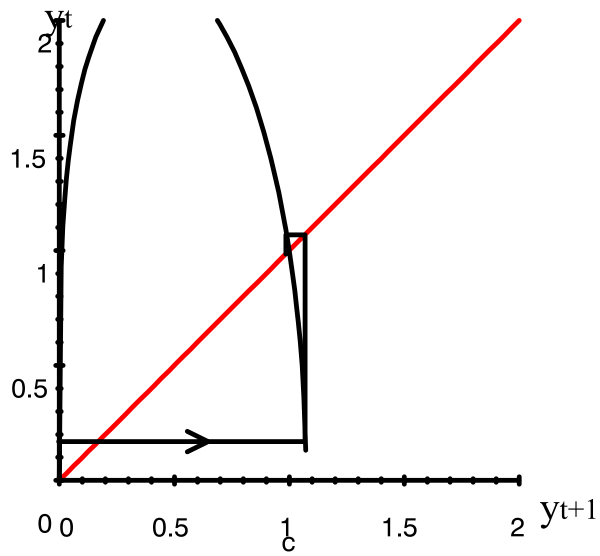

Figure 3: $q_1 = 0.1$

The adopted specification of the functional (21) and (22) allows us to carry out a numerical bifurcation analysis and in particular characterize the direction of the flip bifurcation arising when q goes through q^* and, as a consequence, the stability of the two-period cycle associated to it. This exercise is relevant since, as it is proved e.g. in Grandmont *et al.*(1998), the occurrence of a supercritical flip implies an alternative attractor, namely a the two-period cycle, around which new stochastic sunspot equilibria can be constructed, while a subcritical flip bifurcation individuates an invariant bounded and closed set around the stable steady state in which sunspot fluctuations must remains contained. What we find is summarized in the following proposition.

Proposition 3 *Assume the utility functions to be of the form (21) and (22). Then, the flip bifurcation occurring when q goes through q^* is subcritical, if and only if*

$$3\varepsilon^3\sigma^2\left(4+\varepsilon\right)+4\varepsilon\sigma\left(2+3\varepsilon\right)\left(\sigma-1\right)+\left(8+16\sigma^2-24\sigma\right)>0. \tag{24}$$

and supercritical otherwise. Condition (24) is in particular satisfied for $\sigma > 1$.

Proof. See appendix.

In the light of Proposition 3, we have that for $\sigma < 1$, the bifurcation can be supercritical (for instance if $1/2 < \sigma < 1$ and the labor supply sufficiently elastic, i.e. ε sufficiently low).

7 Conclusion

We have presented a pure exchange infinite horizon model with representative agent and partial liquidity constraint on consumption purchases. We have shown that the scope for indeterminacy increases as soon as the amount of good to be paid cash decreases, and not increases, as one may be reasonably prone to think. Moreover, there exists a threshold for the amplitude of the liquidity constraint below which the steady state is bound to be indeterminate for whatever curvature of the utility of consumption.

The result that indeterminacy does not need any important degree of market imperfection but on the contrary appears for arbitrarily small ones, although does not pretend to be general, nevertheless represents a theoretical example suggesting that the phenomenon of indeterminacy is perhaps much more pervasive than it is commonly though, and can therefore challenge the widespread criticism based on its empirical implausibility.

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8 Appendix

Proof of proposition 3.

The dynamics of the economy are described by system (23). Let us normalize the steady state and the bifurcation value to zero as follows:

$$\begin{aligned} x_t &= \left\{ \frac{\beta}{\alpha + q^*} \left[(x_{t+1} + \bar{y})^{1-1/\sigma} - (1 - \alpha - q^*)\,(x_{t+1} + \bar{y})^{1+\varepsilon} \right] \right\}^{1/(1+\varepsilon)} - \bar{y} \\ &\equiv H\left(x_{t+1}, \alpha\right), \end{aligned}$$

where $x_t \equiv y_t - \bar{y}$ and $\alpha \equiv q - q^*$ and where the function H describes the backward looking dynamics.

In view of the implicit function theorem it is possible to generically define an inverse function G locally describing the dynamics in forward looking, i.e.

$$x_{t+1} \equiv G\left(x_t, \alpha\right) \text{ (locally and generically).}$$

By construction, the new steady state is defined by $x = 0$, while the bifurcation value is now normalized to $\alpha = 0$.

We observe that $0 = x = G(x, \alpha) = G(0, \alpha)$ and that

$$\frac{\partial G}{\partial x_t}(0,0) = -1.$$

Moreover

$$\frac{\partial^2 G}{\partial \alpha \partial x_t}(0,0) < 0,$$

because as α (or q) increases, the steady state becomes unstable. Therefore the family $G(\cdot, \alpha)$ undergoes a flip bifurcation at $\alpha = 0$ (see, for instance, Grandmont, 1988). To characterize the direction of the flip bifurcation, consider the second iterate with respect to x_t:

$$G^2(x_t, \alpha) \equiv G(G(x_t, \alpha), \alpha).$$

Notice that the steady state x remains a steady state for the second iterate.

$$0 = G^2(0, \alpha).$$

The alternative steady state of the second iterate is possibly the two-period cycle of the simple function G. We want to know whether this new steady state is attractive (supercritical) or unstable (subcritical). The necessary and sufficient condition for the flip to be supercritical in forward looking is

$$\frac{\partial^3 G^2}{\partial x_t^3}(0,0) < 0$$

(see again, among the others, Grandmont, 1988). By applying the chain rule, one restates this inequality in terms of G.

$$2\frac{\partial^3 G}{\partial x_t^3}(0,0) + 3\left[\frac{\partial^2 G}{\partial x_t^2}(0,0)\right]^2 > 0.$$

It is possible to analytically show that this condition is perfectly equivalent to that of subcritical flip in backward looking:

$$2\frac{\partial^3 H}{\partial x_{t+1}^3}(0,0) + 3\left[\frac{\partial^2 H}{\partial x_{t+1}^2}(0,0)\right]^2 < 0. \quad (25)$$

So, to characterize the emergence of an attractive two-period cycle in forward looking, we impose condition (25) to the parametric space. Straightforward but very tedious computations show that (25) is equivalent to

$$3\varepsilon^3\sigma^2(4+\varepsilon) + 4\varepsilon\sigma(2+3\varepsilon)(\sigma-1) + \left(8 + 16\sigma^2 - 24\sigma\right) < 0,$$

which becomes the necessary and sufficient condition for the bifurcation to be supercritical. We observe that $3\varepsilon^3\sigma^2(4+\varepsilon)>0$. Moreover $4\varepsilon\sigma(2+3\varepsilon)(\sigma-1)>0$ if and only if $\sigma>1$, and finally $8+16\sigma^2-24\sigma<0$ if and only if $1/2<\sigma<1$. Hence, if $\sigma\geq 1$, the flip bifurcation is always subcritical. If $1/2<\sigma<1$ there exists always a positive and sufficiently low ε such that the flip is supercritical. ■

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