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Liquidity Constraints, Heterogeneous Households and Sunspots Fluctuations

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Abstract

This paper is concerned with the empirical relevance of indeterminacy and sunspots in explaining the business cycle. It argues that limited borrowing opportunities provide a propagation mechanism able to generate business cycle facts observed in data in response to sunspot shocks. This point is demonstrated using an equilibrium business cycle model featuring heterogeneous households, endogenous labor supply and liquidity constraints. We first show that, due to a complementarity between individual labor supplies, the model exhibits indeterminacy for roughly constant returns to scale. We then establish that our model accounts for stylized facts that neither the standard RBC model nor previous sunspots models have been able to capture. More specifically, the model driven by sunspots alone matches the procyclical movements in aggregate consumption, and the positively correlated forecastable changes of basic macroeconomic variables.

Keywords: Heterogeneity, liquidity constraints, sunspots, increasing returns.
JEL code numbers: E32

1 Introduction

This paper is concerned with the empirical relevance of indeterminacy and sunspots in explaining the business cycle. It argues that limited borrowing opportunities provide a propagation mechanism able to transform sunspots shocks into observed business cycle fluctuations. This point is demonstrated in a framework with heterogeneous individuals, endogenous labor supply and liquidity constraints.

Recent years have witnessed the development of *endogenous business cycle* (EBC) models in which fluctuations are triggered by sunspots, *i.e.*, self-fulfilling changes in beliefs.[1] Earlier empirical assessments have underlined the ability of EBC models of replicating key business cycle facts, thereby confirming the relevance of sunspots (see, *e.g.*, Farmer and Guo [1994] and Schmitt-Grohé [1997]).[2] However, earlier EBC models have been widely criticized on the grounds that the existence of sunspots fluctuations requires unrealistically high returns to scale, as evidenced by the empirical work of Basu and Fernald [1997]. More recent work has addressed this issue and added features to the "baseline" framework. Prominent examples include Benhabib and Farmer [1996] two-sector model, and Wen [1998] who incorporates variable capital utilization into the one-sector model. In both cases, the level of returns to scale needed for indeterminacy is drastically reduced to one that falls within the empirically plausible range. However, this comes at the cost of weakening the ability of sunspots models of explaining fluctuations along some dimensions. From our viewpoint, among the various failures that could be mentioned, the most significant ones are actually related to the joint dynamics of output and consumption. More precisely, each model strikingly fails to replicate the pattern of U.S. consumption fluctuations along at least one dimension: Wen [1998] actually predicts an almost zero relative volatility of consumption growth with respect to output growth

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[1]See Benhabib and Farmer [1999] for an excellent survey.

[2]The aforementioned sunspots models share as a common feature increasing returns to scale and differ in the behavior of marginal costs and markups. It is worth stressing that in the empirically oriented sunspots cycles literature, indeterminacy almost exclusively arises in the presence of increasing returns to scale.

(see also Benhabib and Wen [2002]), while Schmitt-Grohé [2001] shows that the model by Benhabib and Farmer [1996] generates countercyclical consumption movements.[3] Schmitt-Grohé [2001] has further established that it also fails to explain the positive correlation of forecastable changes in output and consumption highlighted by Rotemberg and Woodford [1996].

The idea advocated in this paper is that the inconsistency of the propagation mechanisms in EBC models stems from the complete financial markets assumption that is hidden in the representative household framework. In the real world, however, there is evidence suggesting that many households have limited borrowing opportunities. Zeldes [1989], for instance, finds microeconomic evidence that liquidity constraints affect a significant part of U.S. households. More direct evidence is reported by Jappelli [1990] and Gimenez, Quadrini and Rios-Rull [1997] who suggest that approximately 25 percent of U.S. households are liquidity constrained. As Aiyagari [1994] points out, the presence of liquidity constraints has significant effects on individuals consumption and saving behavior. As a matter of fact, the behavior of liquidity constrained households is at odds with the predictions of the Permanent Income Theory: their consumption is more sensitive to labor income. Insofar as endogenous labor supply is considered, liquidity constraints can in addition alter the arbitrage between consumption and leisure. So that one may conjecture that the empirical difficulties encountered by representative household sunspots models might like be resolved with the help of suitable liquidity constraints.

Is limited access to financial market improves the propagation mechanisms of expectations-driven business cycle models? This paper aims at addressing such an issue by considering a model with heterogeneous households and liquidity constraints. This model is an extension of the business cycle model by Barinci and Chéron [2001] who build on the framework of Woodford [1986]. We extend that framework to allow for *all* households to

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[3]Note that these shortcomings can be avoided by adding fundamental - preferences, government spending and/or technology - shocks. However, in such a case fluctuations are no longer purely *endogenous* (see, *e.g.*, Benhabib and Wen [2002]).

supply a variable quantity of labor.[4] Our model economy is inhabited by two types of infinitely-lived households both of whom work. There are two sources of heterogeneity: preferences and access to the credit market. Households of the first type are liquidity constrained in which they are assumed unable to borrow as much as they would like. More specifically, though both types of households work, households of the first type are precluded from borrowing against their future labor income. Our main results can be listed as follows. First, the model exhibits indeterminacy for essentially constant returns, *i.e.*, for arbitrarily close to zero external effects. This result can be understood in terms of complementarity between individual labor supplies that enhances the sensitivity of labor with respect to beliefs, hence output, as if (labor) externalities were stronger. Second, the Keynesian-like behavior of constrained households allows the model driven by sunspots alone not only to account for both the relative volatility and the procyclicality of consumption, but also to replicate the positive correlation of forecastable movements of output, consumption and hours.

The remaining of the paper is organized as follows. Section 2 introduces the model economy. Section 3 deals with the local dynamics. Section 4 presents the predictions of the model. Section 5 concludes.

2 The Model

2.1 Firms

The homogeneous consumption/investment good, Y_t, is produced by competitive firms using a technology permitting growth:

$$Y_t = \left[(\bar{K}_t/z_t)^\alpha (\bar{N}_t)^{1-\alpha} \right]^\theta (K_t)^\alpha (z_t N_t)^{1-\alpha}$$

where $\bar{K}_t$ and $\bar{N}_t$ denote for the economy-wide averages of capital and labor in period t; increasing returns are measured by the parameter $\theta \geq 0$. In addition, z_t is a measure of labor-augmenting, exogenous, technical progress

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[4]Barinci and Chéron [2001], following the lines set out in Woodford [1986], assume that the labor supply of some households is inelastic.

in period t. It is assumed to follow the deterministic process:

$$\log z_t = \log z_{t-1} + \log \gamma$$

for $\gamma \geq 1$ the gross growth rate.

Profit maximization leads to the following set of conditions:

$$w_t = (1-\alpha)z_t \left[(\bar{K}_t/z_t)^\alpha (\bar{N}_t)^{1-\alpha} \right]^\theta (K_t)^\alpha (z_t N_t)^{-\alpha} \tag{1}$$

$$r_t = \alpha \left[(\bar{K}_t/z_t)^\alpha (\bar{N}_t)^{1-\alpha} \right]^\theta (K_t)^{\alpha-1} (z_t N_t)^{1-\alpha} \tag{2}$$

2.2 Households

The economy is inhabited by two types of infinitely-lived households differing in preferences. In particular, they are assumed to have different degrees of impatience, *i.e.*, discount rates. Both types of households supply a variable quantity of labor. Beside a real asset - the productive capital - there is a financial asset: fiat money in constant supply. The timing of events is as follows: households enter each period with money and capital stored from the previous period. Capital and labor services are rented; yet, households are paid their wages and capital returns solely at the end of the period. Households can borrow against their end-of-the-period incomes. The crucial feature of the model is that the access to credit is imperfect and *asymmetric*. Specifically, we make the assumption that some households are not allowed to borrow against future *labor* income.[5]

2.2.1 Constrained Households

Constrained households value consumption and leisure according to (the superscript c stands for constrained)

$$E_0 \sum_{t=0}^{\infty} \lambda^t \left[U_c(C_t^c) + V_c(1 - N_t^c) \right] \tag{3}$$

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[5]We do not attempt to derive such a constraint endogenously. However, this kind of constraint might arise as an equilibrium phenomenon in the presence of information asymmetries regarding, for instance, "work effort".

where C_t^c is consumption in period t, and N_t^c is labor supplied in period t; $\lambda \in (0,1)$ is a discount factor. The momentary utility function is assumed to verify usual properties. Households have the possibility of saving their income by holding two assets, money and capital, so their budget constraint writes as follows:

$$C_t^c + K_{t+1}^c + \frac{M_{t+1}^c}{p_t} = w_t N_t^c + [r_t + 1 - \delta]K_t^c + \frac{M_t^c}{p_t} \tag{4}$$

for p_t the price level, w_t the real wage, r_t the rental rate on capital and δ the rate of capital depreciation.

As discussed above, constrained households are unable to borrow against their end-of-the-period labor income. However, capital can be pledged as collateral so as to secure a loan. It follows that constrained households' current expenditures must be financed either out of money or borrowing against the value of capital held at the beginning of the period:

$$C_t^c + K_{t+1}^c \leq [r_t + 1 - \delta]K_t^c + \frac{M_t^c}{p_t} \tag{5}$$

Constrained households maximize (3) subject to (4) and (5). The necessary first-order conditions are:

$$U_c'(C_t^c) \geq \lambda E_t \left[R_{t+1} U_c'(C_{t+1}^c) \right] \tag{6}$$

$$U_c'(C_t^c) w_t \geq V_c'(1 - N_t^c) \tag{7}$$

$$V_c'(1 - N_t^c) \geq \lambda p_t w_t E_t \left[\frac{U_c'(C_{t+1}^c)}{p_{t+1}} \right] \tag{8}$$

where $R_{t+1} \equiv r_{t+1} + 1 - \delta$ denotes the gross rate of return on capital. Relations (6), (7) and (8) hold with equality if $K_t^c > 0$, the borrowing constraint (5) does not bind and $M_t^c > 0$, respectively.

Before turning to the unconstrained households problem, some comments are in order. In the sequel we will focus on equilibria in which the liquidity constraint is *binding* and, in addition, $K_t^c = 0$ (reasons are discussed below). In such circumstances the constrained households' behavior is described by:

$$V_c'(1 - N_t^c) = \lambda p_t w_t E_t \left[\frac{U_c'(C_{t+1}^c)}{p_{t+1}} \right] \tag{9}$$

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$$w_t N_t^c \;=\; \frac{M_{t+1}^c}{p_t} \tag{10}$$

$$C_t^c \;=\; \frac{M_t^c}{p_t} \tag{11}$$

We can use equations (9), (10) and (11) to obtain:

$$N_t^c V_c'(1 - N_t^c) \;=\; \lambda E_t \left[C_{t+1}^c U_c'(C_{t+1}^c) \right] \tag{12}$$

Recall that the model is designed so as to permit growth. More specifically, we assume that fundamentals are compatible with balanced growth in consumption and wages and at the same time no growth in hours worked. Along such a (deterministic) balanced growth path (hereafter BGP), both sides of equation (9) grow at the same rate. Whenever $V_c'(1 - N_t^c)$ is constant, it must be the case that $U_c(C_t^c) = \log(C_t^c)$. Yet, a glance at the r.h.s. of equation (12) reveals that under this specification, labor supply of constrained households would be inelastic. In order to circumvent this problem, without relaxing the suitable separability assumption, one could allow $V_c'(1 - N_t^c)$ to grow at the constant rate γ. But since the time allocated to non-market activities $(1 - N_t^c)$ is constant along a BGP, it must be the case that:

$$U_c(C_t^c) + V_c(1 - N_t^c) = C_t^c + z_t(1 - N_t^c) \tag{13}$$

where z_t is the level of technology in period t.

The non-standard specification (13) for the utility over market consumption and non-market activities, that will be assumed throughout, can actually be rationalized by allowing the constrained households to have *home* activities along the lines of Benhabib, Rogerson and Wright [1991] (see the Appendix for more details). Making use of the *reduced-form* utility function (13), equation (12) boils down to:

$$z_t N_t^c \;=\; \lambda E_t \left[C_{t+1}^c \right] \tag{14}$$

2.2.2 Unconstrained households

The second type of households do not face the same kind of credit limits. This is equivalent of saying that these households are able to borrow against

their end-of-the-period wage payments. Accordingly, they maximize (the superscript u stands for unconstrained):

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t^u) + V_u(1 - N_t^u) \right] \tag{15}$$

subject to the budget constraint:

$$C_t^u + K_{t+1}^u + \frac{M_{t+1}^u}{p_t} = w_t N_t^u + [r_t + (1 - \delta)]K_t^u + \frac{M_t^u}{p_t} \tag{16}$$

The necessary first-order conditions are:

$$\frac{1}{C_t^u} \geq \beta E_t \left[R_{t+1} \frac{1}{C_{t+1}^u} \right] \tag{17}$$

$$\frac{1}{C_t^u} \geq \beta E_t \left[\frac{p_t}{p_{t+1}} \frac{1}{C_{t+1}^u} \right] \tag{18}$$

$$w_t \frac{1}{C_t^u} = V_u'(1 - N_t^u) \tag{19}$$

Relations (17) and (18) hold with equality if $K_t^u > 0$ and $M_t^u > 0$, respectively. In the sequel we shall focus on equilibria in which (see the discussion below):

$$R_{t+1} > \frac{p_t}{p_{t+1}} \tag{20}$$

so that the real gross return on capital dominates that on money. Consequently, unconstrained households are unwilling to hold outside money, *i.e.*, $M_t^u = 0$. Their behavior is thus merely described by:

$$\beta E_t \left[R_{t+1} \frac{C_t^u}{C_{t+1}^u} \right] = 1 \tag{21}$$

$$w_t = C_t^u V_u'(1 - N_t^u) \tag{22}$$

$$C_t^u + K_{t+1}^u = w_t N_t^u + [r_t + (1 - \delta)]K_t^u \tag{23}$$

2.3 Equilibrium

A symmetric equilibrium consists in a set of prices $\{p_t, r_t, w_t\}_{t=0}^{\infty}$ and external effects $\{\bar{K}_t, \bar{N}_t\}_{t=0}^{\infty}$ such that, given these prices and externalities:

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i) $\{C_t^c, N_t^c, K_t^c, M_t^c, C_t^u, N_t^u, K_t^u, M_t^u, K_t, N_t\}_{t=0}^{\infty}$ solve the households and firms problems,

ii) $\bar{K}_t = K_t$ and $\bar{N}_t = N_t$,

iii) Markets clear.

As mentioned above, the equilibria we shall focus on are such that constrained households choose not to hold capital ($K_t^c = 0$) and are forced to hold money, while unconstrained households own the whole stock of capital and hold no money ($M_t^u = 0$). Such an asymmetric assets holding mainly stems from the assumed heterogeneity in time preferences. To see this, note first that along a BGP unconstrained households are unwilling to hold money. Indeed, as the quantity of money M is constant, the steady real gross return on money is $p_t/p_{t+1} = \gamma$.[6] In addition, conditions (17)-(18) entail $\beta R_{t+1} = \gamma$. The condition (20) is then fulfilled, and accordingly $M_t^u = 0$. To pursue, note that constrained households will choose to hold no capital provided that they are sufficiently impatient. In fact, it is easy to check, by making use of equation (6), that $K^c = 0$ whenever households' discount factors satisfy:

(Assumption 1) $\gamma\lambda < \beta$.[7]

From equations (7) and (8), one sees that Assumption 1 also ensures that the liquidity constraints (5) will be binding along the BGP. To sum up, Assumption 1 and $M_t = M$ at all times guarantee that along the BGP, and therefore by continuity near it, the choices of constrained and unconstrained households are described by the conditions (14) and (21)-(23), respectively.

The remainder of this section gives the equations governing the equilibrium dynamics in the vicinity of the BGP. In the neighborhood of the BGP

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[6]Along a BGP all real variables, except hours, grow at the same rate γ. Thus, $M_{t+1}^c/p_{t+1} = \gamma M_t^c/p_t$ and $M_{t+1}^u/p_{t+1} = \gamma M_t^u/p_t$. Hence, $M_{t+1}^c + M_{t+1}^u = \gamma \frac{p_{t+1}}{p_t}(M_t^c + M_t^u)$. As money is in fixed quantity, in equilibrium, $M_t^c + M_t^u = M$, $\forall t$. It follows that $p_t/p_{t+1} = \gamma$.

[7]Note that the form of Assumption 1 rests upon the specification retained for the constrained households' momentary utility function (13), notably the linearity in consumption. If the latter was logarithmic in consumption rather than linear, Assumption 1 would be $\lambda < \beta$.

the point i) of the definition of an equilibrium merely reads: an equilibrium is a set $\{w_t, r_t, C_t^c, N_t^c, M_t^c, C_t^u, N_t^u, K_t^u, K_t, N_t\}_{t=0}^\infty$ satisfying (1)-(2), (14), and (21)-(23). Besides, as locally $K_t^c = M_t^u = 0$, the market clearing conditions - point iii) of the definition - are particularly simple:

$$N_t^c + N_t^u = N_t \tag{24}$$

$$K_t^u = K_t \tag{25}$$

$$C_t^c = \frac{M}{p_t} = w_t N_t^c \tag{26}$$

$$Y_t = C_t^c + C_t^u + K_{t+1}^u - (1-\delta)K_t^u \tag{27}$$

The money market clearing condition (26) deserves some comments. Recall that the money supply is constant over time. As money is only held by constrained households, equilibrium on the money market at time t means that the real balances M/p_t is equal to their consumption C_t^c and to their wage income $w_t N_t^c$.

To complete the description of the dynamics of the model nearby the BGP it is convenient to consider variables that will be constant along the BGP. Since all the original variables (except hours) will grow at the same rate as the technology level z_t, this can be accomplished by deflating these variables by z_t. Letting $k_t \equiv K_t/z_t$, $c_t^u \equiv C_t^u/z_t$[8]... it is straightforward to see that locally an equilibrium is a set $\{k_t, n_t^c, n_t^u\}_{t=0}^\infty$ satisfying:

$$n_t^c = \lambda E_t \left[(1-\alpha)\gamma k_{t+1}^{\alpha(1+\theta)} n_{t+1}^{\theta-\alpha(1+\theta)} n_{t+1}^c \right] \tag{28}$$

$$\frac{1}{c_t^u} = \beta E_t \left[\left(\alpha k_{t+1}^{\alpha(1+\theta)-1} n_{t+1}^{(1-\alpha)(1+\theta)} + 1 - \delta \right) \frac{1}{\gamma c_{t+1}^u} \right] \tag{29}$$

$$\gamma k_{t+1} + c_t^u = (1-\delta)k_t + k_t^{\alpha(1+\theta)} n_t^{(1-\alpha)(1+\theta)} \left[\alpha + (1-\alpha)\frac{n_t^u}{n_t} \right] \tag{30}$$

where $\gamma \equiv z_{t+1}/z_t$ and

$$c_t^u = \frac{(1-\alpha)k_t^{\alpha(1+\theta)} n_t^{\theta-\alpha(1+\theta)}}{V_u'(1-n_t^u)} \tag{31}$$

$$n_t = n_t^c + n_t^u \tag{32}$$

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[8]For notations homogeneity lowercases are also used for hours

3 Local dynamics

This section characterizes the local dynamics of the economy around the steady state. For that purpose, let $X_t \equiv (\hat{n}_t^c, \hat{n}_t^u, \hat{k}_t)'$, where the hats indicate percentage deviations from the steady state. The log-linear approximation of the equilibrium system (28)-(31) is of the form:

$$M_1 E_t[X_{t+1}] = M_2 X_t \tag{33}$$

where M_1 and M_2 are 3×3 matrices. Now, introduce the vector of Euler equation errors:

$$\Phi_{t+1} = M_1 \left(X_{t+1} - E_t[X_{t+1}] \right)$$

that satisfies $E_t[\Phi_{t+1}] = 0.$[9] Making use of Φ_{t+1}, the equation (33) can be written as follows[10]:

$$X_{t+1} = J X_t + e_{t+1} \tag{34}$$

where $J \equiv M_1^{-1} M_2$ and $e_{t+1} \equiv M_1^{-1} \Phi_{t+1}$.

The properties of the set of stationary solutions of (34) heavily depend upon the value of the eigenvalues of the Jacobian matrix J or, more specifically, upon the dimension of the stable subspace of J. For instance, stables solutions exist if one can choose expectation errors, *i.e.*, the components of e_{t+1}, so as to eliminate the explosive components of X_t.

When the equilibrium is *determinate*, *i.e.*, when J has two eigenvalues located outside the unit circle, one can find *two* linear restrictions on both X_t and e_{t+1}.[11] The restrictions placed on X_t yield the approximate decision rules of the free variables, $\hat{n}_t^c$ and $\hat{n}_t^u$, as linear functions of the predetermined variable, $\hat{k}_t$. Those on e_{t+1} imply that the expectation errors, e_{t+1} and e_{t+1}^2, are equal to zero in every period. Therefore, extrinsic beliefs do not matter.

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[9]The capital being a predetermined variable, the last component of Φ_{t+1} is nil since $E_t[k_{t+1}] = k_{t+1}$.

[10]The matrix M_1 is assumed to be non-singular.

[11]Let $J = Q\Lambda Q^{-1}$ where Λ is a diagonal matrix with the eigenvalues of J on the diagonal and Q is the matrix of eigenvectors of J. The linear restrictions aforementioned are found by setting the rows of $Q^{-1} X_t$ and $Q^{-1} e_{t+1}$ associated with the *two* explosive eigenvalues of J equal to zero.

When instead the equilibrium is *indeterminate*, *i.e.*, when J has one or zero eigenvalue lying outside the unit circle, there is no longer enough restrictions on X_t in order to pin down the non-predetermined variables. Accordingly, the state space now includes $\hat{n}_t^c$ or/and $\hat{n}_t^u$, depending upon the number of unstable eigenvalue. Moreover, e_{t+1}^1 and e_{t+1}^2 can be different from zero as long as $E_t[e_{t+1}^i] = 0$, $i = 1, 2$. In such a case, the forecast errors can be written: $e_{t+1}^i = \eta^i \kappa_{t+1}$, where the zero-mean random variable κ stands for the *sunspot*.[12] The previous considerations clearly illustrate that indeterminacy implies the existence of rational expectations equilibria in which fluctuations are driven by self-fulfilling changes in households' beliefs.

The model exhibits indeterminacy whenever J has at least two eigenvalues of modulus less than one. Since the analytical characterization of the eigenvalues of the 3×3 matrix J is cumbersome, a numerical procedure will be considered. To this end, following the existing literature, we calibrate our model by setting the time interval to be a quarter; Table 1 summarizes the calibration. The unconstrained households elasticity of labor supply evaluated at the steady state, denoted ε_u, is set at $1/3$. This value is in accordance with the bulk of empirical estimates. Diaz-Giménez, Quadrini and Rios-Rull [1997] have evidenced that around 25 percents of U.S. households are virtually liquidity-constrained. Accordingly, we set the constrained households share of labor at $\frac{N^c}{N} = 0.25$. These values constitute our baseline calibration. The externality parameter θ is left free for experiments.

Table 1: Structural parameters

β	δ	α	γ	ε_u
0.99	0.025	0.3	1.004	1/3

Table 2 reports the eigenvalues of J for various values of the externality parameter. It shows that the model, or at least the current calibrated ver-

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[12]When, as it turns out to be the case in this model, the steady state is a *saddle* with *one* dimension on instability, the expectation errors are not linearly independent. We have $e_{t+1}^1 = \nu e_{t+1}^2$, which may rewrites as $e_{t+1}^1 = \eta^1 \kappa_{t+1}$ with $\eta^2 = \nu \eta^1$.

sion, exhibits indeterminacy for arbitrarily close to zero externalities, *i.e.*, for roughly constant returns-to-scale. This finding, which stands in sharp contrast to the results of recent one-sector sunspot models, is actually robust to changes in individuals shares of total hours, as well as labor supply elasticity of unconstrained households as illustrated by tables 3 and 4 (both computed with $\theta = 0.01$).

In order to get some intuitions about the occurrence of indeterminacy, let us consider the effects of optimistic beliefs. Constrained households expecting higher return on money, *i.e.*, waiting for more consumption tomorrow, increase their labor supply. Unconstrained households then expect a decrease in the wage, together with an increase in the return on capital. These expected price dynamics lead them to supply more labor and to invest more instantaneously. As both types of households supply more labor, one observes a large rise of the real return on capital, hence of investment. The latter allows for more output and consumption tomorrow and households' expectations will be self-fulfilling. Indeed, the complementarity between labor supplies, *i.e.*, the fact that optimistic beliefs of constrained households raise the unconstrained households labor supply, enhances the impact of the initial shift of the labor supply on the return on capital and investment. This makes more likely the rise of future output and consumption consistent with initial beliefs. Returns to scale do not have to be high in order for indeterminacy to occur.[13]

4   Business Cycle Properties

This section presents a quantitative evaluation of the model, thereby providing an appraisal of the mechanisms through which variations in beliefs are propagated over time. For comparison purposes, we contrast the predictions of our model (hereafter BCL) with those of two benchmark models: the standard RBC model driven by permanent technology shocks, and the two-sector

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[13]This result highlights the role of the heterogeneity of labor supply since the model used by Barinci and Chéron [2001], in which the labor supply of unconstrained households is inelastic, indeterminacy needs unrealistic aggregate increasing returns to scale around 1.35.

Table 2: Eigenvalues and modulus – Baseline Calibration

Externality (θ)	Eigenvalues			Modulus		
0.00	0.985	1.000	1.097	0.985	1.000	1.097
0.01	0.992± 0.004i		1.095	0.991	0.991	1.095
0.02	0.991± 0.010i		1.094	0.991	0.991	1.094
0.05	0.991± 0.020i		1.091	0.991	0.991	1.091
0.10	0.989± 0.028i		1.087	0.990	0.990	1.087
0.20	0.986± 0.042i		1.079	0.986	0.986	1.079

Table 3: Eigenvalues and modulus – $\theta = 0.01$ and $\epsilon_u = 1/3$

Population $(\frac{N^c}{N})$	Eigenvalues			Modulus		
0.0	0.954	1.000	1.059	0.954	1.000	1.059
0.1	0.969	0.998	1.070	0.969	0.998	1.070
0.2	0.983	0.995	1.086	0.983	0.995	1.086
0.25 (Ref.)	0.992± 0.004i		1.095	0.992	0.992	1.095
0.3	0.994± 0.007i		1.106	0.994	0.994	1.106
0.5	0.999± 0.009i		1.160	0.999	0.999	1.160
0.6	1.001± 0.008i		1.194	1.002	1.002	1.194

Table 4: Eigenvalues and modulus – $\theta = 0.01$ and $\frac{N^c}{N} = 0.25$

Elasticity (ϵ_u)	Eigenvalues			Modulus		
1/10	0.993± 0.005i		1.095	0.993	0.993	1.0957
1/3 (Ref.)	0.992± 0.004i		1.095	0.991	0.991	1.095
1/2	0.990± 0.001i		1.095	0.990	0.990	1.095
1	0.981	0.994	1.095	0.981	0.994	1.095
∞	0.926	0.996	1.095	0.926	0.996	1.095

EBC model driven by sunspot disturbances alone analyzed by Schmitt-Grohé [2001] (hereafter SG).

4.1   Simple measures of comovements

The empirical performances of a business cycle model are commonly judged in light of its ability of matching *unconditional* second moments of key macroeconomic aggregates. The model moments are computed from the stochastic system (34) making use of the baseline calibration discussed above, and setting $\theta = 0.01$.[14] Table 5 reports the predicted second moments for growth rates and their empirical counterparts. The first panel presents the standard deviation of per capita consumption growth relative to the standard deviation of the output growth, and the cross-correlation between these two variables. The second and third panels report the same statistics for per capita investment growth and detrended per capita hours. The first columns of these three panels report the estimates based on U.S. quarterly data covering the period 1948:Q3-1997:Q4; asymptotic standard errors are in parentheses.

Table 5 illustrates the ability of the RBC model of fitting the relative volatilities of consumption and investment growth with respect to output growth. However, the model predicts hours growth that is too smooth relative to output growth. Both EBC models generate relative volatilities of investment and hours growth significantly larger than in U.S. data. The relative volatility of consumption is the only fact matched by the three models.

As regards to contemporaneous correlations between output growth and consumption, investment and hours growth, respectively, the RBC model predicts statistics close to one, while in data such high correlations are always rejected. Nonetheless, it is fair to notice that the sign of these correlations is correctly predicted. On the contrary, and this is probably its main shortcoming, the SG model produces a time series for consumption that is countercyclical.[15] This can be understood from the intratemporal efficiency

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[14]This value implies a degree of increasing returns of 1%, which obviously falls within the estimate range by Basu and Fernald [1997].

[15]Moreover the volatility of hours and investment relative to the output growth is excessive.

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condition: $U_1(c, 1 - n)w = U_2(c, 1 - n)$. Whenever consumption and leisure are normal goods, a spontaneous increase of employment, which entails a decrease in the wage, must be accompanied by a drop in consumption.[16]

Even though the BCL model presents some weaknesses, notably regarding the relative standard deviations of hours and investment, it not faces the well-documented consumption "anomaly" of sunspots models: it successfully predicts the positive instantaneous correlation between output and consumption. Moreover, it succeeds in replicating the lead-lag pattern of consumption along the cycle, while both SG and RBC models fail to account for this fact. The mechanism behind the procyclicality result is simple. As in equilibrium the liquidity constraint binds, the constrained households' intratemporal marginal efficiency condition does not hold with equality. Accordingly, consumption and hours worked are no longer forced to move in opposite directions. In fact, constrained households consume their current labor income (see equation (26)). The keynesian-like behavior of constrained households explains the relevancy of the propagation of sunspot shocks in the BCL model economy.

4.2 Forecastable comovements in main aggregates

Any useful model of the business cycle should provide accurate forecasts of main macroeconomic aggregates. However, Rotemberg and Woodford [1996] argue that the standard RBC model generates counterfactual comovements between the forecastable component of output, consumption, investment and hours fluctuations. Estimating a VAR model between U.S. output, consumption, investment and hours, they are able to recover the expected movements of the aggregate variables. They then compute the correlation between these components at several leads and lags. Table 6–lines 1 to 3 – reports the estimated values obtained by Rotemberg and Woodford [1996]. It shows that predictive changes in output are strongly correlated with the predictive

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[16]Naturally, if instead the wage rises, due for instance to the presence of high increasing returns as in Benhabib and Farmer [1994], consumption could be procyclical.

changes in consumption, investment and hours.[17]

Rotembreg and Woodford [1996] perform the same exercise on simulated series obtained from the standard RBC model, and show that this model can not replicate the data. Schmitt-Grohé [2001] recently reaches the same conclusions regarding the two-sector EBC model (see Table 6). These counterfactual results can be explained as follows.

RBC. Following a positive and permanent technological shock, the inherited capital stock is below its steady state value. Then, its marginal product is above the steady state. Under standard parameterizations, this leads households to enjoy less consumption and leisure than in the steady state. Consequently, consumption is expected to rise while hours worked are expected to decline. Provided that the labor share is sufficiently large (it is set at 0.7 in our calibration) the initial level of hours implies that output approaches its steady state value from above. Overall, the RBC model predicts that when output and hours fall, consumption rises.

SG. Following a sunspot shock, labor supply increases. As long as the labor-demand schedule slopes downward, the wage falls. If consumption and leisure are normal goods, the intratemporal efficiency condition forces consumption to decrease. Consequently, in the transition toward the steady state consumption is forecasted to increase, while output is expected to decline.[18]

Table 6 illustrates that, in sharp contrast to RBC and SG models, the BCL model implies positively correlated forecastable changes in consumption and output. The economic mechanism that creates this result can be understood as follows. If constrained households expect an increase of the real return on money, they wait for more consumption tomorrow and thus supply more labor today. Unconstrained households then anticipate higher return on capital. Under usual utility specifications, they supply more labor and raise their investment. The increase of labor supplies entails a large rise

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[17]A comparison with the unconditional moments reported in Table 5 indicates that the correlation between forecasts is larger than between the overall changes, notably for consumption.

[18]It is worthy to note that the allowance of permanent technological shocks is not sufficient to overcome this shortcoming, whereas it allows the model to capture the positive *unconditional* correlation between consumption and output (see Schmitt-Grohé [2001]).

of the current return on capital, hence of investment, allowing for more output and consumption in the current period. Thereby, along the transition paths output and consumption are expected to decrease.

5 Conclusion

This paper has established that restricted borrowing noteworthy improves the propagation of sunspot impulses. We have proposed a one-sector model with heterogeneous households and liquidity constraints which is able to overcome some of the criticisms that have been addressed to both standard RBC and EBC models. The most significant improvement being that the model exclusively driven by sunspot shocks can account for the joint dynamics of output and consumption. More specifically, it does a pretty good job in matching the unconditional and predictable movements in output, hours and consumption which are defining features of the business cycle fluctuations that available EBC models have not been able to capture.

Table 5: ESTIMATED AND PREDICTED COMOVEMENTS

	Δc_t				Δi_t				Δn_t			
	U.S.	SG	BCL	RBC	U.S.	SG	BCL	RBC	U.S.	SG	BCL	RBC
$\frac{\sigma_{\Delta x_t}}{\sigma_{\Delta y_t}}$	0.50 (0.033)	0.47	0.46	0.53	2.53 (0.114)	5.39	4.53	2.47	0.94 (0.045)	1.41	1.36	0.49
$corr\left(\Delta x_t, \Delta y_t\right)$	0.48 (0.056)	-0.95	0.25	0.99	0.68 (0.062)	0.99	0.94	0.99	0.77 (0.026)	0.99	0.99	0.99
$corr\left(\Delta x_t, \Delta y_{t-1}\right)$	0.29 (0.051)	-0.04	0.56	0.06	0.41 (0.083)	0.14	0.01	-0.04	0.62 (0.064)	0.13	0.13	-0.07
$corr\left(\Delta x_t, \Delta y_{t+1}\right)$	0.45 (0.061)	-0.23	0.30	-0.01	0.40 (0.078)	0.19	0.09	0.00	0.39 (0.056)	0.19	0.16	0.00

Notes: Standard errors are in parentheses.

Δx_t denotes the change in the logarithm of x from $t-1$ to t.

Table 6: ESTIMATED AND PREDICTED CORRELATIONS AMONG FORECAST CHANGES

	Horizon (in quarters)					
	1	2	4	8	12	24
ESTIMATED CORRELATIONS						
$corr\left(\widehat{\Delta c_t^k}, \widehat{\Delta y_t^k}\right)$	0.69	0.78	0.82	0.82	0.79	0.72
	(0.121)	(0.083)	(0.071)	(0.068)	(0.070)	(0.092)
$corr\left(\widehat{\Delta n_t^k}, \widehat{\Delta y_t^k}\right)$	0.88	0.89	0.92	0.97	0.98	0.98
	(0.044)	(0.042)	(0.028)	(0.012)	(0.008)	(0.009)
$corr\left(\widehat{\Delta i_t^k}, \widehat{\Delta y_t^k}\right)$	0.98	0.98	0.98	0.97	0.96	0.89
	(0.010)	(0.011)	(0.011)	(0.014)	(0.023)	(0.061)
PREDICTED CORRELATIONS						
			RBC			
$corr\left(\widehat{\Delta c_t^k}, \widehat{\Delta y_t^k}\right)$	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
$corr\left(\widehat{\Delta n_t^k}, \widehat{\Delta y_t^k}\right)$	1.00	1.00	1.00	1.00	1.00	1.00
$corr\left(\widehat{\Delta i_t^k}, \widehat{\Delta y_t^k}\right)$	1.00	1.00	1.00	1.00	1.00	1.00
			SG			
$corr\left(\widehat{\Delta c_t^k}, \widehat{\Delta y_t^k}\right)$	-0.86	-0.86	-0.87	-0.85	-0.81	-0.77
$corr\left(\widehat{\Delta n_t^k}, \widehat{\Delta y_t^k}\right)$	0.98	0.98	0.98	0.98	0.98	0.98
$corr\left(\widehat{\Delta i_t^k}, \widehat{\Delta y_t^k}\right)$	0.99	0.99	0.99	0.99	0.99	0.99
			BCL			
$corr\left(\widehat{\Delta c_t^k}, \widehat{\Delta y_t^k}\right)$	0.98	0.98	0.98	0.98	0.98	0.98
$corr\left(\widehat{\Delta n_t^k}, \widehat{\Delta y_t^k}\right)$	0.99	0.99	0.99	0.99	0.99	0.99
$corr\left(\widehat{\Delta i_t^k}, \widehat{\Delta y_t^k}\right)$	0.88	0.88	0.88	0.87	0.87	0.85

Notes: Standard errors are in parentheses.

Δx_t^k denotes the expected change in the logarithm of x from t to $t+k$.

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6 Appendix: The reduced-form utility function (13)

The present appendix shows how a home production setup can be used in order to justify the specific functional assumed for the constrained households' utility function. As a matter of fact, since home quantities do not appear in their budget and liquidity constraints, their problem can be solved in two steps: first compute the optimal home production quantities as functions of market consumption and labor, then use market prices to compute home consumption and labor. Let the subscript h denotes home quantities. The momentary utility function over market and home quantities is given by :

$$\frac{\left[(C^c + C_h^c)^\eta \left(1 - N_h^c - N^c\right)^{1-\eta}\right]^{1-\varsigma}}{1 - \varsigma}$$

for $\varsigma \geq 0$ and $\eta \in (0, 1)$.

Now, assume that the consumption of home goods is a simple linear function of time spent in home production: $C_h^c = zN_h^c$, where z is the level of technology. It is easy to show that the optimal home labor choice is:

$$zN_h^c = \eta z(1 - N^c) - (1 - \eta)C^c$$

Substituting the latter into the utility function and making use of $C_h^c = zN_h^c$ lead to the indirect utility function:

$$\left[\eta^\eta(1 - \eta)^{1-\eta}z^{\eta-1}\right]^{1-\varsigma} \frac{[C^c + z(1 - N^c)]^{1-\varsigma}}{1 - \varsigma}$$

Finally, setting $\varsigma = 0$ one finally gets the utility function over market consumption and labor assumed in the main text.

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