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Jean-Guillaume SAHUC

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UNIVERSITE D'EVRY – VAL D'ESSONNE, 4 BD. FRANÇOIS MITTERRAND, 91025 EVRY CEDEX

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Influence of Parameter Estimation Uncertainty on the European Central Banker Behavior: An Extension

Jean-Guillaume Sahuc*
Banque de France
EPEE, Université d'Evry-Val d'Essonne

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Abstract

This paper deals with the linear quadratic stochastic control approach to assessing the effects of parameter estimation uncertainty on the central banker behavior. The treatment of parameter estimation uncertainty is covered by the introduction of the full variance-covariance matrix of the parameters estimates in the optimal control theory. The proposed approach is simple to implement and is applicable to a large class of model. The principle of conservatism of Brainard is found empirically relevant for a Euro area policymaker.

Keywords: European monetary policy, parameter estimation uncertainty, linear quadratic stochastic control.

JEL Classifications: C61, D81, E58

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Contact Address: Banque de France, Centre de Recherche, 31 rue Croix des Petits Champs, F-75049 Paris. E-Mail: jean-guillaume.sahuc@banque-france.fr

1 Introduction

Uncertainty is a characteristic of the real world that plays an important part in the decision-making process of all economic agents. As observers and experts recognize unanimously, this uncertainty applies to central banks which need to take policy decision in an environment of considerable uncertainty regarding current and future economic conditions and the functionning of the economy. Much of the uncertainty facing monetary policy makers is unavoidable. In this regard, the situation of the European Central Bank (ECB) is by no means unique, although the specific features of the euro area create additional challenges. Since it is crucial that monetary policy itself does not become an independant source of additional uncertainty, understanding the implications for the ECB's monetary policy strategy is of very first importance.

While following Poole (1998) or Clements & Hendry (1998), one can distinguish five-fold categorization for the sources of model-based uncertainty. (i) *Future changes in the underlying structure of the economy*: the economic system faces constantly with intrinsic risks like unanticipated shocks (oil crisis) or structural changes (realization of the monetary union). (ii) *Mis-specification of the model:* the model used to analyse the effects of a policy does not well represent the economy. (iii) *Mis-measurement of the data:* the policymaker uses data often provisional and prone to revision; in addition, some concepts are the subject of dissension between economists (the output gap for example). (iv) *The cumulation of future errors (or "shocks") to the economy:* the response of the economy to an action of economic policy depends on the mode of formation of anticipations and the credibility of the central bank, factors difficult to observe, potentially fluctuating. (v) *Inaccuracies in the estimates of model's parameters:* even if the econometrician has the correct specification, the parameters of the model are estimated on samples of small size and are prone to errors in estimation.

In practice, all five sources are important when analysing uncertainty. But the first three sources are unpredictable and unanticipated. If their extent and nature were known, they could be incorporated into the model and they would be predictable and predicted. These sources of uncertainty is beyond the scope of this paper and we focus on the sources of predictable uncertainty - those of which the degree of uncertainty can be anticipated and even calculated - and primarily on item (v).

Main goal of this recent literature on uncertainty[1] is to know how the monetary authorities must adjust their behavior when one introduces an uncertainty on the

[1] See Sack and Wieland (2001), Rudebusch (2001) and, Le Bihan and Sahuc (2002), for recent surveys.

estimate of the parameters: does it have to be shown more careful or on the contrary more aggressive? For that, two approaches clash: a branch of the literature has looked for robust rules that minimize a loss criterion in some worst-case scenario, within a specified set of possible scenarios;[2] a second approach is to use Bayesian methods to determine the policy that minimizes the expected loss, given a prior distribution on the parameters.

This last motive is a revisiting of the classical argument offered by Brainard (1967) who institutes the *"conservatism principle"*. When policy-makers are uncertain on the key parameters which determine the transmission of monetary policy in the adopted structural model of the economy, aggressive policy moves are more likely to have unpredictable consequences on output and inflation, then gradual policy (or equivalently a cautiously policymaker behavior) is optimal to minimize fluctuations of output anf inflation around their targets. To test the veracity of the Brainard principle, majority of papers has used the optimal control theory - detailed in the technical book by Chow (1977) - and have found it to be empirically relevant.[3]

However none of these articles really used all information available in the matrix of variance-covariance and retained (by simplification) only variances of the parameters as a measure of uncertainty. But Amman and Kendrick (2000) have found *"evidence that the potential damage from ignoring the variances and covariances of the parameter estimates is substantial and that taking them into account can improve matters"*. In fact, covariances may play a considerable role in the derivation of the reaction function of the monetary authorities, and since the loss function can be viewed as a quadratic approximation to the level of expected utility of a representative household, the policymaker must choose the rule that lower this welfare criterion.

In this paper, we consider the omitted consideration of all second-moments of the distribution of the estimated parameters in explaining in detail the optimal control methodology. The paper is structured in four sections. The first section presents the general framework of the analysis. The second section details the standard and extended optimal control methodology. The third section contains the empirical illustrations for the Euro Area case, in order to measure the interactions between parameter uncertainty and the behavior of the central banker and more particularly

[2] Hansen and Sargent (2001), Onatski and Stock (2000), Tetlow and von zur Muehlen (2000), Giannoni (1999) and Sahuc(2002) have recently applied robust control theory to derive robust monetary policies. The foundations are found in Knight (1921) who defined the uncertainty as a situation in which no known probability distribution exists.

[3] See Estrella and Mishkin (1999), Söderström (1999), Srour (1999), Rudebusch (2000) or Sack (2000) among others.

the contribution to incorporate all information available. The last section concludes.

2 The Central Bank Problem

2.1 Overview

When taking monetary policy decisions, central banks face considerable uncertainty about transmission mechanism of monetary policy to the price level. In particular, the role played by monetary developments in the euro area transmission mechanism is not well understood. The problems arise from what the policymaker do not know: he must deal with the uncertainty from the base of what he do know. Especially, he must analyse of whether uncertainty about the relationship between the economic variables in the economy could entail a slower, or smoother, policy response to shocks to the economy than otherwise.

This analysis, which follows a proposition first put forward by Brainard (1967), is based on the premise that uncertainty about the relationship between the interest-rate and the rest of the economy creates trade-off for policymakers: the parameter estimation uncertainty may mean that movements in the interest rates themselves increase uncertainty about the future path of the economy. This could lead the policymaker to use their policy instrument more cautiously in order to reduce the chance of missing the target significantly.

2.2 Monetary Policy in Small Structural Models

A standard approach to analysing monetary policy is to specify an objective for the policymaker, and a model of the economy, and then to determine how monetary policy should be operated in response to disturbances to the economy. We are interested by the class of dynamic programming problems in which the return function is quadratic and the transition function is linear. This specification allows a good description of the optimal monetary authorities problem.

Recent empirical studies of optimal monetary policy in closed economies have adopted a simple two-equation framework linking the inflation (π_t), the output gap (y_t) and a short interest rate (i_t):

$$\begin{cases} y_{t+1} = \alpha(L)\, y_t - \beta(L)\,(i_t - \pi_t) + \varepsilon_{y,t+1} \\ \pi_{t+1} = \delta(L)\, \pi_t + \gamma(L)\, y_t + \varepsilon_{\pi,t+1} \end{cases}$$

with L the lag operator, $\varepsilon_{y,t+1}$ and $\varepsilon_{\pi,t+1}$ are zero mean normally distributed shocks.

A typical model in this class is the one estimated by Rudebusch and Svensson (1999) who represent the model economy as follows:

$$y_{t+1} = cst + \alpha_1 y_t + \alpha_2 y_{t-1} - \beta\left(i_t - \pi_t\right) + \varepsilon_{y,t+1} \qquad \text{(IS)}$$

$$\pi_{t+1} = \delta_1 \pi_t + \delta_2 \pi_{t-1} + \delta_3 \pi_{t-2} + \delta_4 \pi_{t-3} + \gamma y_{t-1} + \varepsilon_{\pi,t+1} \qquad \text{(Phillips)}$$

The first equation is an IS curve that relates the output gap to its own lags and to the difference between the short nominal interest rate and inflation, an approximate *ex-post* real interest rate. The constant term provides an estimate for β time r^*, the natural real interest rate. The second equation is a Phillips curve that relates inflation to a lagged output gap and to lags of inflation, which represent an autoregressive or adaptive form of inflation expectations. In our future empirical investigation, we will not reject the hypothesis that the coefficients of the four inflation lags sum to one, which implies a long run vertical Phillips curve. As explained by Rudebusch and Svensson, the simple structure of this model allows the production of benchmark results. We can notice that this two-equations system may be easily written in a state-space form such as,

$$x_{t+1} = Ax_t + Bi_t + \varepsilon_{t+1} \tag{1}$$

where $x_t = (y_t \; y_{t-1} \; \pi_t \; \pi_{t-1} \; \pi_{t-2} \; \pi_{t-3})'$ is the state vector, and matrices A and B are the matrices of the coefficients.

In order to represent the optimal monetary policy rule, we define the stochastic discounted linear optimal regulator problem faces to the central banker. He wants to minimize the following expected intertemporal loss function over choice of his instrument (the interest rate), $\{i_t\}_{t=0}^{\infty}$,

$$E_t \sum_{h=0}^{\infty} \phi^h L\left(y_{t+h}, \pi_{t+h}\right) \tag{2}$$

where in each period the loss function $L\left(.\right)$ is given by

$$L\left(y_t, \pi_t\right) = \left(\pi_t - \bar{\pi}\right)^2 + \lambda y_t^2 \tag{3}$$

where $\bar{\pi}$ is the inflation target. By simplification we choose a zero inflation target, $\bar{\pi} = 0$, such as

$$L\left(y_t, \pi_t\right) = x_t' Q x_t \tag{4}$$

$0 < \phi < 1$ is the society's discount factor, $\lambda \geq 0$ is the weight on output gap with respect to inflation stabilization and Q is a 6×6 matrix with λ on the first diagonal element and 1 on the third diagonal element and zeros elsewhere, which reflects the central bank preferences. This choice reflects the goal of the central bank to minimize each period the variation in inflation, π_t, around its target (here 0 to simplify) and in the output gap y_t. Specifically, it can be shown that as the discount factor approaches unity, the value of the intertemporal loss function becomes proportional to the unconditional expected value of the period loss function (see Appendix A for a discussion on the criterion value): $E(L_t) = Var(\pi_t) + \lambda Var(y_t)$. The central bank's objective is set to a path for i_{t+h}, $(h = 0, ..., \infty)$ so as to minimize function (2) subject to equations (IS) and (Phillips).

The solution of the central bank program has the form of a linear feedback rule:

$$i_t = Fx_t \tag{5}$$

The elements of F can be called response coefficients.

3 Dynamic Programming Methodology

The preceding framework allows us to use the dynamic programming theory to calculate the optimal policy rule. This methodology shows that it is optimal for the policymaker to set its instrument in each period as a function of current and lagged values of y_t and π_t.

Previous works on optimal control and uncertainty (especially those of Amman and Kendrick) consider the problem over finite planning horizon T, and provide no indication of how the lenght of T should be determined. In that case, the resulting optimal policy as well as the final state of the system may vary substancially with T, depending on the controllability properties of the system.

A central result of economic growth theory is the *"turnpike"* of capital accumulation essentially stating that for T large enough, optimal growth is achieved by steering the system from the initial state to an optimal steady-state (or "turnpike"), and then leaving the turnpike towards the end of the horizon to achieve short-term unsustainable superior growth. For infinite-horizon problems a natural consequence of the turnpike property is global asymptotic convergence of the optimal trajectory, and such results have been provided under strong convexity assumptions on the Hamiltonian by Brock and Scheinkman (1976) and others. As a consequence of these results, it is natural to consider the problem of finding an optimal monetary policy rule over an *infinite* horizon, as this avoids unsustainable endpoint effects (for longer horizons) and variations of the optimal trajectory (for shorter horizons).

3.1 The Method in a Certain World

From a technical point of view, the use of the dynamic programming with infinite-horizon optimal control problems rests on the contraction mapping theorem (see Appendix B). With these theoretical definitions, we may apply the method to the baseline model in the certainty case, i-e when the policymaker is supposed to know the structure of the model and the true value of the parameters.

The policymaker solves the following problem

$$J(x_t) = \min \left\{ x_t' Q x_t + \phi E_t J(x_{t+1}) \right\} \tag{6}$$

subject to

$$x_{t+1} = A x_t + B i_t + \varepsilon_{t+1}$$

Since the objective function is quadratic and the constraint linear, the value function will be of the form

$$J(x_t) = x_t' V x_t + d \tag{7}$$

where V is the unique negative semidefinite solution of the discounted algebraic matrix Ricatti equation. The scalar d is given by

$$d = \phi (1-\phi)^{-1} tr V \Sigma_\varepsilon$$

where "tr" denotes the trace of a matrix and Σ_ε is the covariance matrix of the noises ε_t.

Using the transition law to eliminate the next period's state, the Bellman equation is

$$x_t' V x_t + d = \min_{\{i_t\}} \left\{ x_t' Q x_t + \phi (A x_t + B i_t)' V (A x_t + B i_t) + \phi d \right\} \tag{8}$$

The first-order condition for the minimization problem is then

$$B' V B i_t = -B' V A x_t,$$

So the optimal instrument is given by

$$\begin{aligned} i_t &= -\left(B' V B\right)^{-1} \left(B' V A\right) x_t \\ &= F x_t. \end{aligned} \tag{9}$$

Substituting the optimizer into the Bellman equation gives

$$\begin{aligned} x_t' V x_t + d &= x_t' Q x_t + \phi (A x_t + B F x_t)' V (A x_t + B F x_t) + \phi d \\ &= x_t' \left(Q + \phi (A + BF)' V (A + BF) \right) x_t + \phi d \end{aligned}$$

The algebraic matrix Ricatti equation V is

$$V = Q + \phi \left(A + BF\right)' V \left(A + BF\right) \tag{10}$$

where

$$F = -\left(B'VB\right)^{-1}\left(B'VA\right) \qquad \text{(Certainty Case)}$$

3.2 Parameter Estimation Uncertainty

Although the statistical inference of model form and of parameter values from available data obviously entails some degree of arbitrariness and uncertainty, the previous theoretical expressions generally used for deriving the optimal policy do not take into account these sources of approximation. In particular, no account of the effects of uncertainty in parameter estimates (which can be quite large) is commonly made.

Indeed, the introduction of parameter estimation uncertainty into the general framework will have an important effect on optimal policy. When incorporating this type of uncertainty, the certainty equivalence principle ceases to hold, and the error variances and covariances of the estimated state variables will affect the optimal rule.

The state-space formulation of the general model economy is then

$$x_{t+1} = A_{t+1}x_t + B_{t+1}i_t + \varepsilon_{t+1} \tag{11}$$

We now suppose that we know the two first moments of the parameters and that parameter matrices A_t and B_t are then stochastic with means given by the matrix A and B defined in the last section and variance-covariance matrices given by:[4]

$$\Sigma_A = \begin{bmatrix} \sigma^2_{\alpha(0)} & & \sigma_{\alpha(0)\alpha(m)} & \cdots & \sigma_{\alpha(0)\beta(n)} & \cdots & \sigma_{\alpha(0)\gamma(o)} & \cdots & \sigma_{\alpha(0)\delta(p)} \\ & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ - & & \sigma^2_{\alpha(m)} & \cdots & \sigma_{\alpha(m)\beta(n)} & \cdots & \sigma_{\alpha(m)\gamma(o)} & & \sigma_{\alpha(m)\delta(p)} \\ & & & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ - & & - & & \sigma^2_{\beta(n)} & \cdots & \sigma_{\beta(n)\gamma(o)} & & \sigma_{\beta(n)\delta(p)} \\ & & & & & \ddots & \vdots & \ddots & \vdots \\ - & & - & & - & & \sigma^2_{\gamma(o)} & & \sigma_{\gamma(o)\delta(p)} \\ & & & & & & & \ddots & \vdots \\ - & & - & & - & & - & & \sigma^2_{\delta(p)} \end{bmatrix}$$

[4]Note that in our case the lag index on output gap, real interest rate, and inflation are $m = 2$ and $o = 0$, $n = 0$, and $p = 4$, respectively.

$$
\Sigma_B = \begin{bmatrix} \sigma^2_{\beta(0)} & 0 & \cdots & 0 \\ - & 0 & \ddots & \vdots \\ - & - & \ddots & 0 \\ - & - & - & 0 \end{bmatrix}
$$

$$
\Sigma_{AB} = \begin{bmatrix}
\sigma_{\alpha(0)\beta(0)} & 0 & \cdots & 0 & \sigma_{\gamma(0)\beta(0)} & 0 & \cdots & 0 \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
\sigma_{\alpha(m)\beta(0)} & & & & \sigma_{\gamma(o)\beta(0)} & & & \\
\sigma^2_{\beta(0)} & & & & \sigma_{\delta(1)\beta(0)} & & & \\
\sigma_{\beta(1)\beta(0)} & & & & \vdots & & & \\
\vdots & \vdots & & \vdots & \sigma_{\delta(p)\beta(0)} & \vdots & & \vdots \\
\sigma_{\beta(n)\beta(0)} & & & & 0 & & & \\
-\sigma_{\beta(1)\beta(0)} & & & & & & & \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
-\sigma_{\beta(n)\beta(0)} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0
\end{bmatrix}
$$

The policymaker faces the same control problem

$$
J(x_t) = \min_{\{i_t\}} \left\{ x_t' Q x_t + \beta E_t J(x_{t+1}) \right\}
$$

subject to

$$
x_{t+1} = A_{t+1} x_t + B_{t+1} i_t + \varepsilon_{t+1}
$$

The value function will still be

$$
J(x_t) = x_t' \widetilde{V} x_t + \tilde{d},
$$

but now, since the policymaker uses the second moments, the modified expected value is then given by

$$
E_t J(x_{t+1}) = (E_t x_{t+1})' \widetilde{V} (E_t x_{t+1}) + tr\left(\widetilde{V} \Sigma_{t+1|t} \right) + \tilde{d} \tag{12}
$$

where the expected value of x_{t+1} is given by $E_t x_{t+1} = A x_t + B i_t$ and where $\Sigma_{t+1|t}$ is the covariance matrix of x_{t+1}, evaluated at t.

The (i,j)th element of $\Sigma_{t+1|t}$ is defined by

$$
\Sigma^{i,j}_{t+1|t} = x_t' \Sigma^{i,j}_A x_t + 2 x_t' \Sigma^{i,j}_{AB} i_t + i_t' \Sigma^{i,j}_B i_t + \Sigma^{i,j}_{\varepsilon} \tag{13}
$$

where $\Sigma^{i,j}_{AB}$ is the covariance matrix of the ith row of A with the jth row of B.

Since at t, y_{t+1} and π_{t+1} are the only stochastic variables in x_{t+1}, the only non zero entries of $\Sigma_{t+1|t}$ are the matrices $\Sigma_{t+1|t}^{11}$, $\Sigma_{t+1|t}^{nn}$, $\Sigma_{t+1|t}^{1n}$ (with $n = s_y + 1$).

The only non-zero elements are then

$$\begin{aligned} \Sigma_{t+1|t}^{11} &= Var_t\left(y_{t+1}\right) \\ &= x_t'\Sigma_A^{11}x_t + 2x_t'\Sigma_{AB}^{11}i_t + i_t'\Sigma_B^{11}i_t + \Sigma_\varepsilon^{11} \end{aligned} \tag{14}$$

$$\begin{aligned} \Sigma_{t+1|t}^{nn} &= Var_t\left(\pi_{t+1}\right) \\ &= x_t'\Sigma_A^{nn}x_t + \Sigma_\varepsilon^{nn} \end{aligned} \tag{15}$$

$$\begin{aligned} \Sigma_{t+1|t}^{1n} &= cov_t\left(y_{t+1}, \pi_{t+1}\right) \\ &= x_t'\Sigma_{AA}^{1n}x_t + x_t'\Sigma_{AB}^{n1}i_t \end{aligned} \tag{16}$$

Consequently

$$\begin{aligned} tr\left(\widetilde{V}\Sigma_{t+1|t}\right) &= \tilde{v}_{11}\left(x_t'\Sigma_A^{11}x_t + 2x_t'\Sigma_{AB}^{11}i_t + i_t'\Sigma_B^{11}i_t + \Sigma_\varepsilon^{11}\right) \\ &\quad + \widetilde{v}_{nn}\left(x_t'\Sigma_A^{nn}x_t + \Sigma_\varepsilon^{nn}\right) \\ &\quad + \left(\widetilde{v}_{1n} + \widetilde{v}_{n1}\right)\left(x_t'\Sigma_{AA}^{1n}x_t + x_t'\Sigma_{AB}^{n1}i_t\right) \end{aligned}$$

where $\widetilde{v}_{ij}$ is the (i, j)th element of $\widetilde{V}$.

The Bellman equation is

$$x_t'\widetilde{V}x_t + \tilde{d} = \min_{\{i_t\}}\left\{x_t'Qx_t + \phi\left(Ax_t + Bi_t\right)\widetilde{V}\left(Ax_t + Bi_t\right) + \phi tr\left(\widetilde{V}\Sigma_{t+1|t}\right) + \phi\tilde{d}\right\} \tag{17}$$

and

$$\frac{\partial tr\left(\widetilde{V}\Sigma_{t+1|t}\right)}{\partial i_t} = 2\tilde{v}_{11}\left(\Sigma_{AB}^{11}x_t + \Sigma_B^{11}i_t\right) + \left(\widetilde{v}_{1n} + \widetilde{v}_{n1}\right)\Sigma_{AB}^{n1}x_t$$

So the first-order condition is

$$B'\left(\widetilde{V} + \widetilde{V}'\right)\left(Ax_t + Bi_t\right) + 2\tilde{v}_{11}\left(\Sigma_{AB}^{11}x_t + \Sigma_B^{11}i_t\right) + \left(\widetilde{v}_{1n} + \widetilde{v}_{n1}\right)\Sigma_{AB}^{n1}x_t = 0, \tag{18}$$

leading to the optimal interest rate

$$i_t = \widetilde{F}x_t$$

where

$$\widetilde{F} = -\left[B'\left(\widetilde{V} + \widetilde{V}'\right)B + 2\tilde{v}_{11}\Sigma_B^{11}\right]^{-1}\left[B'\left(\widetilde{V} + \widetilde{V}'\right)A + 2\tilde{v}_{11}\Sigma_{AB}^{11} + \left(\widetilde{v}_{1n} + \widetilde{v}_{n1}\right)\Sigma_{AB}^{n1}\right]$$

(Uncertainty Case)

Substituting back into the Bellman equation, we get

$$\begin{aligned} x_t'\widetilde{V}x_t + \tilde{d} &= x_t'Qx_t + \phi\left(Ax_t + B\widetilde{F}x_t\right)\widetilde{V}\left(Ax_t + B\widetilde{F}x_t\right) \\ &+\phi\widetilde{v}_{11}\left(x_t'\Sigma_A^{11}x_t + 2x_t'\Sigma_{AB}^{11}\widetilde{F}x_t + x_t'\widetilde{F}'\Sigma_B^{11}\widetilde{F}x_t + \Sigma_\varepsilon^{11}\right) \\ &+\phi\widetilde{v}_{nn}\left(x_t'\Sigma_A^{nn}x_t + \Sigma_\varepsilon^{nn}\right) \\ &+\phi\left(\widetilde{v}_{1n} + \widetilde{v}_{n1}\right)\left(x_t'\Sigma_{AA}^{1n}x_t + x_t'\Sigma_{AB}^{n1}\widetilde{F}x_t\right) + \phi\tilde{d} \end{aligned}$$

and it can be established that $\widetilde{V}$ is determined by the Ricatti equation

$$\begin{aligned} \widetilde{V} &= Q + \phi\left(A + B\widetilde{F}\right)'\left(A + B\widetilde{F}\right) + \phi\tilde{v}_{11}\left(\Sigma_A^{11} + 2\Sigma_{AB}^{11}\widetilde{F} + \widetilde{F}'\Sigma_B^{11}\widetilde{F}\right) \\ &+\phi\widetilde{v}_{nn}\Sigma_A^{nn} + \phi\left(\widetilde{v}_{1n} + \widetilde{v}_{n1}\right)\left(\Sigma_{AA}^{1n} + \Sigma_{AB}^{n1}\widetilde{F}\right) \end{aligned} \quad (19)$$

4 Influence of Parameter Estimation Uncertainty

4.1 Parameter Estimation

To assess the influence of parameter estimation uncertainty on the central banker behavior, we apply the methodology detailed in section 3 to the european central banker. Since one knows that the results obtained from an exercice of calibration are somewhat dependant upon the choice of parameter values, we choose to estimate the parameters and retrieve their variance-covariance matrix to perform the simulations.

Consequently, we estimate the model (IS and Phillips curves) by the Full Information Maximum Likelihood (FIML) method in order to recover a full covariance matrix of parameters over the 1976-2000 period. The quarterly historical data for the Euro area come from the "augmented" Area Wide Model (AWM) database provided by Fagan, Henry and Mestre (2001). Data definitions are summarized in Table 1 and represented in Figure 1.

< Insert Table 1 here >

< Insert Figure 1 here >

The hypothesis that the sum of the lag coefficients of inflation equals one had a p-value of 0.76 (obtained with the F-statistic), so this restriction was imposed in estimation. The fit and dynamics of this model compare favorably to an unrestricted VAR (Vector AutoRegressive process). Indeed, the model can be interpreted as a restricted VAR, where the restrictions imposed are not at odds with the data.

The estimated equations are shown below (coefficient standard errors are given in parentheses),

$$y_t = \underset{(0.08)}{0.16} + \underset{(0.10)}{0.85} y_{t-1} + \underset{(0.10)}{0.01} y_{t-2} - \underset{(0.02)}{0.08} \left(i_{t-1} - \pi_{t-1}\right) + \hat{\varepsilon}_{y,t}$$

$$\pi_t = \underset{(0.10)}{0.55} \pi_{t-1} + \underset{(0.11)}{0.17} \pi_{t-2} - \underset{(0.11)}{0.03} \pi_{t-3} + \underset{(0.10)}{0.31} \pi_{t-4} + \underset{(0.09)}{0.20} y_{t-1} + \hat{\varepsilon}_{\pi,t}$$

$$\sigma_{\varepsilon_y} = 0.472, \ \sigma_{\varepsilon_y} = 1.057, \ \log L = -210.226, \ -T/\log|\Omega| = 73.561$$

<Insert Figure 2 here>

As shown by the Figure 1, the empirical fit for this simple model is good. Since our purpose is to measure the influence of parameter estimation uncertainty *via* the variance-covariance matrix, we reproduce it in Table 2.

<Insert Table 2 here>

4.2 Analysis of Parameter Estimation Uncertainty

After determining the adjusted system representing the model economy and the standard deviation of their estimation error, the analysis of the effect of parameter estimation uncertainty is carried out along the lines described in Section 3.

Table 3 synthetized the reaction functions of the monetary authorities for three cases depending on the relative weights on stabilizing the output gap (λ). The primary objective of the ECB is price stability although the Maastricht Treaty does not provide for a specific definition of this objective. However, we may assimilate these objective as an inflation targeting policy. As in Svensson (2000), the case when $\lambda = 0$ and only inflation enters the loss function is called strict inflation targeting, whereas the cases when $\lambda > 0$ and the output gap (or concern about stability of the real economy in general) enters the loss function is called flexible inflation targeting. Inflation targeting obviously always involves an attempt to minimize deviations of inflation from the explicit inflation target (here 0). Whereas there may previously have been controversy about whether inflation targeting involves concern about real variability, represented by the output gap variability. This is why we present two other cases: one in which the central banker seeks to stabilize the output gap slightly

$\left(\lambda = \frac{1}{2}\right)$ and another in which the central banker seeks to stabilize inflation and the production with the same intensity ($\lambda = 1$).

<Insert Table 3 here>

The coefficients of the optimal functions are typically larger (in absolute value) than the empirical estimates, especially striking for the response to current output and inflation. We observe that the "Brainard conservatism" - the fact that the central banker becomes cautious or equivalently the fact that the parameters of the reaction function become lower - is empirically relevant in any cases. For example, when $\lambda = \frac{1}{2}$, the coefficient of F_{y_t} is equal to 12.044 under certainty and 11.284 with parameter estimation uncertainty. We also compared these results with those in which only the variances were taken into account in optimization (not reproduced here). The coefficients of the optimal rule under parameter estimation uncertainty (with full variance-covariance matrix) are always lower about 6%. Figure 3 gives the values of F_{y_t} and F_{π_t} according to λ.

<Insert Figure 3 here>

4.2.1 The Policy Response Over Time

Some features of the model are revealed by deriving the impulse response functions, which trace how each variable in the system responds to structural shocks. But these functions naturally depend on how monetary policy responds to the shocks. So we must calculate how policy responds over time to shocks to output and inflation. In the first period, the economy is hit by a 1% shock either to output or to inflation. This shock is then transmitted through the economy by the state equation. The central bank responds optimally in each period according to its reaction function (5).

Figure 4 shows the optimal responses from the model. Still, the initial response to both output and inflation shocks is very strong, whereas the impulse responses are very volatile, probably due to the fourth lags. Consequently, the Rudebusch-Svensson model leads to substantially more aggressive policy behavior than what seems to be observed in practice (but we are far away from a reasonable monetary policy model).

It is noticed first of all that parameter estimation uncertainty makes optimal policy less volatile in response to a shock. And secondly that the larger λ is and the smaller the parameters of the rule become. The responses are less volatile and so more reasonable. Consequently, taking parameter estimation uncertainty into account leads to less aggressive policy.

<Insert Figure 4 here>

4.2.2 The Implied Dynamics of the Interest Rate

As a final experiment, we can calculate the implied optimal path of the short term interest rate over the sample period and under parameter estimation uncertainty by applying the reaction function to the actual data for the euro area economy.

It is immediately clear (Figure 5a) that the model implies considerably more interest rate volatility than the evolution of the historical data. Indeed, even if the evolutions are very close (Figure 5b) on different scales, it is not a reasonable approximation of the true path. As shown by Sack (2000) and Söderström (1999), it is possible to find a relatively similar path with an unrestricted VAR. But in a VAR, a lot of coefficients are note significant and overevaluate the uncertainty (large standard errors). A simple solution in order to cure this problem is to introduce an objective of interest rate smoothing (Δi_t) in the loss function but this assumption doesn't find sufficient microfoundations to be justified, in our opinion.[5]

<Insert Figure 5 here>

5 Conclusion

Monetary policymakers, especially European, face considerable uncertainty. When taking their decisions, policymakers need to be aware of this uncertainty and factor its effects into their interest rate choices.

In this paper, we examine how the traditional linear quadratic programming may be augmented to take into account the problem of parameter estimation uncertainty in which the moments of second order represent a measure of this uncertainty. This method is quite simple to implement and permits an overview of the importance of the parameter estimation uncertainty on the structure of the reaction function.

We illustrate the control problem in analysing the impact of the uncertainty on the optimal monetary policy. Our empirical results on european data suggest that the

[5] It is written nowhere that the central banker has an unspecified objective of interest rate smoothing.

"conservatism principle" holds when we use a Rudebusch-Svensson's type of model and a standard quadratic central bank objective and that the loss function is always weaker.

The only limit of this Bayesian methodology is its impossibility (at this stage) of taking into account some forward-looking variables in the underlying model, and so it is restricted to pure backward-looking models.

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Appendix A: The Criterion Value

We consider the intertemporal loss function in quarter t,

$$\begin{aligned}
E_t(W) &= E_t \sum_{\tau=0}^{\infty} \phi^{\tau} L\left(y_{t+\tau}, \pi_{t+\tau}\right) \\
&\Leftrightarrow \\
E_t(W) &= \left[\sum_{\tau=0}^{\infty} \phi^{\tau} L\left(\bar{y}_{t+\tau}, \bar{\pi}_{t+\tau}\right) + E_t \sum_{\tau=0}^{\infty} \phi^{\tau} L\left(y^*_{t+\tau}, \pi^*_{t+\tau}\right)\right] \qquad (20) \\
&= \left[W_1 + E_t\left(W_2\right)\right]
\end{aligned}$$

where, for a variable z_t, $z^*_t = z_t - \bar{z}_t$.

The discount factor ϕ appears to be problematic. In fact when $\phi \to 1$, the sum in the above equation becomes unbounded. It consists of two elements : one corresponding to the deterministic optimization problem when all shocks are zero, (W_1), and one proportional to the variances of the shocks, $E_t(W_2)$.

If $\phi = 1$, the former component converges (because the terms approach zero quickly), and the decision problem is actually well defined also for that case. But, for $\phi \to 1$, the value of the intertemporal loss function approaches the infinite sum of unconditional means of the period loss function, $E(L_t)$, because $E_t(W) = \frac{1}{1-\phi}\left[\sum_{\tau=0}^{\infty} L\left(\bar{y}_{t+\tau}, \bar{\pi}_{t+\tau}\right) + E_t \sum_{\tau=0}^{\infty} L\left(y^*_{t+\tau}, \pi^*_{t+\tau}\right)\right]$.

Then the scaled loss function $(1-\phi) E_t \sum_{\tau=0}^{\infty} \phi^{\tau} L\left(y_{t+\tau}, \pi_{t+\tau}\right)$ approaches the unconditional mean $E(L_t)$. We follow that we can also define the optimization problem for $\phi = 1$ and then interpret the intertemporal loss function as the unconditional mean of the period loss function, which equals the weighted sum of the unconditional variances of the goal variables,

$$L\left(y_t, \pi_t\right) = Var\left(\pi_t\right) + \lambda Var\left(y_t\right)$$

We see that it is very simple to derive the loss function when $\phi = 1$ but not if $\phi \neq 1$.

If one assumes that both the parameters of the model and the state variables are known, then each of the rule is a linear function of the vector of state variables $(i_t = Fx_t)$. The dynamics of the model and the goal variables, g_t, for a given rule are then given by

$$\begin{aligned}
x_{t+1} &= Mx_t + \varepsilon_t, \\
g_t &= Cx_t
\end{aligned}$$

where $M = A + BF$ and $C = C_x + C_i F$.

For any given rule F that results in finite unconditional variances of the goal variables, the unconditional loss fulfills

$$E\left[L\left(y_t,\pi_t\right)\right] = E\left[g_t' Q g_t\right] = tr\left[Q\Sigma_g\right] \tag{21}$$

where Σ_g is the unconditional covariance matrix of the goal variables and is given by:

$$\Sigma_g = E\left[g_t g_t'\right] = C\Sigma_x C', \tag{22}$$

where Σ_x is the unconditional covariance matrix of the state variables. The latter fulfills the matrix equation

$$\Sigma_x = E\left[x_t x_t'\right] = M\Sigma_x M' + \Sigma_\varepsilon \tag{23}$$

We can use the relation $vec\left(A+B\right) = vec\left(A\right) + vec\left(B\right)$ and $vec\left(ABC\right) = \left(C' \otimes A\right) vec\left(B\right)$ to obtain

$$\begin{aligned} vec\left(\Sigma_x\right) &= vec\left(M\Sigma_x M'\right) + vec\left(\Sigma_\varepsilon\right) \\ &= \left(M \otimes M\right) vec\left(\Sigma_x\right) + vec\left(\Sigma_\varepsilon\right) \end{aligned}$$

$$\Rightarrow vec\left(\Sigma_x\right) = \left[I - \left(M \otimes M\right)\right]^{-1} vec\left(\Sigma_\varepsilon\right) \tag{24}$$

If the state variables are observed, but the parameters of the estimated model are subject to uncertainty, then equation (21) and (22) again describe the loss function and the covariance matrix of the goal variables, but in this case the covariance matrix of the state variables is given by the following equation :

$$vec\left(\Sigma_x\right) = \left[I - E\left[\left(M \otimes M\right)\right]\right]^{-1} vec\left(\Sigma_\varepsilon\right) \tag{25}$$

Remark 1 *The criterion calculus is constrained by the fact that ϕ must be near the unity and, for large models, it is very difficult to program the $M \otimes M$ matrix though the dynamic programming allows a decomposition in smaller matrix and is usable for all the value of ϕ.*

Appendix B: The Contraction Mapping Theorem

This appendix presents the mathematical definitions which allow us to rationalize the use of the numerical dynamic programming. For that, we will use infinite-horizon, discounted, time separable, autonomous dynamic programming problems to analyze the long run economic problem. The time t does not enter into the problem directly; therefore the payoff function is $L(x,i)$, and $D(x)$ is the nonempty set of feasible controls for the current state x. The state in the "next" period will be noted x^{+}, and it depends on the current state and action in a possibly stochastic, but autonomous, fashion.

As shown above, the objective of the controller is to minimize discounted expected returns $E_t \sum_{\tau=0}^{\infty} \phi^{\tau} L(x_{t+\tau}, i_{t+\tau})$ given a fixed initial value, x_0, and $\phi < 1$, the discount factor. The value function is defined by

$$J(x) = \sup_{I(x)} E \left\{ \sum_{gt=0}^{\infty} \phi^{\tau} L(x_{t+\tau}, i_{t+\tau}) \,|x_0 = x \right\}, \tag{26}$$

where $I(x)$ is the set of all feasible strategies starting at x. The value function satisfies the Bellman equation

$$J(x) = \sup_{i \in D(x)} L(x,i) + \phi E \left\{ J\left(x^{+}\right) |x,i \right\} \equiv (MJ)(x), \tag{27}$$

and the policy function, $I(x)$, solves

$$I(x) \in \arg\max_{i \in D(x)} L(x,i) + \phi E \left\{ J\left(x^{+}\right) |x,i \right\} \tag{28}$$

The key theorem in infinite horizon dynamic programming is the contraction mapping theorem applied to Bellman equation. Let use giving two preliminary definitions:

Definition 1 *A map $M : Y \rightarrow Z$ on ordered spaces Y and Z is monotone if and only if $y_1 \geq y_2$ implies $My_1 \geq My_2$.*

Definition 2 *A map $M : Y \rightarrow Z$ on a metric space Y is a contraction with modulus $\phi < 1$ if and only if $\|My_1 - My_2\| \leq \phi \|y_1 - y_2\|$.*

So, with this two definitions, the critical theorem is the contraction mapping theorem for dynamic programming that is underlying for all the results in this paper:

Theorem 1 *If X is compact, $\phi < 1$, and L is bounded above and below, then the map*

$$MJ = \sup_{i \in D(x)} L(x,i) + \phi E \left\{ J\left(x^{+}\right) |x,i \right\} \tag{29}$$

is monotone in J, is a contraction mapping with modulus ϕ in the space of bounded functions, and has a unique fixed point.

Table 1: Quarterly Data, 1976:1-2000:4

Variable	Definition
y_t	Output gap: $100 \times \log(Q_t - Q_{pot})$
π_t	Inflation rate: $400 \times \Delta \log(p_t)$
i_t	Short term nominal interest rate

The output gap is defined as the log deviation of real GDP (Q_t) to potential output (Q_{pot}). Potential output is assumed to be given by a constant-returns-to-scale Cobb-Douglas production function (see Fagan and *al.* (2001)). p_t is the GDP deflator.

Table 2. Variance-Covariance Matrix (coefficient×1000)

	α_1	α_2	β	γ	δ_1	δ_2	δ_3	δ_4
α_1	9.423	-8.280	0.426	0.121	-0.051	0.136	-0.0911	0.013
α_2	-	9.076	-0.257	0.026	0.049	-0.132	0.104	-0.012
β	-	-	0.402	-0.001	-0.029	0.018	0.003	0.005
γ	-	-	-	7.633	-1.055	-1.299	0.626	2.260
δ_1	-	-	-	-	9.352	-5.561	-2.401	-1.237
δ_2	-	-	-	-	-	12.750	-4.502	-2.664
δ_3	-	-	-	-	-	-	12.459	5.506
δ_4	-	-	-	-	-	-	-	9.505

Table 3. Optimal Reaction Function Parameters

Case	F_{y_t}	$F_{y_{t-1}}$	F_{π_t}	$F_{\pi_{t-1}}$	$F_{\pi_{t-2}}$	$F_{\pi_{t-3}}$
$\lambda = 0$						
Certainty-equivalent	17.45	0.113	31.198	3.128	18.383	10.978
Parameter uncertainty	14.860	0.068	22.115	2.864	13.595	7.805
$\lambda = \frac{1}{2}$						
Certainty-equivalent	12.044	0.113	8.118	2.817	2.692	2.347
Parameter uncertainty	11.284	0.068	7.380	2.619	2.627	2.116
$\lambda = 1$						
Certainty-equivalent	11.607	0.113	6.063	2.079	1.778	1.649
Parameter uncertainty	10.890	0.068	5.594	1.940	1.759	1.504

Figure 1. European Data Series

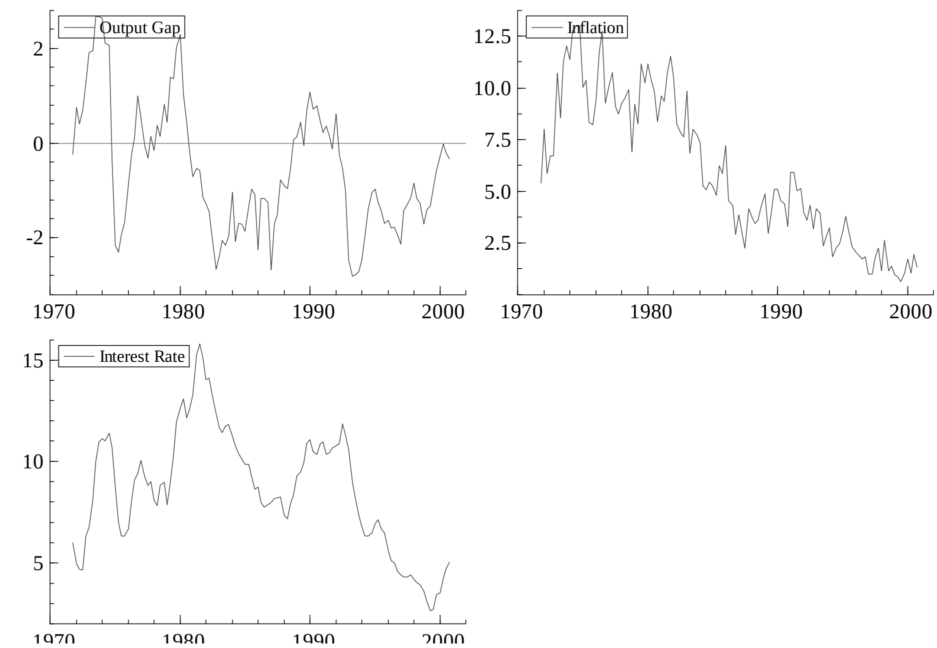


Figure 2. Historical and Estimated Series

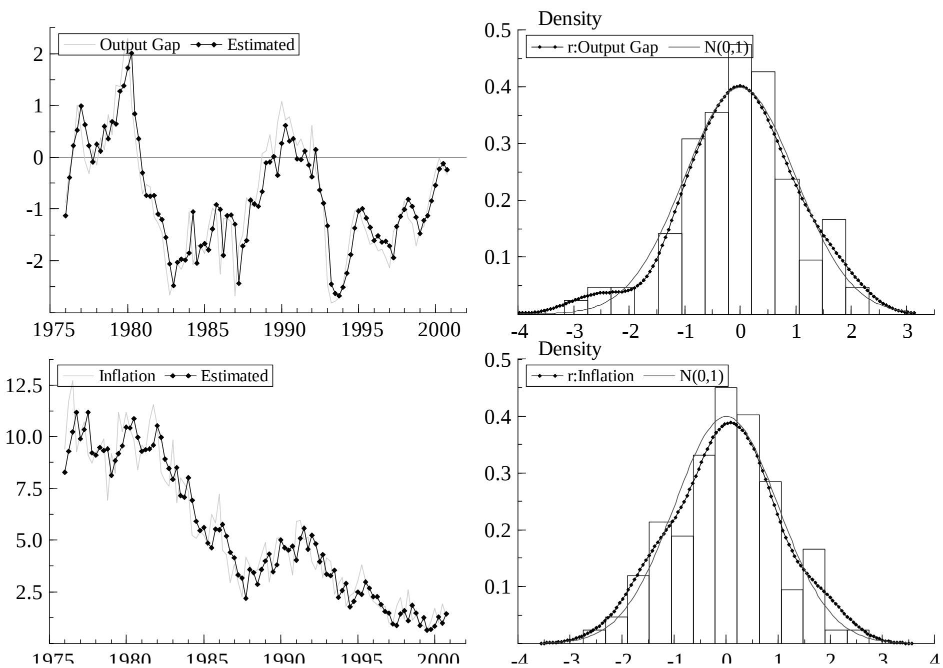


Figure 3. Response to Output and Inflation

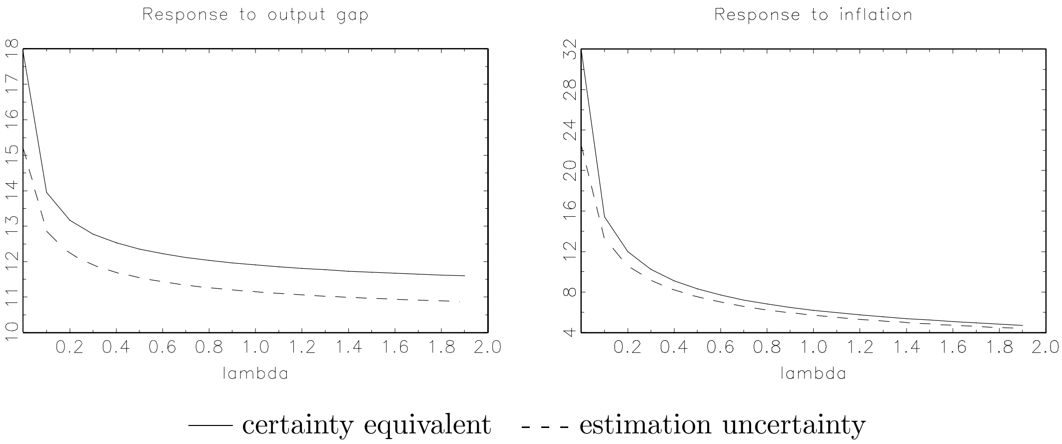


Figure 4. Optimal Policy Response to Shocks (with $\lambda = 0$)

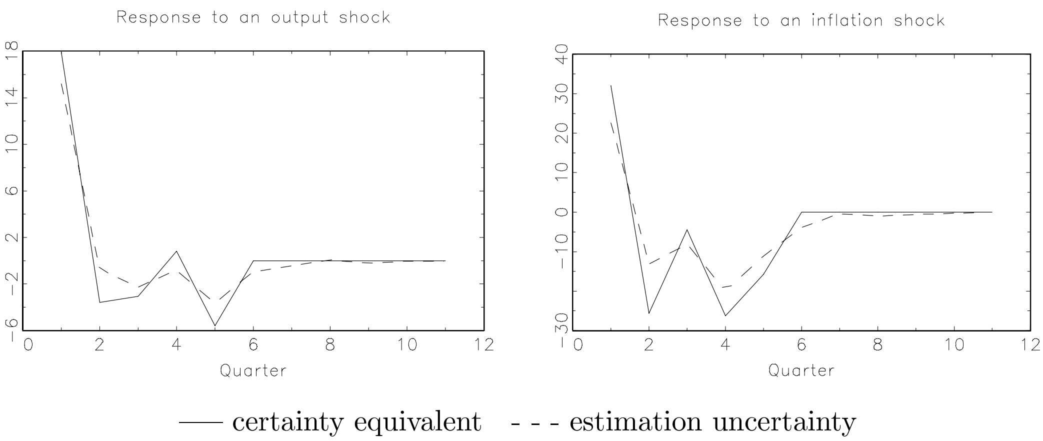
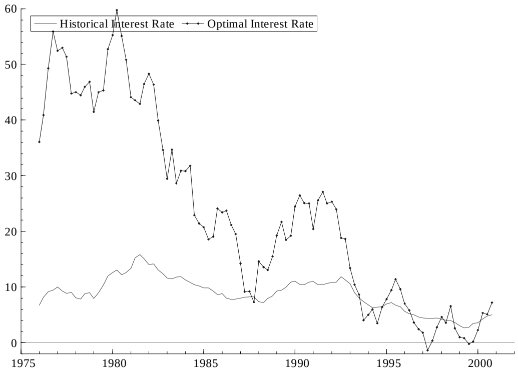
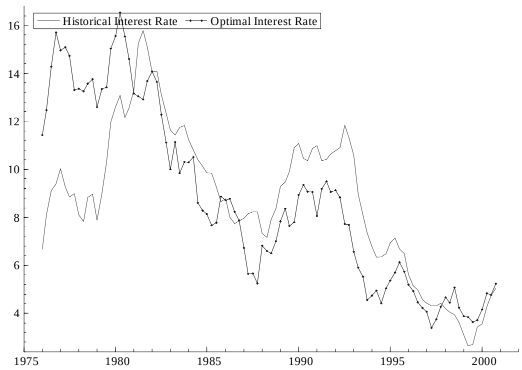


Figure 5. Historical and Optimal Interest Rates



(*a*) Same scale



(*b*) Different scales

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