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Abstract

This note applies H_∞ methods to an estimated hybrid monetary policy model (*cf.* Sahuc, *Applied Economics Letters*, **9**, pp. 949-55, 2002) and derives standard and robust optimal interest rate rules. We find that the "robust" central banker responds more aggressively than it does without concerns for robustness and that the responses of the state variables in this case are not necessarily always stronger than the standard ones.

Keywords: Euro area, robustness, uncertainty, optimal monetary policy

JEL Classifications: C32, E17

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1 Introduction

The advances in the field of monetary policy making have been remarkable in recent years. However some fundamental problems about policy implementation are still present. Even when practitioners tried to build and evaluate realistic stochastic general equilibrium models in the perspective to help the monetary authorities policy choices, these last are continually faced to uncertainty.[1] As explained by Issing *et al.* (2001, chap.7), this is particularly true for the European Central Bank (ECB) which must manage the misspecification about its underlying aggregated model and heterogeneity between the area members when implementing its optimal strategy. The result is a particularly high level of uncertainty in the way the single monetary policy affects the economy.

Diverse literatures on uncertainty have tried to deal with this problem, especially the Bayesian and the robust control approaches. They both consider decision makers who do not know the true model economy, but only the second approach represents model uncertainty by having a decision maker who faces a set of models that he refuses to reduce to a single specification in assigning probabilities over models in set. This non-Bayesian decision maker has a *minimax* behaviour and chooses a robust plan that is optimal against the worst case model specification within the range.

The purpose of this paper is then to derive robust optimal monetary policy rules which are capable of taking into account uncertainty and to discuss their properties. To do so, we resort on the notion of robustness well detailed by Hansen and Sargent (2002). The concept of robustness is defined in section 2 and applied in section 4 to a rational expectations framework of the euro area presented in section 3.

[1]See Rudebusch (2001) or Sack and Wieland (2001) for well documented surveys.

2 Robust decision theory and optimal policy rules

Robust decision theory allows to decompose model uncertainty in structured and unstructured uncertainty. Svensson (2000) or Giannoni (2001), among others, present the structured model uncertainty where the authority is assumed to face Knightian uncertainty about specific parameters within a reference model that is otherwise taken to be approximately correct. In this case the best the decision maker can do is to specify a neighborhood for the uncertainty and to assign boundaries for the true model parameters. But, as Hansen and Sargent (2002) have argued, model uncertainty in the sense of Knight may be generalized and an unstructured model uncertainty may be specified. The assumption that the model misspecification is known *a priori* is then dropped: uncertainty may comes from the unknown parameters, the functional form or the standard errors of the model shocks. This section takes up Hansen and Sargent's definition of a robust Ramsey plan in the case of forward-looking models.

Let x_t be a vector of state variables (predetermined or non-predetermined) and i_t a vector of controls. Define the one-period loss function

$$L(x_t, i_t) = \left(x_t' Q x_t + i_t' R i_t\right) \tag{1}$$

with Q and R both being symmetric and positive semi-definite matrices.

Normally, the policymaker wants to solve

$$\min_{\{i_t\}_0^\infty} E_0 \sum_{t=0}^{\infty} \beta^t L(x_t, i_t) \tag{2}$$

subject to the private sector behaviour summarized by the law of motion (called approximating model),

$$x_{t+1} = A x_t + B i_t + C \xi_{t+1} \tag{3}$$

where β is the discount factor, $\{\xi_{t+1}\}$ is an i.i.d. Gaussian vector process with mean zero and identity contemporaneous covariance matrix.

However, when the policymaker thinks there is some model misspecification, he regards the model (3) as approximating another model that he cannot specify, such that all specification errors appear to the authority as another vector of residuals $\{w_{t+1}\}$ representing the approximation errors variable.[2] The idea is that the misspecification is of unknown origin, but it will show up as outsized and deleterious residuals when the policymaker attempts to exploit the model, conditional on the estimated parameters, to achieve its policy goals. The dynamic misspecification (called distorted model) is then

$$x_{t+1} = Ax_t + Bi_t + C\left(\varepsilon_{t+1} + w_{t+1}\right) \tag{4}$$

where $\varepsilon_{t+1} \sim N(0, I)$ represents the standard vector of residuals.

When equation (4) generates the data, the errors ξ_{t+1} in (3) are distributed as $N(w_{t+1}, I)$ rather than as $N(0, I)$. The assumption that the reference model is taken only to be an approximation of the truth is captured by the added constraint,

$$E_0 \sum_{t=0}^{\infty} \beta^{t+1} w'_{t+1} w_{t+1} \leq \eta_0 \tag{5}$$

The term on the left side is the relative entropy which measures the model misspecification, defined as the ratio of the transition density associated with distorted model and the transition density associated with approximating model (conditional on x_t). The term on the right side represents the magnitude of uncertainty: the smaller η_0 and the better the approximation of the reference model will be.

Finally, this robust Ramsey problem is obtained as the solution of the zero-sum two-player game:

$$\min_{\{i_t\}_0^\infty} \max_{\{w_{t+1}\}_0^\infty} E_0 \sum_{t=0}^{\infty} \beta^t \left(x'_t Q x_t + i'_t R i_t - \beta\theta w'_{t+1} w_{t+1}\right) \tag{6}$$
$$\text{s.t. } x_{t+1} = Ax_t + Bi_t + C\left(\varepsilon_{t+1} + w_{t+1}\right)$$

[2] $\{w_{t+1}\}$ is a vector process that can feed back in a general way on the history of x_t: $w_{t+1} = g_t(x_t, x_{t-1}, ...)$,where $\{g_t\}$ is a sequence of measurable functions.

where $\theta \in (\underline{\theta}, +\infty)$ is a non negative Lagrange multiplier that reflects the agent's preferences of robustness (see the Appendix).

Let define operators $\mathcal{T}$ and $\mathcal{D}$ which emerge respectively from the minimisation over i_t and the maximisation over w_{t+1},

$$\begin{aligned} \mathcal{T}(P) &= Q + \beta AP \left(I + \beta B \left(R + \beta B'PB \right)^{-1} B'P \right) A \qquad (7) \\ \mathcal{D}(P) &= P + PC \left(\theta I - C'PC \right)^{-1} C'P, \qquad (8) \end{aligned}$$

and by noting P the fixed point of operator $\mathcal{T} \circ \mathcal{D}$, the associated Bellman equation leads to the Riccati equation:

$$P = \mathcal{T} \circ \mathcal{D}(P), \qquad (9)$$

The solution of the multiplier problem (6) is then

$$\begin{aligned} i_t &= -\left[\beta \left(R + \beta B' \mathcal{D}(P) B \right)^{-1} B' \mathcal{D}(P) A \right] x_t \qquad (10) \\ &= F_1 x_t \\ w_{t+1} &= \left[\theta^{-1} \left(I - \theta^{-1} C'PC \right)^{-1} C'P \left(A - BF_1 \right) \right] x_t \qquad (11) \\ &= F_2 x_t \end{aligned}$$

3 The Euro area economy model

To perform the robustness analysis, we stand into the solid microfoundations framework called "New-Neoclassical Synthesis" described by Clarida *et al.* (1999) and Woodford (2002). Unfortunately it is common knowledge that these models fail to fit the data well. In order to circumvent this ecueil, Sahuc (2002) derives and estimates a "hybrid" monetary policy model - based on habit formation and rule of thumb firms behaviour - that is more consistent with the data properties. This model is taken as the approximation of the euro area economy and consists of two equations estimated by maximum likelihood using euro area annual data from 1974 to 2000:

$$\begin{aligned} y_t &= \underset{(0.16)}{0.51} E_t y_{t+1} + \underset{(-)}{0.49} y_{t-1} - \underset{(0.03)}{0.06} \left(i_t - E_t \pi_{t+1} \right) + \varepsilon_{y,t} \\ \pi_t &= \underset{(0.08)}{0.46} E_t \pi_{t+1} + \underset{(-)}{0.54} \pi_{t-1} + \underset{(0.03)}{0.06} y_t + \varepsilon_{\pi,t} \end{aligned}$$

where y_t is the output gap measured as the percentage deviation from a linear trend, π_t is the inflation rate at an annual percentage change in GDP deflator and i_t is the annual short-term interest rate. The first aggregate relationship is a hybrid IS curve and the second is a hybrid Phillips curve. In each equation, we supposed that the sums of the coefficients relating to the past and the future of the endogenous variables were equal to one. The two shocks are supposed to follow first-order autoregressive processes with i.i.d. Normal error terms: $\varepsilon_{k,t} = \rho_\varepsilon \varepsilon_{k,t-1} + e_{k,t}$, $k = y, \pi$. At this stage, we retain $\rho_\varepsilon = 0$.

We assume that the European Central Bank objective is to minimise the expected value of a loss criterion which is quadratic in inflation, the output gap and the interest rate,

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda_y y_t^2 + \lambda_i i_t^2 \right) \right\},$$

where the representative agent's discount factor β is fixed to 0.95, $\lambda_y = 0.1$ and $\lambda_i = 0.1$ are weights placed on the stabilisation of the output gap and the nominal interest rate.[3] The welfare theoretic justification for this form of loss function is given in Woodford (2002, chap.6) where parameters are related to those of the model of the structural model. In each period, the problem facing to the central bank is to find the nominal interest plan so as to minimise the intertemporal loss function, given the state of the world and the monetary policy transmission uncertainties.

The intertemporal optimisation of forward-lookingness economic policy is subject to the problem of time inconsistency, that is the policymaker have incentives to

[3] Because price stability is the primary objective adopted by the ECB, we suppose that the policymaker has a little incentive to stabilise the output gap and the interest rate.

deviate, in later periods, from the optimal plan obtained in the first period. In contrast, discretionary policies are obtained through policy optimisation separately in each period and are time consistent; but the resulting decisions plan won't lead to the optimum of the economy and we can quantitatively measured the inflation bias and stabilisation bias that are associated to them.[4] Without tools to determine the true policy, we present the two standard alternatives: commitment and discretion.[5]

4 Simulations and results

We use the above results to study the properties of the robust monetary policy into the euro area. Table 1 shows the outcomes for the proposed euro area model. Several experiments and simulations (available upon demand) have been carried out and provided the following results:

Result 1 Robustness leads to more aggressive policies under commitment or discretion. Robust rules have coefficients that are always larger than those of the standard ones (Table 1).

Result 2 Dynamic responses of the economy to demand and supply shocks (Figures 1 and 2) show that on impact, robust optimal rules don't involve systematically a stronger response than under $\theta = +\infty$. In fact, these responses may be quite

[4] Some work have shown that the gains from committing may be significant and to answer its time inconsistency Woodford (2002) has proposed monetary policy making based on the "timeless perspective". This is a rule based policy that is obtained by respecting the optimality conditions from the full intertemporal optimisation under commitment, except for the current decision-making period. Unfortunately, as shown by Jensen and McCallum (2002), this policy is not unique and not always better than the consistent alternative.

[5] Computation of the commitment policy under robustness is proposed by Hansen and Sargent (2002) though those of the discretionary policy is proposed by Giordani and Söderlind (2002).

smooth, as in the commitment case, or may be larger during the first periods but return very quickly to steady state, as in the discretion case.

Result 3 Less aggressive robust rules can be derived by increasing the persistence of the two shocks. Table 2 shows that when $\rho_\varepsilon \to 1$, the relative variations between parameters of robust and standard rules tend towards one. They generally allow a quicker stabilisation of the economy.

Result 4 Varying the loss function weights doesn't change qualitatively the three preceding results but modifies the time of stabilisation of the economy.

A common intuitive view is that the introduction of uncertainty should make policy makers cautious, however, uncertainty needs not result in cautiousness. Indeed, the typical observation that robustness leads to a more aggressive optimal policy rule is verified in the context of our Euro area model. But a potentially interesting result is that robustness may stabilise the economy very efficiently following persistent shocks since large effects are well taken into account by the robust control methodology, in searching a "safe" policy which secures a guaranteed performance. However, as explained by Onatski and Williams (2002), we must be careful to the structure of uncertainty and how a robust policy rule is designed. In others words, different control rules will lead to different magnitudes of uncertainty and empirical divergences that we may provide from the choice and characteristics of the model economy or the fact that (in our case) all sources of uncertainty are absorbed into an additional shock and collapse uncertainty into shock uncertainty.

5 Conclusion

In order to take into account uncertainty underlying the Euro area, this note applies robust control methodology developed by Hansen and Sargent (2002) to a model based on rational expectations hypothesis. Works on robust theory generally find that the parameters associated to the robust rule are always higher than those of the standard rule and that robustness leads to higher contemporaneous reactions of all variables. We certainly find a more aggressive robust rule (under commitment and discretion) but this aggressive policy doesn't entail stronger reactions to shocks.

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Appendix: Detection error probability

In order to find a reasonable value for θ, Hansen and Sargent design a detection error probability function by a likelihood ratio. Consider a fixed sample of observations on the state x_t, $t = 0, ..., T-1$, and let L_{ij} be the likelihood of that sample for model j assuming that model i generates the data, the likelihood ratio is

$$r_i \equiv \log\left(\frac{L_{ii}}{L_{ij}}\right) \tag{12}$$

where $i \neq j$ and $i = A$ (approximating model), D (distorted model). When model i generates the data , r_i should be positive. Define

$$\begin{aligned} p_A &= prob\,(\text{mistake}|A) = freq\,(r_A \leq 0)\,, \\ p_D &= prob\,(\text{mistake}|D) = freq\,(r_D \leq 0)\,. \end{aligned}$$

Thus p_A is the frequency of negative log-likelihood ratios r_A when model A is true and p_D is the frequency of negative log-likelihood ratios r_D when model D is true. Attach equal prior weights to model A and D, the detection error probability can be defined as

$$p(\theta) = \frac{1}{2}(p_A + p_D) \tag{13}$$

When a reasonable value of $p(\theta)$ is chosen, a corresponding value of θ can be determined by inverting the probability function defined in (13). θ can be defined as the negative inverse value of the so-called risk-sensitive parameter σ, that is $\theta = -\frac{1}{\sigma}$.

The detection error probability is a statistic concept designed to spell out how difficult it is to tell the approximating model apart from the distorted one. The larger the detection error probability, the more difficult to tell the two models apart. In the extreme case, when it is 0.5 ($\theta = +\infty$), the two models are identical. So a policymaker can choose a θ according to how large a detection error probability he wants. If the detection probability is very small, that means, it is quite easy to tell the two models apart, it does not make much sense to design a robust rule.

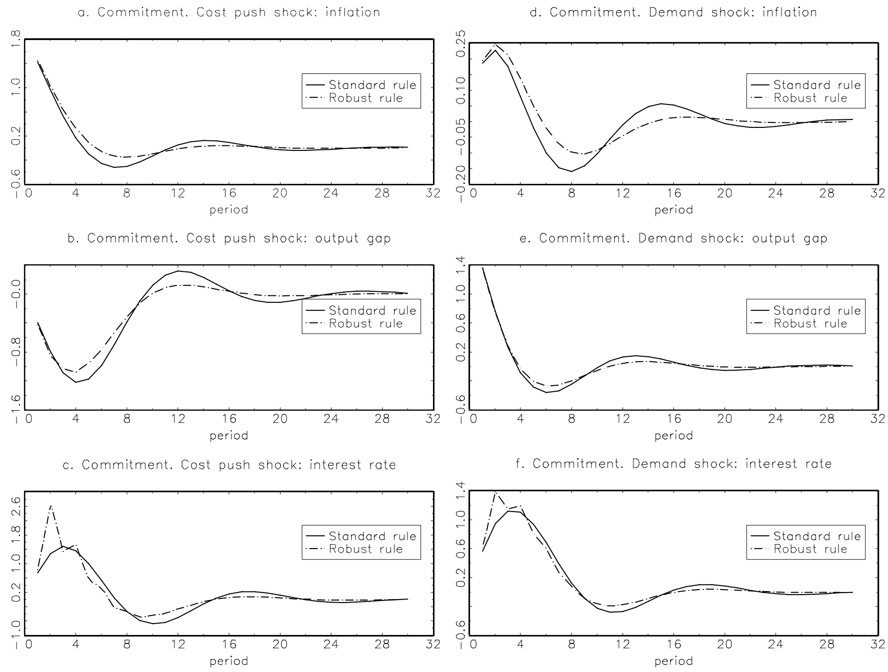

Figure 1. Impulse responses under commitment

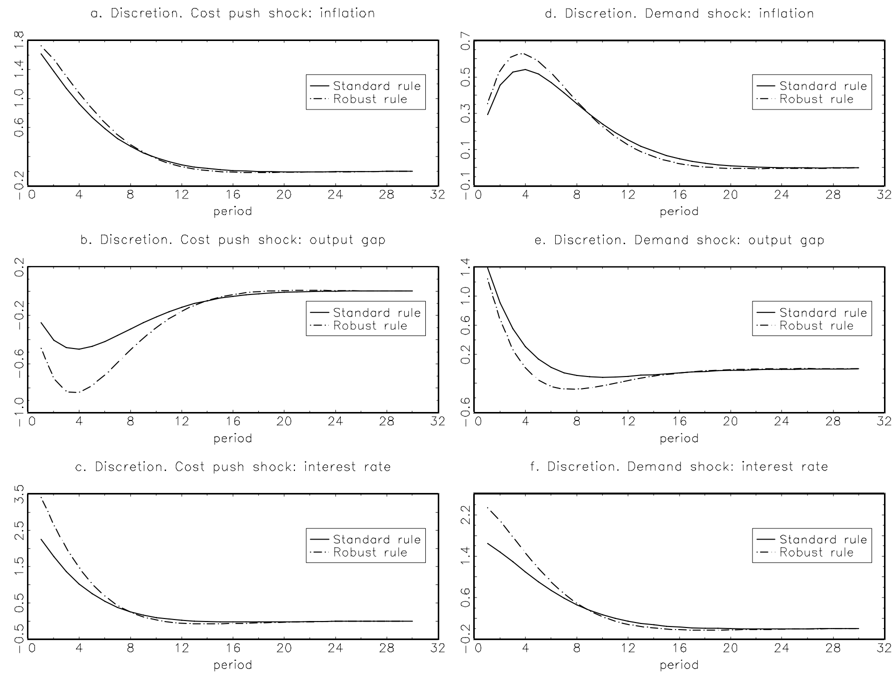

Figure 2. Impulse responses under discretion

Table 1. Standard and robust monetary policy rules

Rules ($i_t =$)	$\varepsilon_{\pi,t}$	$\varepsilon_{y,t}$	π_{t-1}	y_{t-1}	$\mu_{1,t}$	$\mu_{2,t}$
Commitment						
Standard	0.746	0.567	0.403	0.278	-0.118	-0.864
Robust	0.838	0.613	0.453	0.301	-0.124	-0.856
Discretion						
Standard	2.268	1.645	1.224	0.806	-	-
Robust	3.414	2.339	1.844	1.147	-	-

The rules under commitment depend not only on the current state of the economy, but also on the Lagrange multipliers on the forward-looking variables $(\mu_{i,t},\ i = 1, 2)$. In contrast, under discretionary optimisation, the optimal rules depend only on the predetermined variables.

Table 2. Relative variations between parameters of robust and standard rules

Parameters	$\varepsilon_{\pi,t}$	$\varepsilon_{y,t}$	π_{t-1}	y_{t-1}	$\mu_{1,t}$	$\mu_{2,t}$
$\lambda_y = 1, \lambda_i = 1, \rho_\varepsilon = 0$						
Commitment	1.233	1.110	1.234	1.110	1.166	1
Discretion	1.411	1.343	1.411	1.343	-	-
$\lambda_y = 1, \lambda_i = 1, \rho_\varepsilon = 0.5$						
Commitment	1.195	1.106	1.167	1.075	1.083	1
Discretion	1.209	1.251	1.205	1.170	-	-
$\lambda_y = 1, \lambda_i = 1, \rho_\varepsilon = 0.9$						
Commitment	1.068	1.057	1.049	1.030	1.083	1
Discretion	1.017	1.016	1.019	1.014	-	-
$\lambda_y = 0.2, \lambda_i = 0.1, \rho_\varepsilon = 0$						
Commitment	1.159	1.056	1.161	1.057	1.055	1.005
Discretion	1.516	1.330	1.516	1.313	-	-
$\lambda_y = 0.2, \lambda_i = 0.1, \rho_\varepsilon = 0.5$						
Commitment	1.124	1.032	1.097	1.048	1.055	1.003
Discretion	1.286	1.198	1.268	1.169	-	-
$\lambda_y = 0.2, \lambda_i = 0.1, \rho_\varepsilon = 0.9$						
Commitment	1.033	1.029	1.032	1.014	1.012	1
Discretion	1.038	1.026	1.031	1.014	-	-
$\lambda_y = 0.1, \lambda_i = 0.1, \rho_\varepsilon = 0$						
Commitment	1.123	1.103	1.125	1.083	1.059	1.009
Discretion	1.505	1.422	1.506	1.423	-	-
$\lambda_y = 0.1, \lambda_i = 0.1, \rho_\varepsilon = 0.5$						
Commitment	1.070	1.082	1.049	1.055	1.042	1.005
Discretion	1.274	1.231	1.253	1.217	-	-
$\lambda_y = 0.1, \lambda_i = 0.1, \rho_\varepsilon = 0.9$						
Commitment	1.053	1.067	1.042	1.036	1.008	1.005
Discretion	1.043	1.039	1.035	1.031	-	-

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