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**Pace versus Type:
The Effect of Economic Growth
on Unemployment and Wage Patterns?**

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Pace versus Type: The Effect of Economic Growth on Unemployment and Wage Patterns[*]

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Abstract

Much of the literature on growth and unemployment has emphasized the effect of the increasing pace of technological progress on job instability and wage inequality. But less attention has been placed on how the change in the very nature of work, due to technical change, affects the rate of job destruction, and hence the level of unemployment. We argue that technological progress modifies on-the-job learning and, through general equilibrium effects, unemployment and wage dispersion. In the context of the canonical Mortensen and Pissarides [1998] model, we show that, in a routine world, this "on–the-job learning effect" can offset the creative destruction effect induced by an increase in the pace of technological change on unemployment, whereas it can amplify it as jobs become less routine. Moreover, the relationship between wage dispersion and growth can be non monotone. This finding helps explain the wage compression/expansion observed in time series data.

1 Introduction

Economists have been interested in studying the relationship between growth and unemployment at least since the beginning of the industrial revolution. In recent years, an important branch of the literature has examined the influence of the rate of technological change on the level of unemployment. But less attention has been placed on the effect of the *nature* of technological progress on job instability, unemployment and wage patterns. This article addresses this issue in a matching framework (Diamond [1982], Pissarides [1990]).

The standard matching model with technological progress (Pissarides [1990], Postel-Vinay [1998]) predicts that a faster rate of technological progress is likely to reduce unemployment through a "capitalization effect": faster growth raises the returns to firm creation, more firms are encouraged to enter and the job finding rate of unemployed workers rises. In Schumpeterian models of growth (Aghion and Howitt [1992], Aghion and Howitt [1994], Mortensen and Pissarides [1998], Postel-Vinay [2002]), however, another effect works in the opposite direction, namely a "creative destruction effect": faster technological change goes along with faster obsolescence of technologies, inducing more intense labor turnover and higher frictional unemployment. Mortensen and Pissarides [1998] show that the first effect dominates when firms are able to update their technology continuously, whereas the second effect rests on the assumption of irreversibility in the firms' technological choices.

Common to all these articles is the fact that they ignore what happens during the employment relationship, whereas a number of empirical studies emphasize the change of nature of work induced by innovations. Givord and Maurin [2003] for example show that the increase of job instability results from the impact of technical progress on the internal organization of firms, and more precisely from more substitutability between new recruits and seniors. Actually, the introduction of technological innovations within firms may affect the nature of work in many ways[1]. We argue that this change may influence job instability, and hence the level of unemployment.

We focus on one aspect of this process, the impact of technological improvements on learning. Technical change is indeed likely to modify the labor-learning[2]. After taylorism and automatization, technical progress is now associated with the end of routine for some activities. Jobs are on average more and more complex. A worker is now less likely than, say, twenty years ago to perform a unique task throughout his career. He faces new problems for which ready-made solutions do not exist and he has to find original solutions[3]. Analyzing the change of *nature* of technological progress, Lindbeck and Snower [2000]

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[1] For an overview of the evidence, see Lindbeck and Snower [2001]. The contemporary reorganization of work consists mainly in delegating responsibility to production workers, promoting team-work, job-rotation and increasing the average number of tasks per employee.

[2] See for instance Bahk and Gort [1993] for a distinction between labor learning, capital learning and organization learning.

[3] As Autor, Levy, and Murnane [2001] note, computerization is associated with a decline in demand for routine manual and cognitive tasks. In the same line, Maurin and Thesmar [2003] argue that "new technologies make it possible to allocate

separate two types of learning: "intratask" and "intertask" learning. Intratask learning corresponds to learning by doing in the traditional sense (Arrow [1962]): the more time a worker spends at a particular task, the more skillful he becomes at performing that task. Intertask learning, by contrast, arises when a worker can use the expertise accumulated at one task to improve his performance at other tasks. According to these authors, the change of nature of technical progress enhances the intertask learning but slowdowns the intratask learning. Depending on the sector and the organization of firms, the movement away from routinized work can be associated with a faster or a longer on-the-job learning process. We assume that the present change of nature of work is mainly associated with a longer one[4]. This point of view is consistent with the finding of Greenwood, Hercovitz, and Krusell [1997] that, after accounting for improvements in capital-embodied technology, the slowdown in the productivity of other factors has been important.

A number of papers have analyzed the interplay between growth and learning. Chari and Hopenhayn [1991] present an overlapping generation model with ongoing technological improvement and investment in technology-specific human capital. They study the lag between the appearance of a technology and its peak usage. Parente [1994] and Jovanovic and Nyarko [1996] consider the interactions between learning-by-doing and technological choices and the timing of adoption of new technologies in models without labor market frictions[5]. Laing, Palivos, and Wang [1995] develop an endogenous growth model with such frictions. They show how education choices, which determine workers' ability to accumulate on-the-job expertise, interact with wage determination and affect the growth rate in a search economy. As we focus on job stability, we depart from these authors by assuming that the optimal duration of a job, and consequently how long on-the-job learning lasts, is endogenously determined.

Our model has the main following features. First, the duration of jobs is an endogenous variable, which is used as an indicator of instability. We follow Mortensen and Pissarides [1998] formalization. The productivity associated with the technological frontier evolves following an exogenous constant rate of technological change. When a vacant job is filled, the firm equips its employee with the most recent available technology. The crucial assumption is that this technological choice is irreversible. In our model, job destruction is motivated solely by the desire to break-up a persistently unproductive employment relationship, thereby allowing the firm and the worker to seek out more productive new relationships on the matching market. Second, learning is firm-specific and resembles the formulation of Parente [1994].

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more human and material resources to non-routine activities". They test the assumption of an increasing decline of the most programmable activities in favor of those that are the least programmable.

[4] Nevertheless, at a more disaggregated level, we cannot preclude that in some economic sectors, innovations improve accessibility of technologies. Technologies become more user friendly (see for example Galor and Tsiddon [1997]), which means again a change in the learning process, which then becomes faster.

[5] In Parente [1994] and Jovanovic and Nyarko [1996], the transferability of a part of the expertise accumulated by working in a firm has a crucial role. By contrast, in our model, the knowledge accumulated on a job cannot be transferred to another job. We do not enter the debate on the impact of technological change on the transferability of skills (See for example Violante [2002] or Aghion, Howitt, and Violante [2000]).

In Mortensen and Pissarides [1998], the productivity of workers is constant. By contrast, in our model, as long as a firm continues to use a technology, its employee gains expertise in that technology and operates it more efficiently. Job productivity therefore changes overtime, with consequences on the optimal length of a job and on the correlation between growth and unemployment. The introduction of a learning process has moreover profound implications for the life-cycle wage pattern and the aggregate wage distribution.

The impact of a change in the learning process on job security works through several channels. A decrease in the pace of skill acquisition induces a loss in productivity that may lead to a shortening of a job's lifetime. The subsequent reduction in worker productivity discourages firms to create jobs. In turn, this leads to a deterioration of the labor market conditions faced by workers. Crucially, this lowers the equilibrium wage and has an offsetting effect of encouraging entry by firms. One of the main points of this article is to show that this latter effect may dominate and to determine the conditions under which it may do so. For instance, in a routine world, this effect can offset the impact of an increase in the pace of technical change, whereas it can amplify it as jobs become less humdrum. This can explain why job security may remain relatively stable during a period in which technical change evolves.

An increase in the pace of technical change may induce a decrease in wage dispersion, depending on the speed of learning. Indeed, there are two sources of heterogeneity in the model: the different vintage of machines used by workers and the seniority of these workers. New machines are always better than old ones but as long as a firm continues to use its current technology, its employee accumulates expertise in that technology. We then have two effects: first, an increase in the pace of technical change means faster obsolescence of installed technologies. The productivity gap between two vintage technologies widens. Therefore, the heterogeneity of wages increases among jobs. Second, any increase in the pace of technical change means a shorter learning time for all jobs, just because it is then more profitable to shorten the lifetime of jobs in order to take advantage of new technologies. Therefore, the impact of the second source of wage heterogeneity is reduced. Indeed, we observe a non-monotone relationship between technical change and wage gap. For either high or low speeds of the learning process, an increase in the pace of technical change increases the wage inequalities as usual in the literature (see Violante [2002]). But, when the speed of learning is in a middle range, an increase in the rhythm of technical change decreases the wage inequalities. This may rationalize the finding of Katz and Murphy [1992], that the wage differential between skilled and unskilled labor narrowed between 1940 and 1960 in the United States, and has been widening in the seventies and eighties[6].

The paper proceeds as follows. The setup of the model is described in the second section. Section 3 analyzes the case of the adoption of new technology through job destruction only. We focus there on the impact of a change in the rhythm or nature of the technical progress on unemployment and the length of job relations. Section 4 presents a study of the evolution of wage inequality in this framework. Section 5

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[6]In a very different framework, Laing, Palivos, and Wang [2003] present an alternative insight on the compression and expansion of the aggregate wage distribution observed in time series data. See also Acemoglu [2003].

4

presents a more general case, where technologies can be retooled. A last section offers some concluding remarks.

2   The framework

The basic framework is similar in many respects to Mortensen and Pissarides [1998]. We consider an economy populated by a continuum of firms and workers. The total measure of workers is equal to one. We focus our attention on the case of a particular industry and we assume that the total measure of firms is larger than one. All workers have the same level of education (general human capital), denoted by x, but at each point in time, workers can differ in terms of productivity level, as will be clear below.

2.1   Production

Production takes place within single worker/firm pairs. At the time of a match between a worker and a firm, the firm equips its employee with a machine embodying the technological frontier. The productivity of the technological frontier evolves at an exogenous constant rate, g. A crucial assumption is that the technological choice is irreversible. The output produced by a worker hired at date τ on a job of tenure T is denoted:

$$y\left(\tau,T\right) = p\left(\tau\right)\chi\left(T\right)$$

where $p\left(\tau\right) = e^{g\tau}$ is the level of the technological frontier at date τ, and $\chi\left(T\right)$ represents the productivity of the worker in a job of tenure T. As long as a firm continues to use its current technology, its employee accumulates expertise in that technology, as a result of learning, so that the productivity of this worker rises when time elapses. We borrow the specification of learning from Parente [1994]. The increase of productivity due to learning is assumed to occur at a decreasing rate [7], depending on the value of the parameter γ. The law of motion governing the increase in productivity is

$$\dot{\chi}\left(T\right) = \gamma s - \gamma\chi\left(T\right), \quad \gamma \geq 0 \quad \text{and } \chi\left(0\right) = x$$

Hence,

$$\chi\left(T\right) = s - \left(s - x\right)e^{-\gamma T} \tag{1}$$

Any worker starts at a level x of productivity, and is able to reach at best a level s of productivity. In this section, a firm keeps the same technology until the job is destroyed. The parameter γ reflects the rhythm of learning within job. The higher γ is, the shorter the period of learning.

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[7] See also Laing, Palivos, and Wang [1995]. In their framework, γ depends positively upon the level of schooling, which is homogenous for all workers in this paper.

Note that the knowledge accumulated on a job can not be transferred to another job, or machine. This knowledge is a specific skill gained by the memorization, routine and automation of tasks. We depart then from the debate on the impact of technical progress on the transferability of skill. We focus here on a change of the degree of automation of tasks.

2.2 The labor market

We use a matching process in the spirit of Diamond [1982] and Pissarides [1990]. At each moment in time a mass u of unemployed workers and a mass v of vacant jobs coexist on the labor market. Then jobs and workers meet pairwise at a Poisson rate $M(u,v)$, where $M(u,v)$ is the matching function. It represents the number of matches per unit time. The function $M : \Re_+^2 \to \Re_+$ is traditionally assumed to be strictly increasing and concave, exhibiting constant returns to scale. Furthermore it has the following properties : $M(0,v) = M(u,0) = 0$ and satisfies the Inada conditions. With these assumptions, we are able to define the Poisson probability for a firm posting a vacancy to interview a worker as follows: $M(u,v)/v = M(\theta,1)/\theta \equiv m(\theta)/\theta$, decreasing with respect to θ, where $\theta \equiv v/u$ represents the labor market tightness ratio. Symmetrically, the Poisson probability for any unemployed worker to meet a firm is given by: $M(u,v)/u = M(\theta,1) \equiv m(\theta)$, increasing with respect to θ.

We assume that there are two sources of job destruction, either in response to an exogenous shock that arrives with frequency δ, or in response to an endogenous motive : when the job ceases to be profitable, the firm chooses to close down.

2.3 Firm's valuations

In this section, we describe the asset price equations. We restrict attention to stationary equilibria. A job owned by a firm is either filled or vacant. We denote by $J(\tau,T)$ the value of an existing firm at date $t = \tau + T$ with a job of tenure T, filled at date τ. As long as the partners do not mutually decide to quit their relation, the unique source of destruction is exogenous with frequency δ per unit time. Then, $J(\tau,T)$ solves

$$rJ(\tau,T) = Max\left\{ y(\tau,T) - w(\tau,T) + \delta\left[V(t) - J(\tau,T)\right] + \dot{J}(\tau,T); 0 \right\} \qquad (2)$$

where r is the interest rate, $w(\tau,T)$ the current wage and $V(t)$ is the value of a job vacancy at date t. The right side of this equation takes account of the fact that the job is destroyed once the expected return of its value to the employer falls below zero. $V(t)$ is governed by the following asset equation:

$$rV(t) = -c(t) + \frac{m(\theta)}{\theta}\left[J(t,0) - V(t)\right] + \dot{V}(t) \qquad (3)$$

where $c(t)$ is the flow cost per unit time paid by firms until it meets a job searcher.

Next, consider the workers' decisions. They can be employed or unemployed at any date. In this model, only unemployed workers search for a job. We denote by $W(\tau, T)$ the present value for a worker of being employed in a job of tenure T, filled at date τ. $W(\tau, T)$ is governed by the following asset equation:

$$rW(\tau, T) = Max\left\{w(\tau, T) + \delta[U(t) - W(\tau, T)] + \dot{W}(\tau, T); rU(t)\right\} \tag{4}$$

where $U(t)$ is the value of search for an unemployed agent. $U(t)$ solves the following equation:

$$rU(t) = b(t) + m(\theta)[W(t, 0) - U(t)] + \dot{U}(t) \tag{5}$$

In order to ensure the existence of a balanced growth path, as usually we assume that all the exogenous parameters follow the pace of productivity growth. More formally, we define $c(t) = p(t)c$ and $b(t) = p(t)b$ where c and b are two positive exogenous parameters.

In any equilibrium, firms enter the market until all opportunities from new creation are exhausted. This free entry condition yields $V(t) = 0$ for all t. Equation (3) and the free-entry condition imply:

$$\frac{\theta c}{m(\theta)} = \frac{J(t, 0)}{p(t)} \tag{6}$$

2.4 Wages

As long as the total surplus $S(\tau, T) = J(\tau, T) - V(t) + W(\tau, T) - U(t)$ at date t associated with a job of tenure T, created at date τ remains positive, the firm and its employee always find a mutually profitable contract to share that surplus in fixed proportions. Thus, $w(\tau, T)$ satisfies:

$$(1 - \beta)[W(\tau, T) - U(t)] = \beta[J(\tau, T) - V(t)] \tag{7}$$

where β represents the relative weight of the employee in the bargaining[8].

From (2)-(6) with (7), we deduce that the wage at time t on a job of tenure T is given by:

$$w(\tau, T) = \beta p(\tau)\chi(T) + (1 - \beta)p(t)\omega(\theta) \tag{8}$$

where we define:

$$\omega(\theta) \equiv b + \frac{\beta\theta c}{(1 - \beta)} \tag{9}$$

$\omega(\theta)$ represents the outside option of any worker. Because of the growth of the productivity in new jobs due to productivity growth of the technological frontier, the outside option of workers grows.

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[8]This wage scheme implies a continuous bargaining of wages, which might seem hard to justify. More fundamentally, such a scheme leads to excessive wage volatility in response to an aggregate shock. It could be then interesting to combine all the other elements of the model with a wage as a fixed share of the product at each date. Nevertheless, this modification would lead to the same intuitions and to the same results in terms of wage inequality. We think that it is easier to understand the specific role played by the learning by adopting the same wage scheme as Mortensen and Pissarides [1998].

This increase of the outside option is a source of growth for wages in existing job. Another source is the improvement of the productivity of existing jobs induced by learning. Nevertheless, the increase of productivity occurs at a decreasing rate while the increase of the outside option takes place at a constant rate. Intuitively, the increase of wage induced by these two forces will eventually lead to job destruction.

3   Endogenous job destruction

3.1   The optimal age of job destruction

The partners choose to continue their relation as long as the surplus associated with the job, $S(\tau, T) = J(\tau, T) - V(t) + W(\tau, T) - U(t)$, remains positive. In any equilibrium, we have $V(t) = 0$ and according to the rule of determination of wages, we deduce $S(\tau, T) = (1 - \beta)^{-1} J(\tau, T)$. Thus, it is profitable for a firm to voluntarily cease its relation with its current worker at the date $\tau + T^*$, at which the surplus is negative or equal to zero, i.e. $J(\tau, T^*) \leq 0$, where T^* is the optimal duration of job. And it is also optimal for the worker to quit since $J(\tau, T) = \beta^{-1}(1 - \beta)[W(\tau, T) - U(t)]$.

As the function J is continuous, T^* satisfies $J(\tau, T^*) = 0$. By virtue of (2) the maximal value of a filled job is:

$$J(\tau, T) = \underset{T}{Max} \left\{ \int_t^{\tau + T} e^{-(r+\delta)(\xi - t)} [p(\tau) \chi(\xi - \tau) - w(\tau, \xi - \tau)] d\xi \right\}$$

Using (8) in the previous equation gives

$$J(\tau, T) = \underset{T}{Max} \left\{ (1 - \beta) p(t) \int_t^{\tau + T} e^{-(r+\delta)(\xi - t)} \left[e^{g(\tau - t)} \chi(\xi - \tau) - e^{g(\xi - t)} \omega(\theta) \right] d\xi \right\}$$

After a variable switch, $\nu = \xi - t$, when $\tau = t$, the above equation can be rewritten as:

$$J(t, 0) = \underset{T}{Max} \left\{ (1 - \beta) p(t) \int_0^T e^{-(r+\delta)\nu} [\chi(\nu) - e^{g\nu} \omega(\theta)] d\nu \right\}$$

Thus, $J(t, 0) = p(t) J$ where

$$J = \underset{T}{Max} (1 - \beta) \int_0^T e^{-(r+\delta)\nu} [\chi(\nu) - e^{g\nu} \omega(\theta)] d\nu \tag{10}$$

is the value of a job on the technological frontier. This value is independent of the current date. The optimal age of job destruction, denoted by T^*, is the solution to the problem defined by (10). The first-order condition gives the following condition for T^*:

$$\chi(T^*) = e^{g T^*} \omega(\theta)$$

or [9]

$$s - (s - x) e^{-\gamma T^*} = e^{g T^*} \omega(\theta), \quad \gamma > 0 \tag{11}$$

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[9] The second order condition implies : $\dot{\chi}(T^*)/\chi(T^*) \leq g$ equivalent to $\chi(T^*) \geq \gamma s/(g + \gamma)$.

The left side of (11) is strictly increasing and concave in T while the right side is strictly increasing and convex. In the appendix, we show that if b, c and/or β are sufficiently small, there is a unique solution, denoted by T^*. These conditions are assumed to be satisfied throughout the remainder of the analysis. Moreover, (11) yields T^* as a decreasing function of θ. We denote this relation by $T^* = \Upsilon(\theta)$ with $\Upsilon'(\theta) < 0$. When we introduce the optimal value of T in (10), we obtain:

$$J^* = (1 - \beta) \int_0^{T^*} e^{-(r+\delta)\nu} \left[\chi(\nu) - e^{g(\nu - T^*)} \chi(T^*) \right] d\nu \tag{12}$$

The endogenous variables are θ, J, T and u, the unemployment rate. u^* is determined once we know the optimal values of T^* et θ^*. Using (6) with $J(t, 0) = p(t) J$, and (10) with $T^* = \Upsilon(\theta)$, we can finally represent J^* and θ^* in the plane (θ, J) thanks to the two following equations

$$\frac{\theta c}{m(\theta)} = J \tag{13}$$

with

$$J = (1 - \beta) \int_0^{\Upsilon(\theta)} e^{-(r+\delta)\nu} \left[\chi(\nu) - e^{g\nu} \omega(\theta) \right] d\nu \tag{14}$$

(13) gives a positively sloped relation in the plane (θ, J) while (14) gives a negatively sloped relation. Consequently, there is at most a unique solution with θ and J positive provided that b is not too large (see appendix).

We finally turn to the long-run unemployment rate. In any steady state, the flows of workers in and out of the unemployment pool must be equal. As only unemployed workers search for a job, the flow of workers out of unemployment is given by $m(\theta^*) u^*$. At each date t, among filled jobs, a fraction $(1 - u^*) \delta$ is hit by a negative exogenous shock, leading to their destruction and among the jobs occupied for T^* units of time, those not yet destroyed, i.e. a fraction $exp(-\delta T^*)$, become non-profitable and therefore are endogenously destroyed. Therefore, we deduce the following relation:

$$u^* = \frac{\delta}{\delta + \left[1 - e^{-\delta T^*} \right] m(\theta^*)} \tag{15}$$

3.2 Pace of technological progress, duration of jobs and unemployment

In this section, we focus our attention on the effects of the growth rate on the optimal destruction age. We notice that (13) is not affected by g while (14) is negatively affected. Therefore, when g increases, the labor market tightness declines in equilibrium. We substitute θ^* in the first-order condition to obtain the value of T^*. We define $\theta^* \equiv \theta(g)$ with $\theta'(g) < 0$, introduce in (11), and then differentiate. Finally, we obtain:

$$\frac{dT^*}{dg} g = \frac{\varepsilon_{\omega(\theta^*)/g} + g T^*}{\dot{\chi}(T^*) / \chi(T^*) - g} \tag{16}$$

9

where $\varepsilon_{\omega(\theta^*)/g}$ is the elasticity of $\omega(\theta^*)$ with respect to g, which is always negative. The denominator is always negative while the numerator has *a priori* an ambiguous sign. In fact, we show in appendix that it is possible to solve this indetermination. We obtain the following results:

$$\frac{dJ^*}{dg} < 0 \ , \ \frac{d\theta^*}{dg} < 0 \ , \ \frac{dT^*}{dg} < 0$$

The intuitive economic interpretation is as follows. Any increase of g has two direct effects which work in the direction of decreasing the life-time of production units : an increase of the growth rate of the outside option as well as a faster rhythm of obsolescence of existing technologies. The productivity of workers, $\chi(T)$, is however not affected by g. An indirect effect works through a decrease of labor market tightness, even though this effect seems to be of second-order. Thus, the destruction of jobs occurs faster when the growth rate is higher.

Therefore, from (15) and previous results, we deduce that the unemployment rate unambiguously increases with g. Two forces work in the same direction : the decline of the optimal destruction age and the faster rate of growth of wages. These effects are the direct and indirect creative destruction effects described in Aghion and Howitt [1994].

3.3 Nature of technological progress, duration of jobs and unemployment

The results are somewhat more difficult to establish regarding the effect of a variation of γ on T^*. We know that (13) is not affected by γ while (14) is positively affected so that the labor market tightness increases. Hence, we can deduce:

$$\frac{dJ^*}{d\gamma} > 0 \ , \ \frac{d\theta^*}{d\gamma} > 0$$

According to the previous results, we know that the equilibrium values of J^* and θ^* raise with γ because of the increase of firm's incentives to open new vacancies induced by the improvement of the productivity of workers. We introduce the value of θ^* in (11) and we differentiate this equation. This yields:

$$\frac{dT^*}{d\gamma}\gamma = \frac{\varepsilon_{\omega(\theta^*)/\gamma} - T^*\dot{\chi}(T^*)/\chi(T^*)}{\dot{\chi}(T^*)/\chi(T^*) - g} \tag{17}$$

where $\varepsilon_{\omega(\theta^*)/\gamma}$ is the elasticity of $\omega(\theta^*)$ with respect to γ, which is always positive. The denominator is always negative while the sign of the numerator remains ambiguous. Hence, the effect of γ on T^* seems to be ambiguous.

In fact, this effect is likely to be non-monotone (For a more formal discussion, see the appendix). We can see it by examining the first-order condition, (11). A firm decides to continue its relation with its employee as long as:

$$s - (s - x)e^{-\gamma T} \geq e^{gT}\omega(\theta), \quad \gamma > 0$$

Then, we distinguish between two effects induced by an increase in γ:

10

- First, the effects of learning are faster. The consequence is that we observe an instantaneous increase of production level, but the decreasing effects of learning are stronger. This effect works in the direction of increasing the optimal length of jobs.

- second, an increase in the value of γ increases the incentives of firms to open new vacancies because of the previous effect, namely the increased productivity. As a result, the outside option for workers increases, pushing towards an increase in wages. This effect works in the direction of decreasing the optimal duration of jobs.

So, the effects of an increase in the value of γ are as follows. When γ is relatively high, the output level produced by a worker comes quickly near its ceiling-level. Hence, the second effect is likely to dominate in order to entail $dT^*/d\gamma < 0$. By contrast, when γ is sufficiently low, the first effect is likely to dominate.

Observe that some cross-effects between g and γ are at stake. For instance, when g is low, the effects working through the outside option might be less relevant.

Finally, we study the effects of a variation of the value of γ on unemployment. From (15) with the preceding results, two cases have to be distinguished. First, when the initial value of γ is high, any decrease of this parameter yields ambiguous effects on the unemployment rate : on the one hand, the incentives for firms to create vacancies decrease, but on the other hand, the lifetime of filled jobs raises. Hence, the unemployment rate may decrease. In this case, the effects of γ might compensate the effects of a rise of g in order to keep the unemployment rate unchanged. Second, when γ is initially sufficiently low, any decrease of the value of γ induces an increase of the unemployment rate. So, the effects of a decrease in γ reinforces the effects of an increase in g. We can then summarize the results within a tabular:

	After an increase of g	After a decrease of γ	
		Initial high level of γ	Initial low level of γ
Impact on the expect. wait of the unemployed	+	+	+
Impact on the unemployment rate	+	-	+
Impact on the expected job lifespan	-	+	-

4 Wage inequality

4.1 Productivity of workers and wages

The wage of a worker at date t, with a seniority equal to T ($T = t - \tau \geq 0$), is given by (8). In order to analyze the effects of tenure on wages, we rewrite wages as adjusted to productivity level. Thus, in stationary equilibrium:

$$\tilde{w}(T) \equiv \frac{w(\tau, T)}{p(t)} = \beta e^{-gT} \left[s - (s - x) e^{-\gamma T} \right] + (1 - \beta) \omega (\theta^*)$$

11

According to this equation, we have to distinguish between two components in the wages : a component linked to the own productivity of a worker on the job and a component related to labor market tightness. The higher the labor market tightness, the higher the outside option for each worker, pushing towards a higher individual wage. The expression in brackets in the above equation is the productivity level reached by any worker after T units of time of expertise on a technology. The higher this level, the higher the wage. A faster growth rate makes the obsolescence of existing technologies faster. We recall that the initial investment of any firm is irreversible. Thus, the factor e^{-gT} in the above equation measures the gap between the technology currently operated by a worker on a job created for T units of time and the current technological frontier. This is the "*obsolescence effect*".

So, the adjusted wage raises with the level of the outside option as well as with the productivity level reached by a worker. But, two forces may entail a collapse of the adjusted wage : first, the learning technology has decreasing returns and second, the assumption of irreversibility of the initial investment implies an increasing technological gap as just mentioned.

More formally, the growth rate of the adjusted wage can be expressed as:

$$\frac{\dot{\tilde{w}}\left(T\right)}{\tilde{w}\left(T\right)} = \beta e^{-gT} \frac{\left[\dot{\chi}\left(T\right) - g\chi\left(T\right)\right]}{\tilde{w}\left(T\right)}$$

This growth rate depends positively on the difference between two terms : $\dot{\chi}\left(T\right)$ and $g\chi\left(T\right)$. The former measures the ability of a worker to improve his own productivity, which is decreasing in T, and the latter is simply the level of productivity reached by a worker after T units of time weighted by the growth rate of the technological frontier. In order to analyse the evolution of wages with respect to T, we distinguish between, on the one hand, two extreme values of γ, namely $\gamma \to 0$ and $\gamma \to +\infty$, and on the other hand, what we will call intermediate values of γ.

Let us first examine what happens when γ is low and $\gamma \to +\infty$. Intuitively, these cases may yield some similar results because the growth rate of the individual productivity is similar, and to be more precise, it is close to zero. More formally, γ low enough, and more precisely[10] $\gamma \leq gx/\left(s - x\right)$, or $\gamma \to +\infty$, is likely to lead to $\dot{\chi}\left(T\right)/\chi\left(T\right) < g \ \forall T > 0$. Then, the growth rate of adjusted wage is negative throughout the career of the worker in a production unit, as in Mortensen and Pissarides [1998] framework. Similarly to their work, the highest wage in the economy is associated with the more recent technology.

By contrast, for some intermediate values of γ, what we will simply denote by $\gamma > gx/\left(s - x\right)$, the growth rate of adjusted wage is positive up to a certain time, denoted by $\tilde{T}$, and afterwards it becomes negative. This U-inverted shape is due to the fact that at the outset of the career, the "*learning effect*" dominates the "*obsolescence effect*", while it is the contrary when the seniority raises. The highest wage,

---

[10]From the equation of individual productivity, we show that the growth rate of productivity, $\dot{\chi}\left(T\right)/\chi\left(T\right)$, is lower than g provided that the productivity level is higher than the threshold $\gamma s/\left(\gamma + g\right)$. As $\chi\left(T\right)$ is an increasing function, if this property is true for the initial date for which we have $\chi\left(0\right) = x$, then it is also true for all following dates.

in cross-section, is reached by the worker for who the growth rate of his own productivity is just equal to the growth rate of technical change ($\tilde{T}$ is such that $\dot{\chi}\left(\tilde{T}\right)/\chi\left(\tilde{T}\right) = g$).

From the learning technology, we deduce:

$$\chi\left(\tilde{T}\right) = \frac{\gamma s}{(\gamma + g)}$$

Hence,

$$\tilde{T} = \frac{\ln\left(\gamma + g\right)\left(s - x\right) - \ln gs}{\gamma}$$

which is decreasing in g. Any increase of g leads to a decrease of $\tilde{T}$, leading to a decrease of the period of increasing of the adjusted-wage.

For $\tilde{T}$ to be positive, γ must satisfy $\gamma > gx/\left(s - x\right) \equiv \hat{\gamma}_2$. We also have[11]

$$\frac{d\tilde{T}}{d\gamma}\begin{cases} \geq 0 & \text{if } \gamma \in [\hat{\gamma}_2, \bar{\gamma}] \\ \leq 0 & \text{if } \gamma \geq \bar{\gamma} \end{cases} \quad \text{and} \quad \lim_{\gamma \to +\infty} \tilde{T} = 0$$

These results indicate that the evolution of $\tilde{T}$ as a function of the parameter γ describes a U-inverted curve. We deduce that there are two ranges of values of γ for which $\tilde{T}$ gets nearer to zero, namely for all values in the interval $[0; \hat{\gamma}_2]$ or when γ approaches the infinity. In the next section, we will show that these analytical results are of great value for the study of inequality.

Finally, the growth rate of (non-adjusted) wage, $w\left(\tau, T\right)$, is

$$\frac{\dot{w}\left(\tau, T\right)}{w\left(\tau, T\right)} = g + \beta\frac{\left[\dot{\chi}\left(T\right) - g\chi\left(T\right)\right]}{e^{g\tau}w\left(\tau, T\right)} = g + \frac{\dot{\tilde{w}}\left(T\right)}{\tilde{w}\left(T\right)}$$

which is always positive. Thus, within a particular relation, the growth rate of wage is higher than the growth rate of technological progress as long as the duration of implementation of a technology is below $\tilde{T}$ units of time. After a higher duration, the growth rate of wage becomes lower than that of technological progress.

4.2   Dispersion of wages

We are also interested in studying inequality through the comparison between the highest and the lowest wage. This indicator is usual in literature. We know that the lowest wage is paid by the oldest firm. Hence, this wage is $\tilde{w}\left(T^*\right)$. According to the previous results, the highest wage depends on the value of γ. It is given by $\max\left\{\tilde{w}\left(0\right); \tilde{w}\left(\tilde{T}\right)\right\}$. In traditional matching model, the highest wage is always associated with the newest technology. It is not necessarily the case in our model. More precisely, when the learning

---

[11]When γ is sufficiently high in comparison with g, we are able to approximate the value of $\bar{\gamma}$: we find $\bar{\gamma} = \hat{\gamma}_2 + \left[gs\left(e^{1/2} - 1\right)\right]/\left(s - x\right)$.

process is fast, a non-monotone evolution of the adjusted-wage level throughout the career of a worker emerges. We denote by I_w the dispersion index:

$$I_w = \frac{\max\left\{\tilde{w}\left(0\right); \tilde{w}\left(\tilde{T}\right)\right\}}{\tilde{w}\left(T^*\right)}$$

So, we have to distinguish between three cases. Intuitively, according to the previous section, since $\tilde{T}$ gets near zero for two ranges of values of γ, we expect to obtain similar results for these values of the learning parameter. But, for some intermediate values, the results should be different. We indeed establish the following results:

- If $\gamma < gx/\left(s - x\right) \equiv \hat{\gamma}_2$, then,

$$I_{w0} = \frac{\tilde{w}\left(0\right)}{\tilde{w}\left(T_0^*\right)} = \frac{\beta x + \left(1 - \beta\right)\omega\left(\theta_0^*\right)}{\omega\left(\theta_0^*\right)}$$

which is always above one provided that $x > \omega\left(\theta^*\right)$. This ratio is decreasing with θ^*. Since θ^* is a decreasing function of g, the higher the growth rate of technological frontier is, the higher wage inequality between a beginner and a worker on a mature job is. And as θ^* is an increasing function of γ, the lower γ is, the higher wage inequality is.

- If $\gamma > gx/\left(s - x\right) \equiv \hat{\gamma}_2$, then,

$$I_w = \frac{\tilde{w}\left(\tilde{T}\right)}{\tilde{w}\left(T^*\right)} = \frac{\beta e^{-g\tilde{T}}\chi\left(\tilde{T}\right) + \left(1 - \beta\right)\omega\left(\theta^*\right)}{\omega\left(\theta^*\right)}$$

It is straightforward to show that the term $e^{-g\tilde{T}}\chi\left(\tilde{T}\right)$ is increasing (respect. decreasing) with respect to γ (respect. g). Moreover, we know that the tightness parameter is increasing (respect. decreasing) with respect to γ (respect. g). Nevertheless, the final impact of γ and g on the inequality indicator when $\gamma > \hat{\gamma}_2$ is more difficult to determine. The next subsection discuss this question.

- If $\gamma \to +\infty$,

$$I_{w\infty} = \frac{\tilde{w}\left(0\right)}{\tilde{w}\left(T_\infty^*\right)} = \frac{\beta s + \left(1 - \beta\right)\omega\left(\theta_\infty^*\right)}{\omega\left(\theta_\infty^*\right)}$$

which is very similar to the first indicator I_{w0} expect that x is replaced with s. Obviously, this result comes from the fact that the individual productivity is equal to s when $\gamma \to +\infty$. So, we have the same conclusions about the evolution of inequality with respect to g and γ. However, we have to notice that we cannot compare the index $I_{w\infty}$ with the index I_{w0}, since on the one hand, $s > x$, and on the other hand, $\omega\left(\theta_\infty^*\right) > \omega\left(\theta_0^*\right)$.

Impact of the rate of technical change

In this section, we argue that the effects of the rate of technological change on wage inequality are not monotone, depending on the learning process. Compared to the existing literature, this is a new result.

14

According to the previous section, we know that when $\gamma \to 0$ or $\gamma \to +\infty$, the index of wage inequality is an increasing function of g. But, for some intermediate values of the parameter γ, the conclusion becomes less clear. Formally, we have:

$$\frac{dI_w}{dg} \begin{cases} > 0 & \text{if } \dot{\chi}\left(\tilde{T}\right)/\chi\left(\tilde{T}\right)\tilde{T} = g\tilde{T} < \left|\varepsilon_{\omega(\theta^*)/g}\right| \\ < 0 & \text{if } \dot{\chi}\left(\tilde{T}\right)/\chi\left(\tilde{T}\right)\tilde{T} = g\tilde{T} > \left|\varepsilon_{\omega(\theta^*)/g}\right| \end{cases}$$

where $\varepsilon_{\omega(\theta^*)/g}$ is the elasticity of $\omega\left(\theta^*\right)$ with respect to g, which is always negative. According to the section 4.1, for extreme values of γ, $\tilde{T} \to 0$, and then, the sign of dI_w/dg is likely to be positive. But, for intermediate values of γ, we know that $\tilde{T}$ can reach some high values and the sign of dI_w/dg may be negative. This result indicates that if we want to say something about on how the rate of technological change affects wage inequality, we have to take into account another dimension of technological change, namely its impact on the learning process on the job.

What are the intuitions? Recall that there are two sources of heterogeneity in this model : the different vintage of machines used by workers and the seniority of these workers. New machines are always better than old ones but as long as a firm continues to use its current technology, its employee accumulates expertise in that technology. To understand the effects on wage inequality induced by an increase of g, we must take two effects into account.

- Firstly, any increase of g means an increase of the obsolescence of installed technologies. The productivity gap between two vintage technologies increases. Therefore, the heterogeneity of wages increases among jobs.

- Secondly, any increase of g means a shorter learning process for the highest wage (associated to $\tilde{T}$), and the lowest wage (associated with T^*) which correspond to the outside option, namely $\omega\left(\theta^*\right)$. Both wages decrease but, as the return of learning is decreasing, the highest wage is more reduced than the other. Therefore, the heterogeneity of wages decreases among jobs.

Finally, it can emerge a non-monotone relation between the rate of technical change and the dispersion of wages, depending on the speed of the learning. For extreme values of γ, that is to say, when the learning process is very high or very low, an increase in the pace of technical change increases the wage dispersion. But, when the speed of learning is average, an increase in the pace of technical change decreases the wage inequalities.

In a very different framework, Lloyd-Ellis [1999] considers the interplay between learning and technological change for wage inequality. According to his study, when the rate of absorption is relatively low, the rate of technical change may exceed it in the short run, leading to increasing wage inequality. In our model, the technological change acts directly on the length of learning. Depending on the speed of learning, this effect can induce a increasing or a declining wage inequality.

15

5 Renovation of technology

A firm might find profitable to update its technology rather than destroy its job. This will be the case if there is a date of updating such that $T^o \leq T^*$. First, we calculate this date. We assume, like Mortensen and Pissarides [1998] that there is a fixed cost, I, paid by a firm when it updates its technology. Hence, the value of a filled job can be written as follows:

$$
\begin{aligned}
J\left(\tau,T\right) &= \underset{T}{Max} \int_t^{\tau+T} e^{-(r+\delta)(\nu-t)} \left[p\left(\tau\right)\chi\left(\nu-\tau\right)-w\left(\tau,\nu-\tau\right)\right]d\nu \\
&\quad +e^{-(r+\delta)(\tau+T-t)}\left[J\left(\tau+T;0\right)-p\left(\tau+T\right)I\right] \\
&= \underset{T}{Max}\left(1-\beta\right)\int_t^{\tau+T} e^{-(r+\delta)(\nu-t)}\left[p\left(\tau\right)\chi\left(\nu-\tau\right)-p\left(\nu\right)\omega\left(\theta\right)\right]d\nu \\
&\quad +e^{-(r+\delta)(\tau+T-t)}\left[J\left(\tau+T;0\right)-p\left(\tau+T\right)I\right]
\end{aligned}
$$

At a date $\tau = t$, we obtain

$$
\begin{aligned}
J &= \underset{T}{Max}\left(1-\beta\right)\int_0^T e^{-(r+\delta)\nu}\left[\chi\left(\nu\right)-e^{g\nu}\omega\left(\theta\right)\right]d\nu \\
&\quad +e^{-(r+\delta-g)T}\left[J-I\right]
\end{aligned}
\tag{18}
$$

The first-order condition is then

$$
\left(1-\beta\right)\left[\chi\left(T^o\right)-e^{-gT^o}\omega\left(\theta\right)\right] = \left(r+\delta-g\right)e^{gT^o}\left[J-I\right]
$$

We introduce this expression in (18) in order to eliminate the wage:

$$
I = \left(1-\beta\right)\int_0^{T^o} e^{-(r+\delta)\nu}\left[\chi\left(\nu\right)-e^{g(\nu-T^o)}\chi\left(T^o\right)\right]d\nu
\tag{19}
$$

Two reasons can explain the diffusion of new technologies : on the one hand, the collapse of the price of new equipment leads to a lower renovation cost and on the other hand, for some industries, the decrease of the pace of the learning process is a factor which has magnified the effects of the collapse of prices. We can graphically illustrate this result: on the figure 1, we represent the function $\Gamma\left(T\right)$ with two different value of the parameter γ. We define (the study of this function is reported in the appendix):

$$
\Gamma\left(T\right) \equiv \left(1-\beta\right)\int_0^T e^{-(r+\delta)\nu}\left[\chi\left(\nu\right)-e^{g(\nu-T)}\chi\left(T\right)\right]d\nu
$$

On the figure 1, we see that for a relatively low renovation cost, a slow learning process leads to a shorter renovation date. The intuition is straightforward. When γ is high, a firm wish to take advantage of the possibility for a worker to accumulate expertise on its current technology rather than paying the renovation cost. When γ is low, the accumulation of expertise takes place too slowly such that a firm prefers to increase its productivity by changing its technology. In this case, firms update more frequently.
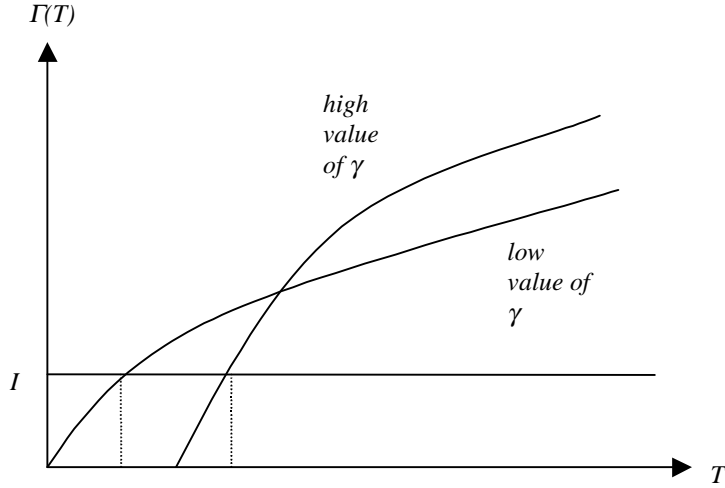
16

Figure 1: T^0 according to the value of γ

6 Conclusion

Our model suggests a new view concerning the effects of the speed of emergence of innovations on the job instability as well as on wage inequality. More precisely, we show that the effects of faster technological progress vary with on-the-job learning process. Whenever the accumulation of expertise on a particular task is relatively easy, any decrease in the speed of learning process leads to an increase of the optimal job lifetime. Indeed, so long as the learning rate is high enough, firms expect to take advantage of a relatively high productivity level within a relatively short period of time. So, when this rate decreases, as the job creation and therefore the outside option of workers decreases, it might be profitable for firms to increase the lifetime of jobs. In other words, the learning process is slower, but fast enough to make firms think it does pay to allow workers to reach a high level of productivity. This is why the optimal destruction date is further away. Under these circumstances, jobs become less unstable, wage inequality between workers as well as the unemployment rate might decrease.

Without taking the learning process into account, the traditional matching model cannot explain why instability can remain relatively stable when technical change evolves. In our model, such a phenomenon can emerge. When the rate of technical change increases, the unemployment rate and job instability should increase. But the adjustments in the learning process, of the sort considered in this paper, can offset this latter effect. We indeed showed that from high initial value of the learning rate, any decrease of this rate might induce a decrease of the unemployment rate. In this type of economy, the negative effects on stability of an acceleration in the emergence of innovations are offset by the positive effects due to the learning process. By contrast, if the accumulation of expertise by workers is slow, the effects of faster technological progress and slower learning progress can reinforce each other and lead to an increase

in the instability of jobs.

So, we argue that if instability of jobs is increasing today, it is because the past cumulative decrease in the rate of learning results in a relatively low speed of accumulation of expertise on some jobs. And it is only in this context that the rate of emergence of innovations has a negative effect on stability of jobs.

We have the same kind of conclusions concerning the increase of wage inequality, namely it is worthwhile to study the interaction between the pace and the type of technological progress to have a better understanding of increasing wage inequality. In fact, a non-monotone relationship between the rate of technical change and the dispersion of wages can emerge, depending on the speed of the learning process. When the learning process is very high or very low, an increase in the pace of technical change increases the wage dispersion. But, when the speed of learning is average, it decreases it.

An interesting extension of this model could be the study of different sources of learning. More particularly, as Laing, Palivos, and Wang [1995] do, we could assume that schooling and formal education enhance ability to acquire additional skills once employed. The level of schooling also acts as a key determinant of job instability and wage inequality. Reciprocally, job instability acts on the returns to education, and on the investment in schooling when young. Such an interaction between schooling and job instability is left for further research.

Appendix

Existence of a unique equilibrium

We want to establish the conditions to have a unique equilibrium from the system $(13) - (14)$. In the plane (θ, J), the first equation is positively sloped, concave and for $\theta = 0$, then $J = 0$. The second equation is negatively sloped. To have a solution with $\theta > 0$ and $J > 0$, from (14), we must have $J > 0$ for $\theta = 0$ and this is true if b is not too large.

Once we have a solution for θ, we can check that the solution for T, i.e. the endogenous length of job, is unique. To do so, we introduce the solution of θ in the equation (11) and we obtain the equilibrium value of T, denoted by T^* and given by:

$$se^{-gT^*} - (s - x) e^{-(g+\gamma)T^*} = \omega(\theta^*) > 0$$

First, we can notice that we have $se^{-gT} > (s - x) e^{-(g+\gamma)T} \ \forall T \geq 0$. Second, we have to distinguish two separate cases depending on the values of the parameters. Let us define the function $G(T) \equiv se^{-gT} - (s - x) e^{-(g+\gamma)T}$. This function is concave with $G(0) = x$ and $\lim_{T \to +\infty} G(T) = 0$. The highest value is obtained for a particular value of T, denoted by $\hat{T}$ with $\hat{T} = \gamma^{-1} \ln \left[\frac{gs}{(g+\gamma)(s-x)} \right]$. We deduce that we have $\hat{T} < 0$ if and only if $x > \gamma s / (g + \gamma)$ and $\hat{T} > 0$ if and only if $x < \gamma s / (g + \gamma)$.

In the first case, i.e. when $x > \gamma s / (g + \gamma)$, the function G is strictly decreasing in T for all $T > 0$ and there is a unique solution T^* such that $se^{-gT^*} - (s - x) e^{-(g+\gamma)T^*} = \omega(\theta^*)$ provided that $\omega(\theta^*) < x$, i.e. provided that b and/or c and/or β are sufficiently small.

In the second case, i.e. $x > \gamma s / (g + \gamma)$, the function G is increasing in T and then decreasing for all $T > \hat{T}$. So, if $\omega(\theta^*) < x$, there is a unique solution like in the previous case. But if $\omega(\theta^*) > x$, there are two potential solutions. In fact, it is straightforward to show that the second order condition $(\chi(T^*) \geq \gamma s / (g + \gamma)$, see the footnote $(4))$ is satisfied only in the decreasing part of the curve representative of G. However, to have a solution $\omega(\theta^*)$ doesn't have to be too large. More precisely, the highest value of G is given by $G\left(\hat{T}\right)$:

$$G\left(\hat{T}\right) = \left[\frac{(g + \gamma)(s - x)}{gs} \right]^{\frac{g}{\gamma}} \left[x - \frac{(s - x)}{gs} [s\gamma - (g + \gamma)x] \right]$$

So, we have a solution if $\omega(\theta^*) \leq G\left(\hat{T}\right)$, i.e. if b and/or c and/or β are sufficiently small.

Impact of g on T*

First, we differentiate the system formed by (13) and (14). We obtain

$$\begin{bmatrix} 1 & -\eta(\theta^*) c \\ 1 & (1 - \beta) \int_0^{T^*} e^{-(r+\delta-g)v} \omega'(\theta^*) dv \end{bmatrix} \begin{bmatrix} \frac{dJ^*}{dg} \\ \frac{d\theta^*}{dg} \end{bmatrix}$$

$$
= \begin{bmatrix} 0 \\ -\left(1-\beta\right) \int_0^{T^*} e^{-\left(r+\delta-g\right)v} \omega\left(\theta^*\right) v dv \end{bmatrix}
$$

where we denote by A, the Jacobian matrix, (2×2), of this system and $\eta\left(\theta^*\right) \equiv \left(1 - \varepsilon_{m(\theta)/\theta}\right)/m\left(\theta\right)$ with $\varepsilon_{m(\theta)/\theta}$, the elasticity of $m\left(\theta\right)$ with respect to θ. This elasticity is smaller than one. It is straightforward to see that we have $\det A > 0$ for all positive values of T^*. Then, by Cramer's rule, we deduce the sign of $d\theta^*/dg$:

$$
\frac{d\theta^*}{dg} = \frac{-\left(1-\beta\right) \int_0^{T^*} e^{-\left(r+\delta-g\right)v} \omega\left(\theta^*\right) v dv}{\det A} < 0
$$

Second, we introduce the expression in (16), and after some calculus, we finally obtain:

$$
e^{-gT^*} \frac{dT^*}{dg} \left[\gamma\left(s-x\right) e^{-\gamma T^*} - g e^{gT^*} \omega\left(\theta\right)\right]
$$
$$
= \omega\left(\theta^*\right) T^* \left\{ \frac{\frac{\beta c}{\left(r+\delta-g\right)} \left[1 - \frac{1-e^{-\left(r+\delta-g\right)T^*}}{\left(r+\delta-g\right)T^*}\right] + \eta\left(\theta^*\right) c}{\frac{\beta c}{\left(r+\delta-g\right)} \left[1 - e^{-\left(r+\delta-g\right)T^*}\right] + \eta\left(\theta^*\right) c} \right\}
$$

The term in brackets to the left of just above equality is negative. Hence, dT^*/dg has the same sign as the opposite of the term to the right of this equality. A sufficient condition for the term in brackets to be positive is

$$
1 - e^{-\left(r+\delta-g\right)T^*} < \left(r + \delta - g\right) T^*
$$

It turns out that this inequality is always true provided that T^* has any positive value. We conclude that we have

$$
\frac{dT^*}{dg} < 0
$$

Impact of γ on T^*

We study the equilibrium effects of γ on the following equality:

$$
\omega\left(\theta^*\right) = e^{-gT^*} \chi\left(T^*\right)
$$

We know that the left-hand side is increasing with respect to γ. Hence, any decrease of γ induces a decrease of the right-hand side. We have to take two effects into account. First, a direct effect and second, an indirect effect of γ. The direct effect on the right-hand side is always positive. The indirect effect works through T^* and the right-hand side is decreasing with respect of T^* (we can check that we have $\partial e^{-gT^*} \chi\left(T^*\right)/\partial T^* < 0$). So, the overall effect of a decrease of γ on T^* depends on the extent of the direct effect of γ.

Then, we have to evaluate the extent of the direct effect of γ on $e^{-gT^*} \chi\left(T^*\right)$ according to the initial value of γ. We find that, for any given value of T^*, the first derivative is positive and the second derivative

is negative. This implies that when γ decreases, the right-hand side of the above equality decreases, but the lower γ is, the higher this decrease is. In other words, when γ has an high initial value, the direct effect seems to be negligible but becomes more significant for lower initial values of γ.

These results allow to conjecture the following general result: T^* is likely to be decreasing (respect. increasing) with respect to γ for high (respect. low) initial value of γ.

Study of $\Gamma(T)$

$$\Gamma(T) \equiv (1 - \beta) \int_0^T e^{-(r+\delta)(\nu)} \left[\chi(\nu) - e^{g(\nu - T)} \chi(T) \right] d\nu$$

First, notice that we have $\Gamma(0) = 0$. Second, the derivative gives the following result

$$
\begin{aligned}
\frac{\partial \Gamma(T)}{\partial T} &= \frac{(1 - \beta) \left[e^{-gT} - e^{-(r+\delta)T} \right]}{(r + \delta - g)} \left[gs - (g + \gamma)(s - x) e^{-\gamma T} \right] \\
&= \frac{(1 - \beta) \left[e^{-gT} - e^{-(r+\delta)T} \right]}{(r + \delta - g)} \left[g\chi(T) - \gamma(s - x) e^{-\gamma T} \right]
\end{aligned}
$$

For $r > g$, $\text{sign} \partial \Gamma(T) / \partial T = \text{sign} \psi(T)$ where we define $\psi(T) \equiv \left[gs - (g + \gamma)(s - x) e^{-\gamma T} \right]$. Moreover, for some values of parameters, we can have $\psi(0) < 0$, and in this case, the slope of $\Gamma(T)$ is negative for $T \in \left[0, \widetilde{T} \right]$. The value for $\widetilde{T}$ is given by

$$\widetilde{T} = \frac{-1}{\gamma} \log \left(\frac{gs}{(g + \gamma)(s - x)} \right)$$

For $T \in \left] \widetilde{T}, \infty \right[$, the slope of $\Gamma(T)$ is strictly positive with an horizontal asymptote:

$$\lim_{T \to +\infty} \Gamma(T) = \frac{(1 - \beta)}{(r + \delta)(r + \delta + \gamma)} \left[\gamma s + (r + \delta) x \right] > 0$$

The value of this asymptote is strictly increasing in γ.

We have $\psi(0) < 0$ if $(g + \gamma) x - \gamma s < 0$, i.e. if γ is sufficiently high. We also can calculate the slope in a neighbourhood of zero. We obtain:

$$\lim_{T \to 0^+} e^{-gT} - e^{-(r+\delta)T} = 0^+$$

and

$$\lim_{T \to 0^+} g\chi(T) - \gamma(s - x) e^{-\gamma T} = (g + \gamma) x - \gamma s$$

Hence,

$$
\begin{aligned}
\frac{\partial \Gamma}{\partial T}(T = 0) \quad &> 0 \quad \text{for any sufficiently low value of } \gamma \\
&< 0 \quad \text{otherwise.}
\end{aligned}
$$

21

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23

2003

0301 Basic Income/ Minimum Wage Schedule and the Occurrence of Inactivity Trap: Some Evidence on the French Labor Market	
Thierry LAURENT & Yannick L'HORTY	
0302 Exonérations ciblées de cotisations sociales et évolution de l'emploi : un bilan à partir des études disponibles	
Philippe DE VREYER	
0303 Incorporating Labour Market Frictions into an Optimizing-Based Monetary Policy Model	
Stéphane MOYEN & Jean-Guillaume SAHUC	
0304 Indeterminacy in a Cash-in-Advance Two-Sector Economy	
Stefano BOSI, Francesco MAGRIS & Alain VENDITTI	
0305 Competitive Equilibrium Cycles with Endogenous Labor	
Stefano BOSI, Francesco MAGRIS & Alain VENDITTI	
0306 Robust European Monetary Policy	
Jean-Guillaume SAHUC	
0307 Reducing Working Time and Inequality: What Has the French 35-Hour Work Week Experience Taught Us?	
Fabrice GILLES & Yannick L'HORTY	
0308 The Impact of Growth, Labour Cost and Working Time on Employment: Lessons from the French Experience	
Yannick L'HORTY & Christophe RAULT	
0309 Inflation, Minimum Wage and Other Wages: an Econometric Study on French Macroeconomic Data	
Yannick L'HORTY & Christophe RAULT	
0310 Exogeneity in Vector Error Correction Models with Purely Exogenous Long-Run Paths	
Jacqueline PRADEL & Christophe RAULT	
0311 Retraite choisie et réduction des déficits : quelles surcotes proposer?	
Jean-Olivier HAIRAULT, François LANGOT & Thepthida SOPRASEUTH	
0312 Indeterminacy in a Two-Sector Finance Constrained Economy	
Stefano BOSI, Francesco MAGRIS & Alain VENDITTI	
0313 La nouvelle économie irlandaise	
Nathalie GREENAN & Yannick L'HORTY	
0314 Pace versus Type: The Effect of Economic Growth on Unemployment and Wage Patterns	
Martine CARRE & David DROUOT	

2002