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Eric JONDEAU & Jean-Guillaume SAHUC

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Optimal Monetary Policy in an Estimated DSGE Model of the Euro Area with Cross-country Heterogeneity*

Eric Jondeau†
HEC Lausanne and FAME

Jean-Guillaume Sahuc‡
Banque de France
and EPEE, University of Evry

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Abstract

We investigate the implications of cross-country heterogeneity within the euro area for the design of optimal monetary policy. We build an optimizing-based multi-country model (MCM) describing the euro area in which differences between structural parameters across countries are allowed. Using Bayesian techniques, we estimate the MCM and its area-wide counterpart (AWM) and compare their empirical performances. We then question which model is the most appropriate for monetary policy purposes. We find that using an AWM induces relatively large and significant welfare losses. Our results also suggest that this is not the use of a rule based on aggregated variables that is costly in terms of welfare, but rather the use of a sub-optimal forecasting model.

Keywords: Euro area, heterogeneity, optimal monetary policy, Bayesian econometrics

JEL Classification: C51, E52, F41

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† *Correspondence:* HEC Lausanne, Institute of Banking and Finance, Route de Chavannes 33, CH-1007 Lausanne-Vidy, Switzerland. E-mail: eric.jondeau@unil.ch.

‡ *Correspondence:* Banque de France, Research Unit, 31 rue Croix des Petits Champs, 75049 Paris, France. E-Mail: jean-guillaume.sahuc@banque-france.fr.

1 Introduction

The Maastricht Treaty (Art. 105) states that the primary objective of the European Central Bank (ECB) is to maintain price stability within the European Monetary Union. In order to fulfill this aim and although it may use a battery of economic indicators, including country-specific ones, decisions are taken on the basis of aggregate developments, while national idiosyncrasies are left to the care of national governments. The consequences of such a constraint on the monetary policy of the euro area are obviously related to the extent and the nature of heterogeneity of countries within the area. Even if we acknowledge that the decisions have to be taken on the basis of aggregate developments only, an additional difficulty for the central bank is that it is not clear a priori what type of forecasting model (multi-country or area-wide) should be used for implementing an optimal monetary policy. It may be argued that, since objectives are defined in terms of aggregate variables, an area-wide model (AWM) would be sufficient for capturing most characteristics of the euro-area economy. On the other hand, a multi-country model (MCM) may help capturing the heterogeneity of countries and therefore bring valuable information about the state of the euro-area economy. Consequently, it would allow to define a more appropriate monetary policy rule. Once again, the choice between the two types of model should depend on the extent and the nature of heterogeneity. Essentially, heterogeneity may come from (i) the asymmetry in the behavior of private agents across countries, (ii) the asymmetry in the transmission mechanisms of country-specific policies, or (iii) the asymmetry of shocks across countries. To evaluate the effect of heterogeneity on the area welfare, it is crucial to disentangle these different sources of heterogeneity.

Until now, a few studies have investigated the role devoted to country-specific information in the decision process of the Eurosystem.[1] They follow the standard approach to policy evaluation recently revived by a growing literature on monetary policy rules (see the contributions in Taylor, 1999): The optimal policy rule is determined so as to minimize the expected value of an intertemporal loss function, under the constraint provided by a simplified multi-country model (MCM) of the euro area. Assuming that the monetary authority is exclusively interested in area-wide objectives, the performance of two classes of simple optimal reaction functions, based on an MCM and an AWM respectively, are compared. Comparisons between the minimized expected loss under the two alternative policy rules generally reveal that the loss associated with the neglect of country-specific information might be large. However, the underlying macroeconomic models are not designed in an optimization-based framework. Consequently, the optimal monetary policy deduced from such models is subject to the Lucas critique, since it is based on reduced-form, not structural, parameters. This is a serious limitation when the welfare resulting from an optimal

[1] The literature includes Aksoy, De Grauwe and Dewachter (2002), De Grauwe (2000), De Grauwe and Piskorki (2001), Angelini *et al.* (2002), and Monteforte and Siviero (2003), among others.

policy rule has to be evaluated.

The objective of this paper is to reassess and generalize the preceding results in investigating how heterogeneity of agents across euro-area countries is likely to affect the optimal monetary policy into an *optimizing-based framework.* More precisely, we measure the cost in terms of welfare of using an AWM instead of an MCM to evaluate the optimal monetary policy. The basic idea is that the MCM is more likely to capture heterogeneity across countries and thus to describe more accurately the way monetary policy affects the economy. Consequently, a welfare-maximizing central bank may be able to implement a more efficient monetary policy, even if the policy rule is assumed to be based on aggregate variables only. An obvious shortcoming of the MCM is that the estimation of the joint dynamics of the various national economies is much more demanding, since it requires modeling international transmission mechanisms. In addition, the MCM is likely to induce a large amount of country-specific uncertainty, while an AWM may average these errors. Conversely, the estimation of an AWM is likely to induce an aggregation bias, if structural parameters actually differ across countries. Such a bias has already been highlighted in the context of the Phillips curve (Demertzis and Hugues Hallett, 1998).

Our approach comprises several challenges both on theoretical and empirical grounds. From a theoretical point of view, we derive a simple but complete MCM which resorts to the "New Open Economy Macroeconomics" literature (initiated by Obstfeld and Rogoff, 2000). By incorporating significant frictions in the form of nominal rigidities, Dynamic Stochastic General Equilibrium (DSGE) models have been shown to provide a sufficiently rich dynamics to fit the actual data fairly well (Christiano, Eichenbaum, and Evans, 2005, or Smets and Wouters, 2003, SW thereafter). However, in our open-economy context, additional mechanisms must be introduced: (i) cross-country differences in the structural parameters are allowed, since we are primarily interested in the effect of such heterogeneity on the design of the optimal monetary policy, (ii) perfect risk sharing and a home bias in preferences are incorporated in the model to deal with exchange-rate indeterminacy, and (iii) cross-country correlations between shocks are introduce to capture co-movement in the joint dynamics of national conditions. From an empirical point of view, pure Full Information Maximum Likelihood (FIML) estimation turned out to be very sensitive to the specification of medium- or large-scale macroeconomic models and in many cases resulted in unrealistic parameter estimates. Consequently, we resort to Bayesian econometrics, which introduces priors on unknown parameters in an FIML framework. This avenue has been followed for instance by Schorfheide (2003), SW and Onatski and Williams (2004).

Following the strategy described above, we first estimate two models, mimicking the way the ECB forecasts macroeconomic developments within the Eurosystem. In the first one, we model the dynamics of area-wide macroeconomic data. In the second one, we adopt an open-economy framework and model the joint dynamics of the data for the major countries in the euro area (Germany, France and Italy). Our empirical evidence suggests that there exists some significant heterogeneity within the euro area, even among core countries. First, we obtain some large and significant differences between estimates of the structural parameters at euro-area level and at country level, suggesting an aggregation bias. But more importantly, the main root of heterogeneity is the weak correlation between shocks across countries.[2]

Then, we investigate how cross-country heterogeneity affects the design of optimal monetary policy within the euro area. We consider two alternative modeling approaches. In both of them, the central bank is assumed to define its preferences and its loss function at the area-wide level. In addition, the reaction function is designed in terms of aggregate variables only. In the first approach, the model used for computing the loss function is an AWM, estimated using aggregated data, while in the second approach an MCM is used. Then, we evaluate the optimal monetary policy that maximizes the aggregate welfare, both under the AWM and the MCM, and we measure the welfare cost of using the AWM (sub-optimal) forecasting model. In order to investigate the specific consequences of interest-rate smoothing in the policy rule, we also consider several *ad hoc* loss functions, in which we vary the relative weights of inflation, output-gap and interest-rate variances.

The remainder of the paper is organized as follows. In section 2, we describe the theoretical MCM. In section 3, we present the data and the estimates of the AWM and MCM. In section 4, we determine the optimal monetary policy under the two forecasting models and evaluate the welfare implications of using the (sub-optimal) AWM model. Section 5 summarizes our main findings and concludes.

2 Structure of the multi-country model

The euro area is modelled as the aggregate of several economies. For each country, we formulate an open-economy sticky-price model, which is inspired by recent theoretical models derived from the "New Open Economy Macroeconomics", and which has a sufficiently rich dynamics to fit actual data fairly well.[3] The model is enriched in several dimensions, to offer a comprehensive framework that encompasses and generalizes other previous contributions. Most elements of this model are individually already present in the closed or open economy macroeconomic literature, but they have not been brought together in a single framework as is done here. In terms of dynamics, first key modifications are the explicit incorporation of habit formation in the households' preferences and partial indexation in a price-setting

[2] In a companion paper (Jondeau and Sahuc, 2005), we investigate the different sources of heterogeneity more thoroughly.

[3] See, among others, Obstfeld and Rogoff (2000), Corsetti and Pesenti (2000), Devereux and Engel (2000), Monacelli (2001), Clarida, Gali and Gertler (2002), Smets and Wouters (2002), Benigno and Benigno (2003), Benigno (2004), Galí and Monacelli (2004). For additional references on the new open-economy macroeconomics literature, see Brian Doyle's Web site on the topic at http://www.geocities.com/brian_m_doyle/open.html.

framework *à la* Calvo (1983). These assumptions provide us with microfounded "hybrid" versions of the IS and Phillips curves. Second, contrary to most recent studies on DSGE models, we do not assume that preferences and technologies are the same across countries, since we are interested in measuring the effect of heterogeneity on the optimal monetary policy of the area. In addition, domestic and foreign shocks are allowed to be imperfectly correlated. Third, to cope with the indeterminacy of the exchange rate, we resort to the perfect risk sharing assumption. Although this assumption is admittedly heroic in empirical work, it avoids assuming non-rational expectations of exchange rate that has been shown to be an alternative way of dealing with non-stationarity.[4] Finally, households are assumed to have a taste bias towards home-produced goods. Since preferences differ across countries, the price of consumption bundles will differ when expressed in a common currency. The real exchange rate thus deviates from purchasing power parity (PPP).[5] This assumption is crucial, because it allows the perfect risk-sharing equation to determine uniquely the dynamics of the terms of trade.

In order to lighten the notations, we assume that there are two countries in the euro area, denoted H(ome) and F(oreign). Since commercial links are much stronger between countries within the area than with countries outside the area, we neglect trade with the rest of the world. The population of the euro area is a continuum of agents on the interval $[0,1]$. The population of country H belongs to $[0,n)$, while the foreign population belongs to $[n,1]$. Therefore, n is the relative measure of the home country size into the area. An agent in the home country is indexed $h \in [0,n)$, while a foreign agent is indexed $f \in [n,1]$. Variables in the home country are denoted X_t while foreign variables are denoted X_t^*. The home economy produces a continuum of differentiated goods indexed on the interval $[0,n)$. Foreign goods (or, equivalently, goods produced in the rest of the area) are indexed on the interval $[n,1]$. All goods are tradeable.

2.1 Households

The home economy is populated by infinitively-living households, consuming Dixit-Stiglitz aggregates of domestic and imported goods. A home household h owns a firm producing goods h and receives dividends from it. We assume that households in a given country have the same preferences and endowments. Although there may be idiosyncratic shocks among households, we assume that households have access to complete markets for state-contingent claims, so that there is no heterogeneity among agents in a given country. Consequently, all households in the same country behave in the same manner and then we consider the optimization problem of a representative household. The representative household in country H maximizes the following expected sequence of present and future utility flows

[4] See, e.g., Lubik and Schorfheide (2003).

[5] An earlier contribution that introduced home bias in preferences is due to Warnock (2000).

that depends positively on consumption (C_t) and negatively on labor (hours worked, L_t):[6]

$$\mathcal{U}_t = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \varepsilon_{p,t+k} \left[\frac{1}{1-\sigma} \left(C_{t+k} - \gamma \mathcal{H}_{t+k} \right)^{1-\sigma} - \frac{1}{1+\varphi} \left(L_{t+k} \right)^{1+\varphi} \right] \tag{1}$$

where $\mathbb{E}_t$ denotes the expectation operator conditional on the information set at time t, β is the intertemporal discount factor, with $0 < \beta < 1$, σ is the inverse of the intertemporal elasticity of substitution of consumption, and φ is the inverse of the elasticity of labor disutility with respect to hours worked. $\varepsilon_{p,t}$ denotes a country-specific preference shock that affects the inter-temporal substitution of all households in the same manner in the home economy.[7] Preferences display external habit formation as in Abel (1990). The habit stock is supposed to equal the level of aggregate consumption in the previous period ($\mathcal{H}_t = C_{t-1}$), and γ represents the habit persistence parameter, measuring the effect of past consumption on current utility ($0 \leq \gamma < 1$). Including habit formation in a macroeconomic model results in a better fit of the data and captures the "hump-shaped" gradual responses of spending (see Fuhrer, 2000).

The aggregate consumption index for home households and the corresponding consumption index for foreign households are defined by[8]

$$C_t = \frac{\left(C_{H,t}\right)^{\omega} \left(C_{F,t}\right)^{1-\omega}}{\omega^{\omega} \left(1-\omega\right)^{1-\omega}} \qquad \text{and} \qquad C_t^* = \frac{\left(C_{H,t}^*\right)^{\omega^*} \left(C_{F,t}^*\right)^{1-\omega^*}}{\left(\omega^*\right)^{\omega^*} \left(1-\omega^*\right)^{1-\omega^*}} \tag{2}$$

where ω and ω^* denote the share of home goods in the consumption of home and foreign households respectively. $C_{H,t}$ (resp. $C_{F,t}$) is the sub-index of consumption of imperfectly substitutable, home (resp. foreign) goods, which is in turn given by the following CES aggregators:

$$C_{H,t} = \left[\left(\frac{1}{n} \right)^{1/\theta} \int_0^n C_t \left(h \right)^{\frac{\theta-1}{\theta}} \mathrm{d}h \right]^{\frac{\theta}{\theta-1}} \qquad \text{and} \qquad C_{F,t} = \left[\left(\frac{1}{1-n} \right)^{1/\theta} \int_n^1 C_t \left(f \right)^{\frac{\theta-1}{\theta}} \mathrm{d}f \right]^{\frac{\theta}{\theta-1}} \tag{3}$$

where $C_t(h)$ (resp. $C_t(f)$) is consumption of the generic good h (resp. f) produced in country H (resp. F). Parameter θ denotes the elasticity of substitution across goods produced within a given country. The corresponding consumption price indexes (CPI) are given by:

$$P_t = \left(P_{H,t}\right)^{\omega} \left(P_{F,t}\right)^{1-\omega} \qquad \text{and} \qquad P_t^* = \left(P_{H,t}^*\right)^{\omega^*} \left(P_{F,t}^*\right)^{1-\omega^*}.$$

[6] We abstract from money in this model since the central bank adjusts money supply to satisfy money demand with a simple feedback rule.

[7] We assume that $\varepsilon_{p,t}$ follows an AR(1) process: $\varepsilon_{p,t} = \left(1-\rho_p\right) \bar{\varepsilon}_p + \rho_p \varepsilon_{p,t-1} + \eta_{p,t}$.

[8] As shown by Corsetti and Pesenti (2000), the Cobb-Douglas consumption index is a necessary condition for the trade to be invariably balanced.

Here, $P_{H,t}$ (resp. $P_{F,t}$) is the price sub-index for home- (resp. foreign-) produced goods expressed in the home currency, defined as

$$P_{H,t} = \left[\frac{1}{n}\int_0^n P_{H,t}(h)^{1-\theta}\,\mathrm{d}h\right]^{\frac{1}{1-\theta}} \qquad \text{and} \qquad P_{F,t} = \left[\frac{1}{1-n}\int_n^1 P_{F,t}(f)^{1-\theta}\,\mathrm{d}f\right]^{\frac{1}{1-\theta}},$$

where $P_{H,t}(h)$ (resp. $P_{F,t}(f)$) is the price in units of country H of a generic good h (resp. f) produced in country H (resp. F).

We also assume that prices are set in the producer currency and that the law of one price holds. We then have $P_{H,t}(h) = P^*_{H,t}(h)\,S_t$ and $P_{F,t}(f) = P^*_{F,t}(f)\,S_t$, where S_t is the nominal exchange rate expressed as units of domestic currency needed for one unit of foreign currency.[9] Since we assume the same elasticity of substitution among goods in a given country, we also have $P_{H,t} = P^*_{H,t}S_t$, and $P_{F,t} = P^*_{F,t}S_t$. Yet, from the definition of the CPI, we obtain that

$$P_t = P^*_t S_t \left(\frac{P_{H,t}}{P_{F,t}}\right)^{\omega-\omega^*}.$$

Therefore, if we assume that there exists a home bias in preferences ($\omega \neq \omega^*$), PPP does not necessarily hold, i.e. $P_t \neq P^*_t S_t$. We expect $\omega > \omega^*$, so that home households put a higher weight on home goods than foreign households.

As indicated above, we assume complete markets for state-contingent claims. Consequently, households can transfer wealth to the next period by holding B_{t+1} unit of the one-period nominal bond denominated in the domestic currency.[10] We thus obtain the following home household's budget constraint:

$$P_t C_t + \frac{B_{t+1}}{1+i_t} = W_t L_t + B_t + \Pi_t - TR_t \tag{4}$$

where W_t is the nominal wage income, Π_t is the dividend received from home firms, TR_t are lump sum government transfers, and i_t is the nominal interest rate.

The maximization problem of the home household consists in maximizing equation (1) subject to constraint (4), yielding the optimal profile of consumption, holdings of domestic bond and labor supply. The first-order conditions imply:[11]

$$\mathcal{U}_{C,t} = \varepsilon_{p,t}\left(C_t - \gamma\mathcal{H}_t\right)^{-\sigma}, \tag{5}$$

[9] Although it has been investigated in a number of recent papers, we do not consider here the presence of imperfect exchange rate pass-through. A reason is that it is not likely to be an important feature across countries within the euro area. In addition, this feature is obviously irrelevant from the euro-area point of view.

[10] More precisely, at date t, home households hold $B\left(s^{t+1}\right) = B_{t+1}$ units of the one-period bond denominated in home currency that pay 1 at date $t+1$ if state s_{t+1} occurs and 0 otherwise, where $s^t = (s_0, \cdots, s_t)$ denotes the story of events up to date t. Foreign households hold $B^*_t\left(s^{t+1}\right) = B^*_{t+1}$ units of such bond. The price of this bond in home currency is denoted $Q\left(s^t, s^{t+1}\right) = Q_{t,t+1}$. The price at date t of the portfolio held by home households is thus given by $E_t\left[Q_{t,t+1}B_{t+1}\right]$. We define the one-period interest rate as $1 + i_t = 1/E_t\left[Q_{t,t+1}\right]$. Note that, since bonds are state-contingent, including bonds denominated in foreign currency would be redundant. For more details, see Chari, Kehoe, and McGrattan (2002).

[11] We abstract here from the optimal intra-temporal allocations between domestic and foreign goods.

$$(1+i_t)^{-1} = \beta \mathbb{E}_t \left[\frac{\mathcal{U}_{C,t+1}}{\mathcal{U}_{C,t}} \frac{P_t}{P_{t+1}} \right], \tag{6}$$

$$\frac{\mathcal{U}_{L,t}}{\mathcal{U}_{C,t}} = \frac{W_t}{P_t}, \tag{7}$$

where $\mathcal{U}_{X,t}$ denotes the derivative of utility $\mathcal{U}$ with respect to variable X at the period t. Equation (5) defines the marginal utility of consumption. Equation (6) is the usual Euler equation for inter-temporal consumption flows. It establishes that the ratio of marginal utility of future and current consumption is equal to the inverse of the real interest rate. Equation (7) is the condition for the optimal consumption-leisure arbitrage, implying that the marginal rate of substitution between consumption and labor is equated to the real wage.

2.2 Firms

There is a continuum of infinitely living and monopolistically competitive firms indexed by h on the interval $[0, n)$ for the home country and by f on the interval $[n, 1]$ for the foreign country. They produce differentiated goods which are bundled into homogeneous home and foreign goods by a constant returns to scale of the Dixit-Stiglitz form:

$$Y_t = \left[\left(\frac{1}{n} \right)^{1/\theta} \int_0^n Y_t(h)^{\frac{\theta-1}{\theta}} \mathrm{d}h \right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad Y_t^* = \left[\left(\frac{1}{1-n} \right)^{1/\theta} \int_n^1 Y_t^*(f)^{\frac{\theta-1}{\theta}} \mathrm{d}f \right]^{\frac{\theta}{\theta-1}}.$$

The production technology of the representative home firm h combines labor as primary input and a country-specific productivity shock.[12]

$$Y_t(h) = A_t L_t(h). \tag{8}$$

Output is normalized by population size, so that it is expressed in per capita terms. We thus deduce that total home labor demand is given by

$$L_t = \int_0^n L_t(h)\, \mathrm{d}h = \frac{Y_t V_t}{A_t} \tag{9}$$

where $V_t = \int_0^n \frac{Y_t(h)}{Y_t} \mathrm{d}h$ represents the dispersion of production across firms in the home economy.

Since input markets are perfectly competitive are country specific, the standard static first-order condition for cost minimization implies that all domestic firms have identical real marginal cost, MC_t, given by,

$$MC_t = \frac{1}{(1+\vartheta)} \frac{W_t}{P_{H,t} A_t} \tag{10}$$

[12] We assume that the productivity shock A_t follows an AR(1) process: $A_t = (1-\rho_a)\bar{A} + \rho_a A_{t-1} + \eta_{a,t}$.

where ϑ is a subsidy for output that offsets the effect on imperfect competition in goods markets on the steady-state level of output $(0 \leq \vartheta < 1)$.

Firms price setting decision is modelled through a modified version of the Calvo's (1983) staggering mechanism. In addition to the baseline mechanism, we allow for the possibility that firms that do not optimally set their prices may nonetheless adjust it to keep up with the previous period increase in the general price level (see Sbordone, 2003, and Christiano, Eichenbaum, and Evans, 2005, for details concerning this assumption). In each period, a firm faces a constant probability, $1-\alpha$, of being able to re-optimize its price and chooses the new price $\tilde{P}_{H,t}(h)$ that maximizes the expected discounted sum of profits

$$\mathbb{E}_t \sum_{k=0}^{\infty} \alpha^k \Upsilon_{t,t+k} \left[\frac{\tilde{P}_{H,t}(h) \Psi^H_{t,t+k}}{P_{H,t+k}} - MC_{t+k} \right] Y_{t+k}(h) \tag{11}$$

subject to the sequence of demand equations:

$$Y_{t+k}(h) = \left(\frac{\tilde{P}_{H,t}(h) \Psi^H_{t,t+k}}{P_{H,t+k}} \right)^{-\theta} Y_{t+k} \tag{12}$$

where $\Upsilon_{t,t+k} = \beta^k \mathcal{U}_C(C_{t+k}) / \mathcal{U}_C(C_t)$ is the discount factor between time t and $t+k$, and

$$\Psi^H_{t,t+k} = \begin{cases} \prod_{\nu=0}^{k-1} (\bar{\pi}_H)^{1-\xi} (\pi_{H,t+v})^{\xi} & k > 0 \\ 1 & k = 0, \end{cases} \tag{13}$$

where $\bar{\pi}_H$ is the domestic trend inflation and the coefficient $\xi \in [0,1]$ indicates the degree of indexation to past prices, during the periods in which firm is not allowed to re-optimize. $\Psi^H_{t,t+k}$ is a correcting term that accounts for the fact that, if the firm h does not re-optimize its price, it updates it according to the rule:

$$P_{H,t}(h) = (\bar{\pi}_H)^{1-\xi} (\pi_{H,t-1})^{\xi} P_{H,t-1}(h). \tag{14}$$

Consequently, the first-order condition associated to the profit maximization implies that firms set their price equal to the discounted stream of expected future real marginal costs:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \alpha^k \Upsilon_{t,t+k} \left[(\bar{\pi}_H)^{(1-\xi)k} \left(\frac{P_{H,t+k-1}}{P_{H,t-1}} \right)^{\xi} \frac{\tilde{P}_{H,t}(h)}{P_{H,t+k}} - \frac{\theta}{\theta-1} MC_{t+k} \right] Y_{t+k}(h) = 0. \tag{15}$$

If flexible prices is assumed $(\alpha = 0)$, this expression gives the optimal relative price $\tilde{P}_{H,t}(h) / P_{H,t} = \mu MC_t$, where $\mu \equiv \theta / (\theta - 1)$ is the optimal markup in a flexible-price economy. As there are no firm-specific shocks in this economy, all firms that are allowed to re-optimize their price at date t select the same optimal price $\tilde{P}_{H,t}(h) = \tilde{P}_{H,t}$, $\forall h$.

Staggered price setting under partial indexation implies the following expression for the evolution of the domestic price index:

$$P_{H,t} = \left[\alpha \left((\bar{\pi}_H)^{1-\xi} (\pi_{H,t-1})^{\xi} P_{H,t-1} \right)^{1-\theta} + (1-\alpha) \left(\tilde{P}_{H,t} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{16}$$

The price setting problem solved by firms in the foreign country is similar and leads to an optimal rule analogous to equation (15). Yet, we allow foreign structural parameters (α^*, ξ^*) and country-specific shocks (A_t^*) to differ from their home country counterparts.

2.3 Real exchange rate dynamics

Under the assumption of complete markets, domestic and foreign households trade in state-contingent claims denominated in the home currency. This implies the following perfect risk-sharing condition (cf. Chari, Kehoe, and McGrattan, 2002):[13]

$$Q_t = \kappa \frac{\mathcal{U}^*_{C^*,t}}{\mathcal{U}_{C,t}} \tag{17}$$

where the real exchange rate, defined as $Q_t \equiv S_t P_t^* / P_t$, is proportional to the ratio of the marginal utility of consumption between the two countries.[14] The assumption of international market completeness insures that, in our model, the real exchange rate and consumption are stationary variables (see also Benigno, 2004).

Since the real exchange rate deviates from PPP because of home bias in preferences, we also have

$$Q_t = \left(\frac{S_t P^*_{H,t}}{P_{H,t}}\right)^{\omega^*} \left(\frac{S_t P^*_{F,t}}{P_{F,t}}\right)^{1-\omega^*} \left(\frac{P_{F,t}}{P_{H,t}}\right)^{\omega-\omega^*} = (\mathcal{T}_t)^{\omega-\omega^*} \tag{18}$$

where $\mathcal{T}_t$ is the home terms of trade, i.e. the relative price between foreign and home bundles of goods as perceived by the home resident. It is defined as[15]

$$\mathcal{T}_t = \frac{P_{F,t}}{P_{H,t}} = \frac{S_t P^*_{F,t}}{P_{H,t}}. \tag{19}$$

This definition implies, using equations (5), (17), and (18):

$$(\mathcal{T}_t)^{\omega-\omega^*} = \kappa \frac{\varepsilon^*_{p,t} \left(C_t - \gamma C_{t-1}\right)^{\sigma}}{\varepsilon_{p,t} \left(C_t^* - \gamma^* C^*_{t-1}\right)^{\sigma^*}}. \tag{20}$$

Equation (20) provides a rather elegant way to escape the exchange rate non-stationarity and model indeterminacy issues. Note that, when there is no home bias in preferences ($\omega = \omega^*$), the perfect risk sharing assumption does not allow to determine the terms of trade anymore.

[13] It is worth emphasizing that the exchange rates between the countries within the euro area have experienced significant changes over the estimation period. For instance, the French Franc and the Italian Lira have been depreciated several times with respect to the German Mark, notably for compensating loss of competitiveness. It is hence necessary to incorporate the relation between exchange rate and price differential in the structural model, even though the exchange rate is now fixed within the area.

[14] $\kappa = \left[S_0 P_0^* \mathcal{U}_{C,0}\right] / \left[P_0 \mathcal{U}^*_{C^*,0}\right]$ is a constant that depicts initial condition.

[15] The foreign terms of trade are simply given by $\mathcal{T}_t^* = P^*_{H,t}/P^*_{F,t} = 1/\mathcal{T}_t$, because the law of one price holds.

Combining Euler equation (6) with the perfect risk sharing equation (17), we obtain the following dynamics for the real exchange rate and the terms of trade:

$$\mathbb{E}_t\left[\frac{Q_{t+1}}{Q_t}\right] = \mathbb{E}_t\left[\frac{\mathcal{U}_C^*\left(C_{t+1}^*\right)\mathcal{U}_C\left(C_t\right)}{\mathcal{U}_C^*\left(C_t^*\right)\mathcal{U}_C\left(C_{t+1}\right)}\frac{P_t^*P_{t+1}}{P_tP_{t+1}^*}\right] = \frac{1+i_t}{1+i_t^*} \tag{21}$$

$$\mathbb{E}_t\left[\frac{\mathcal{T}_{t+1}}{\mathcal{T}_t}\right] = \mathbb{E}_t\left[\frac{P_{F,t+1}^*P_{H,t}}{P_{H,t+1}P_{F,t}^*}\frac{1+i_t}{1+i_t^*}\right]. \tag{22}$$

Equation (21) is the Uncovered Interest rate Parity (UIP) condition, which states that the expected change in the exchange rate is exactly compensated by the real interest rate differential. It is worth emphasizing that the UIP condition is not an additional implication in the model, but rather a redundant relation.

2.4 Market clearing conditions

Demands for goods are given by the sub-index of consumption (3), the allocation of demand across each of the goods produced within a given country for consumers H, F are given by

$$C_t\left(h\right) = \frac{1}{n}\left(\frac{P_{H,t}\left(h\right)}{P_{H,t}}\right)^{-\theta}C_{H,t} \quad \text{and} \quad C_t^*\left(h\right) = \frac{1}{n}\left(\frac{P_{H,t}^*\left(h\right)}{P_{H,t}^*}\right)^{-\theta}C_{H,t}^*$$

$$C_t\left(f\right) = \frac{1}{1-n}\left(\frac{P_{F,t}\left(f\right)}{P_{F,t}}\right)^{-\theta}C_{F,t} \quad \text{and} \quad C_t^*\left(f\right) = \frac{1}{1-n}\left(\frac{P_{F,t}^*\left(f\right)}{P_{F,t}^*}\right)^{-\theta}C_{F,t}^*$$

The consumption aggregator (2) implies that home and foreign demands for composite home and foreign are given by

$$C_{H,t} = \omega\left(\frac{P_t}{P_{H,t}}\right)C_t \quad \text{and} \quad C_{H,t}^* = \omega^*\left(\frac{P_t^*}{P_{H,t}^*}\right)C_t^*$$

$$C_{F,t} = \left(1-\omega\right)\left(\frac{P_t}{P_{F,t}}\right)C_t \quad \text{and} \quad C_{F,t}^* = \left(1-\omega^*\right)\left(\frac{P_t^*}{P_{F,t}^*}\right)C_t^*$$

Then, goods market clearing in the home and foreign countries implies:

$$\begin{aligned} Y_t\left(h\right) &= nC_t\left(h\right) + \left(1-n\right)C_t^*\left(h\right) \\ &= \left(\frac{P_{H,t}\left(h\right)}{P_{H,t}}\right)^{-\theta}\left(\frac{P_t}{P_{H,t}}\right)\left(\omega C_t + \mathcal{T}_t^{\omega-\omega^*}\frac{1-n}{n}\omega^* C_t^*\right) \end{aligned}$$

and

$$\begin{aligned} Y_t^*\left(f\right) &= nC_t\left(f\right) + \left(1-n\right)C_t^*\left(f\right) \\ &= \left(\frac{P_{H,t}\left(f\right)}{P_{F,t}}\right)^{-\theta}\left(\frac{P_t}{P_{F,t}}\right)\left(\frac{n}{1-n}\left(1-\omega\right)C_t + \left(1-\omega^*\right)\mathcal{T}_t^{\omega-\omega^*}C_t^*\right) \end{aligned}$$

so that aggregate outputs in home and foreign goods are:

$$Y_t = \omega \left(\mathcal{T}_t\right)^{1-\omega} C_t + \frac{1-n}{n}\omega^* \left(\mathcal{T}_t\right)^{1-\omega^*} C_t^* \tag{23}$$

and

$$Y_t^* = \left(1-\omega\right)\left(\mathcal{T}_t\right)^{-\omega} \frac{n}{1-n} C_t + \left(1-\omega^*\right)\left(\mathcal{T}_t\right)^{-\omega^*} C_t^*. \tag{24}$$

2.5 Log-linear equilibrium

In order to rewrite the model in a tractable form for conducting estimation and policy simulation, we approximate the model using a first-order Taylor development around the steady state. The resulting system, expressed in terms of percentage deviation around the steady state is presented in Appendix A.[16]

Before looking at the link between heterogeneity and optimal monetary policy, our first objective is to estimate the model. We close the model by specifying an interest rate rule for each country. We adopt feedback rules such that the nominal interest rate adjusts to the deviation of inflation to its steady-state value and to the deviation of domestic aggregate output to its flexible-price equilibrium (or natural) value.[17] In addition, we allow for partial adjustment to capture the interest-rate smoothing found in actual data. The log-linearized home feedback rule is then given by:

$$\hat{\imath}_t = \psi_i \hat{\imath}_{t-1} + \left(1-\psi_i\right)\left[\psi_\pi \hat{\pi}_{H,t} + \psi_y \left(\hat{y}_t - \hat{y}_t^n\right)\right] + \hat{\varepsilon}_{i,t} \tag{25}$$

where $(\hat{y}_t - \hat{y}_t^n)$ denotes the log-deviation of home output to its natural value, $\hat{\varepsilon}_{i,t}$ is the monetary policy shock, $\psi_i \in (0,1)$, $\psi_\pi > 1$ and $\psi_y > 0$.[18]

At this level, we do not pay particular attention to the specification of the monetary policy reaction function. In particular, we do not try to determine the optimal timing for inflation and output in equation (25) or to incorporate the exchange rate or the terms of trade in the policy rule. The reason is that the historical policy rule has not been necessarily optimal, so that parameters of the reaction function cannot be viewed as structural ones. Consequently, we focus for the moment on a widely-accepted specification, in order to estimate structural parameters reflecting the behavior of private agents. The determination of the optimal monetary policy consistent with our structural model is performed in Section 4. (See also Dieppe, Küster, and McAdam, 2004, for a comparison of several policy rules using an AWM.)

[16] $\hat{x}_t$ denotes the log-deviation from the steady-state value $\bar{x}$, i.e. $\hat{x}_t = \log\left(x_t/\bar{x}\right)$.

[17] See Appendix B for the derivation of the flexible-price equilibrium.

[18] We assume that $\hat{\varepsilon}_{i,t}$ follows an AR(1) process: $\hat{\varepsilon}_{i,t} = \rho_i \hat{\varepsilon}_{i,t-1} + \eta_{i,t}$. We also estimated a specification with a time-varying inflation objective and an i.i.d. monetary policy shock, along the lines of SW. As in Onatski and Williams (2004), however, we obtained that the variance of the monetary policy shock is essentially null. Consequently, we kept specification (25) that does not resort to a shock with a zero variance.

In the case of an area with more than two countries, the broad structure of the model remains essentially unchanged. The major change is that, in an N-country model, international transmission mechanisms pass through $(N-1)$ independent terms of trade. Consequently, since the Phillips curve depends on the terms of trade through movements in real marginal cost, inflation dynamics is affected by demand conditions in all countries. Moreover, domestic consumption is affected by the average of real interest rates prevailing in all countries of the area.

3 Estimation

We now concentrate on the two forecasting models that will be used to evaluate the optimal monetary policy rules. The first one is an AWM that implicitly assumes that the heterogeneity of behaviors and the asymmetry of shocks across countries can be neglected in the design of monetary policy. For this purpose, we resort to the closed-economy version of the model described above, estimated over aggregated data of the euro area. The second model is an MCM that incorporates information on individual countries, allowing model parameters to differ from one country to another.

3.1 Data

The AWM is estimated for the euro area, while the MCM is estimated for the three largest countries of the area (Germany, France, and Italy). The sample period runs from 1970:1 to 1998:4 at a quarterly frequency. The data are drawn from OECD Business Sector Data Base for individual countries.[19] As regards the euro area, we used two kinds of data. The first data set corresponds to the weighted average of series pertaining to the three countries under study, which cover some 70% of the area-wide GDP. Aggregation is performed in the same way as in Fagan, Henry and Mestre (2001). The second data set is the updated Area-Wide Model database from Fagan, Henry and Mestre. We primarily focus on the first data set in the estimation of the AWM, because this data is consistent with that used for the MCM.

The estimation of the model is based ultimately on three key macroeconomic variables for each country: real consumption, the inflation rate, and the nominal short-term interest rate. Consumption is defined as real consumption expenditures, linearly detrended.[20] We measure inflation as the annualized quarterly percent change in the implicit GDP deflator. The interest rate is the three-month money-market rate. Figure 1 displays the historical path of the various series under consideration for each country or area. We first notice

[19] Note that, in the case of Germany, we corrected for the mechanical impact of re-unification on GDP and GDP deflator data using data for West Germany for the year 1991.

[20] We also examined a detrended consumption computed using the regression on a quadratic time trend or a Hodrick-Prescott filter, and we obtained very similar results.

that the two data sets for the euro area look very similar. We also observe a downward trend in inflation and interest rate, which mainly corresponds to the convergence process of economic conditions within the euro area. The structural model presented above is clearly not designed to capture such an empirical feature. Therefore, inflation and the nominal interest rate are detrended by the same quadratic trend in inflation. It should be noticed that neither the terms of trade nor the real marginal cost are necessary for the estimation of model, since they are exact functions of the other macroeconomic variables.

3.2 Econometric approach

For estimating the DSGE model described above, we adopt the Bayesian strategy proposed, among others, by Fernandez-Villaverde and Rubio-Ramirez (2003), Schorfheide (2003), and SW.[21] Most alternative approaches are precluded in our context. On one hand, calibration is not a promising avenue, because we focus on the effect of heterogeneity between countries within the euro area. The choice of distinct parameters for the various countries would be largely arbitrary, since the economic differences between these countries are not always clearly established. On the other hand, the FIML has proved to be rather tricky to implement in medium- or large-scale models and resulted in unrealistic estimation of some important structural parameters. In particular, the Calvo's probability α that a firm is not allowed to re-optimize its price was found to reach its upper bound of 1, a value that has to be rules out for theoretical reasons. Consequently, we resort to the Bayesian econometrics to incorporate some prior information on structural parameters and render the estimation procedure more stable.

Let $\hat{x}_t = \left(\hat{x}_t^k, k = 1, \cdots, N\right)$ denote the vector of observable variables, where $\hat{x}_t^k = \left(\hat{c}_t^k, \hat{\pi}_{H,t}^k, \hat{\imath}_t^k\right)'$ contains the country-k observable variables (consumption, inflation and interest rate). The log-linearized MCM is cast in a state-space representation for $\hat{x}_t$ in order to form the likelihood function of the data:

$$\hat{x}_t = C(\Theta)\,\hat{s}_t \tag{26}$$

$$\hat{s}_t = A(\Theta)\,\hat{s}_{t-1} + B(\Theta)\,\eta_t \tag{27}$$

where $\hat{s}_t$ is the vector of state variables. In addition to observable variables, it includes unobservable variables such as marginal cost, natural output, terms of trade or shock processes. Last, η_t is a vector of i.i.d. variables with mean zero and covariance matrix $\Sigma(\Theta)$. The system matrices $C(\Theta)$, $A(\Theta)$, $B(\Theta)$ and $\Sigma(\Theta)$ are all functions of the parameter vector Θ.

A Kalman filter is used to estimate the system (26)–(27). The algorithm preliminary evaluates the number of explosive eigenvalues. Consequently, indeterminate models (that

[21] Procedures to compute Bayesian econometrics are available in GAUSS software (see Schorfheide, 2003) and MATLAB software (the pre-processor DYNARE – developed by M. Juillard – includes now a module for estimation. See http://www.cepremap.cnrs.fr/dynare/).

do not satisfy the Blanchard-Kahn conditions) are directly ruled out during the course of the estimation.

For a given structural model $\mathcal{M}_i$ and a set of parameters Θ, we denote $\Gamma(\Theta|\mathcal{M}_i)$ the prior distribution of Θ and $\mathcal{L}(X_T|\Theta,\mathcal{M}_i)$ the likelihood function associated to the observable variables $X_T = \{\hat{x}_t\}_{t=1}^{T}$. Then, from Bayes rule, the posterior distribution of the parameter vector is proportional to the product of the likelihood function and the prior distribution of Θ,

$$\Gamma(\Theta|X_T,\mathcal{M}_i) \propto \mathcal{L}(X_T|\Theta,\mathcal{M}_i)\,\Gamma(\Theta|\mathcal{M}_i). \tag{28}$$

Given the specification of the model, the posterior distribution cannot be recovered analytically. However, it can be evaluated numerically, using a Monte-Carlo Markov Chain (MCMC) sampling approach. More specifically, we rely to the Metropolis-Hastings (MH) algorithm to obtain a random draw of size 100,000 from the posterior distribution of the parameters.[22] The mode and the Hessian of the posterior distribution evaluated at the mode are used to initialize the MH algorithm.

3.3 Prior distribution

In this section, we describe how we selected the prior distribution for unknown parameters. In most cases, priors have been chosen to be very close to those adopted by SW for the euro area, but we also incorporate some information drawn from Onatski and Williams (2004).[23] Priors are reported in the first column of Table 1. The habit persistence parameter, γ, the fraction of firms that are not allowed to re-optimize their price, α, and the degree of price indexation, ξ, are assumed to follow a beta distribution, with a mean of 0.7 and a standard error of 0.1. The mean value of 0.7 is close to values found in other studies in the literature. The inverse of the inter-temporal elasticity of substitution of consumption, σ, and the inverse of the elasticity of labor disutility, φ, are assumed to follow a normal distribution, because they may theoretically take rather large values. They have a mean of 2 with a standard error of 0.25. This choice is based on evidence provided by Onatski and Williams (2004) who stress that these parameters may actually be larger than those reported by SW. Parameters pertaining to the monetary policy reaction function are standard: the long-term parameter on inflation ψ_π is 1.5 and the long-term parameter in output gap ψ_y is 0.5, with a standard error of 0.1, corresponding to the plain vanilla Taylor rule (they follow a normal distribution). The smoothing parameter ψ_i follows a beta distribution (we do not show concern about super inertial rules), with a mean of 0.7 and a standard error of 0.1. The persistence parameters (ρ_p, ρ_a, and ρ_i) are also assumed to follow a beta

[22] The first 50,000 observations are discarded to eliminate any dependence on the initial values.

[23] The latter authors provide an interesting investigation of some shortcomings of the standard Bayesian approach in the context of DSGE models. In particular, they put forward that parameter estimates are very sensitive to the way priors are introduced. In the estimation of the model, we took advantage of some of their results.

distribution, with a mean of 0.6 and a standard error of 0.1. We opt for a prior uniform distribution between $[0, 2]$ for all standard deviations of the stochastic shocks, σ_p, σ_a, and σ_i.

While the shocks in a given country are assumed to be uncorrelated, we allow a non-zero correlation between a given shock in two countries. We thus denote δ_p, δ_a, and δ_i the correlations between domestic and foreign preference shocks, technology shocks, and monetary policy shocks, respectively. Correlations across countries have a normal distribution with a mean of 0.2 and a standard error of 0.1. We use the same priors for all countries and the euro area in turn.

Finally, we imposed dogmatic priors over the discount factor β and the elasticity of substitution across goods produced in a given country, θ. The values we use ($\beta = 0.99$ and $\theta = 10$) are conventional in the literature. The consumption/output ratio s is set equal to 1 for all countries, assuming that commercial trade is broadly balanced. The selection of the parameters of home bias in preferences (ω) is more tricky since the three countries under study are far from covering the whole external trade. We therefore set these parameters as follows, in order to reflect the weight of each country in the external trade of the others: the weights of German, French and Italian goods in the consumption of German households are $(0.8; 0.11; 0.09)$. For French and Italian households, the weights are $(0.13; 0.8; 0.07)$ and $(0.13; 0.07; 0.8)$ respectively. We checked that marginally altering these values would not change our results in any significant way.

3.4 Parameter estimates

3.4.1 Results for the AWM

Table 1 provides two sets of information regarding parameter estimates. The first set reports the posterior mode of parameters, that is obtained directly by maximizing the log of the posterior distribution with respect to parameters.[24] The second set contains the 5, 50, and 95 percentiles of the posterior distribution of parameters. Figure 2 summarizes this information visually by plotting the prior and posterior distributions. As it appears clearly from the figure, the posterior distribution of some parameters (namely, σ, φ and ψ_π) is rather close to the prior distribution. This suggests that these parameters do not strongly affect the likelihood and translates in the rather large associated standard deviations.

As regards the behavior of households, our estimate of the inverse of the consumption elasticity of substitution (σ) is equal to 2.08, while the inverse of the elasticity of labor disutility (φ) is equal to 1.98. The habit persistence parameter γ is high to 0.87, indicating that the reference for current consumption is about 90% of past aggregate consumption. These estimates are somewhat different from the estimates reported in SW. Their estimate are $\gamma = 0.6$, $\sigma = 1.37$, and $\varphi = 2.49$ respectively. We found that using their parameter

[24] Reported standard errors are computed using the Hessian of the log-likelihood function.

estimates in our set-up would significantly deteriorate the ability of the model to reproduce the auto-covariances of the actual consumption.

Focusing on the behavior of firms, we obtain estimates that are close to those of SW and Onatski and Williams (2004). The parameter of price indexation is $\xi = 0.48$, while the probability that firms are not allowed to re-optimize their price is $\alpha = 0.93$. The degree of price stickiness is rather large, since the average duration of price contracts is about 15 quarters. This figure is somewhat larger than microeconomic evidence, but it is in the range of previous macroeconomic estimates.

Another sizeable difference our estimates and those obtained by SW lies in the serial correlation of shocks. Our median estimates of the serial correlation of shocks range between 0.42 and 0.6, while they range between 0.81 and 0.87 in SW. This result suggests that our structural model is able to reproduce most persistence in the data without resorting too heavily to the serial correlation of shocks.

Finally, our estimate of the monetary policy rule is only indicative of how short-term interest rates reacted to macroeconomic developments over the sample period. In the absence of a common central bank over the sample, this estimate cannot be taken as reflecting plausibly the behavior of monetary authorities. The long-run response to inflation is $\psi_\pi = 1.48$ while the reaction to output gap is $\psi_y = 0.11$.

3.4.2 Results for the MCM

We now comment our estimates of the MCM parameters. The joint dynamics of the whole system is estimated simultaneously for Germany, France, and Italy. This is actually a rather time-consuming task, since it involves 9 observable series and 51 unknown parameters. Table 2 reports the parameter estimates of the MCM model and Figure 3 displays the prior and posterior distributions.

As regards the behavior of households, our estimates of the consumption elasticity of substitution (σ) range between 1.5 and 2, while the elasticity of labor disutility (φ) is close to 2. Although we select the same priors for all countries, we obtain significant differences for the habit persistence parameter γ. This parameter is estimated to be medium in Germany (0.63) and France (0.69), and large in Italy (0.78). We reject the null hypothesis that the three parameters are equal across countries, suggesting that there is some heterogeneity of structural parameters across countries. These estimates differ slightly from the estimates of the AWM since the area-wide habit persistence parameter is found to be significantly larger (0.87). Turning to the behavior of firms, we obtain some disparity in the parameters of price indexation ξ, that range between 0.28 for Germany and 0.43 for Italy, although the difference does not turn out to be significant.

Reaction function parameters display rather similar patterns across countries. The long-run reaction of short-term interest rate to inflation and output gap are close to 1.5 and

0.5 respectively in the three countries. The interest rate persistence ψ_i is about 0.85. The volatilities of the preference and technology shocks are very close for the three countries, although they are smaller than the area-wide counterparts. In contrast, some large differences in the variability of the monetary policy shock are found. While the volatility is low in Germany and Italy (around 0.23%), it is large in France (at 0.42%). This result may be related to some aspects of the French monetary policy, not incorporated in the model, such as the implicit anchoring to the German monetary policy from 1983 on.

Concerning the serial correlation of shocks, the table reveals some significant differences across countries for the preference shock ($\rho_p = 0.51$ in France and 0.80 in Italy) and for the technology shock ($\rho_a = 0.66$ in France and 0.86 in Italy). In contrast, the estimates of ρ_i are all very close to 0.45. Most cross-country correlations between shocks are significantly positive. Note however that shocks are far from being perfectly correlated across countries. This result is of importance, because it suggests that the asymmetry of shocks may be rather large across countries. It appears as the main source of heterogeneity within the euro area.

The second interesting result lies in the differences in the parameter estimates between countries and the euro area as a whole. The area-wide estimation of parameters describing the behavior of households appears to suffer from an aggregation bias. Such an aggregation bias has already been pointed out as a possible undesirable outcome of estimating an AWM (Demertzis and Hugues Hallett, 1998). Our results suggest that it operates at the levels of both households and firms.

3.5 Assessing the performances of the estimated models

There are several ways to assess the empirical performances of an estimated DSGE model. Most evaluations rely on the comparison with an a-theoretical VAR model.[25] Such a reference to a VAR model is rather natural, because the reduced form of log-linearized DSGE models can be viewed as a constrained VAR model. Thus, the test is based on whether the constraints imposed by the DSGE model to the VAR model are rejected by the data.

3.5.1 Posterior odds

A first way to assess empirical performances consists in comparing the posterior distributions of the DSGE and VAR models (see Geweke, 1999). Once the likelihood function and the prior distribution are given, the marginal likelihood of a given model $\mathcal{M}_i$ is obtained using the following expression

$$\mathcal{L}\left(X_T|\mathcal{M}_i\right) = \int_\Theta \mathcal{L}\left(X_T|\Theta, \mathcal{M}_i\right) \Gamma\left(\Theta|\mathcal{M}_i\right) \mathrm{d}\Theta. \tag{29}$$

[25] See, e.g., Geweke (1999), Fernandez-Villaverde and Rubio-Ramirez (2003), or Schorfheide (2003).

Multiple integration is required to obtain the marginal likelihood, making exact computation infeasible. However, using random draws from the posterior distribution, it is possible to evaluate the expression (29) numerically, as shown in Geweke (1999).

Let $\hat{\mathcal{L}}\left(X_T|\mathcal{M}_i\right)$ denote the estimated marginal likelihood of model $\mathcal{M}_i$. Then, we compute the Bayes factor between two models $\mathcal{M}_i$ and $\mathcal{M}_j$ as

$$\mathcal{B}_{i,j}\left(X_T\right)=\frac{\hat{\mathcal{L}}\left(X_T|\mathcal{M}_i\right)}{\hat{\mathcal{L}}\left(X_T|\mathcal{M}_j\right)}.$$

Now, assume that there are $m+1$ competing models $\mathcal{M}_i$ for $i=0,\cdots,m$, with $\mathcal{M}_0$ denoting the reference model. If we denote $\mathcal{P}_{i,0}$ the prior probability of model $\mathcal{M}_i$ (with $\sum_{j=0}^{m}\mathcal{P}_{j,0}=1$), we obtain the posterior odds, which incorporates information on priors, as follows:

$$\mathcal{PO}_{i,T}=\frac{\mathcal{P}_{i,0}\hat{\mathcal{L}}\left(X_T|\mathcal{M}_i\right)}{\sum_{j=0}^{m}\mathcal{P}_{j,0}\hat{\mathcal{L}}\left(X_T|\mathcal{M}_j\right)}.$$

A widely-accepted approach to assess the empirical performances of an estimated DSGE model relies on the comparison of the DSGE models with an a-theoretical VAR model. Such a reference to a VAR model is rather natural, since the reduced form of log-linearized DSGE models can be viewed as a constrained VAR model. Thus, the test is based on whether the constraints imposed by the DSGE model to the VAR model are rejected by the data.

Such an empirical assessment is performed in Table 3, which reports, for the DSGE models and the competing VAR models (from one to 4 lags), the prior probabilities $\mathcal{P}_{i,0}$, the marginal likelihood $\hat{\mathcal{L}}\left(X_T|\mathcal{M}_i\right)$, the Bayes factor $\mathcal{B}_{i,j}\left(X_T\right)$ and the posterior odds $\mathcal{PO}_{i,T}$. The reference model is the AWM in Panel A and the MCM in Panel B. Since the marginal likelihood cannot be computed analytically due to the complexity of the model, it is approximated using the simulation-based modified harmonic mean estimator proposed by Geweke (1999). We assign a prior probability of 1/5 to the five models under consideration.

We notice that the optimal VAR model is the VAR(4) model, since the posterior odds is equal to 87% for the area-wide data and essentially 100% for the multi-country data. This empirical evidence indicates that the data do not support the strong restrictions imposed by the DSGE, as compared to VAR models. A similar observation was made by Schorfheide (2000). As the posterior odds reported in the table indicate, the MCM is strongly rejected even with respect to the corresponding VAR(1) model. It should be emphasized, however, that the MCM is not designed to capture all statistical characteristics in the data. In particular, the joint dynamics of shocks is overly constrained, since most cross-correlations are imposed to be zero. In addition, it imposes several constraints on the contemporaneous relationships between variables, in particular across countries: For instance, international transmission mechanisms do not involve any additional parameter as compared to a closed economy set-up. From this point of view, a more relevant way to assess the performances of the DSGE would be whether the model is able to replicate some important stylized facts estimated on actual data, such as cross-covariances.

Another important explanation for the poor ability of the DSGE models to reproduce some features of the actual data is the likely occurrence of structural shifts over the sample period. In particular, it may be argued that monetary policy rules have experienced significant changes during this period. We did not pursue this way, however, because our focus is on the implementation of an optimal rule by the central bank. Consequently, the identification of possible break in the historical policy rules would be an interesting, yet distracting, additional results. Moreover, we expect that shifts in monetary policy rules would not affect too dramatically the structural parameters of the model. A reason is that structural parameters are not likely to be altered by changes in agents' expectations.

3.5.2 Comparison of cross-covariances

We now compare the DSGE-based cross-covariances with those obtained from a VAR model. For this purpose, we follow the approach adopted by SW and proceed as follows: on one hand, we compute the empirical cross-covariances by estimating a VAR(2) model on the actual data. On the other hand, we simulate a large number of random samples using the DSGE model. Then, we estimate a VAR(2) model on the simulated data and deduce the model-based cross-covariances for the VAR model. The comparison of the two sets of cross-covariances is displayed in Figure 4 for the AWM and in Figure 5 for the MCM. The solid line represent the median and the dashed lines represent the 5% and 95% intervals for the covariance sample of the DSGE models. The dotted line gives the empirical cross-covariances based on the VAR(2) model estimated on the actual data.

An important point is that model-based error bands are in general rather large, so that most cross-covariances are in fact insignificantly different from 0. This suggests that there is a large amount of uncertainty in model-based as well as in empirical cross-covariances. In addition, in most cases, empirical cross-covariances are well within the error bands. The DSGE models perform particularly well in reproducing the empirical auto-covariances (along the diagonal) and the output-inflation cross-covariances. An exception is the positive cross-covariances between interest rate and past output that is severely under-estimated by the DSGE. Overall, our results suggest that both the AWM and the MCM are able to reproduce most dynamics of the data.

4 Optimal monetary policy

In this section, we turn to the evaluation of the optimal monetary policy in the context of the euro area. Therefore, we acknowledge that there is a unique central bank within the euro area, and we keep the nominal exchange rate constant and equal to one within the area.[26] An advantage of having developed a structural model based on optimizing behaviors

[26] This hypothesis does not affect the structure of terms of trade in the theoretical model.

is that it provides a natural objective for monetary policy, namely the maximization of the welfare, defined as the expected utility of the representative household. Following Woodford (2003, chap. 6), we compute the second-order Taylor series approximation to this objective function as a quadratic function of variables and shocks. Various aspects of our model, such as inflation inertia and external habit formation, require that we derive an appropriate welfare-based stabilization objective.

Several important issues arise when considering the evaluation of welfare in the context of an open economy with habit formation. First, as discussed in Rotemberg and Woodford (1998), under the assumption that the constant subsidy for output ϑ neutralizes the distortion associated with firm's market power, it can be shown that in a closed economy the optimal monetary policy is the one that replicates the flexible-price equilibrium allocation.[27] In an open economy, as noted by Corsetti and Pesenti (2001) and Galí and Monacelli (2004), a second source of distortion comes from the fact that the transmission of monetary policy affects demand not only through the relative cost of borrowing, but also through its effect on the terms of trade. This possibility is a consequence of the imperfect substitutability between home and foreign goods, combined with sticky prices. As in Benigno and Benigno (2003), we assume that the subsidy for output exactly offsets the combined effects of market power and the terms-of-trade distortions in the steady state.

Second, in an open economy framework, most previous studies investigated the way the optimal monetary policy may be designed, for a given type of monetary arrangement between central banks. Typical extreme cases are non-cooperation and full cooperation (see, e.g., Benigno, 1999, Clarida, Galí, and Gertler, 2002, Tchakarov, 2004). Our evaluation of the optimal monetary policy obviously presumes full cooperation, since only one central bank is involved. More specifically, our focus is not on whether coordination may improve the global welfare, but rather on whether the fully cooperative monetary policy should be based on an aggregated model or on a multi-country model.

In addition to the utility-based welfare criterion, we also investigate the implications of heterogeneity in the context of *ad hoc* loss functions. A motivation for considering such *ad hoc* functions is that the utility-based welfare does not incorporate any concern for interest rate. Since interest-rate smoothing has been found to be an important empirical feature of monetary policy, we investigate how it may affect to role plaid by heterogeneity on the optimal monetary policy.

[27] The intuition is straightforward: with the subsidy in place, there is only one distortion left in the economy, namely sticky price. By stabilizing markups at their frictionless level, nominal rigidities cease to be binding, since firms do not feel any desire to adjust their price.

4.1 The welfare objective

4.1.1 Expression for the welfare

A DSGE model delivers a natural measure of welfare based on the representative household's utility. It is defined as the conditional expectation of the current and discounted future values of the approximated utility function. In Appendix C, we derive the welfare for the two-country model. In the closed-economy version, that corresponds to our AWM, the aggregated welfare at date 0 can be approximated by

$$\begin{aligned}\mathcal{W}_0^{AWM} \approx\; & -\frac{\bar{\mathbb{U}}_{\bar{C}}\bar{C}}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big\{\left(\hat{c}_t-\Phi_c+\beta\gamma\right)^2+\left(\beta\gamma\left(1+\rho_p\right)-\sigma-1\right)\hat{c}_t^2 \\ & +\frac{\sigma}{(1-\gamma)}\left(\hat{c}_t-\gamma\hat{c}_{t-1}\right)^2+\left(\frac{\sigma}{1-\gamma}+\varphi\right)\left(\hat{c}_t-\hat{c}_t^n\right)^2-\frac{\sigma\gamma}{(1-\gamma)}\left(\hat{c}_t-\hat{c}_{t-1}^n\right)^2 \\ & -\beta\gamma\rho_p\left(\hat{c}_t-\hat{\varepsilon}_{p,t}\right)^2+\frac{\theta\alpha}{(1-\alpha)(1-\beta\alpha)}\left(\hat{\pi}_t-\xi\hat{\pi}_{t-1}\right)^2\Big\}+t.i.p. \end{aligned} \qquad (30)$$

where all variables denote area-wide variables and parameters are those pertaining to the AWM. $\hat{c}_t^n$ is the natural value of aggregate consumption and $t.i.p.$ regroups terms independent of the actual policy. Φ_c is a measure of inefficiency in the economy at steady state as compared to the economy at the flexible-price equilibrium (see Woodford, 2003, and Appendix B). Expression (30) combines features implied by the introduction of inflation inertia and external habit formation. Interestingly, we notice that, in our estimated model, it is optimal for the central bank to put a much larger weight (about 100 times more) on the stabilization of goods price inflation than on the stabilization of the other variables. In addition, as indicated above, no concern about interest-rate stabilization is present in this expression.

The aggregated welfare in the multi-country approach (taking account of the heterogeneity across countries) is defined as the weighted average of the national welfare functions:[28]

$$\mathcal{W}_0^{MCM}=n\mathcal{W}_0+(1-n)\,\mathcal{W}_0^{*} \qquad (31)$$

where $\mathcal{W}_0$ and $\mathcal{W}_0^{*}$ are detailed in Appendix C.

4.1.2 Evaluation of the optimal policy rule

We evaluate the optimal monetary policy by taking the unconditional expectation of expressions (30) and (31) with respect to the distribution of exogenous shocks, and under the assumption that all endogenous variables in the initial period are at their unconditional expectation of zero. This assumption ensures that the desirability of the chosen plan does

[28] In the N-country case, the total welfare is given by $\mathcal{W}_0^{MCM}=\sum_{j=1}^{N}n_j\mathcal{W}_0^{j}$, where n_j is the weight of the country j in the euro-area GDP and $\sum_{j=1}^{N}n_j=1$. In our evaluation, we hold the following weights: 0.4 for Germany and 0.3 for France and Italy.

not depend upon initial conditions at time 0 (see Woodford, 2003). We thus define the unconditional expectation of the welfare as $\breve{\mathcal{W}}_0 = (1-\beta)\,\mathbb{E}\sum_{t=0}^{\infty}\beta^t\mathbb{E}_0\mathbb{W}_t$, where $\mathbb{W}_t$ is the household period utility function defined in Appendix C.[29]

Since our aim is to compare the welfare consequences of adopting as forecasting model the (sub-optimal) AWM instead of the MCM, we proceed as follows, considering the two following approaches in turn:

- in the *aggregated approach*, the central bank forecasts area-wide variables (using the AWM) and adopts a policy rule designed in terms of aggregate variables only, in the form

$$\hat{\imath}_t = F_{AWM} \times \hat{s}_t^{AWM}$$

where $\hat{s}_t^{AWM}$ is the vector of state variables under the AWM, i.e. $\hat{s}_t^{AWM} = (\hat{\varepsilon}_{p,t},\, \hat{a}_t,\, \hat{c}_t^n,\, \hat{c}_{t-1}^n,\, \hat{c}_t,\, \hat{c}_{t-1},\, \hat{\pi}_t,\, \hat{\imath}_{t-1}')$. The optimal monetary policy rule is then obtained by maximizing the aggregated welfare (expression (30)), assuming homogeneity of behaviors across countries. The maximal value of welfare is denoted $\breve{\mathcal{W}}_0^{AWM}$.

- in the *multi-country approach*, the central bank uses the MCM to forecast national variables. The policy rule is still assumed to be defined in terms of aggregate variables, since the policy rule is designed on the basis of area-wide developments only. Its expression is given by

$$\hat{\imath}_t = F_{MCM}^{const} \times \Xi \times \hat{s}_t^{MCM}$$

where $\hat{s}_t^{MCM}$ is the vector of state variables under the MCM, i.e., in a two-country set-up, $\hat{s}_t^{MCM} = (\hat{\varepsilon}_{p,t},\, \hat{\varepsilon}_{p,t}^*,\, \hat{a}_t,\, \hat{a}_t^*,\, \hat{c}_t^n,\, \hat{c}_t^{*n},\, \hat{c}_{t-1}^n,\, \hat{c}_{t-1}^{*n},\, \hat{c}_t,\, \hat{c}_t^*,\, \hat{c}_{t-1},\, \hat{c}_{t-1}^*,\, \hat{\pi}_{H,t},\, \hat{\pi}_{F,t}^*,\, \hat{\imath}_{t-1},\, \hat{\imath}_{t-1}^{*\prime})$ and Ξ is an aggregation matrix that defines the area-wide aggregates as functions of country variables. Then, the constrained optimal monetary rule (F_{MCM}^{const}) is obtained by maximizing the weighted average of national welfares (expression (31)), allowing heterogeneity of behaviors across countries. It should be noticed that this rule is not in general fully optimal under the MCM, since it imposes several constraints on the parameters of the rule. An important consequence is that it cannot be computed using the standard approach, based on solving the Bellman equation. Rather, the constrained rule F_{MCM}^{const} is obtained by numerically maximizing the welfare among all policy rules that include aggregate variables only. The maximal value of welfare is denoted $\breve{\mathcal{W}}_0^{const}$. For further use, we also define the fully-optimal policy rule as F_{MCM}^{opt} and the corresponding welfare as $\breve{\mathcal{W}}_0^{opt}$.

It is worth emphasizing that the two welfare functions ($\breve{\mathcal{W}}_0^{AWM}$ and $\breve{\mathcal{W}}_0^{const}$) cannot be directly compared, since they are evaluated under two different sets of assumptions. For the two functions to be comparable, we assume that the correct model for describing the dynamics of the economy within the euro area is the MCM, and we evaluate the welfare associated with the two policy rules using the MCM. Under the MCM, the welfare of the

[29] By maximizing unconditional welfare, we are implicitly maximizing welfare in the steady state. The welfare comparison ignores the possibility of losses in welfare on the transition path from one steady state to another (see Schmitt-Grohé and Uribe, 2004).

area is computed as the weighted average of national welfares, and this expression collapses to the aggregated welfare under full homogeneity only. The maximal value of welfare associated with the AWM policy rule (F_{AWM}) but evaluated under the MCM is denoted $\breve{\mathcal{W}}_0^{aggr}$. We then deduce the cost of using the (sub-optimal) aggregated approach from the comparison of $\breve{\mathcal{W}}_0^{aggr}$ and $\breve{\mathcal{W}}_0^{const}$.

4.2 Welfare implications of heterogeneity

The constrained optimal rule evaluated under the multi-country approach (F_{MCM}^{const}) is expected to induce a higher welfare than the optimal rule under the aggregated approach (F_{AWM}). The reason is that, although both rules are defined in terms of aggregate variables only, the parameters F_{MCM}^{const} are obtained by maximizing the welfare under the "true" model. Assessing whether the central bank should be concerned about heterogeneity therefore requires that the welfare cost of using the AWM be economically significant. For this purpose, we compute two measures that provide some information on the welfare reduction due to the use of the AWM.

The first measure gives the cost of using the sub-optimal forecasting model AWM as a permanent percentage shift in steady-state aggregate consumption. It is defined by scaling the welfare loss $\left(\breve{\mathcal{W}}_0^{aggr} - \breve{\mathcal{W}}_0^{const}\right)$ by $\bar{\mathbb{U}}_{\bar{C}}\bar{C}$:

$$\delta_1 = -\frac{\breve{\mathcal{W}}_0^{aggr} - \breve{\mathcal{W}}_0^{const}}{\bar{\mathbb{U}}_{\bar{C}}\bar{C}}. \tag{32}$$

Such measure has been previously investigated for instance by Erceg, Henderson, and Levin (2000), Benigno and López-Salido (2002), Amato and Laubach (2004), or Tchakarov (2004).[30]

The second measure is the fraction of the gap (in terms of welfare) between the AWM-based rule and the fully optimal MCM-based rule that is filled by the constrained MCM-based rule. It is defined as

$$\delta_2 = \frac{\breve{\mathcal{W}}_0^{const} - \breve{\mathcal{W}}_0^{aggr}}{\breve{\mathcal{W}}_0^{opt} - \breve{\mathcal{W}}_0^{aggr}}. \tag{33}$$

This measure allows to compare our evaluations with those performed for instance by Angelini *et al.* (2002) and Monteforte and Siviero (2003) in the context of *ad hoc* loss functions.

Table 4 reports the welfare obtained for the various policy rules considered, using the median of the posterior distribution of estimated parameters. The first row gives the welfare under the AWM, the constrained MCM and the fully optimal MCM as well as the

[30] Since $\breve{\mathcal{W}}_0 = (1-\beta)\,\mathbb{E}(\mathcal{W}_0)$, expression (32) is also equivalent to

$$\delta_1 = -(1-\beta)\,\frac{\mathbb{E}(\mathcal{W}_0^{aggr}) - \mathbb{E}(\mathcal{W}_0^{const})}{\bar{\mathbb{U}}_{\bar{C}}\bar{C}}.$$

two measures of welfare cost. We obtain that the use of the AWM to define the monetary policy rule implies a welfare reduction as compared to the use of the constrained MCM. If we measure the welfare cost as the permanent percentage shift in steady-state aggregate consumption, we obtain that a cost of using the AWM is equal to $\delta_1 = 0.0037$. This suggests that the steady-state aggregate consumption level obtained using the AWM is almost 0.37 percent lower than the steady-state aggregate consumption obtained using the constrained MCM. This evaluation of the cost of using a sub-optimal forecasting model is rather large as compared to previous welfare evaluations.[31] Note, however, that our measure δ_2 provides additional insight on the source of welfare loss in using an AWM. Indeed, δ_2 indicates that the constrained MCM-based rule makes up for 98 percent of the distance between the AWM-based rule and the fully optimal MCM-based rule. This result suggests that, consistently with previous evidence, this is not the use of a restricted policy rule based on aggregate variables that is costly, but rather the use of a sub-optimal forecasting model.

As a robustness check, we also investigated the role of the two sources of endogenous persistence mechanisms we introduced in the model to reproduce the properties of the data, namely external habit formation and price indexation. We measure how varying both of these assumptions affects the value of the cost of using an AWM rather than an MCM. To this end, we estimate the AWM and the MCM under alternative assumptions, with and without habit formation and with and without price indexation. The second row of Table 4 reports the results for the two measures of welfare cost for the model without habit formation, the third one for the model without price indexation and the last one without habit formation or price indexation. As it may be expected, removing these friction mechanisms reduces the difference in welfare between the AWM and the MCM. Indeed, the welfare cost falls from 0.37 percent under the full specification to only 0.04 percent in absence of habit formation and price indexation. We also notice that the welfare cost of using the AWM is more widely reduced when we assume no price indexation than when we assume no habit formation. In the former case, we obtain $\delta_1 = 0.0012$ while we have $\delta_1 = 0.0024$ in the latter case. The main reason is that the price indexation parameter (ξ) affects the welfare through the expression $(\hat{\pi}_t - \xi\hat{\pi}_{t-1})^2$, which has a weight in the aggregate welfare 100 times larger than the weights on the other variables. Therefore, the rather large welfare cost of using the AWM appears to be mainly attributable to the introduction in our model of price indexation rather than to habit formation.

4.3 *Ad hoc* loss functions

Although it is theoretically appealing, a limitation of the approach based on the welfare maximization is that it does not involve any interest-rate smoothing, a feature that has

[31] Benigno and López-Salido (2002) estimate the cost of monetary policies in the context of heterogeneous Phillips curves within the euro area. They obtain that the cost of using an HICP-targeting policy rule instead of the optimal monetary policy is about 0.02 percent of steady state consumption.

been found to be necessary to reproduce the observed monetary policy rules. However, introducing a micro-founded concern for the interest-rate smoothing is rather complicated (Woodford, 2003). An alternative widely-used approach consists in using *ad hoc* loss functions to evaluate the optimal monetary policy.

Ad hoc loss functions are typically constructed by taking a weighted sum of the variances of inflation, output gap and the change in nominal interest rate. The latter component allows to incorporate some concern for interest-rate smoothing in the objective function of the central bank.

In the case of the AWM, the inter-temporal loss function is given by:

$$\Lambda_0^{AWM} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\hat{\pi}_t^2 + \Lambda_y \left(\hat{y}_t - \hat{y}_t^n \right)^2 + \Lambda_i \left(\hat{\imath}_t - \hat{\imath}_{t-1} \right)^2 \right], \tag{34}$$

and in the case of the MCM,

$$\begin{aligned} \Lambda_0^{MCM} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[\left(n \hat{\pi}_{H,t} + (1-n) \hat{\pi}_{F,t}^* \right)^2 \\ &\quad + \Lambda_y \left(n \left(\hat{y}_t - \hat{y}_t^n \right) + (1-n) \left(\hat{y}_t^* - \hat{y}_t^{*n} \right) \right)^2 + \Lambda_i \left(\hat{\imath}_t - \hat{\imath}_{t-1} \right)^2 \Big], \end{aligned} \tag{35}$$

where Λ_y and Λ_i are positive weights. We consider a wide range of values for the relative weights on output-gap stabilization and interest-rate smoothing. We will refer to different loss functions by their pair of weights, denoting $\Lambda = (\Lambda_y, \Lambda_i)$. Notice that contrary to some previous studies (e.g., Angelini *et al.*, 2002, or Monteforte and Siviero, 2003), we do not assume that the central bank has a feedback policy rule in terms of inflation, output gap and past interest rate only. Rather, we allow the policy rule to depend on any aggregate state variables, i.e. $\hat{s}_t^{AWM}$ and $\Xi \times \hat{s}_t^{MCM}$ for the AWM and the MCM respectively.

Table 5 reports the optimal loss for several pairs of weights of the *ad hoc* criterion.[32] We focus on a grid over the weights $\Lambda \in [0,1] \times [0,1]$, starting the grid from a small positive value to avoid the "singularity" of the loss function that may arise with zero weights. For all experiments, we obtain that the cost of using the AWM is noticeable. The constrained MCM-based rule makes up at least for 82 percent of the distance between the AWM-based rule and the fully optimal MCM-based rule. This result indicates that introducing some concern for interest-rate volatility in the welfare measure would not affect our main result that the use of the (sub-optimal) AWM is costly as compared to a model that incorporates cross-country heterogeneity.

[32] Only the measure δ_2 is used to compare loss functions, because δ_1 is meaningless in the context of the *ad hoc* criterion.

5 Conclusion

In this paper, we evaluate the cost of ignoring the cross-country heterogeneity within the euro area when implementing the optimal monetary policy. To address this issue, we develop a multi-country DSGE model, which is used to estimate the dynamics of national economies within the euro area. This model incorporates frictions required to reproduce the persistence of the actual data, including the presence of sticky-price setting and external habit formation in consumption. An additional characteristic of the model is the introduction of heterogenous behaviors across countries that allows to investigate the cost of using an AWM instead of an MCM.

Using Bayesian techniques, we estimate the AWM and MCM and provide evidence that the behavioral parameters in Germany, France, and Italy display some significant differences, and that shocks affecting the different economies are only very weakly correlated. Our results therefore highlight that heterogeneity can be mainly attributable to the asymmetry of shocks across countries rather than to differences in behavioral parameters.

Since our model is suitable for the analysis of optimal monetary policy, we then compare the two models on the basis of their ability to maximize the welfare of the area-wide representative household. The welfare associated to the two optimal rules are then compared allowing heterogeneity of behaviors. We find that using an AWM generates a relatively large welfare loss that corresponds to a permanent decrease in steady-state aggregate consumption by around 0.37 percent. Moreover, our results suggest that this is not the use of a rule based on aggregate variables that is costly in terms of welfare, but rather the use of a sub-optimal forecasting model. In addition to the utility-based welfare criterion, we also investigate the implications of heterogeneity in the context of *ad hoc* loss functions, that allow to incorporate some concern about interest-rate stabilization. We obtain that, for all loss functions with a concern about output-gap stabilization and interest-rate smoothing, the loss associated to the AWM is always economically higher than the loss associated to the MCM.

However, it may be argued that the difference between the AWM and the MCM in terms of welfare is rather limited, as compared to the cost of designing, estimating and using an MCM. It is worth emphasizing that our evaluation is based on the three largest countries of the area that may be viewed as very similar economies. It is likely that including additional economies would widen the discrepancies between the two models.

Appendix A: The log-linearized dynamic equilibrium

This Appendix displays the log-linearized dynamic equilibrium in the case of a two-country model.

- Home IS curve

$$\begin{aligned}\hat{c}_t &= \frac{\gamma}{1+\gamma}\hat{c}_{t-1} + \frac{1}{1+\gamma}\mathbb{E}_t\hat{c}_{t+1} - \frac{(1-\gamma)}{(1+\gamma)\sigma}\left(\hat{\imath}_t - \mathbb{E}_t\hat{\pi}_{H,t+1}\right) \\ &\quad + \frac{(1-\gamma)(1-\omega)}{(1+\gamma)\sigma}\mathbb{E}_t\Delta\hat{\tau}_{t+1} + \frac{\left(1-\rho_p\right)(1-\gamma)}{(1+\gamma)\sigma}\hat{\varepsilon}_{p,t}\end{aligned} \tag{36}$$

- Home Phillips curve

$$\hat{\pi}_{H,t} = \frac{\xi}{1+\beta\xi}\hat{\pi}_{H,t-1} + \frac{\beta}{1+\beta\xi}\mathbb{E}_t\hat{\pi}_{H,t+1} + \frac{(1-\beta\alpha)(1-\alpha)}{(1+\beta\xi)\alpha}\widehat{mc}_t \tag{37}$$

- Home marginal cost

$$\begin{aligned}\widehat{mc}_t &= \left(\frac{\sigma}{1-\gamma} + \varphi\omega s\right)\hat{c}_t - \frac{\gamma\sigma}{1-\gamma}\hat{c}_{t-1} + \varphi(1-\omega s)\,\hat{c}_t^* - (1+\varphi)\,\hat{a}_t \\ &\quad + \left[(1-\omega)(1+\varphi\omega s) + \varphi(1-\omega^*)(1-\omega s)\right]\hat{\tau}_t\end{aligned} \tag{38}$$

- Home aggregate output

$$\hat{y}_t = \left[(1-\omega)\,\omega s + (1-\omega^*)(1-\omega s)\right]\hat{\tau}_t + \omega s\hat{c}_t + (1-\omega s)\,\hat{c}_t^* \tag{39}$$

- Home preference shock

$$\hat{\varepsilon}_{p,t} = \rho_p\hat{\varepsilon}_{p,t-1} + \eta_{p,t} \tag{40}$$

- Home productivity shock

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \eta_{a,t} \tag{41}$$

- Foreign IS curve

$$\begin{aligned}\hat{c}_t^* &= \frac{\gamma^*}{1+\gamma^*}\hat{c}_{t-1}^* + \frac{1}{1+\gamma^*}\mathbb{E}_t\hat{c}_{t+1}^* - \frac{(1-\gamma^*)}{(1+\gamma^*)\sigma^*}\left(\hat{\imath}_t^* - \mathbb{E}_t\hat{\pi}_{F,t+1}^*\right) \\ &\quad - \frac{(1-\gamma^*)\,\omega^*}{(1+\gamma^*)\sigma^*}\mathbb{E}_t\Delta\hat{\tau}_{t+1} + \frac{\left(1-\rho_p^*\right)(1-\gamma^*)}{(1+\gamma^*)\sigma^*}\hat{\varepsilon}_{p,t}^*\end{aligned} \tag{42}$$

- Foreign Phillips curve

$$\hat{\pi}_{F,t}^* = \frac{\xi^*}{1+\beta\xi^*}\hat{\pi}_{F,t-1}^* + \frac{\beta}{1+\beta\xi^*}\mathbb{E}_t\hat{\pi}_{F,t+1}^* + \frac{(1-\beta\alpha^*)(1-\alpha^*)}{(1+\beta\xi^*)\alpha^*}\widehat{mc}_t^* \tag{43}$$

- Foreign marginal cost

$$\begin{aligned}\widehat{mc}_t^* &= \left(\frac{\sigma^*}{1-\gamma^*}+\varphi^*\left(1-\omega^*\right)s^*\right)\hat{c}_t^*-\frac{\gamma^*\sigma^*}{1-\gamma^*}\hat{c}_{t-1}^*+\varphi^*\left[1-\left(1-\omega^*\right)s^*\right]\hat{c}_t \\ &\quad -\left(1+\varphi^*\right)\hat{a}_t^*-\left[\omega\left[1+\varphi^*\left(1-\left(1-\omega^*\right)s^*\right)\right]+\omega^*\varphi^*\left(1-\omega^*\right)s^*\right]\hat{\tau}_t\end{aligned} \tag{44}$$

- Foreign aggregate output

$$\hat{y}_t^* = \left[1-\left(1-\omega^*\right)s^*\right]\hat{c}_t+\left(1-\omega^*\right)s^*\hat{c}_t^*-\left(\omega\left[1-\left(1-\omega^*\right)s^*\right]+\omega^*\left(1-\omega^*\right)s^*\right)\hat{\tau}_t \tag{45}$$

- Foreign preference shock

$$\hat{\varepsilon}_{p,t}^* = \rho_p^*\hat{\varepsilon}_{p,t-1}^*+\eta_{p,t}^* \tag{46}$$

- Foreign productivity shock

$$\hat{a}_t^* = \rho_a^*\hat{a}_{t-1}^*+\eta_{a,t}^* \tag{47}$$

- Terms of trade

$$\hat{\tau}_t = \frac{1}{\omega-\omega^*}\left[\frac{\sigma}{1-\gamma}\hat{c}_t-\frac{\gamma\sigma}{1-\gamma}\hat{c}_{t-1}-\frac{\sigma^*}{1-\gamma^*}\hat{c}_t^*+\frac{\gamma^*\sigma^*}{1-\gamma^*}\hat{c}_{t-1}^*+\hat{\varepsilon}_{p,t}^*-\hat{\varepsilon}_{p,t}\right]. \tag{48}$$

Notice that $s=\bar{C}/\bar{Y}$ and $s^*=\bar{C}^*/\bar{Y}^*$.

Appendix B: The log-linearized flexible-price output

The so-called natural output is obtained as the level of output that would prevail under flexible price in the absence of cost-push shocks. In this case, the optimal pricing decision for the firm h, i.e., the price that would maximize profits at each period is given by

$$P_{H,t}(h) = \frac{\mu}{(1+\vartheta)} \frac{W_t}{A_t},$$

where $\mu = \theta/(\theta - 1)$ is the optimal mark-up, and ϑ is the subsidy for output that offsets the effect on imperfect competition in goods markets on the steady-state level of output. Using the demand for good h, $Y_t(h) = \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta} Y_t$, we note that the relative supply of good h must in turn satisfy

$$\left(\frac{Y_t(h)}{Y_t}\right)^{-1/\theta} = \frac{\mu}{(1+\vartheta)} \frac{W_t}{P_{H,t}} \frac{1}{A_t}.$$

Note also that, in steady state,

$$\frac{\bar{\mathcal{U}}_{\bar{L}}}{\bar{\mathcal{U}}_{\bar{C}}} = \frac{(1+\vartheta)}{\mu} = 1 - \Phi_y$$

where Φ_y is a measure of inefficiency in the economy at steady state as compared to the economy at the flexible-price equilibrium (see Woodford, 2003).

Because all wages are the same in the case of flexible wages, we have $W_t(h) = W_t$ and $L_t(h) = L_t$ for all h. This implies that all sellers supply a quantity Y_t^n satisfying

$$1 - \Phi_y = \frac{\mathcal{U}_{L^n,t}}{\mathcal{U}_{C^n,t}} \frac{P_t}{P_{H,t}} \frac{1}{A_t} = \frac{(L_t^n)^{\varphi}}{(C_t^n - \gamma C_t^n)^{-\sigma}} \frac{(\mathcal{T}_t^n)^{1-\omega}}{A_t} = \frac{(Y_t^n/A_t)^{\varphi}}{\left(C_t^n - \gamma C_{t-1}^n\right)^{-\sigma}} \frac{(\mathcal{T}_t^n)^{1-\omega}}{A_t}.$$

Log-linearizing this expression yields,

$$\hat{y}_t^n = -\frac{\sigma}{(1-\gamma)\varphi}\hat{c}_t^n + \frac{\sigma\gamma}{(1-\gamma)\varphi}\hat{c}_{t-1}^n - \frac{(1-\omega)}{\varphi}\hat{\tau}_t^n + \frac{(1+\varphi)}{\varphi}\hat{a}_t$$

In using the terms of trade expression

$$\hat{\tau}_t^n = \frac{1}{\omega - \omega^*}\left[\frac{\sigma}{1-\gamma}\hat{c}_t^n - \frac{\gamma\sigma}{1-\gamma}\hat{c}_{t-1}^n - \frac{\sigma^*}{1-\gamma^*}\hat{c}_t^{*n} + \frac{\gamma^*\sigma^*}{1-\gamma^*}\hat{c}_{t-1}^{*n} + \hat{\varepsilon}_{p,t}^* - \hat{\varepsilon}_{p,t}\right],$$

with the definition of the aggregate output

$$\hat{y}_t^n = \omega s \hat{c}_t^n + (1-\omega s)\hat{c}_t^{n*} + \left[(1-\omega)\omega s + (1-\omega^*)(1-\omega s)\right]\hat{\tau}_t^n,$$

we obtain

$$\left(\frac{\sigma}{1-\gamma} + \varphi\omega s + \frac{\sigma\Psi}{(1-\gamma)}\right)\hat{c}_t^n = \frac{\gamma\sigma}{1-\gamma}(1+\Psi)\hat{c}_{t-1}^n - \left(\varphi(1-\omega s) - \frac{\sigma^*\Psi}{(1-\gamma^*)}\right)\hat{c}_t^{*n}$$
$$-\frac{\gamma^*\sigma^*\Psi}{(1-\gamma^*)}\hat{c}_{t-1}^{*n} - \Psi\left(\hat{\varepsilon}_{p,t}^* - \hat{\varepsilon}_{p,t}\right) + (1+\varphi)\hat{a}_t \qquad (49)$$

and

$$\hat{y}_t^n = \left(\omega s + \frac{\sigma\Psi}{(1-\gamma)}\right)\hat{c}_t^n - \left(\frac{\gamma\sigma\Psi}{(1-\gamma)}\right)\hat{c}_{t-1}^n + \left(1 - \omega s - \frac{\sigma^*\Psi}{(1-\gamma^*)}\right)\hat{c}_t^{*n} + \left(\frac{\gamma^*\sigma^*\Psi}{(1-\gamma^*)}\right)\hat{c}_{t-1}^{*n} + \Psi\left(\hat{\varepsilon}_{p,t}^* - \hat{\varepsilon}_{p,t}\right) \tag{50}$$

where $\Psi = \left[(1-\omega)(1+\varphi\omega s) + \varphi(1-\omega^*)(1-\omega s)\right]/(\omega-\omega^*)$.

The same calculations for the foreign country yield,

$$\left(\frac{\sigma^*}{1-\gamma^*} + \varphi^*(1-\omega^*)s^* + \frac{\sigma^*\Psi^*}{(1-\gamma^*)}\right)\hat{c}_t^{*n} = \frac{\gamma^*\sigma^*}{1-\gamma^*}(1+\Psi^*)\hat{c}_{t-1}^{*n} - \left(\varphi^*\left[1-(1-\omega^*)s^*\right] - \frac{\sigma\Psi^*}{(1-\gamma)}\right)\hat{c}_t^n - \frac{\gamma\sigma\Psi^*}{(1-\gamma)}\hat{c}_{t-1}^n + \Psi^*\left(\hat{\varepsilon}_{p,t}^* - \hat{\varepsilon}_{p,t}\right) + (1+\varphi^*)\hat{a}_t^* \tag{51}$$

and

$$\hat{y}_t^{*n} = \left[1-(1-\omega^*)s^* - \frac{\sigma\Psi^*}{(1-\gamma)}\right]\hat{c}_t^n + \frac{\gamma\sigma\Psi^*}{(1-\gamma)}\hat{c}_{t-1}^n + \left((1-\omega^*)s^* + \frac{\sigma^*\Psi^*}{(1-\gamma^*)}\right)\hat{c}_t^{*n} - \frac{\gamma^*\sigma^*\Psi^*}{(1-\gamma^*)}\hat{c}_{t-1}^{*n} - \Psi^*\left(\hat{\varepsilon}_{p,t}^* - \hat{\varepsilon}_{p,t}\right) \tag{52}$$

where $\Psi^* = \left[\omega\left[1+\varphi^*(1-(1-\omega^*)s^*)\right] + \omega^*\varphi^*(1-\omega^*)s^*\right]/(\omega-\omega^*)$.

Appendix C: Approximation of the welfare criterion

The second-order approximation of the home representative household's utility is derived in this section, using methods discussed in more detail in Woodford (2003). The average utility flow of the representative household at date t is given by

$$\mathbb{W}_t = \mathbb{U}\left(C_t, \mathcal{H}_t, \varepsilon_{p,t}\right) - \frac{1}{n}\int_0^n \mathbb{V}\left(L_t\left(h\right), \varepsilon_{p,t}\right) \mathrm{d}h \tag{53}$$

where

$$\mathbb{U}\left(C_t, \mathcal{H}_t, \varepsilon_{p,t}\right) = \frac{\varepsilon_{p,t}}{1-\sigma}\left(C_t - \gamma\mathcal{H}_t\right)^{1-\sigma} \qquad \text{and} \qquad \mathbb{V}\left(L_t\left(h\right), \varepsilon_{p,t}\right) = \frac{\varepsilon_{p,t}}{1+\varphi}\left(L_t\left(h\right)\right)^{1+\varphi}.$$

C.1 Taylor expansion of the utility function

The second-order Taylor expansion of $\mathbb{U}\left(C_t, \mathcal{H}_t, \varepsilon_{p,t}\right)$ around the steady state $\bar{\mathbb{U}} = \mathbb{U}\left(\bar{C}, \bar{\mathcal{H}}, \bar{\varepsilon}_p\right)$ yields

$$\begin{aligned}\mathbb{U}\left(C_t, \mathcal{H}_t, \varepsilon_{p,t}\right) \approx\; & \bar{\mathbb{U}} + \bar{\mathbb{U}}_{\bar{C}}\tilde{C}_t + \bar{\mathbb{U}}_{\bar{\mathcal{H}}}\tilde{\mathcal{H}}_t + \bar{\mathbb{U}}_{\bar{\varepsilon}_p}\tilde{\varepsilon}_{p,t} + \frac{1}{2}\bar{\mathbb{U}}_{\bar{C}\bar{C}}\tilde{C}_t^2 \\ & + \frac{1}{2}\bar{\mathbb{U}}_{\bar{\mathcal{H}}\bar{\mathcal{H}}}\tilde{\mathcal{H}}_t^2 + \frac{1}{2}\bar{\mathbb{U}}_{\bar{\varepsilon}_p\bar{\varepsilon}_p}\left(\tilde{\varepsilon}_{p,t}\right)^2 + \bar{\mathbb{U}}_{\bar{C}\bar{\mathcal{H}}}\tilde{C}_t\tilde{\mathcal{H}}_t \\ & + \bar{\mathbb{U}}_{\bar{C}\bar{\varepsilon}_p}\tilde{C}_t\tilde{\varepsilon}_{p,t} + \bar{\mathbb{U}}_{\bar{\mathcal{H}}\bar{\varepsilon}_p}\tilde{\mathcal{H}}_t\tilde{\varepsilon}_{p,t} + \mathcal{O}\left(\|\zeta\|^3\right)\end{aligned} \tag{54}$$

where $\tilde{X}_t = X_t - \bar{X}$, $\mathcal{O}\left(\|\zeta\|^3\right)$ denotes the order of residual and $\|\zeta\|$ is a bound on the amplitude of exogenous disturbances.

Applying a second-order Taylor expansion ($\tilde{X}_t/\bar{X} = \hat{x}_t + \frac{1}{2}\hat{x}_t^2 + \mathcal{O}\left(\|\zeta\|^3\right)$, where $\hat{x}_t = \ln X_t - \ln\bar{X}$), we obtain

$$\begin{aligned}\mathbb{U}\left(C_t, \mathcal{H}_t, \varepsilon_{p,t}\right) \approx\; & \bar{\mathbb{U}} + \bar{\mathbb{U}}_{\bar{C}}\bar{C}\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right) + \bar{\mathbb{U}}_{\bar{\mathcal{H}}}\bar{\mathcal{H}}\left(\hat{h}_t + \frac{1}{2}\hat{h}_t^2\right) + \bar{\mathbb{U}}_{\bar{\varepsilon}_p}\bar{\varepsilon}_p\left(\hat{\varepsilon}_{p,t} + \frac{1}{2}\hat{\varepsilon}_{p,t}^2\right) \\ & + \frac{1}{2}\bar{\mathbb{U}}_{\bar{C}\bar{C}}\bar{C}^2\hat{c}_t^2 + \frac{1}{2}\bar{\mathbb{U}}_{\bar{\mathcal{H}}\bar{\mathcal{H}}}\bar{\mathcal{H}}^2\hat{h}_t^2 + \frac{1}{2}\bar{\mathbb{U}}_{\bar{\varepsilon}_p\bar{\varepsilon}_p}\bar{\varepsilon}_p^2\hat{\varepsilon}_{p,t}^2 + \bar{\mathbb{U}}_{\bar{C}\bar{\mathcal{H}}}\bar{C}\bar{\mathcal{H}}\left(\hat{c}_t\hat{h}_t\right) \\ & + \bar{\mathbb{U}}_{\bar{C}\bar{\varepsilon}_p}\bar{C}\bar{\varepsilon}_p\left(\hat{c}_t\hat{\varepsilon}_{p,t}\right) + \bar{\mathbb{U}}_{\bar{\mathcal{H}}\bar{\varepsilon}_p}\bar{\mathcal{H}}\bar{\varepsilon}_p\left(\hat{h}_t\hat{\varepsilon}_{p,t}\right) + \mathcal{O}\left(\|\zeta\|^3\right)\end{aligned} \tag{55}$$

with

$\bar{\mathbb{U}}_{\bar{C}} = \bar{\varepsilon}_p\left(\bar{C} - \gamma\bar{\mathcal{H}}\right)^{-\sigma}$,
$\bar{\mathbb{U}}_{\bar{C}\bar{C}} = -\sigma\bar{\varepsilon}_p\left(\bar{C} - \gamma\bar{\mathcal{H}}\right)^{-\sigma-1} = \frac{-\sigma}{\left(\bar{C}-\gamma\bar{\mathcal{H}}\right)}\bar{\mathbb{U}}_{\bar{C}}$,
$\bar{\mathbb{U}}_{\bar{\mathcal{H}}} = -\gamma\bar{\varepsilon}_p\left(\bar{C} - \gamma\bar{\mathcal{H}}\right)^{-\sigma} = -\gamma\bar{\mathbb{U}}_{\bar{C}}$,
$\bar{\mathbb{U}}_{\bar{\mathcal{H}}\bar{\mathcal{H}}} = -\gamma^2\sigma\bar{\varepsilon}_p\left(\bar{C} - \gamma\bar{\mathcal{H}}\right)^{-\sigma-1} = \frac{-\gamma^2\sigma}{\left(\bar{C}-\gamma\bar{\mathcal{H}}\right)}\bar{\mathbb{U}}_{\bar{C}}$,
$\bar{\mathbb{U}}_{\bar{C}\bar{\mathcal{H}}} = \sigma\gamma\bar{\varepsilon}_p\left(\bar{C} - \gamma\bar{\mathcal{H}}\right)^{-\sigma-1} = \frac{\sigma\gamma}{\left(\bar{C}-\gamma\bar{\mathcal{H}}\right)}\bar{\mathbb{U}}_{\bar{C}}$,
$\bar{\mathbb{U}}_{\bar{\varepsilon}_p} = \frac{1}{1-\sigma}\left(\bar{C} - \gamma\bar{\mathcal{H}}\right)^{1-\sigma} = \frac{\left(\bar{C}-\gamma\bar{\mathcal{H}}\right)}{(1-\sigma)\bar{\varepsilon}_p}\bar{\mathbb{U}}_{\bar{C}}$,
$\bar{\mathbb{U}}_{\bar{\varepsilon}_p\bar{\varepsilon}_p} = 0$,

$\bar{\mathbb{U}}_{\bar{C}\bar{\varepsilon}_p} = \left(\bar{C} - \gamma\bar{\mathcal{H}}\right)^{-\sigma} = \frac{\bar{\mathbb{U}}_{\bar{C}}}{\bar{\varepsilon}_p}$,
$\bar{\mathbb{U}}_{\bar{\mathcal{H}}\bar{\varepsilon}_p} = -\gamma\left(\bar{C} - \gamma\bar{\mathcal{H}}\right)^{-\sigma} = \frac{-\gamma}{\bar{\varepsilon}_p}\bar{\mathbb{U}}_{\bar{C}}$.

Replacing $\mathcal{H}_t$ by C_{t-1}, the utility of consumption simplifies to

$$\begin{aligned}\mathbb{U}\left(C_t, C_{t-1}, \varepsilon_{p,t}\right) \approx\ & \bar{\mathbb{U}}_{\bar{C}}\bar{C}\left\{\left(\hat{c}_t - \gamma\hat{c}_{t-1}\right) + \frac{1}{2}\left(\hat{c}_t^2 - \gamma\hat{c}_{t-1}^2\right) - \frac{\sigma}{2\left(1-\gamma\right)}\left(\hat{c}_t - \gamma\hat{c}_{t-1}\right)^2\right.\\ & \left.+\hat{c}_t\hat{\varepsilon}_{p,t} - \gamma\hat{c}_{t-1}\hat{\varepsilon}_{p,t}\right\} + t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right) \qquad (56)\end{aligned}$$

where "$t.i.p.$" denotes terms independent of the actual policy such as constant terms involving only exogenous variables.

C.2 Taylor expansion of the disutility of work

The second-order Taylor expansion for $\mathbb{V}\left(L_t\left(h\right), \varepsilon_{p,t}\right)$ around the steady state $\bar{\mathbb{V}} = \mathbb{V}\left(\bar{L}, \bar{\varepsilon}_p\right)$ is

$$\begin{aligned}\mathbb{V}\left(L_t\left(h\right), \varepsilon_{p,t}\right) \approx\ & \bar{\mathbb{V}} + \bar{\mathbb{V}}_{\bar{L}}\bar{L}\left(\hat{l}_t\left(h\right) + \frac{1}{2}\hat{l}_t^2\left(h\right)\right) + \bar{\mathbb{V}}_{\bar{\varepsilon}_p}\bar{\varepsilon}_p\left(\hat{\varepsilon}_{p,t} + \frac{1}{2}\left(\hat{\varepsilon}_{p,t}\right)^2\right)\\ & +\frac{1}{2}\bar{\mathbb{V}}_{\bar{L}\bar{L}}\bar{L}^2\hat{l}_t^2\left(h\right) + \frac{1}{2}\bar{\mathbb{V}}_{\bar{\varepsilon}_p\bar{\varepsilon}_p}\bar{\varepsilon}_p^2\hat{\varepsilon}_{p,t}^2 + \bar{\mathbb{V}}_{\bar{L}\bar{\varepsilon}_p}\bar{L}\bar{\varepsilon}_p\left(\hat{l}_t\left(h\right)\hat{\varepsilon}_{p,t}\right)\\ & +\mathcal{O}\left(\|\zeta\|^3\right) \qquad (57)\end{aligned}$$

with
$\bar{\mathbb{V}}_{\bar{L}} = \bar{\varepsilon}_p\bar{L}^{\varphi}$,
$\bar{\mathbb{V}}_{\bar{\varepsilon}_p} = \frac{1}{1+\varphi}\left(\bar{L}\right)^{1+\varphi} = \frac{\bar{L}}{(1+\varphi)\bar{\varepsilon}_p}\bar{\mathbb{V}}_{\bar{L}}$,
$\bar{\mathbb{V}}_{\bar{L}\bar{L}} = \varphi\bar{\varepsilon}_p\bar{L}^{\varphi-1} = \frac{\varphi}{\bar{L}}\bar{\mathbb{V}}_{\bar{L}}$,
$\bar{\mathbb{V}}_{\bar{\varepsilon}_p\bar{\varepsilon}_p} = 0$,
$\bar{\mathbb{V}}_{\bar{L}\bar{\varepsilon}_p} = \bar{L}^{\varphi} = \frac{\bar{\mathbb{V}}_{\bar{L}}}{\bar{\varepsilon}_p}$.

The disutility of work becomes

$$\mathbb{V}\left(L_t\left(h\right), \varepsilon_{p,t}\right) \approx \bar{\mathbb{V}}_{\bar{L}}\bar{L}\left\{\hat{l}_t\left(h\right) + \frac{1+\varphi}{2}\hat{l}_t^2\left(h\right) + \hat{l}_t\left(h\right)\hat{\varepsilon}_{p,t}\right\} + t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right). \qquad (58)$$

C.3 Individual labor to composite labor

Now define the composite labor index:

$$L_t = \int_0^n L_t\left(h\right)\mathrm{d}h = \int_0^n \frac{Y_t\left(h\right)}{A_t}\mathrm{d}h = \frac{Y_t}{A_t}\int_0^n\left(\frac{\tilde{P}_t\left(h\right)}{P_t}\right)^{-\theta}\mathrm{d}h.$$

Taking a second-order Taylor expansion of the logarithm of this equation yields:

$$\hat{l}_t = \hat{y}_t - \hat{a}_t + \hat{u}_t \qquad (59)$$

with $\hat{u}_t = \ln \int_0^n \left(\frac{\tilde{P}_t(h)}{P_t}\right)^{-\theta} \mathrm{d}h$ is of second order. As shown by Woodford (2003, chap. 6), one has

$$\hat{u}_t = \frac{\theta\alpha}{2(1-\alpha)(1-\beta\alpha)} \left(\hat{\pi}_{H,t} - \xi\hat{\pi}_{H,t-1}\right)^2 + \mathcal{O}\left(\|\zeta\|^3\right). \tag{60}$$

C.4 Welfare expressions

We first integrate equation (58) over h and replace $\int_0^n L_t(h)\,\mathrm{d}h$ and $\hat{u}_t$ by their respective expressions. We then take the present discounted sum of equations (56) and (58) and subtract the second expression to the first one to obtain

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t\mathbb{W}_t &= \bar{\mathbb{U}}_{\bar{C}}\bar{C}\sum_{t=0}^{\infty}\beta^t\Bigg\{\left(\hat{c}_t - \gamma\hat{c}_{t-1}\right) + \frac{1}{2}\left(\hat{c}_t^2 - \gamma\hat{c}_{t-1}^2\right) - \frac{\sigma}{2(1-\gamma)}\left(\hat{c}_t - \gamma\hat{c}_{t-1}\right)^2 \\ &\quad + \hat{c}_t\hat{\varepsilon}_{p,t} - \gamma\hat{c}_{t-1}\hat{\varepsilon}_{p,t} - \frac{(1-\Phi_y)}{s}\hat{y}_t - \frac{1+\varphi}{2s}\left(\hat{y}_t - \hat{a}_t\right)^2 - s^{-1}\hat{y}_t\hat{\varepsilon}_{p,t} \\ &\quad - \frac{\theta\alpha}{2(1-\alpha)(1-\beta\alpha)s}\left(\hat{\pi}_{H,t} - \xi\hat{\pi}_{H,t-1}\right)^2\Bigg\} + t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right). \end{aligned} \tag{61}$$

Recall that $\bar{\mathbb{V}}_{\bar{L}} = \bar{\mathbb{U}}_{\bar{C}}(1-\Phi_y)$, $s = \bar{C}/\bar{Y}$ and that Φ_y is of order $\mathcal{O}(\|\zeta\|)$. Given that

$$\sum_{t=0}^{\infty}\beta^t x_{t-1} = x_{-1} + \beta\sum_{t=0}^{\infty}\beta^t x_t = \beta\sum_{t=0}^{\infty}\beta^t x_t + t.i.p.$$

and in using the fact that

$$(1+\varphi)\,\hat{a}_t = A_1\hat{c}_t^n + A_2\hat{c}_{t-1}^n + A_3\hat{c}_t^{*n} + A_4\hat{c}_{t-1}^{*n} + A_5\hat{\varepsilon}_{p,t} + A_6\hat{\varepsilon}_{p,t}^*$$

where parameters A_j ($j = 1, \cdots, 6$) find their counterparts in equation (49), it yields

$$\begin{aligned}\mathcal{W}_0 &= \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\mathbb{W}_t = -\bar{\mathbb{U}}_{\bar{C}}\bar{C}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg\{-(1-\beta\gamma)\,\hat{c}_t + \frac{(1-\Phi_y)}{s}\hat{y}_t \\ &\quad + \frac{\sigma}{2(1-\gamma)}\left(\hat{c}_t - \gamma\hat{c}_{t-1}\right)^2 - \frac{(1-\beta\gamma)}{2}\hat{c}_t^2 + \frac{1+\varphi}{2s}\hat{y}_t^2 + \left(\gamma\beta\rho_p - 1\right)\hat{c}_t\hat{\varepsilon}_{p,t} \\ &\quad - s^{-1}\left(A_1\hat{c}_t^n + A_2\hat{c}_{t-1}^n + A_3\hat{c}_t^{*n} + A_4\hat{c}_{t-1}^{*n} + A_5\hat{\varepsilon}_{p,t} + A_6\hat{\varepsilon}_{p,t}^*\right)\hat{y}_t \\ &\quad + s^{-1}\hat{y}_t\hat{\varepsilon}_{p,t} + \frac{\theta\alpha}{2(1-\alpha)(1-\beta\alpha)s}\left(\hat{\pi}_{H,t} - \xi\hat{\pi}_{H,t-1}\right)^2\Bigg\} \\ &\quad + t.i.p. + \mathcal{O}\left(\|\zeta\|^3\right). \end{aligned} \tag{62}$$

Finally, replacing the cross-product $x_{1,t}x_{2,t}$ by $\left(x_{1,t}^2 + x_{2,t}^2 - (x_{1,t} - x_{2,t})^2\right)/2$, we can rewrite the home welfare criterion as

$$\begin{aligned}
\mathcal{W}_0 &= -\frac{\bar{\mathbb{U}}_{\bar{C}}\bar{C}}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left\{(\hat{c}_t-\Psi_c)^2+\frac{1}{s}(\hat{y}_t-\Psi_y)^2\right.\\
&+\left(\beta\gamma\left(1+\rho_p\right)-3\right)\hat{c}_t^2\\
&+\frac{1+\varphi-A_1-A_2-A_3-A_4-A_5-A_6}{s}\hat{y}_t^2\\
&+\frac{\sigma}{(1-\gamma)}(\hat{c}_t-\gamma\hat{c}_{t-1})^2-\left(\beta\gamma\rho_p-1\right)(\hat{c}_t-\hat{\varepsilon}_{p,t})^2\\
&+\frac{A_1}{s}(\hat{y}_t-\hat{c}_t^n)^2+\frac{A_2}{s}\left(\hat{y}_t-\hat{c}_{t-1}^n\right)^2+\frac{A_3}{s}(\hat{y}_t-\hat{c}_t^{*n})^2\\
&+\frac{A_4}{s}\left(\hat{y}_t-\hat{c}_{t-1}^{*n}\right)^2-\frac{1-A_5}{s}(\hat{y}_t-\hat{\varepsilon}_{p,t})^2+\frac{A_6}{s}\left(\hat{y}_t-\hat{\varepsilon}_{p,t}^*\right)^2\\
&\left.+\frac{\theta\alpha}{(1-\alpha)(1-\beta\alpha)s}(\hat{\pi}_{H,t}-\xi\hat{\pi}_{H,t-1})^2\right\}+t.i.p.+\mathcal{O}\left(\|\eta\|^3\right)
\end{aligned}$$

where $\Psi_c=(1-\beta\gamma)$ and $\Psi_y=-(1-\Phi_y)$.

Same calculations for the welfare of the foreign representative household yield:

$$\begin{aligned}
\mathcal{W}_0^* &= -\frac{\bar{\mathbb{U}}_{\bar{C}^*}^*\bar{C}^*}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left\{(\hat{c}_t^*-\Psi_c^*)^2+\frac{1}{s^*}\left(\hat{y}_t^*-\Psi_y^*\right)^2\right.\\
&+\left(\beta\gamma^*\left(1+\rho_p^*\right)-3\right)\hat{c}_t^{*2}\\
&+\frac{1+\varphi^*-A_1^*-A_2^*-A_3^*-A_4^*-A_5^*-A_6^*}{s^*}\hat{y}_t^{*2}\\
&+\frac{\sigma^*\gamma^*}{(1-\gamma^*)}\left(\hat{c}_t^*-\gamma^*\hat{c}_{t-1}^*\right)^2-\left(\beta\gamma^*\rho_p^*-1\right)\left(\hat{c}_t^*-\hat{\varepsilon}_{p,t}^*\right)^2\\
&+\frac{A_1^*}{s^*}(\hat{y}_t^*-\hat{c}_t^n)^2+\frac{A_2^*}{s^*}\left(\hat{y}_t^*-\hat{c}_{t-1}^n\right)^2+\frac{A_3^*}{s^*}(\hat{y}_t^*-\hat{c}_t^{*n})^2\\
&+\frac{A_4^*}{s^*}\left(\hat{y}_t^*-\hat{c}_{t-1}^{*n}\right)^2+\frac{A_5^*}{s^*}(\hat{y}_t^*-\hat{\varepsilon}_{p,t})^2-\frac{1-A_6^*}{s^*}\left(\hat{y}_t^*-\hat{\varepsilon}_{p,t}^*\right)^2\\
&\left.+\frac{\theta\alpha^*}{(1-\alpha^*)(1-\beta\alpha^*)s^*}\left(\hat{\pi}_{F,t}^*-\xi^*\hat{\pi}_{F,t-1}^*\right)^2\right\}+t.i.p.+\mathcal{O}\left(\|\eta\|^3\right)
\end{aligned}$$

where $\Psi_c^*=(1-\beta\gamma^*)$ and $\Psi_y^*=-\left(1-\Phi_y^*\right)$.

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Table 1. Parameter estimates for the AWM

		Prior distribution			Estimated ML		Posterior distribution			Smets-Wouters	Onatski-Williams
		Type	Mean	Std.error	Mode	Std dev.	5%	Median	95%	Median	Median
Consumption habit	γ	Beta	0.700	0.100	0.867	0.040	0.800	0.871	0.932	0.595	0.400*
Consumption elast. of subst.	σ	Normal	2.000	0.250	2.074	0.242	1.674	2.078	2.465	1.371	2.178
Labour desutility	φ	Normal	2.000	0.250	1.972	0.227	1.600	1.979	2.350	2.491	3.000*
Price indexation	ξ	Beta	0.700	0.100	0.485	0.102	0.310	0.478	0.646	0.472	0.323
Calvo probability	α	Beta	0.700	0.100	0.929	0.020	0.900	0.933	0.956	0.905	0.930*
RF lagged interest rate	ψ_i	Beta	0.700	0.100	0.855	0.026	0.814	0.858	0.897	0.958	0.962
RF inflation	ψ_π	Normal	1.500	0.100	1.480	0.098	1.310	1.480	1.632	1.688	1.684
RF output gap	ψ_y	Normal	0.500	0.100	0.163	0.157	-0.032	0.108	0.407	0.095	0.099
Corr. preference shock	ρ_p	Beta	0.600	0.100	0.436	0.103	0.270	0.426	0.610	0.842	0.876
Corr. productivity shock	ρ_a	Beta	0.600	0.100	0.591	0.101	0.429	0.599	0.757	0.815	0.957
Corr. interest rate	ρ_i	Beta	0.600	0.100	0.553	0.081	0.413	0.551	0.681	0.865	0.582
Vol. preference shock	σ_p	Uniform	0.000	2.000	0.114	0.036	0.068	0.106	0.161	0.336	0.240
Vol. productivity shock	σ_a	Uniform	0.000	2.000	0.127	0.063	0.048	0.106	0.219	0.598	0.343
Vol. interest rate	σ_i	Uniform	0.000	2.000	0.210	0.017	0.181	0.208	0.237	0.081	1.000*

Table 2. Parameter estimates for the MCM

		Germany					France					Italy				
		Mean	Std dev.	5%	Median	95%	Mean	Std dev.	5%	Median	95%	Mean	Std dev.	5%	Median	95%
Consumption habit	γ	0.630	0.050	0.553	0.632	0.714	0.688	0.045	0.617	0.691	0.765	0.777	0.029	0.730	0.777	0.823
Consumption elast. of subst.	σ	1.542	0.232	1.162	1.533	1.922	1.851	0.226	1.482	1.851	2.228	2.009	0.218	1.656	2.009	2.373
Labour desutility	φ	1.934	0.253	1.522	1.929	2.349	2.015	0.252	1.595	2.019	2.428	1.922	0.247	1.511	1.919	2.316
Price indexation	ξ	0.290	0.078	0.157	0.283	0.406	0.324	0.083	0.191	0.318	0.455	0.436	0.102	0.257	0.428	0.593
Calvo probability	α	0.839	0.019	0.809	0.840	0.869	0.822	0.017	0.794	0.823	0.848	0.794	0.022	0.759	0.795	0.830
Policy rule: lagged interest rate	ψ_i	0.871	0.020	0.841	0.873	0.901	0.820	0.027	0.778	0.822	0.864	0.906	0.014	0.885	0.908	0.929
Policy rule: inflation	ψ_π	1.507	0.100	1.340	1.510	1.666	1.517	0.101	1.353	1.518	1.681	1.497	0.094	1.344	1.500	1.648
Policy rule: output gap	ψ_y	0.458	0.104	0.288	0.462	0.627	0.482	0.102	0.314	0.480	0.645	0.522	0.091	0.375	0.522	0.670
Serial-corr. preference shock	ρ_p	0.640	0.065	0.531	0.643	0.741	0.509	0.077	0.380	0.510	0.633	0.793	0.036	0.739	0.795	0.851
Serial-corr. productivity shock	ρ_a	0.740	0.067	0.635	0.741	0.854	0.660	0.075	0.536	0.661	0.780	0.854	0.035	0.796	0.855	0.911
Serial-corr. mon. policy shock	ρ_i	0.506	0.067	0.395	0.508	0.617	0.447	0.067	0.337	0.445	0.557	0.414	0.071	0.300	0.412	0.534
Vol. preference shock	σ_p	0.048	0.008	0.035	0.047	0.061	0.063	0.010	0.047	0.062	0.078	0.055	0.008	0.043	0.054	0.068
Vol. productivity shock	σ_a	0.037	0.006	0.026	0.036	0.047	0.038	0.007	0.028	0.038	0.050	0.035	0.006	0.026	0.035	0.045
Vol. mon. policy shock	σ_i	0.244	0.020	0.211	0.243	0.276	0.426	0.034	0.372	0.423	0.482	0.228	0.021	0.196	0.226	0.261
Cross-correlations across countries																
Preference shock - 1/2	$\delta_{p\,12}$	0.311	0.063	0.201	0.313	0.410										
Preference shock - 1/3	$\delta_{p\,13}$	0.166	0.067	0.059	0.168	0.273										
Preference shock - 2/3	$\delta_{p\,23}$	0.279	0.071	0.166	0.279	0.397										
Productivity shock - 1/2	$\delta_{a\,12}$	0.194	0.067	0.077	0.196	0.300										
Productivity shock - 1/3	$\delta_{a\,13}$	-0.032	0.076	-0.156	-0.032	0.096										
Productivity shock - 2/3	$\delta_{a\,23}$	0.135	0.075	0.018	0.138	0.258										
Monetary policy shock - 1/2	$\delta_{i\,12}$	0.198	0.070	0.087	0.200	0.317										
Monetary policy shock - 1/3	$\delta_{i\,13}$	0.124	0.066	0.016	0.127	0.229										
Monetary policy shock - 2/3	$\delta_{i\,23}$	0.239	0.069	0.132	0.237	0.355										

Table 3. Performance evaluation

	DSGE	VAR(1)	VAR(2)	VAR(3)	VAR(4)
		Panel A: AWM			
Prior probability	*0.2*	*0.2*	*0.2*	*0.2*	*0.2*
Posterior probability	1435.818	1473.587	1483.960	1484.057	1486.572
Bayes factor relative to the DSGE	1	2.5E+16	8.1E+20	8.9E+20	1.1E+22
Posterior odds	0.00	0.00	0.06	0.07	0.87
		Panel B: MCM			
Prior probability	*0.2*	*0.2*	*0.2*	*0.2*	*0.2*
Posterior probability	3975.823	4183.968	4208.429	4210.514	4234.233
Bayes factor relative to the DSGE	1	2.5E+90	1.0E+101	8.4E+101	1.7E+112
Posterior odds	0.00	0.00	0.00	0.00	1.00

Table 4. Results for the utility-based criterion

Model	Values of welfare: AWM	Values of welfare: constrained MCM	Values of welfare: optimal MCM	Measures of welfare cost: δ_1	Measures of welfare cost: δ_2
With habit formation and price indexation	-1.470	-1.102	-1.097	0.0037	0.986
Without habit formation	-2.233	-1.998	-1.989	0.0024	0.963
Without price indexation	-1.737	-1.621	-1.619	0.0012	0.983
Without habit formation and price indexation	-2.820	-2.783	-2.782	0.0004	0.976

Table 5. Results for the *ad hoc* criterion

Weight on…		Loss function			Measure
output gap	interest rate	AWM	constrained MCM	optimal MCM	δ_2
0.01	0.01	1.250	0.925	0.910	0.956
0.01	0.25	1.677	1.488	1.447	0.822
0.01	0.50	2.800	1.967	1.887	0.912
0.01	0.75	3.015	2.187	2.153	0.961
0.01	1.00	3.244	2.327	2.189	0.869
0.25	0.01	3.652	3.414	3.395	0.926
0.25	0.25	3.773	3.272	3.269	0.994
0.25	0.50	3.881	3.346	3.338	0.985
0.25	0.75	3.968	3.415	3.400	0.974
0.25	1.00	4.179	3.476	3.453	0.968
0.50	0.01	4.030	3.576	3.546	0.938
0.50	0.25	4.115	4.006	3.995	0.908
0.50	0.50	4.139	4.076	4.069	0.900
0.50	0.75	4.180	4.135	4.128	0.865
0.50	1.00	4.249	4.185	4.177	0.889
0.75	0.01	4.145	3.740	3.693	0.896
0.75	0.25	4.397	3.990	3.979	0.974
0.75	0.50	4.411	4.062	4.053	0.975
0.75	0.75	4.551	4.398	4.369	0.841
0.75	1.00	4.582	4.498	4.487	0.884
1.00	0.01	4.236	3.834	3.770	0.863
1.00	0.25	4.974	4.056	4.022	0.964
1.00	0.50	5.343	4.165	4.143	0.982
1.00	0.75	4.563	4.241	4.225	0.953
1.00	1.00	4.679	4.301	4.287	0.964

Figure 1. Data

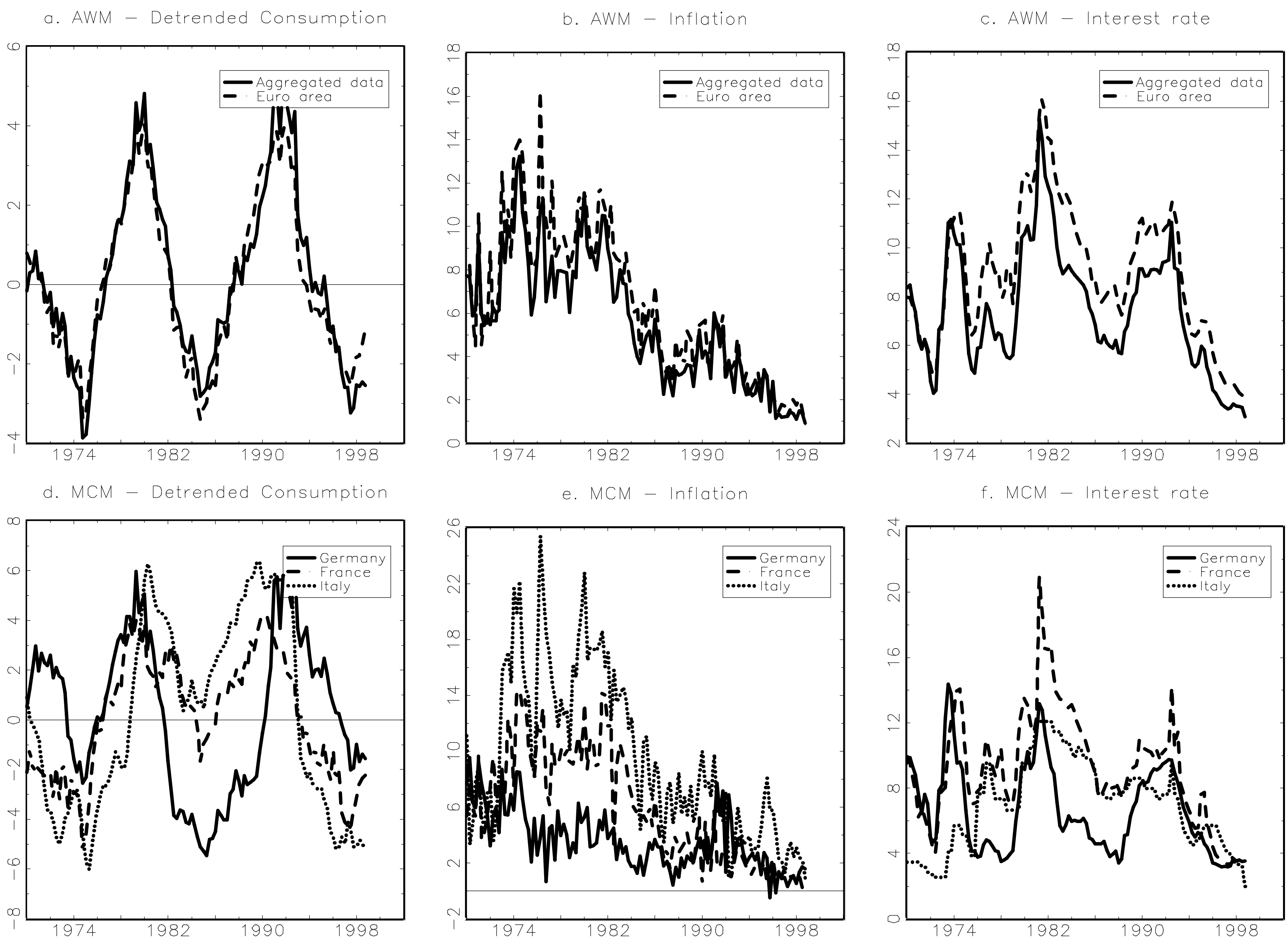


Figure 2. Estimated parameter distribution for the AWM

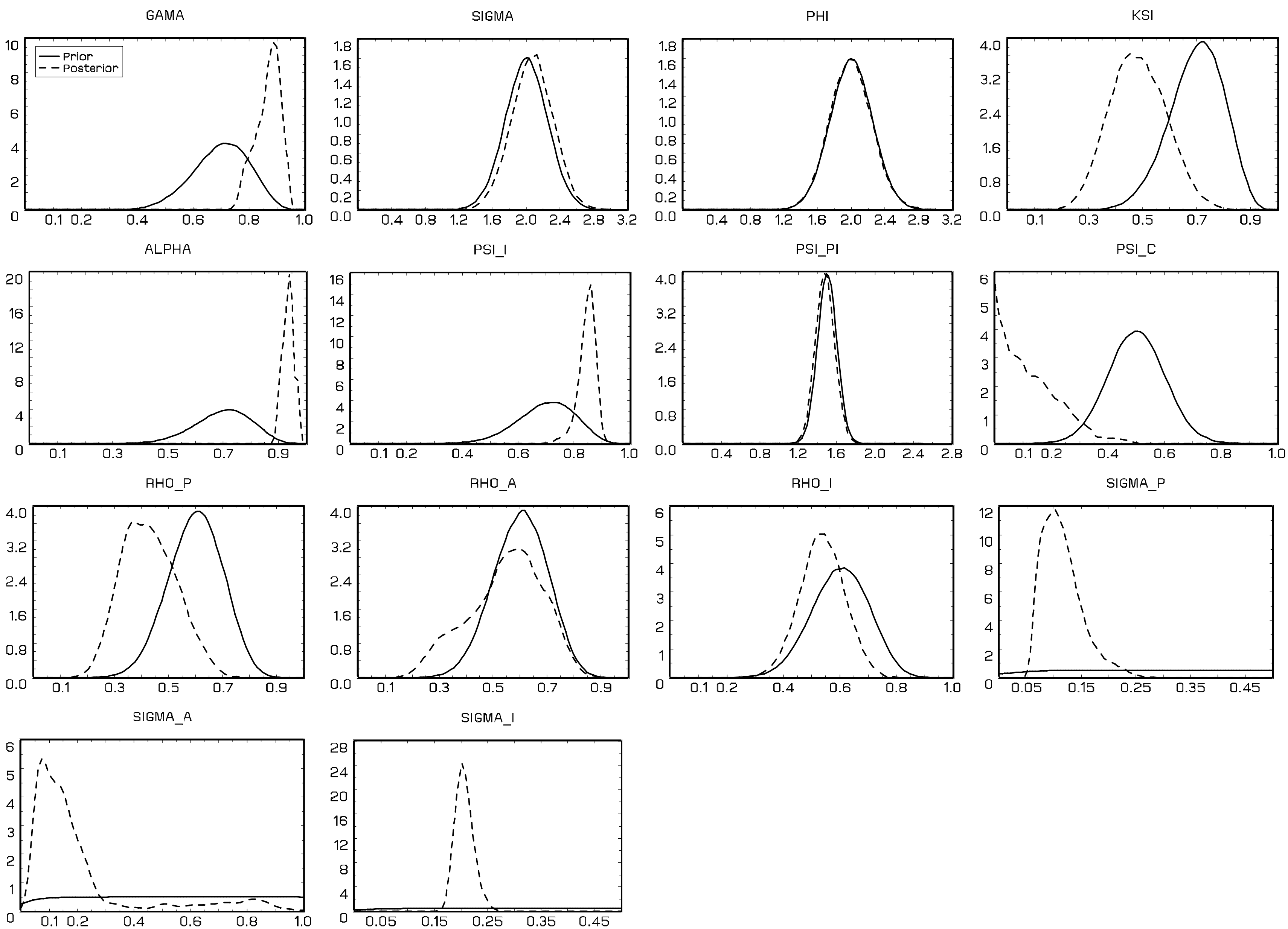


Figure 3. Estimated parameter distribution for the MCM

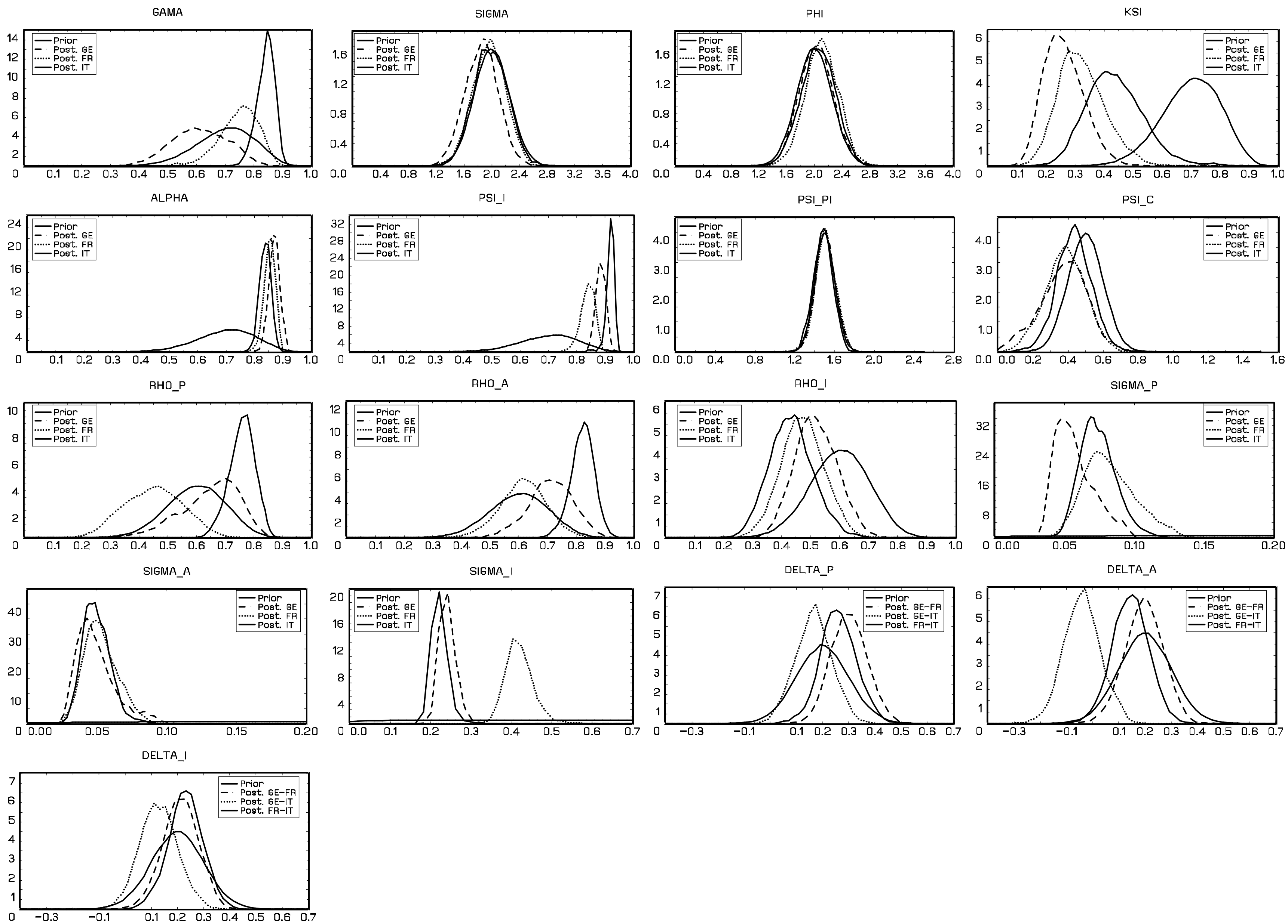


Figure 4. Comparison of cross-covariances of the AWM and the data

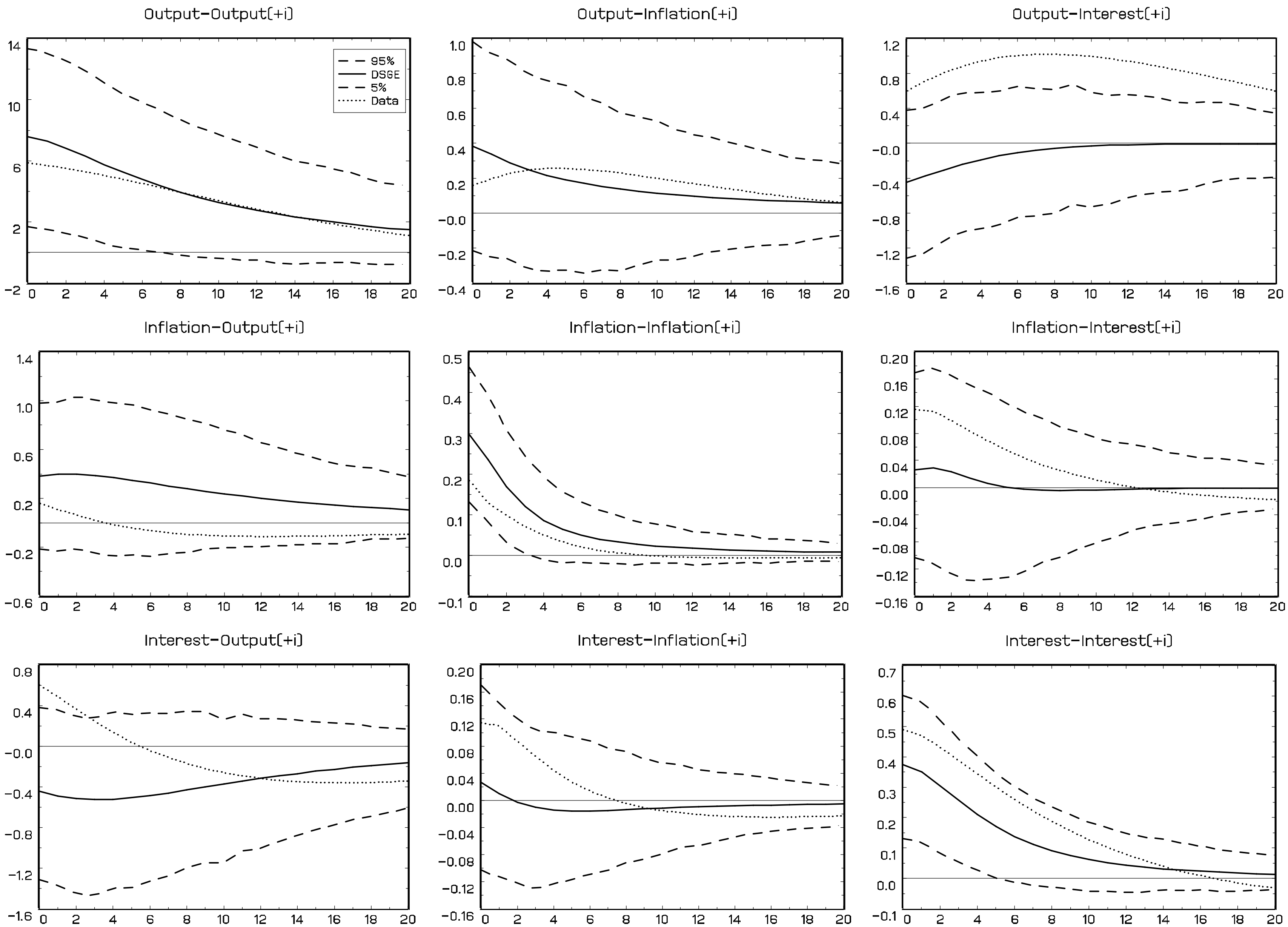
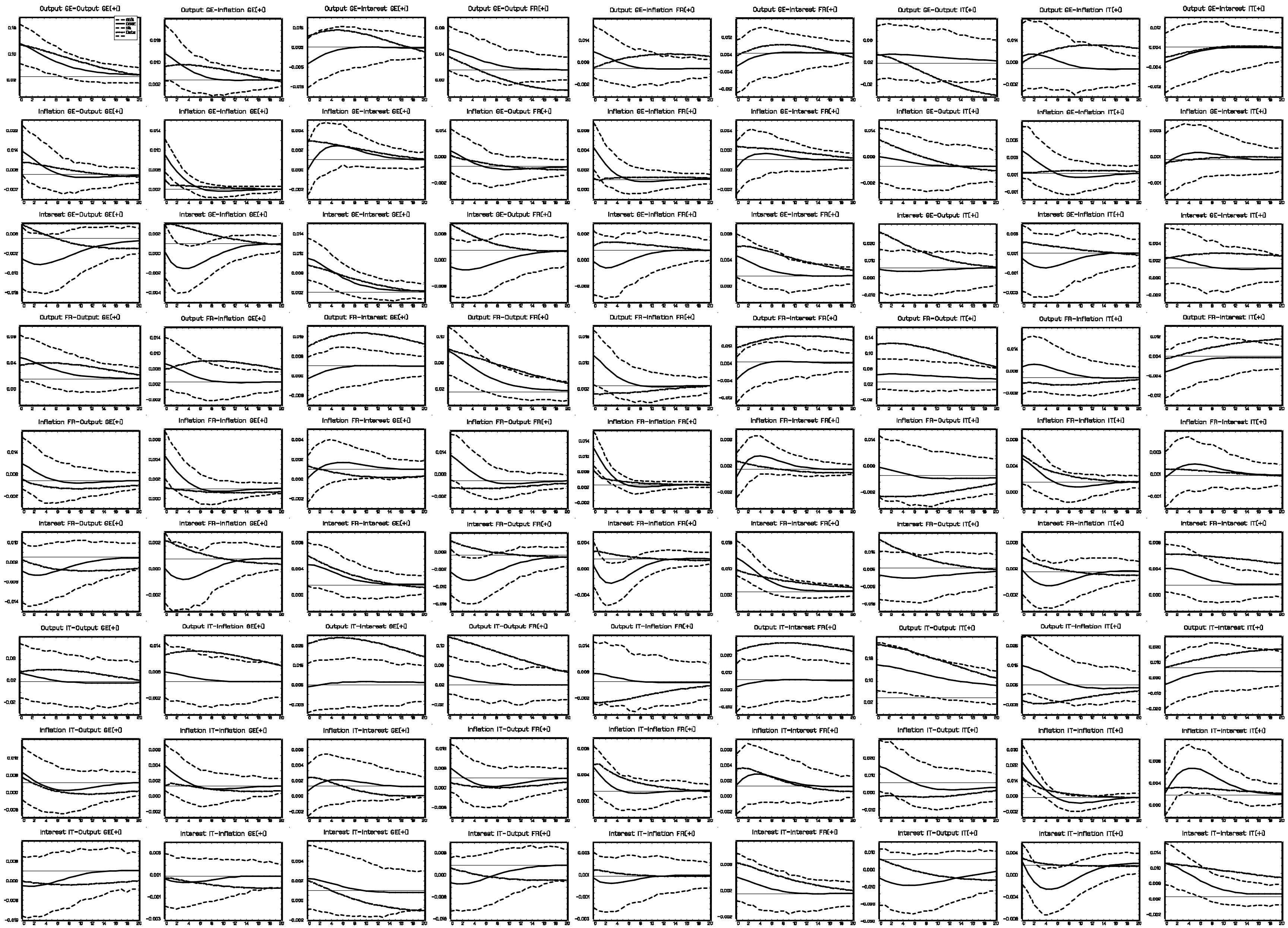


Figure 5. Comparison of cross-covariances of the MCM and the data



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