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Animal Spirits in Woodford and Reichlin Economies: The Representative Agent Does Matter

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Animal spirits in Woodford and Reichlin economies: The representative agent does matter*

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Abstract

Macroeconomic dynamic models are often criticized because the households' behavior is summarized by a representative agent and, then, they do not take into account consumers' heterogeneity. Reconsidering the Woodford (1986) infinite-horizon model and the Reichlin (1986) OLG model, we show that the representative agent can be a good representation of heterogeneity in preferences, in order to study the local dynamics and the occurrence of endogenous fluctuations.

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1 Introduction

A commonly shared criticism addressed to the representative agent is the conjecture that the introduction of a certain degree of heterogeneity in the model could have drastic effects on the equilibrium, in particular on its stability properties (see among others Herrendorf, Waldmann and Valentiny (2000), Ghiglino and Olszak-Duquenne (2001), Ghiglino (2003)).

In this paper we challenge this widespread point of view, by showing that the heterogeneity of preferences can have no effect on the dynamic properties of the equilibrium. The robustness of the representative agent is underlined in two classes of models. On the one hand we take into account an infinite

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horizon model *à la* Woodford (1986), on the other hand an overlapping generations model *à la* Reichlin (1986). The conclusion is the same: The use of a representative agent is quite justified. Indeed, in both the models, endogenous fluctuations, *i.e.* sunspots and deterministic cycles, occur under similar conditions, either if workers have heterogeneous preferences or if a representative agent represents a weighted average of different types of households.

2 The production sector

We consider, at first, the Woodford (1986) model, augmented with factor substitutability as in the version of Grandmont, Pintus and de Vilder (1998). Then we develop the Reichlin (1986) model. In both the cases, we substitute a representative agent, embodying a population of identical workers, with a population of different workers endowed with heterogeneous and rather general preferences. In order to compare the models and to simplify the notation, the production sector is supposed to be the same. Markets are perfectly competitive and the final good is produced by a representative firm with a constant returns to scale technology: $y_t = AF(k_{t-1}, l_t)$, where k_{t-1} and l_t denote, respectively, the aggregate capital and labor employed at period t. $A > 0$ is a scaling parameter. Setting $a_t = k_{t-1}/l_t$, the capital intensity, a reduced production function is defined: $y/l = Af(a)$, the properties of which are now made explicit.

Assumption 1 *The function $f(a)$ is continuous for $a \geq 0$, positive-valued and differentiable as many times as needed for $a > 0$, with $f'(a) > 0 > f''(a)$.*

Profit maximization determines the real interest rate r/p and the real wage w/p:

$$r/p = A\rho(a) \text{ and } w/p = A\omega(a) \tag{1}$$

where p is the price of the final good, $\rho(a) \equiv f'(a)$ and $\omega(a) \equiv f(a) - af'(a)$. The share of capital on total income is denoted by $s(a) \equiv af'(a)/f(a) \in (0,1)$. The elasticity of capital-labor substitution $\sigma(a)$ is defined, as usual, by $1/\sigma(a) = a\omega'(a)/\omega(a) - a\rho'(a)/\rho(a)$. The new parameters are simply related.

$$a\rho'(a)/\rho(a) = [s(a) - 1]/\sigma(a) \tag{2}$$

$$a\omega'(a)/\omega(a) = s(a)/\sigma(a) \tag{3}$$

3 The Woodford model with heterogeneous workers

Two kinds of agents live in a world *à la* Woodford: Capitalists and workers. In our model the workers are no longer identical, but they are allowed to have different preferences. More precisely, there are n types of workers and the weight of the ith class of workers within the production system is given by $\alpha_i \geq 0$ with $\sum_{i=1}^{n} \alpha_i = 1$.

The infinite-lived worker of type i maximizes a specific utility functional: $\sum_{t=1}^{\infty}\beta^t\left(U_i\left(c_{it}/B_i\right)-\beta V_i\left(l_{it}\right)\right)$, under a budget and a liquidity constraint:

$$\begin{aligned} p_t c_{it}+p_t\left[k_{it}-(1-\delta)k_{it-1}\right]+m_{it} &\leq r_t k_{it-1}+w_t l_{it}+m_{it-1} \qquad (4)\\ p_t c_{it}+p_t\left[k_{it}-(1-\delta)k_{it-1}\right] &\leq r_t k_{it-1}+m_{it-1} \qquad (5)\end{aligned}$$

with $t=1,\ldots,\infty$. We denote by c_i, k_i, m_i and l_i, respectively, the individual demands for consumption, capital and nominal balances, and the labor supply. δ is the capital depreciation rate and $B_i>0$ represents a scaling parameter. The time preference $\beta\in(0,1)$ is commonly shared and the heterogeneity of preferences is shaped in terms of the per-period utility U_i and the labor supply disutility V_i.

Inequality (4) means that the individual buys the consumption and the investment good as well as the balances he needs, by spending the capital and the labor income and the disposable money. Moreover he faces an imperfect credit market, where collaterals matter: The financial constraint (5) simply means that agents are not allowed to borrow against their present and future labor income. The heterogeneous utility functions are well behaved:

Assumption 2 *The functions $U_i\left(x_i\right)$ and $V_i\left(l_i\right)$ are continuous for all $x_i,l_i\geq 0$. They have continuous derivatives of every order, as far as needed, for $x_i,l_i>0$, with $U_i'\left(x_i\right)>0>U_i''\left(x_i\right)$, $V_i'\left(l_i\right)>0$, $V_i''\left(l_i\right)\geq 0$ and $\varepsilon_{U_i'}>-1$, where $\varepsilon_{U_i'}$ is the elasticity of $U_i'(x_i)$.*

Putting (4) and (5) in a Lagrangian functional and deriving with respect to $\{c_{it},k_{it},l_{it},m_{it}\}_{t=1}^{\infty}$ and the multipliers, we obtain the first order conditions:

$$\begin{aligned} U_i'\left(c_{it}/B_i\right) &\geq \beta\left(1-\delta+r_{t+1}/p_{t+1}\right)U_i'\left(c_{it+1}/B_i\right) \qquad (6)\\ B_i V_i'\left(l_{it}\right) &\geq \left(w_t/p_{t+1}\right)U_i'\left(c_{it+1}/B_i\right) \qquad (7)\end{aligned}$$

jointly with the constraints.

On the other side, the representative capitalist[1] maximizes a logarithmic utility $\sum_{t=1}^{\infty}\gamma^t\ln c_t$ under the budget constraint:

$$p_t c_t+p_t\left[k_t-(1-\delta)k_{t-1}\right]+m_t\leq r_t k_{t-1}+m_{t-1} \qquad (8)$$

We assume that the capitalist is more patient than workers: $\gamma\in(\beta,1)$, does not work and, in consequence, is not financially constrained.

We set the Lagrangian functional with (8) and we derive it with respect to $\{c_t,k_t,m_t\}_{t=1}^{\infty}$ to obtain:

$$\begin{aligned} 1/c_t &\geq \gamma\left(1-\delta+r_{t+1}/p_{t+1}\right)/c_{t+1} \qquad (9)\\ 1/c_t &\geq \gamma\left(p_t/p_{t+1}\right)/c_{t+1} \qquad (10)\end{aligned}$$

[1] To keep things as simple as possible and to obtain a two-dimensional dynamic system, we consider, as in the seminal paper by Woodford (1986), a logarithmic utility, entailing homogeneous preferences.

In a neighborhood of a steady state, the discount factor of the more patient agent γ determines the real interest factor $1-\delta+r/p=1/\gamma$. Thus, $\beta<\gamma<1$ entails that inequalities (6) and (10) become strict and, in turn, $k_i=m=0$ and $m_i, k>0$, while (7) and (9) hold with equality. Then, the budget constraint (8) boils down to $c_t+k_t=(1-\delta+r_t/p_t)\,k_{t-1}$. This equation jointly with (9) are explicitly solved by the policy function:

$$k_t=\gamma\,(1-\delta+r_t/p_t)\,k_{t-1} \tag{11}$$

Around the steady state (4) and (5) simplify too: $p_{t+1}c_{it+1}=w_tl_{it}=m_{it}$, and (7) becomes

$$z_{it}U_i'\,(z_{it}l_{it})=V_i'(l_{it}) \tag{12}$$

where $z_{it}\equiv w_t/\,(p_{t+1}B_i)$. Labor market and money market clear: $\sum_{i=1}^n\alpha_il_{it}=l_t$, $\sum_{i=1}^n\alpha_im_{it}=M$, where M is constant money supply, entailing $M=w_tl_t=w_{t+1}l_{t+1}$ and, finally, $z_{it}=(w_{t+1}/p_{t+1})\,(l_{t+1}/l_t)\,/B_i$. The implicit function (12), defines the labor supply $l_{it}\equiv\lambda_i\,(z_{it})$, which is invertible under Assumption 2. Let now $\varepsilon_{U_i'}$ and $\varepsilon_{V_i'}$ be the elasticities, respectively, of the utility U_i' and the disutility V_i'. We obtain from (12) the elasticity of the labor supply: $\varepsilon_{\lambda_i}=\left(1+\varepsilon_{U_i'}\right)/\left(\varepsilon_{V_i'}-\varepsilon_{U_i'}\right)$, which is positive under Assumption 2. Noticing that B_iz_i no longer depends on i, we have

$$B_1\lambda_1^{-1}\,(l_{1t})=B_i\lambda_i^{-1}\,(l_{it}) \tag{13}$$

for every i and we define new functions $l_{it}=\phi_i\,(l_{1t})\equiv\lambda_i\left(\lambda_1^{-1}\,(l_{1t})\,B_1/B_i\right)$ for $i=2,...,n$. Arranging equations (1), (11) and the expression of λ_1, we obtain a two-dimensional dynamic system.

Proposition 1 *An intertemporal equilibrium is a sequence* $\{k_t,l_{1t}\}_{t=1}^{\infty}$ *such that:*

$$\begin{aligned} k_t &= \gamma\,[1-\delta+A\rho\,(k_{t-1}/l_t)]\,k_{t-1} && (14)\\ l_{1t} &= \lambda_1\,((A/B_1)\,\omega\,(k_t/l_{t+1})\,(l_{t+1}/l_t)) && (15)\end{aligned}$$

where $l_t=\alpha_1l_{1t}+\sum_{i=2}^n\alpha_i\phi_i\,(l_{1t})$.

We observe that the equilibrium aggregate labor supply l becomes a function of the first agent's labor supply l_1.

3.1 Existence of a steady state

By setting appropriately the scaling parameters $A,B_1,\ldots,B_n$, not only we prove the existence of the steady state, but also we normalize it to $(k,l_1)=(1,1)$ and, in addition, $l_i=1$, $i=2,...,n$. We solve the system of equations (13), (14), (15), evaluated at the steady state, with respect to the unknown A,B_1,B_i $(i=2,...,n)$, taking into account $k=l_1=l_i=1$ and therefore $l=1$. We obtain $A=\theta/\,[\gamma\rho\,(1)]>0$, $B_1=\theta\omega\,(1)\,/\left[\gamma\rho\,(1)\,\lambda_1^{-1}\,(1)\right]>0$, where $\theta\equiv1-\gamma\,(1-\delta)$, and $B_i=B_1\lambda_1^{-1}\,(1)\,/\lambda_i^{-1}\,(1)>0$.[2]

[2] One can further notice that the existence of B_1 requires

$$\lim_{x\to0}xU_1'\,(x)<V_1'\,(1)<\lim_{x\to+\infty}xU_1'\,(x) \tag{16}$$

3.2 Local dynamics

The behavior of the economy in a neighborhood of the steady state $(k, l_1) = (1, 1)$ depends on the eigenvalues of the Jacobian matrix of system (14-15), evaluated at the steady state. To compute the trace and the determinant of the matrix, we use the properties of the steady state and formulas (2) and (3), we set $s \equiv s(1)$, $\sigma \equiv \sigma(1)$ and we note ε_{λ_i} the elasticity of λ_i:

$$T = 1 + \frac{\sigma - \theta(1-s)}{\sigma - s} + \frac{\sigma}{\sigma - s}\frac{1}{\varepsilon_\lambda} \tag{17}$$

$$D = \frac{\sigma - \theta(1-s)}{\sigma - s}\left(1 + \frac{1}{\varepsilon_\lambda}\right) \tag{18}$$

where $\varepsilon_\lambda \equiv \sum_{i=1}^{n} \alpha_i \varepsilon_{\lambda_i}$ represents, now, the aggregate elasticity of the labor supply with respect to the real wage. Expressions (17) and (18) are the same we find in Grandmont, Pintus and de Vilder (1998). In other terms, endogenous fluctuations occur under analogous conditions. ε_λ is indifferently interpreted as the elasticity of a representative worker's labor supply or as the heterogeneous workers' aggregate elasticity.

4 The Reichlin model with heterogeneous consumers

One of the most influential paper in the OLG literature of the last decades has been written by Reichlin (1986).[3] In this model, population is constant and agents live two periods. Young agents work, while old ones consume. We aim at showing how consumers' heterogeneity has no effect on the stability properties of the economy and, namely, the bifurcation parameter values. To understand the role of heterogeneity, we substitute the representative agent of each generation with n different types of individuals. To keep things as simple as possible and compare Reichlin with Woodford, we use the same notation and the same functional forms.

Preferences are rationalized by a separable utility function: $U_i(c_{it+1}/B_i) - V_i(l_{it})$, where U_i and V_i satisfy Assumption 2. In the first part of his life (period t) the individual supplies an amount l_{it} of labor and buys the capital good k_{it} to ensure the intertemporal transfer of wealth and the future consumption c_{it+1}. Formally:

$$k_{it} \leq \Omega_t l_{it} \tag{19}$$

$$c_{it+1} \leq R_{t+1} k_{it} \tag{20}$$

where Ω_t and R_{t+1} denotes the real wage and the real interest factor. The utility maximization under (19) and (20) still gives (12), where z_i is, now, differently

[3] See also Cazzavillan (2001) for a more recent presentation.

interpreted: $z_{it} \equiv R_{t+1}\Omega_t/B_i$. As above, the individual labor supply λ_i is obtained from (12) under Assumption 2. Capital and consumption are provided by the constraints (19) and (20), which are now binding. Aggregating across the types we get $k_t = \sum_{i=1}^{n} \alpha_i k_{it}$ and $l_t = \sum_{i=1}^{n} \alpha_i l_{it}$, and we deduce from (19): $k_t = \Omega_t l_t$. The functional forms ϕ_i are defined exactly as in the revisited Woodford model, to have $l_{it} = \phi_i(l_{1t})$. Since the production sector is also the same, profit maximization gives $\Omega = A\omega(a)$ and $R = 1 - \delta + A\rho(a)$. From $k_t = \Omega_t l_t$ and the definition of the labor supply λ_1, noticing that still $B_i z_i$ no longer depends on i, we find a two-dimensional dynamic system.

Proposition 2 *An intertemporal equilibrium is a sequence* $\{k_t, l_{1t}\}_{t=1}^{\infty}$ *such that:*

$$\begin{aligned} k_t &= A\omega(k_{t-1}/l_t)\, l_t && (21) \\ l_{1t} &= \lambda_1\left((A/B_1)\,\omega(k_{t-1}/l_t)\left[1 - \delta + A\rho(k_t/l_{t+1})\right]\right) && (22) \end{aligned}$$

where $l_t = \alpha_1 l_{1t} + \sum_{i=2}^{n} \alpha_i \phi_i(l_{1t})$.

4.1 Existence of a steady state

As above, we choose the scaling parameters $A, B_1, \ldots, B_n$ to establish the existence of a normalized steady state $(k, l_1) = (1, 1)$, with $l_i = 1$ for $i = 2, ..., n$. We solve the system of equations (13), (21), (22), evaluated at the steady state, under $k = l_1 = l_i = 1$, with respect to the unknown A, B_1, B_i $(i = 2, ..., n)$. We notice that, as above, $l = 1$, and we obtain $A = 1/\omega(1) > 0$, $B_1 = [1 - \delta + \rho(1)/\omega(1)]/\lambda_1^{-1}(1) > 0$ and $B_i = B_1\lambda_1^{-1}(1)/\lambda_i^{-1}(1) > 0$.[4]

4.2 Local dynamics

As above, the dynamic system (14-15) is linearized around the steady state $(k, l_1) = (1, 1)$. Formulas (2) and (3) jointly with $A\rho(1) = s/(1-s)$, enable us to compute the trace and the determinant of the Jacobian matrix:

$$\begin{aligned} T &= 1 + \frac{1}{1-s} - \delta + \frac{\sigma}{s}\left(\frac{1}{1-s} - \delta\right)\frac{1}{\varepsilon_\lambda} && (23) \\ D &= \left(\frac{1}{1-s} - \delta\right)\left(1 + \frac{1}{\varepsilon_\lambda}\right) && (24) \end{aligned}$$

where ε_λ, as in the augmented version of Woodford, represents the aggregate elasticity of labor supply with respect to real wage. Expressions (23) and (24) are as in Reichlin (1986) or Cazzavillan (2001) when he considers constant returns to scale, provided that we interpret ε_λ as a weighted average of individual behaviors. Consequently as in the Woodford model, fluctuations due to animal spirits, or deterministic cycles occur under similar conditions in the model with heterogeneous consumers and in the one where a representative agent represents a weighted sum of different types of households.

[4] As in the Woodford model, the existence of B_1 requires inequalities (16).

5 Conclusion

In our generalization of Woodford with factor substitutability and heterogeneous workers, we find the same local dynamics of Grandmont, Pintus and de Vilder (1998), where the elasticity of the labor supply ε_λ has to be interpreted as a weighted average of the individual elasticities ε_{λ_i} appearing in our extension. This conclusion is robust, after taking in account other models from the OLG literature, like those studied by Reichlin (1986) and more recently by Cazzavillan (2001). In these frameworks, the representative agent is then a good approximation of heterogeneity of consumer preferences, in order to study local dynamics.

Finally, it is interesting to notice that one can easy show that this equivalence still applies as long as one introduces externalities and increasing returns in both the classes of economies.

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