DOCUMENT DE RECHERCHE

EPEE

CENTRE D'ETUDE DES POLITIQUES ECONOMIQUES DE L'UNIVERSITE D'EVRY

Animal Spirits and Public Production in Slow Growth Economies

Stefano BOSI & Carine NOURRY

05 - 16

www.univ-evry.fr/EPEE

UNIVERSITE D'EVRY – VAL D'ESSONNE, 4 BD. FRANÇOIS MITTERRAND, 91025 EVRY CEDEX

Animal Spirits and Public Production in Slow Growth Economies*

Stefano BOSI† and Carine NOURRY‡

July 21, 2005

Abstract

In this paper, we consider a discrete time version of the endogenous growth model developed by Barro [1], but augmented in order to consider public production. Policy solutions are characterized to maximize the social welfare and stabilize the economy. More intervention is required under a higher elasticity of intertemporal substitution, because the public good is a productive externality. As soon as the growth rate of the economy lowers, indeterminacy can be generated by higher degrees of private production, and, then, even a small proportion of public production can have a stabilizing effect. Moreover, the way the public dividends are used matters for the stability properties of the economy: public dividends are used to provide flows of public services and/or long-run infrastructures. According to the value of the elasticity of intertemporal substitution and the proportion of public and private production, a higher share of public dividends used to install infrastructures can stabilize the economy. Endogenous fluctuations are possible within the model for plausible calibrations.

Keywords: *public spending, growth cycles, sunspots.*
JEL classification: *N1, H4.*

1 Introduction

During the last three decades the end of planned regimes and the waves of massive privatization either in transition economies or in western countries have raised many fundamental economic questions. Among other tricky puzzles or urgent policy matters, the main concern probably remains to understand how desirable the private ownership of the productive system is. The usual arguments for privatization point out the sources of economic inefficiency intrinsic to

*We are grateful to Leonor Modesto, Martial Dupaigne and Alain Venditti for very helpful comments and suggestions. We wish to thank also participants to PET 2004 in Peking, anglo-french meeting 2004 in Oxford and to T2M 2005 in Lyon. Any remaining errors are our own.

†EPEE, University of Evry. stefano.bosi@univ-evry.fr.
‡GREQAM, University of Aix-Marseille II. nourry@ehess.vcharite.univ-mrs.fr.

public production. More precisely the economists evoke the difference of goals between private and public firms (profit versus political and electoral targets), the weigh of bureaucracy in the public sector jointly with the power of unions and the more general lack of incentives (under the State wing, managers care less about the risk of bankruptcy, and workers about the risk of firing), public managers' corruption and sensitivity to political pressures, private managers' finer awareness of the market.

By contrast, other arguments justify a public ownership of the productive system as a policy instrument. Returns on public production enter the State budget jointly with taxes, inflationary taxes and issue of public bonds. However tax implementation requires the fiscal target to be observable, while seigniorage and bonds generate current and future inflation.

To keep matters of public finance as simple as possible, it is assumed that the public dividends finance the provision of a pure public good, an externality that improves the productivity and works as growth engine.[1] To shed a light on this specific mechanism, we focus on a competitive economy where the government is as efficient as the private sector, and exerts no market power (in some strategic activities the State plays the monopolist, but we assume the competitive sectors to weigh more and public firms to be generally price-takers).

In practice we modify a discrete time version of the endogenous growth model developed by Barro (1990) to take in account the public production.[2]

We deal with the occurrence of endogenous fluctuations dues to market imperfections (externalities and taxation/public production). In Barro (1990) the economy jumps from the very beginning on the balanced growth path, while in our context there is room for (possibly indeterminate) transitional equilibria. We study the size of the public sector jointly with the impact on the emergence of endogenous fluctuations, and we prove that a level of public production could stabilize the economy (in the sense of saddle-path stability). One may be concerned with in what our model actually differs from a version of Barro (1990), where only the capital taxation should be taken in account and the tax should put at once on dividends and capital gains. The rather surprising variety of our dynamics comes essentially from the assumption that the public spending is only financed by public dividends.

We prove in our paper that for low levels of economic growth, indeterminacy can be generated by high levels of private production. Then a proportion of public production has a stabilizing effect and when the private production stays beyond a given level, endogenous fluctuations are no longer possible. Furthermore, the way the public dividends are used matters for economic stability. Actually, for low levels of public production and low elasticities of intertemporal substitution, indeterminacy is possible for any use of dividends but providing

[1]In Gibson and Dutt (1993) state sector profit are used to finance the public spending. However consumers' preferences are roughly captured by an exogenous saving rate and the authors miss out on the dynamic richness of the model. On the dynamics of privatization within an endogenous growth setup, see also Zou (1994).

[2]See also Cazzavillan (1996) where the presence of public goods creates positive externalities both in the production and consumption sectors

infrastructures can stabilize the economy as soon as the elasticity of intertemporal substitution increases. Moreover, a large proportion of public services has a stabilizing effect for higher shares of public production and for low values, sufficiently close to zero, of intertemporal substitution. Beyond a critical level of public production, the fact that the government mainly provides services from public dividends avoids endogenous fluctuations. A plausible calibration stresses the destabilizing power of a privately owned production (indeterminacy and cycles), at least in economies with low growth level.

The paper is organized as follows. Section 2 presents the model. Section 3 defines the competitive equilibrium, while section 4 characterizes the existence of a stationary equilibrium. In section 5 the optimal degree of privatization is computed. Section 6 studies the occurrence of multiple equilibria and cycles and provides general conditions for indeterminacy. Section 7 characterizes a slow growth regime and applies a geometrical method to analyze the endogenous fluctuations. In section 8 calibrations confirm how plausible the emergence of endogenous fluctuations is, when associated to a proportion of public production. Section 8 contains some concluding comments, while all the proofs are gathered in the Appendix.

2 A model of growth

The economy is populated by a large number of identical infinite-lived households. The representative agent maximizes a utility function $\sum_{t=0}^{\infty} \beta^t u(c_t)$, where c_t is his consumption level at period t, and the positive discount factor $\beta < 1$ captures the time preference.

Assumption 1 *The elasticity of intertemporal substitution $\sigma > 0$ is constant:*

$$u(c) \equiv c^{1-1/\sigma} / (1 - 1/\sigma) \tag{1}$$

In each period the agent supplies inelastically one unit of labor and intends his labor and capital income to buy and smooth consumption over the lifetime. The capital income comes from a portfolio of firms shares and public bonds. The period t budget constraint he faces, is thus:

$$c_t + q_t (z_t - z_{t-1}) + b_t - b_{t-1} \leq d_t z_{t-1} + (R_t - 1) b_{t-1} + w_t \tag{2}$$

where z_t, q_t and b_t are respectively the demand of firms shares, their price and the demand of public bonds, whose price is normalized to one. d_t denotes the dividend per-share distributed by firms. R_t and w_t are the interest factor paid on bonds, and the wage.

Utility maximization subject to (2) gives a no-arbitrage condition: $R_{t+1} = (q_{t+1} + d_{t+1}) / q_t$, jointly with the usual Euler equation:

$$u'(c_t) / u'(c_{t+1}) = \beta R_{t+1} \tag{3}$$

and the budget constraint, now binding. As usual the transversality condition ensures the convergence of the series: $\lim_{t \to +\infty} \lambda_t k_{t+1} = 0$.

In the following $g \equiv G/N$ will be the amount of productive public good per worker, where N is the population size. For simplicity public services are provided with no user's charges and are assumed to be free from congestion effects. In line with Barro (1990), these services enter the production as external inputs. However, we crucially differs in the ownership of the competitive production sector, now supposed to be partially public. More exactly a part ν of the firms shares is owned by the government. Think $1-\nu$ as the privately owned fraction of the firms, or, with some misuse of language, the privatization degree of the economy.

Capital depreciates from a period to another at a constant factor $\Delta_1 \in [0,1]$. Given the sequence $\{g_j\}_{j=t}^{\infty}$, at time t, the firm chooses the forward-looking factor demands to maximize the value:

$$\max_{\{K_{j+1},N_j\}_{j=t}^{\infty}} D_t + \sum_{i=t+1}^{\infty} \frac{D_i}{\prod_{h=t+1}^{i} R_h}$$

where $D_t = Zd_t = e(g_t) AF(K_t, N_t) - (K_{t+1} - \Delta_1 K_t) - w_t N_t$ is the total amount of dividends at time t. Z denotes the number of shares. $e(g_t) AF(K_t, N_t)$ is the constant returns to scale production function, where e is the externality of public spending. In the following $k \equiv K/N$ will denote respectively the capital per worker. Production is shaped as follows.

Assumption 2 *The intensive production function $y(k,g) \equiv e(g) Af(k)$, with $A > 0$, $f \equiv F/N$ and $e, f \in C^2$, is strictly increasing, concave in both arguments, and homogenous of degree one. Each input is necessary, $y(k,0) = y(0,g) = 0$, for all $k > 0, g > 0$. Moreover, $\lim_{g \to +\infty} e(g) = +\infty$, and $\lim_{g \to +\infty} [g - e(g)] \geq 0$.*

In equilibrium the factor productivity equals its price:

$$R_t = \Delta_1 + e(g_t) Af'(k_t) \tag{4}$$

$$w_t = e(g_t) A[f(k_t) - k_t f'(k_t)] \tag{5}$$

3 Competitive equilibrium

In order to simplify the model, but without loss of generality, we set the supply of bonds equal to zero. Moreover, if ν denotes the nationalization degree of the economy, νZ is the endowment of shares in the government hands.

We suppose that the public dividends at time t are the only way to finance the public good. The latter is provided at time $t+1$. More precisely the real returns $\nu Z d_t$ are transformed, by means of a one-to-one technology, in public good. The government pays no interests on bonds and faces a simple budget constraint: $G_{t+1} - \Delta_2 G_t = \nu Z d_t$. The public good depreciation factor Δ_2 gets different values, according to the shape of the model. If we think of the public good as a flow of services, then Δ_2 is close to zero. If we interpret the public good more in terms of long-run infrastructures, then Δ_2 approaches one. A composite good will get an intermediate depreciation value.

Workers' population is constant over time and, for the sake of simplicity, normalized to one: $N_t = N = 1$ for every t. Since the dividends in one period are what remains from the production after the net investment in capital and the payment of current wages, the government constraint $g_{t+1} - \Delta_2 g_t = \nu Z d_t$ can be rewritten $g_{t+1} - \Delta_2 g_t = \nu \left[e(g_t) Af(k_t) - (k_{t+1} - \Delta_1 k_t) - w_t\right]$, or, equivalently, from (4) and (5):

$$g_{t+1} - \Delta_2 g_t = \nu \left(R_t k_t - k_{t+1}\right) \tag{6}$$

Eventually the good market equilibrium requires the production supply to be equal to the demand for future public good and net investment, and current consumption.

$$e(g_t) Af(k_t) = k_{t+1} - \Delta_1 k_t + g_{t+1} - \Delta_2 g_t + c_t \tag{7}$$

In the following the dynamic properties of the economy are more easily characterized in terms of the capital growth factor $\gamma_{t+1} \equiv k_{t+1}/k_t$ and the consumption-capital ratio $x_t \equiv c_t/k_t$. From (1) and (3) we have:From (1) and (3) we have:

$$\gamma_{t+1} = \left(\beta R_{t+1}\right)^{\sigma} x_t / x_{t+1} \tag{8}$$

Setting $h_t \equiv g_t/k_t$, from equations (4), (6) we obtain:

$$h_{t+1}\gamma_{t+1} - \Delta_2 h_t = \nu \left(R_t - \gamma_{t+1}\right) \tag{9}$$

Under Assumption 2 (homogeneity of the production function in g and k), (4) becomes: $R_t = \Delta_1 + e(h_t) Af'(1)$. From equations (9), (7) we obtain the dynamic system

$$\left[\nu + e^{-1}\left(\frac{R_{t+1} - \Delta_1}{Af'(1)}\right)\right]\gamma_{t+1} = \nu R_t + \Delta_2 e^{-1}\left(\frac{R_t - \Delta_1}{Af'(1)}\right) \tag{10}$$

$$(1-\nu)\left(\gamma_{t+1} - \Delta_1\right) = \left(1/s - \nu\right)\left(R_t - \Delta_1\right) - x_t \tag{11}$$

jointly with (8), where

$$s(k) \equiv kf'(k)/f(k) \tag{12}$$

is now the share of capital on total income and $s \equiv s(1) \in (0,1)$. Assumption 2 ensures e to be invertible.

Definition 1 *A competitive equilibrium with public good externalities and a degree $0 < \nu < 1$ of public ownership is a path $\left\{\left(R^e_{t+1}, x^e_t\right)\right\}_{t=0}^{+\infty}$ that satisfies the system (10-11) for every $t \geq 0$, given $h_0 = g_0/k_0$.*

4 Steady state analysis

Definition 2 *A stationary equilibrium is a constant sequence $\{(R,x)\}_{t=0}^{+\infty}$, which satisfies the dynamic system (10-11):*

$$R = \Delta_1 + e\left(\nu \frac{R-\gamma}{\gamma - \Delta_2}\right) Af'(1) \tag{13}$$

$$x = (1-\nu)(R-\gamma) + \frac{1-s}{s}\left(R - \Delta_1\right) \tag{14}$$

where

$$\gamma = (\beta R)^{\sigma} \tag{15}$$

is the balanced growth factor.

The first equation gives R, the second one gives x. As usual growth is positive, iff $R > \beta^{-1}$. Equation (13) determines independently the stationary value of R. The series of the intertemporal utility evaluated along the balanced path $c_t = c_0 \gamma^t$ converges (to $c_0^{1-1/\sigma} \left[(1-1/\sigma)\left(1-\beta\gamma^{1-1/\sigma}\right)\right]^{-1}$), iff $\beta\gamma^{1-1/\sigma} < 1$, that is iff the stationary transversality condition holds

$$R > \gamma \tag{16}$$

At the steady state the dividend is equal to $d_t = (R/\gamma - 1)\, k_t$ and (16) ensures dividends and public spending positivity. Under the transversality condition, equation (13) implies that $R > \Delta_1$ and that the RHS of (14) is always positive, that is $x > 0$.

To characterize the existence of a steady state, we need to focus exclusively on equation (13), or equivalently on $\varphi(R) \equiv R - \Delta_1 - e(\nu\xi(R))\, Af'(1) = 0$, where $\xi(R) \equiv [R - (\beta R)^{\sigma}] / [(\beta R)^{\sigma} - \Delta_2]$. There are as many steady states as the intersections of φ with the positive axis of abscissas.

Proposition 3 *Let Assumptions 1 and 2 hold. (i) If $\sigma \geq 1$, then a steady state exists, iff*

$$\varphi\left(\beta^{\sigma/(1-\sigma)}\right) > 0 \tag{17}$$

Moreover it is unique. (ii) Let now $\sigma < 1$. A stationary equilibrium always exists and the number of steady states is odd.

5 Optimal nationalization degree

We are interested now in finding the optimal nationalization degree ν, that is the degree of public ownership of the productive system, ensuring the maximization of a welfare index, here represented by the representative agent's utility function.

For simplicity, we focus on the case of balanced growth and we compute the optimal nationalization degree ν.

Proposition 4 *There exists $\sigma^* \in (0,1)$, such that, if $\sigma < \sigma^*$, then the optimal nationalization degree is given by*

$$\nu^* = \frac{(\beta R^*)^{\sigma} - \Delta_2}{R^* - (\beta R^*)^{\sigma}} e^{-1}\left(\frac{R^* - \Delta_1}{Af'(1)}\right) \tag{18}$$

where R^* is solution of

$$\frac{R - \Delta_2}{(\beta R)^{\sigma} - \Delta_2} = \frac{1}{\sigma}\frac{R}{(\beta R)^{\sigma}}\left[1 - \frac{1}{1-s}\frac{R - (\beta R)^{\sigma}}{R - \Delta_1}\right] \tag{19}$$

If $\sigma > \sigma^*$, then the full nationalization $\nu^* = 1$ is the optimal policy rule ensuring the welfare maximization.

Proof. See the Appendix. ■

Under a sufficiently high elasticity of intertemporal substitution, individuals prefer to renounce to private dividends (ensuring an extra-consumption today beyond the labor income), to obtain an extra-consumption tomorrow provided by higher externalities of public spending and a subsequent higher future labor income.

6 Multiple equilibria and cycles in a general framework

To study the local behavior of the economy around the steady state, we linearize the two-dimensional dynamic system (10-11), whose variables are r_t and x_t. Depending on two predetermined variables (k_t and g_t in equation (4)), r_t is predetermined as well. By contrast $x_t \equiv c_t/k_t$ inherits the nature of non-predetermined variable from c_t. Therefore to observe indeterminacy, a sink configuration is required around the steady state (both the eigenvalues inside the unit circle).

Lemma 5 *Under Assumptions 1 and 2, the characteristic polynomial of system (10-11) is* $P(\lambda) = \lambda^2 - T\lambda + D$*, with*

$$\begin{aligned} T &= 1 + \frac{1}{z_0}\left(z_1 + (\sigma + z_0)\left[\frac{R}{\gamma}\left(1 + \frac{1-s}{s-\nu s}\frac{R-\Delta_1}{R}\right) - 1\right] - \frac{R}{\gamma}\frac{1-\nu s}{s-\nu s}\right) \\ D &= \frac{z_1\left[(1-\nu s)R - (1-s)\Delta_1\right] - R(1-\nu s)}{s\gamma z_0(1-\nu)} \end{aligned}$$

where

$$z_0 \equiv \frac{R}{R-\Delta_2}\frac{1}{1-s}\frac{R-\gamma}{R-\Delta_1} > 0 \tag{20}$$

$$z_1 \equiv \frac{R}{R-\Delta_2}\left[1 - \frac{\Delta_2}{\gamma}\left(1 - \frac{1}{1-s}\frac{R-\gamma}{R-\Delta_1}\right)\right] > 0 \tag{21}$$

We observe that under the transversality condition (16) and $\gamma \geq \Delta_2$, then $z_0, z_1 > 0$. As seen above, the equilibrium is locally indeterminate, iff the steady state is a sink.

Proposition 6 *The steady state is a sink and local indeterminacy occurs, iff*

$$\begin{aligned} &\max\left\{\sigma + z_0, 2\frac{(1-\nu s)R + s(1-\nu)\gamma\sigma}{(1-\nu s)R - (1-s)\Delta_1 + s(1-\nu)\gamma} - (\sigma + z_0)\right\} \\ &\leq \ z_1 \leq \frac{(1-\nu s)R + s(1-\nu)\gamma z_0}{(1-\nu s)R - (1-s)\Delta_1} \end{aligned} \tag{22}$$

where R and γ satisfy, respectively, (13) and (15). Moreover, we have a transcritical bifurcation at

$$\sigma = z_1 - z_0 \tag{23}$$

a flip bifurcation (jointly with the emergence two-period cycles) at

$$\nu = \frac{2(R + s\sigma\gamma) - (\sigma + z_0 + z_1)[R + s\gamma - (1 - s)\Delta_1]}{2(sR + s\sigma\gamma) - (\sigma + z_0 + z_1)(sR + s\gamma)} \tag{24}$$

and a Hopf bifurcation at

$$\nu = \frac{z_1(1 - s)\Delta_1 + (1 - z_1)R + s\gamma z_0}{s(1 - z_1)R + s\gamma z_0} \tag{25}$$

The existence of stochastic sunspot equilibria around the steady state requires equilibrium indeterminacy. A constructive proof, inherent to a general two-dimensional system with one state variable, and then applicable to our particular context, is provided in Grandmont, Pintus and de Vilder (1998). We observe that there is room for sunspot equilibria also along the saddle path, provided the flip bifurcation is supercritical. In this case, we require the two-period cycle to be stable to attract stochastic equilibria around, but faraway from the steady state. Sunspot equilibria can be also constructed around the limit cycle generated by a Hopf bifurcation.

6.1 On the saddle-path stability

The left-side inequality in (22) is violated by sufficiently high σ's and we obtain the following result.

Corollary 7 *The equilibrium is unique (saddle point) for* $\sigma > z_1 - z_0$.

6.2 On the plausibility of the Hopf bifurcation

From now on, we will assume that the public spending depreciates more than the physical capital: $\Delta_2 \leq \Delta_1$. The rationale is that the public spending is now viewed as a mixture of long-run infrastructures with a depreciation rate close to the capital one, and short-run services.

Corollary 8 *If* $\Delta_2 \leq \Delta_1$,

$$R \leq 2\gamma - \Delta_1 \tag{26}$$

$$\nu < (1 - s)/s \tag{27}$$

and the Hopf bifurcation is no longer achievable.

7 Endogenous fluctuations with low growth rate

As seen above, public production can generate endogenous fluctuations when the growth rate is sufficiently small. In fact, between 1966 and 1990, growth in real GDP per capita in the OECD countries averaged only around 2% (Obstfeld and Rogoff, 1999). But several western countries experienced in the last decade milder growth rates. A local analysis of the growth rate in a neighborhood of zero is, in this respect, justified.

If we consider the extreme case with no growth, the model becomes tractable to use a geometrical method popularized by Grandmont, Pintus and de Vilder [7]. This method allow us to focus on how some fundamental parameters modify the dynamic properties of the steady state. We will characterize the local dynamics for a zero growth economy and we will extend the relevant conclusions to economies with small growth rates, using a continuity argument.

7.1 Existence of a normalized steady state

In order to simplify the analysis, we follow the procedure introduced by Cazzavillan, Lloyd-Braga and Pintus [5] and use the scaling parameter A to normalize the steady state (R^*, x^*) under a zero growth requirement $(\gamma^* = 1)$.

Proposition 9 *Under Assumptions 1-2,*

$$(R^*, x^*) = (1/\beta, (1-\nu)(1/\beta - 1) + (1/s - 1)(1/\beta - \Delta_1))$$

is a steady state of dynamic system (10-11), iff

$$A^* \equiv \frac{1/\beta - \Delta_1}{s\left[\nu\left(1/\beta - 1\right)/\left(1-\Delta_2\right)\right]^{1-s}} \tag{28}$$

We notice that now A^* no longer depends on σ and that, since the growth is null, equation (15) becomes the exogenous growth modified golden rule.

In the following, the headline is that, as long as we remain in a small neighborhood of A^*, by continuity, the growth factor remains in a small neighborhood of $\gamma = 1$, and the qualitative dynamic properties of the model persist. In other words, there is a (possibly small) neighborhood of one, where the degree of stability remains the same. More precisely, sufficiently small growth rates $(\gamma > 1)$ are associated to the same number of eigenvalues inside the unit circle as in the case $\gamma = 1$.

From now on, we denote SSS (Slow (Growth) Steady State) the stationary equilibrium corresponding to $\gamma = 1 + \varepsilon$, with $\varepsilon > 0$, and sufficiently close to the normalized one in order to keep the same qualitative dynamic properties.

7.2 Characteristic polynomial and geometrical method

In the rest of the paper, in order to ensure the existence of a normalized steady state, the productivity parameter A is set equal to A^*.

Assumption 3 $A \equiv A^*$.

Let us consider the characteristic polynomial from Lemma 5 around (R^*, x^*).

Proposition 10 *Under Assumptions 1-3, the qualitative properties of the SSS* *depend on the shape of the characteristic polynomial* $\mathcal{P}(\lambda) = \lambda^2 - \mathcal{T}\lambda + \mathcal{D}$*, where*

$$\begin{aligned} \mathcal{T} &= \omega + (1-\omega)\Delta_2 \\ &\quad + [1 + \varkappa(1-\beta\Delta_1) - \omega(1-\beta\Delta_2)(1 + \varkappa - \sigma[1-\beta+\varkappa(1-\beta\Delta_1)])]/\beta \\ \mathcal{D} &= \Delta_2(1+\varkappa)[\omega + (1-\omega)/\beta] - \Delta_1\varkappa[\omega + (1-\omega)\Delta_2] \end{aligned}$$

with $\omega \equiv (1-s)(1-\beta\Delta_1)/(1-\beta)$ *and* $\varkappa \equiv (1-s)/[s(1-\nu)]$.

Now, our aim is to discuss the local dynamics around the normalized steady state instead of those around the SSS.

7.2.1 The Δ-line

As in Grandmont, Pintus and de Vilder [7], we will analyze the local stability of (R^*, x^*) by studying the variations of the trace $\mathcal{T}$ and the determinant $\mathcal{D}$ in the $(\mathcal{T}, \mathcal{D})$-plane, when some parameters of interest vary continuously. This methodology allows also to easily study the occurrence of local bifurcations.

We are mainly interested in how the dynamic properties of the economy are affected by the levels of the parameters ν and Δ_2. In our model, the proportion of public and private production can generate dynamic disturbances through two channels. The first one is the share of the production sector owned by government, which is given by ν, a proxy of the "size" of the State. The second one is the way the dividends from public bonds are used, which is related to the nature of the public good, captured in our model by Δ_2. When Δ_2 is small, say close to zero, the good can be thought as a flow of public services. When Δ_2 is large, the good shares the same nature of the private capital and can be viewed as an infrastructure. In fact, $0 \leq \Delta_2 \leq \Delta_1$, since the public good is always a mix of infrastructures and services. Thus, the higher admissible depreciation rate will be $\Delta_2 = \Delta_1$.

In our model, $\mathcal{T}$ and $\mathcal{D}$ are linear in the public depreciation rate Δ_2 and, consequently, the locus $\{(\mathcal{T}(\Delta_2), \mathcal{D}(\Delta_2))\}$ obtained as Δ_2 varies, is represented by a straight line $\mathcal{D} = \Delta(\mathcal{T})$ in the $(\mathcal{T}, \mathcal{D})$-plane.[3]

$\Delta(\mathcal{T})$ is illustrated in Figure 1, where three other important lines are represented. Along the line AC, that is $\mathcal{D} = \mathcal{T} - 1$, one characteristic root is equal to 1. Along the line AB, that is $\mathcal{D} = -\mathcal{T} - 1$, one eigenvalue is equal to -1, while along the segment BC, that is $\mathcal{D} = 1$, $|\mathcal{T}| < 2$, the roots are complex and conjugate with modulus equal to 1. These lines partition the $(\mathcal{T}, \mathcal{D})$-plane into three regions according to the number of eigenvalues with modulus less than 1. When Δ_2 goes through $\Delta_2^F \in (0, \Delta_1)$, the Δ-line crosses the line AB and a flip bifurcation is generically expected to occur. When Δ_2 goes through $\Delta_2^T \in (0, \Delta_1)$, the Δ-line crosses the line AC and one root crosses $+1$. From Proposition 3, a steady state always exists and a saddle-node bifurcation never

[3] From Proposition 10, the slope of Δ is given by

$$\frac{1 + \varkappa\omega + (1-\omega)[1-\beta+\varkappa(1-\beta\Delta_1)]/\beta}{1 + \varkappa\omega - \sigma\omega[1-\beta+\varkappa(1-\beta\Delta_1)]}$$

occurs. Therefore, the critical value Δ_2^T is always associated with an exchange of stability between the normalized steady state and another or two other steady states through a transcritical or, respectively, a pitchfork bifurcation. However, pitchfork bifurcations require some non-generic condition[4]. In order to simplify the exposition we then concentrate on the generic case and we associate in the rest of the paper the existence of one eigenvalue equal to 1 to a transcritical bifurcation.

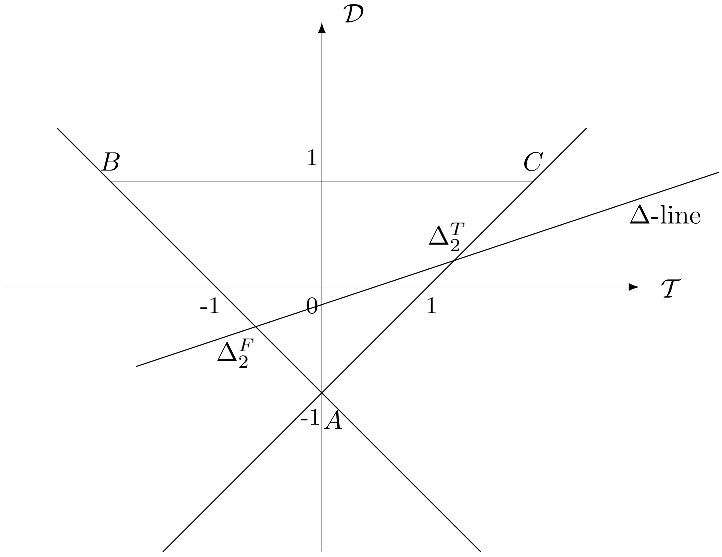

Figure 1: Stability triangle and Δ-line.

As $\mathcal{D}$ and $\mathcal{T}$ get finite values for $\Delta_2 \in [0, \Delta_1]$, then only a segment of the line $\Delta(\mathcal{T})$ really matters in our analysis. The following Lemma tells us what extreme is a starting point.

Lemma 11 *The determinant $\mathcal{D}$ is an increasing function of Δ_2.*

Let $(\mathcal{T}_0, \mathcal{D}_0)$ and $(\mathcal{T}_1, \mathcal{D}_1)$ be the starting and the ending point on such a Δ-segment, corresponding, respectively, to $\Delta_2 = 0$ and $\Delta_2 = \Delta_1$. We observe that the determinant does not depend on the elasticity of intertemporal substitution σ, while the trace is a linear function of σ. Since $\Delta_2 \leq \Delta_1 < 1/\beta$, the trace $\mathcal{T}$ is increasing with σ, from a finite value to infinity[5]. Thus the Δ-segment goes right as σ increases, since both the starting and ending point move to right along horizontal half-lines. More precisely, as soon as σ increases, the initial extreme $(\mathcal{T}_0, \mathcal{D}_0)$ covers the half-line $\{(\mathcal{T}_0(\sigma), \mathcal{D}_0) : \sigma \in [0, +\infty)\}$, while the ending point $(\mathcal{T}_1, \mathcal{D}_1)$ covers the parallel $\{(\mathcal{T}_1(\sigma), \mathcal{D}_1) : \sigma \in [0, +\infty)\}$. For

[4]Some second derivative of the map which defines the dynamical system has to be equal to zero. As shown in Ruelle [15], this requirement is non-generic.

[5]We notice that $\partial\mathcal{T}/\partial\sigma = (1 - \beta\Delta_2)[1 - \beta + \varkappa(1 - \beta\Delta_1)]\omega/\beta$.

simplicity, with some notational misuse, we call Δ_0-half-line the former (with $\Delta_2 = 0$) and Δ_1-half-line the latter (with $\Delta_2 = \Delta_1$).

In order to apply the geometrical method presented in Grandmont, Pintus and de Vilder [7], we require more information about the location of the half-lines Δ_0 and Δ_1. Furthermore, to prove the existence of endogenous fluctuations, we need to know how the Δ-segment moves in response to changes of the nationalization degree ν. To rule out implausible cases, we make use of a new assumption.

Assumption 4 *(i) The value of the parameter s satisfies the inequality:*[6] $s < 1 - (1-\beta)/(1-\beta\Delta_1)$. *(ii) The depreciation factor of the capital goods Δ_1 and the discount factor β are required to satisfy: $2/3 < \Delta_1 < \beta$.*

A first general property is that the determinant decreases with the public production degree ν.[7] Then the half-lines Δ_0 and Δ_1 move downwards as ν increases. In the sequel, we will refer to four critical degrees of nationalization:

$$\nu_1 \equiv 1 + \frac{1-s}{s} \frac{(1-\omega)(1-\beta\Delta_1) - 2\omega\beta\Delta_1}{1-\omega+(1+\omega)\beta} \tag{29}$$

$$\nu_3 \equiv 1 - \frac{1-s}{s}\omega\Delta_1 \tag{30}$$

$$\nu_4 \equiv 1 + \frac{1-s}{s} \frac{(1-\omega)(1-\beta\Delta_1)}{1-\omega+(1+\omega)\beta} \tag{31}$$

$$\nu_5 \equiv 1 + \frac{1-s}{s} \frac{(1-\omega)(1-\beta\Delta_1)}{1-\omega+(1/\Delta_1+\omega)\beta} \tag{32}$$

7.2.2 Δ_0-half-line

We know that the half-line Δ_0, obtained for $\Delta_2 = 0$ as σ varies increases away from 0, is horizontal. Moreover, since the determinant of its origin is $\mathcal{D}_0 = -\omega\varkappa\Delta_1$, the half-line Δ_0 lies below the axis of abscissas. According to the degree of nationalization ν and the elasticity of intertemporal substitution in consumption, there are three possible degrees of stability of the economic system. The origin $(\mathcal{T}_0(0), \mathcal{D}_0)$ of the half-line can lie inside the stability triangle ABC, on the left or below.

Lemma 12 *Under Assumptions 1-4: (i) the Δ_0-half-line is above A, iff $\nu < \nu_3$, (ii) the Δ_0-half-line crosses the line AC, (iii) the Δ_0-half-line crosses the line AB, iff $\nu > \nu_1$, (iv) $\nu_1 < \nu_3$.*

Property (iv) means that when ν increases, the Δ_0-half-lines cross the line AB first and, after, move from above to below. The main locations of the half-line Δ_0 are represented in Figure 2, according to the values of ν.

[6] If, for instance, $\beta \geq 0.98$ and $\Delta_1 \geq 0.9$, we obtain $s < 0.83$, which is a mild and empirically plausible assumption.

[7] Under Assumption 4, we have $\partial\mathcal{D}/\partial\nu = -\varkappa[\omega(\Delta_1-\Delta_2) + (\omega-1)(1/\beta - \Delta_1)\Delta_2] / (1-\nu) < 0$.

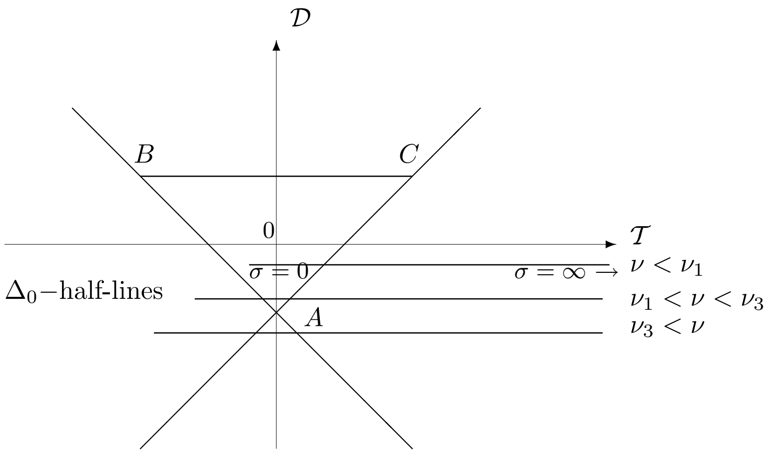

Figure 2: Δ_0-half-line

7.2.3 Δ_1-half-line

We now study how the locus of the ending points of the Δ-segments behaves, when the elasticity of intertemporal substitution, σ, varies from 0 to infinity. Δ_1 is an horizontal half-line, lying above Δ_0, because of lemma 11. We first prove that a sufficiently high degree of nationalization makes the normalized steady state determinate.

Proposition 13 *Under Assumptions 1-4, the local indeterminacy of the normalized steady state is ruled out by $\nu > \nu_5$, where ν_5 is given by (32).*

Thus, a not too high level of private ownership of the firms immunizes the economy against the occurrence of endogenous fluctuations. To study the way indeterminacy appears, we will assume in the sequel a nationalization degree low enough.
Assumption 5 *The value of the nationalization degree $\nu < \nu_5$.*
The properties of the Δ_1-half-line are summarized in the following lemma.

Lemma 14 *Under Assumptions 1-5, the Δ_1-half-line (i) lies below the line BC, (ii) goes through the line AC and (iii) crosses the line AB, iff $\nu > \nu_4$, where ν_4 is given by (31).*

The main locations of the Δ_1-half-line are represented in Figure 3. From Lemma 14, we are able to order ν_4 and ν_5. We notice that the Δ_1-half-lines cross the line AC for any proportion of public production. Hence, at $\nu = \nu_4$, since the Δ_1-half-line goes through A, it also crosses the line AB and, consequently, $\nu_4 < \nu_5$.

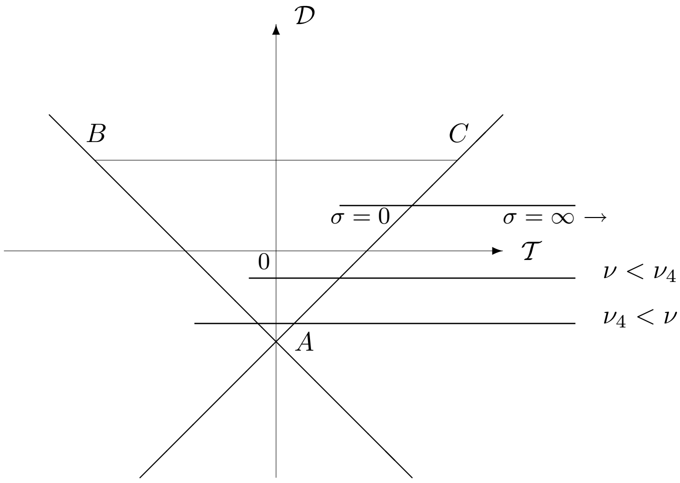

Figure 3: Δ_1-half-lines for $\nu < \nu_5$.

7.3 Indeterminacy in a slow growth regime

In order to focus on the cases with a sufficiently low nationalization degree, we impose the restriction $0 < \nu < \nu_5$ through Assumption 5. Straightforward computations show that the slope of the Δ-segment is positive for low values of σ and rotates in a counterclockwise sense, becoming first infinite and then negative as soon as σ increases. However, this property, even if enables us to provide a correct geometrical representation, doesn't affect the qualitative properties of the normalized steady state and is of no use in the analytical proofs.

In contrast, what really matters is understanding, for a given ν, whether the Δ_0-half-line crosses the line AC for lower or higher values of σ with respect to the Δ_1-half-line. In the following, σ_0 and σ_1 will denote the values of the elasticity of intertemporal substitution for which, respectively, the half-lines Δ_0 and Δ_1 cross the line AC.

If $\sigma = \sigma_1 < \sigma_0$, the starting point of the Δ-segment is located on the left of the line AC, while the ending point lies on AC. In other words, the Δ-segments cross AC with a slope weaker than one.

σ_2 is the critical value of the elasticity of intertemporal substitution, such that the Δ_0-half-line crosses the line AB. When $\sigma_2 < \sigma_1$, the Δ-segment begins on the right of the line AB, inside the ABC triangle, and ends on the line AC for $\sigma = \sigma_1$. Otherwise, when $\sigma_2 > \sigma_1$, the Δ-segment begins on the left of the triangle ABC. This ranking depends on the nationalization degree ν and the two cases have to be discussed in terms of ν.

Eventually, we rank the critical values of the share of public production, ν_1,

ν_3 and ν_4, in order to understand the simultaneous location of the half-lines Δ_0 and Δ_1. The next lemma sums up and clarifies our remarks.

Lemma 15 *Under Assumptions 1-5, (i) $\sigma_1 < \sigma_0$, for any $\nu \in [0,1]$, (ii) there exists $\nu_2 \in (\nu_1, \nu_3)$, such that $\sigma_1 < \sigma_2$, iff $\nu > \nu_2$, and (iii) $\nu_3 < \nu_4$.*

(i) means that, given ν, the Δ_1-half-line crosses the line AC for a lower value of σ than the Δ_0-half-line. Hence, the Δ_1-half-line leaves earlier the triangle. Property (iii) means that $\nu_1 < \nu_4$, or, in other words, that the Δ_0-half-line crosses the line AB for lower values of ν than the Δ_1-half-line. Moreover, the Δ_1-half-line goes through the line AB when the half-line Δ_0 lies below A.

According to Lemmas 11-15, we need to distinguish five cases, depending on the position of ν with respect to ν_1, ν_2, ν_3, ν_4.

7.3.1 Case $\nu < \nu_1$

Figure 1 helps us to discuss the local stability of (R^*, x^*), the normalized steady state: we locate the pair $(\mathcal{T}, \mathcal{D})$ on the $\Delta(\mathcal{T})$ as the public good depreciation factor Δ_2 varies over $(0, \Delta_1)$. Two critical values of Δ_2 are of interest: at Δ_2^F the line $\Delta(\mathcal{T})$ crosses AB and generates a flip bifurcation; at Δ_2^H the line $\Delta(\mathcal{T})$ crosses the line AC and generates a transcritical bifurcation.
In the light of lemma 11, considering the position of the Δ_0-half-line, when $0 < \nu < \nu_1$, and, in the light of lemmas 14 and 15, the location of the Δ_1-half-line, when $0 < \nu < \nu_4$, we are able to provide a complete picture of the local dynamic properties of the normalized steady state (see Figure 4).

Focus, for instance, on the line corresponding to an intertemporal substitution $\sigma < \sigma_1$. The steady state is locally indeterminate for any $\Delta_2 \in (0, \Delta_1)$. For a Δ-line corresponding to $\sigma_1 < \sigma < \sigma_0$, the steady state is locally indeterminate for any $\Delta_2 \in \left(0, \Delta_2^T\right)$. Then a transcritical bifurcation occurs when Δ_2 crosses Δ_2^T from below and the steady state is saddle point stable, whatever $\Delta_2 \in \left(\Delta_2^T, \Delta_1\right)$. If, conversely, we consider a Δ-line associated to an intertemporal substitution $\sigma > \sigma_0$, the normalized steady state turns out to be saddle-point stable for $\Delta_2 \in (0, \Delta_1)$.

All the details of the bifurcation analysis are gathered in the next Proposition.

Proposition 16 *Let Assumptions 1-4 hold and $\nu < \nu_1$, where ν_1 is given by (29).*

(i) If $0 < \sigma < \sigma_1$, then the SSS is locally indeterminate for every $\Delta_2 \in (0, \Delta_1)$.

(ii) If $\sigma_1 < \sigma < \sigma_0$, then the SSS is locally indeterminate for $\Delta_2 \in \left(0, \Delta_2^T\right)$, undergoes a transcritical bifurcation at $\Delta_2 = \Delta_2^T$, and becomes a saddle point for $\Delta_2 \in \left(\Delta_2^T, \Delta_1\right)$.

(iii) If $\sigma > \sigma_0$, then the SSS is a saddle point for every $\Delta_2 \in (0, \Delta_1)$.

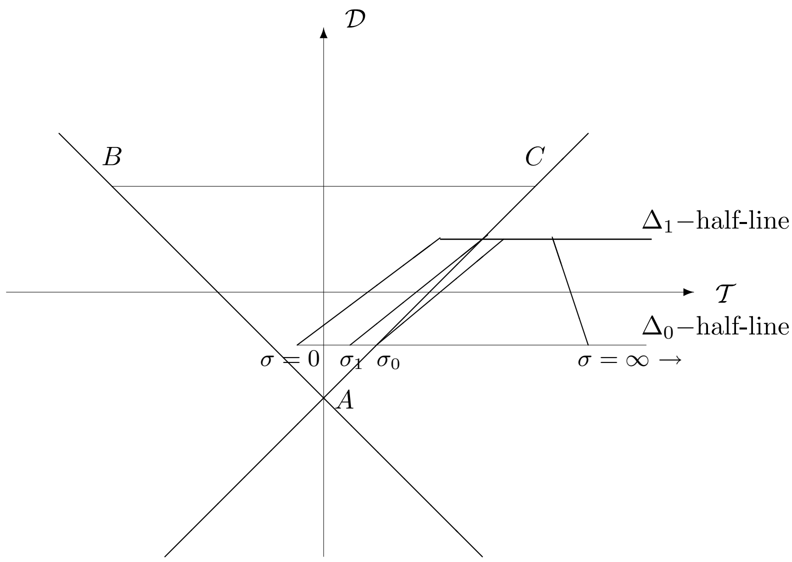

Figure 4: Δ-lines for $\nu < \nu_1$.

Proposition 16 shows that under low nationalization degrees, local indeterminacy requires a sufficiently small elasticity of intertemporal substitution ($\sigma < \sigma_0$) and, if $\sigma_1 < \sigma < \sigma_0$, a public good depreciation weak enough. Indeed, the location of the Δ_1-half-line clearly proves that the steady state is never indeterminate for $\sigma > \sigma_1$, when the public good is constituted only by long-run infrastructures.

7.3.2 Case $\nu_1 < \nu < \nu_2$

Now, consider an economy with a higher share of state-owned production. Since $\nu < \nu_2$ and, as above, $\sigma_2 < \sigma_1$, the configuration is quite close to the case $\nu < \nu_1$. However, since $\nu > \nu_1$, the Δ_0-half-line starts on the left of the triangle ABC and a new dynamic feature arises for low values of the intertemporal substitution ($\sigma < \sigma_2$). The geometrical findings of Figure 5 are formalized in the next Proposition.

Proposition 17 *Let $\nu_1 < \nu < \nu_2$. Under Assumptions 1-5, there exist σ_1 and σ_2, such that the following generically holds.*

(i) If $\sigma < \sigma_2$, then the SSS is saddle-point stable for $\Delta_2 \in \left(0, \Delta_2^F\right)$, undergoes a flip bifurcation at $\Delta_2 = {\Delta_2}^F$, becomes locally indeterminate for $\Delta_2 \in \left(\Delta_2^F, \Delta_1\right)$.

(ii) If $\sigma_2 < \sigma < \sigma_1$, then the SSS is indeterminate for every $\Delta_2 \in (0, \Delta_1)$.

(iii) If $\sigma_1 < \sigma < \sigma_0$, then the SSS is locally indeterminate for $\Delta_2 \in \left(0, \Delta_2^T\right)$, undergoes a transcritical bifurcation at $\Delta_2 = \Delta_2^T$, and becomes saddle-point stable for $\Delta_2 \in \left(\Delta_2^T, \Delta_1\right)$.

(iv) If $\sigma > \sigma_0$, then the SSS is saddle-point stable for every $\Delta_2 \in (0, \Delta_1)$.

For $\sigma < \sigma_0$, indeterminacy is still possible. When $\sigma_2 < \sigma < \sigma_1$, the steady state is locally indeterminate, whatever use the government makes of public firms' dividends. The occurrence of endogenous fluctuations due to a provision of public services instead of public infrastructures, depends essentially on the value of the intertemporal substitution σ. If $\sigma < \sigma_2$, higher shares of public dividends to finance public infrastructures can destabilize the economy; whereas, if $\sigma_0 > \sigma > \sigma_1$, expectations-driven fluctuations occur, when public services are mainly provided.

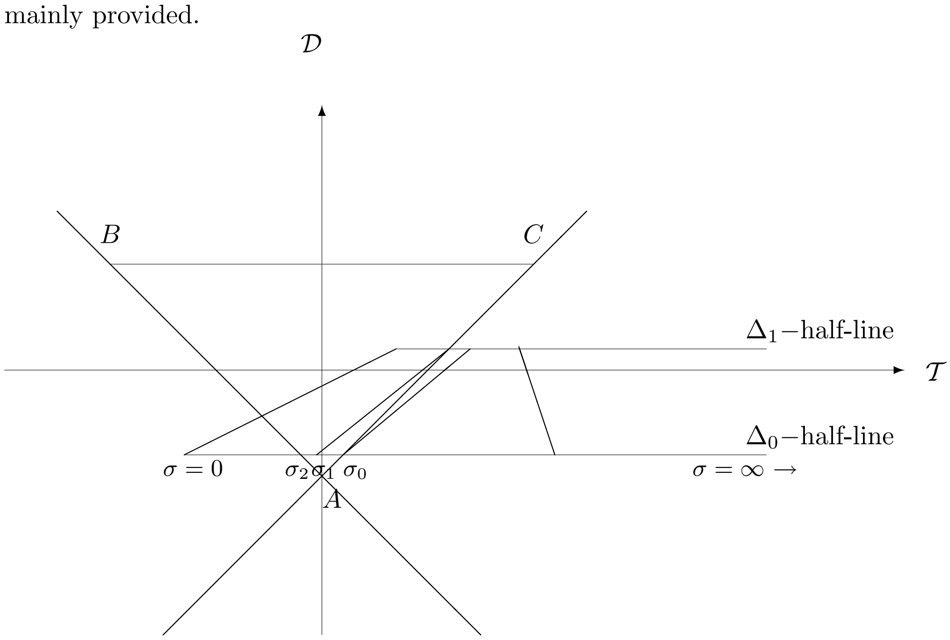

Figure 5: Δ-lines for $\nu_1 < \nu < \nu_2$.

7.3.3 Case $\nu_2 < \nu < \nu_3$

When the degree of nationalization increases, local indeterminacy occurs under conditions similar to those of Proposition 17. However, the surprising case in which indeterminacy occurs whatever the nature of the public good (case (ii) in Proposition 17), is no longer attainable. Moreover, the ranges of σ such that a kind of public spending generates indeterminacy, are, now, reversed with respect to σ_1 and σ_2. More precisely, local indeterminacy now occurs for $\sigma < \sigma_1$, if the government uses the dividends to finance mainly long-run infrastructures. Conversely, if $\sigma_2 < \sigma < \sigma_0$, a large provision of services turns out to be destabilizing. A formal proposition sums up the qualitative information in Figure 6.

Proposition 18 *Let $\nu_2 < \nu < \nu_3$. Under Assumptions 1-5, there exist σ_1 and σ_2, such that the following generically holds.*

(i) If $\sigma < \sigma_1$, then the SSS is saddle-point stable for $\Delta_2 \in \left(0, \Delta_2^F\right)$, undergoes a flip bifurcation at $\Delta_2 = \Delta_2^F$, becomes locally indeterminate for $\Delta_2 \in \left(\Delta_2^F, \Delta_1\right)$.

(ii) If $\sigma_1 < \sigma < \sigma_2$, then the SSS is saddle-point stable for $\Delta_2 \in \left(0, \Delta_2^F\right)$, undergoes a flip bifurcation at $\Delta_2 = \Delta_2^F$, becomes locally indeterminate for $\Delta_2 \in \left(\Delta_2^F, \Delta_2^T\right)$, undergoes a transcritical bifurcation at $\Delta_2 = \Delta_2^T$, and becomes saddle-point stable for $\Delta_2 \in \left(\Delta_2^T, \Delta_1\right)$.

(iii) If $\sigma_2 < \sigma < \sigma_0$, then the SSS is locally indeterminate for $\Delta_2 \in \left(0, \Delta_2^T\right)$, undergoes a transcritical bifurcation at $\Delta_2 = \Delta_2^T$, and becomes saddle-point stable for $\Delta_2 \in \left(\Delta_2^T, \Delta_1\right)$.

(iv) If $\sigma > \sigma_0$, then the SSS is saddle-point stable for every $\Delta_2 \in (0, \Delta_1)$.

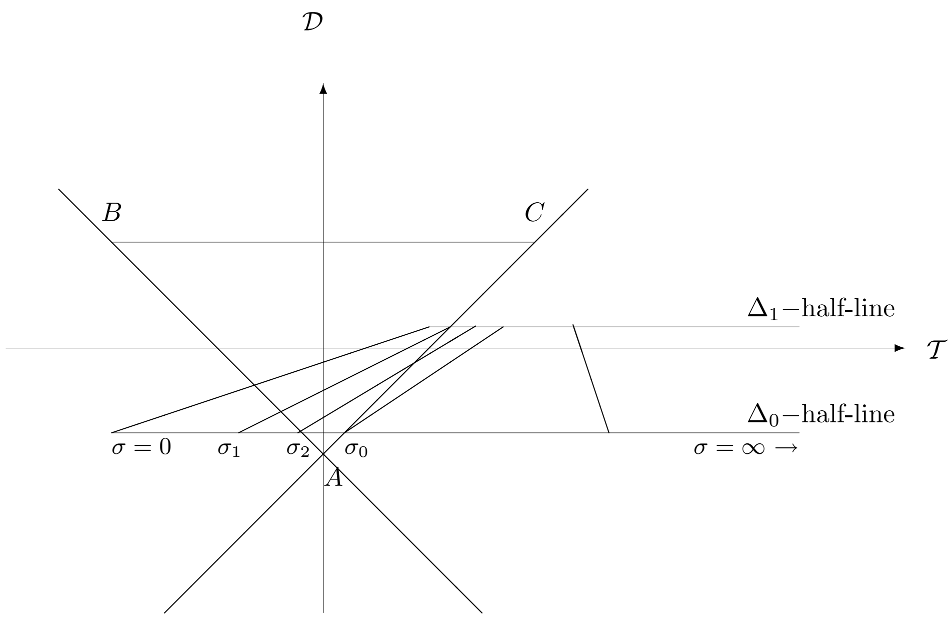

Figure 6: Δ-lines for $\nu_2 < \nu < \nu_3$.

7.3.4 Case $\nu_3 < \nu < \nu_4$

Raising the nationalization degree, we obtain a new dynamic case. Now, the Δ_0-half-line is above A and, therefore, local indeterminacy is no longer possible, whenever a large share of public dividends is devoted to provide services. The next proposition characterizes the subcases represented in Figure 7.

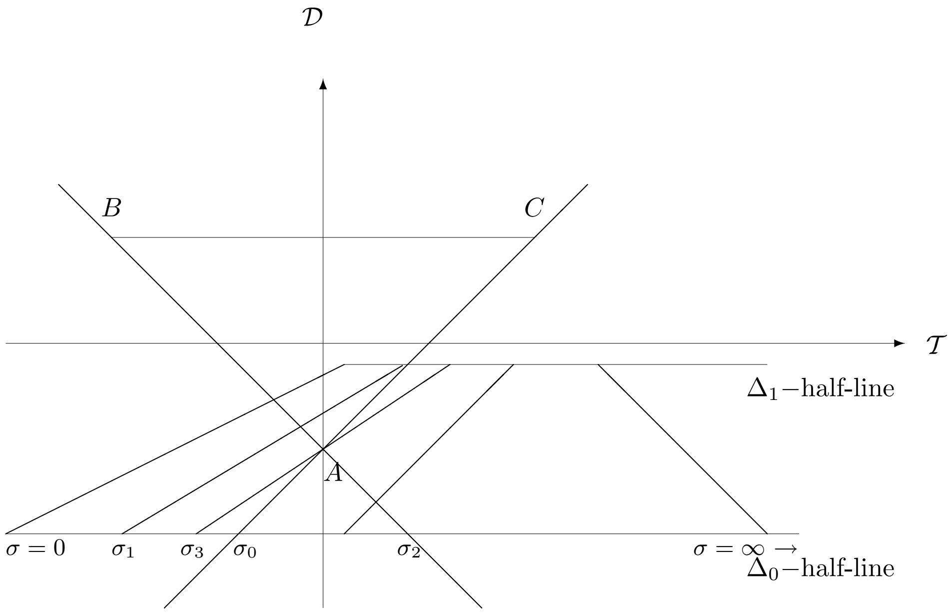

Figure 7: Δ-lines for $\nu_3 < \nu < \nu_4$.

Proposition 19 *Let $\nu_3 < \nu < \nu_4$. Under Assumptions 1-5, there exists a value of the elasticity of intertemporal substitution $\sigma_3 \in (\sigma_1, \sigma_0)$, such that the following generically holds.*

(i) If $\sigma < \sigma_1$, then the SSS is saddle-point stable for $\Delta_2 \in \left(0, \Delta_2^F\right)$, undergoes a flip bifurcation at $\Delta_2 = \Delta_2^F$, becomes locally indeterminate for $\Delta_2 \in \left(\Delta_2^F, \Delta_1\right)$.

(ii) If $\sigma_1 < \sigma < \sigma_3$, then the SSS is saddle-point stable for $\Delta_2 \in \left(0, \Delta_2^F\right)$, undergoes a flip bifurcation at $\Delta_2 = \Delta_2^F$, becomes locally indeterminate for $\Delta_2 \in \left(\Delta_2^F, \Delta_2^T\right)$, undergoes a transcritical bifurcation at $\Delta_2 = \Delta_2^T$, and becomes saddle-point stable for $\Delta_2 \in \left(\Delta_2^T, \Delta_1\right)$.

(iii) If $\sigma_3 < \sigma < \sigma_0$, then the SSS is saddle-point stable for $\Delta_2 \in \left(0, \Delta_2^F\right)$, undergoes a flip bifurcation at $\Delta_2 = \Delta_2^F$, becomes unstable for $\Delta_2 \in \left(\Delta_2^F, \Delta_2^T\right)$, undergoes a transcritical bifurcation at $\Delta_2 = \Delta_2^T$, and becomes saddle-point stable for $\Delta_2 \in \left(\Delta_2^T, \Delta_1\right)$.

(iv) If $\sigma_0 < \sigma < \sigma_2$, then the SSS is unstable for $\Delta_2 \in \left(0, \Delta_2^T\right)$, undergoes a transcritical bifurcation at $\Delta_2 = \Delta_2^T$, and becomes saddle-point stable for $\Delta_2 \in \left(\Delta_2^T, \Delta_1\right)$.

(v) If $\sigma > \sigma_1$, then the SSS is saddle-point stable for every $\Delta_2 \in (0, \Delta_1)$.

In this case, indeterminacy can still occur only for a large spending in public infrastructure and for low values of the intertemporal substitution, that is large income effects.

7.3.5 Case $\nu_4 < \nu < \nu_5$

Now, for $\sigma = 0$, the Δ_1-half-line begins outside the triangle. As a consequence, the conditions are the same as in the previous case, except for σ close to zero, where indeterminacy is no more possible ($\sigma < \sigma_3 < \sigma_0$).

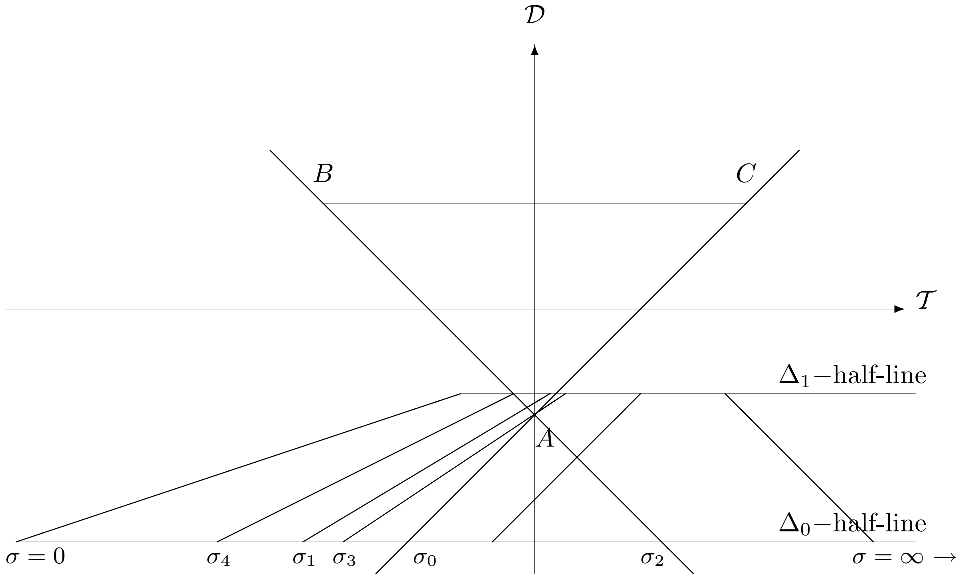

Figure 8: Δ-lines for $\nu_4 < \nu < \nu_5$.

Proposition 20 *Let $\nu > \nu_4$. Under Assumptions 1-5, there exist values σ_3, σ_4 of the intertemporal elasticity of substitution, with $\sigma_4 < \sigma_1 < \sigma_3 < \sigma_0$, such that the following generically holds.*

(i) If $\sigma < \sigma_4$, then the SSS is saddle-point stable for every $\Delta_2 \in (0, \Delta_1)$.

(ii) If $\sigma_4 < \sigma < \sigma_1$, then the SSS is saddle-point stable for $\Delta_2 \in \left(0, \Delta_2^F\right)$, undergoes a flip bifurcation at $\Delta_2 = \Delta_2^F$, becomes locally indeterminate for $\Delta_2 \in \left(\Delta_2^F, \Delta_1\right)$.

(iii) If $\sigma_1 < \sigma < \sigma_3$, then the SSS is saddle-point stable for $\Delta_2 \in \left(0, \Delta_2^F\right)$, undergoes a flip bifurcation at $\Delta_2 = \Delta_2^F$, becomes locally indeterminate for $\Delta_2 \in \left(\Delta_2^F, \Delta_2^T\right)$, undergoes a transcritical bifurcation at $\Delta_2 = \Delta_2^T$, and becomes saddle-point stable for $\Delta_2 \in \left(\Delta_2^T, \Delta_1\right)$.

(iv) If $\sigma_3 < \sigma < \sigma_0$, then the SSS is saddle-point stable for $\Delta_2 \in \left(0, \Delta_2^F\right)$, undergoes a transcritical bifurcation at $\Delta_2 = \Delta_2^T$, becomes unstable for $\Delta_2 \in \left(\Delta_2^T, \Delta_2^F\right)$, undergoes a flip bifurcation at $\Delta_2 = \Delta_2^F$, and becomes saddle-point stable for $\Delta_2 \in \left(\Delta_2^F, \Delta_1\right)$.

(v) If $\sigma_0 < \sigma < \sigma_2$, then the SSS is unstable for $\Delta_2 \in \left(0, \Delta_2^F\right)$, undergoes either a flip bifurcation at $\Delta_2 = \Delta_2^F$, and becomes saddle-point stable for $\Delta_2 \in \left(\Delta_2^T, \Delta_1\right)$.

(vi) If $\sigma > \sigma_1$, then the SSS is saddle-point stable for $\Delta_2 \in (0, \Delta_1)$.

In this case, the stationary equilibrium is always a saddle point, when $\sigma < \sigma_4$ or $\sigma > \sigma_2$. In the interval (σ_4, σ_3) expectations-driven fluctuations arise, when the state mainly provides long-run infrastructures.

8 A calibrated example

Let $y(k, g) \equiv g^{1-s} A k^s$ be the explicit production function with, now, a constant elasticity s. The implicit equation of the steady state becomes $R = \Delta_1 +$

$[\nu (R-\gamma) / (\gamma - \Delta_2)]^{1-s} As$.

The majority of calibrated simulations based on the U.S. post-war yearly data, agree on assigning the discount factor value β close to 0.98 and the depreciation value Δ_1 close to 0.9 (see among the others: Hansen, 1985; Maddison, 1987; and Summers and Heston, 1988). The capital share in total income s, which also determines the magnitude of the public externality, is set equal to 0.7, to be consistent with the calibrations of Baxter and King (1991) and Mankiw, Romer and Weil (1992).

The literature does not provide a clear picture concerning the admissible values for the elasticity of intertemporal substitution in consumption. While many standard RBC models such that Hansen (1985), or King, Plosser and Rebelo (1988) have assumed a relatively high value (i.e. around unity), recent empirical estimates taken from Campbell (1999), and Kocherlakota (1996) suggest a plausible elasticity of intertemporal substitution close to 1/3.

8.1 Public services

Assume no public spending accumulation: $\Delta_2 = 0$. As above, first we consider a zero growth economy ($\gamma = 1$). The transcritical value for σ is $\sigma_T = 0.4350$. Let for instance $\sigma = 1/3$. According to formula (24), we compute the flip bifurcation: $\nu_F = 0.3114$. Set now $\nu < \nu_F$. For instance $\nu = 0.3$. We get $\lambda_1 = -0.9837$ and $\lambda_2 = 0.9915$. In other words, we observe a slow, but oscillating convergence.

Eventually, we consider a small growth economy. We compute the value A_0 ensuring a zero growth. Formula (28) gives $A_0 = 0.7934$. Then we adjust A in a small neighborhood of A_0 to obtain a slightly positive growth. We set, for instance, $A = 0.79$ and we obtain $\gamma = 1.0014 < R = 1.0246$. The transversality condition is respected, dividends are positive as well as the growth rate. However, the equilibrium is indeterminate and close to a flip bifurcation: $\lambda_1 = -0.8883$, $\lambda_2 = 0.9962$.

8.2 Public durable goods

Set now $\Delta_2 = 0.9$. First we consider a zero growth economy ($\gamma = 1$). The transcritical value for σ is $\sigma_T = 0.3687$. Let, for instance, $\sigma = 1/3$. We compute the (flip) bifurcation value: $\nu_F = 0.9809$. Let now $\nu < \nu_F$. For instance: $\nu = 1/2$. We get $\lambda_1 = 0.8184$ and $\lambda_2 = 0.9950$. In other words, we observe a slow monotonic convergence.

Finally, we consider now a low growth economy. We compute the value A_0 ensuring a zero growth. Formula (28) gives $A_0 = 0.3411$. Then we adjust A in a small neighborhood of A_0 to obtain a slightly positive growth. Setting, for instance, $A = 0.34087$, we obtain $R = 1.0226 > \gamma = (\beta R)^{\sigma} = 1.0007$. The transversality condition is respected, dividends are positive as well as the growth rate. However, the equilibrium is indeterminate: $\lambda_1 = 0.8225$, $\lambda_2 = 0.9991$.

9 Conclusion

Our model has aimed at underlining how the level and the nature of public production can affect the dynamic properties of the economy. In the paper we are concerned by the way a public good, viewed as an externality that enhances productivity and plays the role of growth engine, is financed. On the one side tax collection is costly and applies to imperfectly observable revenues, the inflation tax is accompanied by monetary disorder, bonds issues call for future taxes or inflation. On the other side state production is an additional source of public revenue, but a widespread commonplace stresses the efficiency gains of privatization. If we set aside the inefficiency of the public sector and we assume that the public dividends are channelled to finance a productive good, the economy experiences higher and endogenous growth rates. However, the quantity and the quality of the public spending has an impact on the shape of the intertemporal general equilibrium. More precisely, we are naturally leaded to evaluate the optimal involvement of the state in the economy as well as the stabilizing power of a mix of long-run infrastructure or short-run services. We apply a geometrical method in order to study the occurrence of endogenous cycles, either deterministic or stochastic, and we provide policy suggestions to eliminate such inefficient fluctuations.

More explicitly, we find that a state participation to productive activities is always desirable, enhancing growth through the positive externalities, and more desirable for higher intertemporal substitution effects.

Moreover, under dominant income effects, the government is required mainly to provide infrastructures, in order to rule out the fluctuations, while the provision of short-run services is recommended in presence of stronger intertemporal substitution effects.

10 Appendix

Proof of Proposition 3. First, we notice that $\varphi(\Delta_1) < 0$. Second, we observe that $\varphi'(R) = 1 - \nu\xi'(R)\, e'(\nu\xi)\, Af'(1)$ with

$$\xi'(R) = \frac{1}{\gamma - \Delta_2}\left(1 - \sigma\frac{\gamma}{R}\frac{R - \Delta_2}{\gamma - \Delta_2}\right)$$

Moreover, $\xi'(R) \leq 0$, if and only if $\sigma > \sigma_1(R)$, where

$$\sigma_1(R) \equiv (R/\gamma) / [(\gamma - \Delta_2) / (R - \Delta_2)]$$

Eventually, we notice that $f'(1) > 0$ implies that R is bounded from below by $\Delta_1 \geq \Delta_2$. Consider the case $\sigma \geq 1$. The transversality condition (16) entails $\sigma_1(R) < 1$ and, therefore, $\xi'(R) \leq 0$: under Assumption 2, φ is strictly increasing. Then, if the steady state exists, it is unique. To provide a condition for the existence, we observe that the transversality condition sets an upper bound $\beta^{\sigma/(1-\sigma)}$ for R. Then a necessary and sufficient condition is (17). When $\sigma < 1$, Assumption 2 ensures that $\lim_{R\to+\infty}\varphi(R)=+\infty$. By continuity, φ crosses the axis of abscissas an odd number of times. ■

Proof of Proposition 4. Since we deal with a representative agent, the welfare function to maximize with respect to ν, is simply given by his utility functional:

$$W=\sum_{t=0}^{\infty}\beta^t\frac{c_t^{1-1/\sigma}}{1-1/\sigma}=\frac{\sigma}{\sigma-1}\frac{1}{1-\gamma/R}c_0^{1-1/\sigma} \tag{33}$$

where the second equality comes from (15) and the assumption of regular growth ($c_t=c_0\gamma^t$). Using equations (4) and (6) in (7) and definition $\gamma_{t+1}\equiv k_{t+1}/k_t$, we obtain $e(g_t)Af(k_t)=[\gamma_{t+1}-\Delta_1+\nu(R_t-\gamma_{t+1})]k_t+c_t$ and in particular at time 0:

$$c_0=e(g_0)Af(k_0)+[\Delta_1-\nu R_0-(1-\nu)\gamma_1]k_0 \tag{34}$$

We observe that the initial condition (k_0,g_0) determines the interest factor R_0 according to equation (4), evaluated at time 0. If the initial condition $R_0\neq R$, then the initial demand c_0 will adjust to set the economy on the stable manifold in the (R_t,x_t)-plane. If the sable manifold is a saddle, there is a unique c_0 entailing the convergence to the steady state. Il the stable manifold is two-dimensional, then a continuum of initial demands c_0's will be compatible with the equilibrium. Since, for simplicity, we refer to a balanced growth, we consider, at first, a pair (k_0,g_0) implementing the steady state interest factor

$$R=\Delta_1+e(g_0)Af'(k_0) \tag{35}$$

(we observe that there are infinitely many pairs $(k_0,g_0(k_0))$, where now the function g_0 is implicitly defined by (35)) and, at second, the initial consumption c_0 setting x_t equal to x. More explicitly, since, by assumption of balanced growth, $R_0=R$ and $\gamma_1=\gamma$, we have from (34): $c_0=e(g_0(k_0))Af(k_0)+[\Delta_1-\nu R-(1-\nu)\gamma]k_0$. In other words, we are computing the policy rule (optimal nationalization degree) for a long-run equilibrium: approximately the initial conditions can be supposed to be sufficiently close to those of steady state. Fortunately, we notice that the steady state (13) does not depend on (k_0,g_0). Therefore, given k_0, we have to maximize (33):

$$W=\frac{\sigma}{\sigma-1}\frac{1}{1-\gamma/R}[e(g_0(k_0))Af(k_0)+[\Delta_1-\nu R-(1-\nu)\gamma]k_0]^{1-1/\sigma} \tag{36}$$

with respect to ν, where R, γ and g_0 are respectively given by (13), (15) and (35). We observe that (13), (15) locally define a function $R=R(\nu)$ and that, from (35), we obtain

$$g_0=e^{-1}\left(\frac{R-\Delta_1}{Af'(k_0)}\right) \tag{37}$$

Therefore, substituting (37) in (36), we have to solve the following program:

$$\max_{\nu}\frac{\sigma}{\sigma-1}\left[\left((R-\Delta_1)\frac{1-s(k_0)}{s(k_0)}+(1-\nu)[R-(\beta R)^{\sigma}]\right)k_0\right]^{1-1/\sigma}\frac{R}{R-(\beta R)^{\sigma}}$$

where now $R = R(\nu)$ and $s(k_0)$ is given by (12). Setting $\varsigma \equiv s(k_0)^{-1} - 1$ and $S(\nu) \equiv R(\nu) - [\beta R(\nu)]^\sigma$, we get

$$W(\nu) = \frac{\sigma}{\sigma - 1} k_0^{1-1/\sigma} \left([R(\nu) - \Delta_1] \varsigma + (1-\nu) S(\nu) \right)^{1-1/\sigma} \frac{R(\nu)}{S(\nu)}$$

Noticing that $S = R - \gamma$ and $S' = (1 - \sigma\gamma/R) R'$, we compute the impact of ν on W:

$$\begin{aligned} W'(\nu) &= \frac{(R-\gamma) R (\varsigma + 1 - \nu) + \sigma\gamma\varsigma (R - \Delta_1)}{[\varsigma (R - \Delta_1) + (1-\nu)(R-\gamma)](R-\gamma)} \\ &\quad (k_0 [(R - \Delta_1)\varsigma + (1-\nu) S])^{1-1/\sigma} R'/S \end{aligned}$$

Since $R > \Delta_1$ (positivity of h), $R > \gamma$ (transversality condition) and $S > 0$, we have that $W'(\nu) > 0$, iff $R'(\nu) > 0$. Moreover, applying the implicit function theorem to

$$R = \Delta_1 + e\left(\nu \frac{R - (\beta R)^\sigma}{(\beta R)^\sigma - \Delta_2}\right) A f'(1) \tag{38}$$

using (9), evaluated at the steady state, and observing that $e'(h) A f'(1) h = (1-s)(R - \Delta_1)$, we obtain

$$R'(\nu) = \frac{1}{\nu} \frac{(1-s)(R-\Delta_1)}{1 - (1-s)\frac{R-\Delta_1}{R-\gamma}\left(1 - \sigma\frac{\gamma}{R}\frac{R-\Delta_2}{\gamma-\Delta_2}\right)} \tag{39}$$

Hence $W'(\nu) > 0$, iff the denominator of (39) is strictly positive, that is, iff

$$\frac{R - \Delta_2}{\gamma - \Delta_2} > \frac{1}{\sigma}\frac{R}{\gamma}\left(1 - \frac{1}{1-s}\frac{R-\gamma}{R-\Delta_1}\right) \tag{40}$$

Let now $\nu^* \equiv \arg\max W(\nu)$. First case: $\sigma > 1$. Since $(R - \Delta_2)/(\gamma - \Delta_2) > R/\gamma$ (provided that $\gamma > \Delta_2$), then the inequality (40) is always satisfied and $W'(\nu) > 0$. In this case the optimal rule becomes $\nu^* = 1$ (full nationalization). Second case: $\sigma < 1$. By continuity, there exists a critical $\sigma^* \leq 1$, such that $\sigma > \sigma^*$, implies that $W'(\nu) > 0$ for every $\nu \in (0,1]$ and then $\nu^* = 1$ (full nationalization as optimal policy). If $\sigma < \sigma^*$, then there exists an interior optimal $\nu \in (0,1)$. To compute ν^*, we solve (19), obtained from (40), to find at first the optimal interest factor R^* and then we substitute it in (18), obtained from (38). ■

Proof of Lemma 5. Under Assumption 2 (productive homogeneity in g and k) we have the elasticity of the externality of public spending $e'(h) h/e(h) = 1 - s(h)$, and we get the following linearized system.

$$\begin{aligned} (\sigma + z_0)\frac{dR_{t+1}}{R} - \frac{dx_{t+1}}{x} &= z_1 \frac{dR_t}{R} - \frac{dx_t}{x} \\ \sigma\frac{dR_{t+1}}{R} - \frac{dx_{t+1}}{x} &= \frac{R}{\gamma}\frac{1-\nu s}{s-\nu s}\frac{dR_t}{R} - \frac{R}{\gamma}\left(1 + \frac{1-s}{s-\nu s}\frac{R-\Delta_1}{R}\right)\frac{dx_t}{x} \end{aligned}$$

T and D are, respectively, the trace and the determinant of the Jacobian matrix. ■

Proof of Proposition 6. Let λ_1, λ_2 be the eigenvalues, with $\lambda_1 \leq \lambda_2$, if real. The characteristic polynomial is a convex parabola. If $P(-1) < 0$, then $\lambda_1 < -1$. If $P(1) < 0$ (that is $\sigma > z_1 - z_0$), then $\lambda_2 > 1$. If $D > 1$, then $\max\{|\lambda_1|, |\lambda_2|\} > 1$. Let now $P(-1), P(1) > 0$ and $D < 1$. If $D < 0$, then $-1 < \lambda_1 < 0 < \lambda_2 < 1$. If $D \geq 0$ and the eigenvalues are real, then $\lambda_1, \lambda_2 \in (-1, 0]$ or $\lambda_1, \lambda_2 \in [0, 1)$; if they are complex, $|\lambda_1| = |\lambda_2| < 1$. We observe that

$$\begin{aligned} P(1) &\geq 0, \text{ iff } \sigma + z_0 \leq z_1 \\ P(-1) &\geq 0, \text{ iff } z_1 \geq 2\frac{(1-\nu s) R + s(1-\nu)\gamma\sigma}{(1-\nu s) R - (1-s)\Delta_1 + s(1-\nu)\gamma} - (\sigma + z_0) \\ D &\leq 1, \text{ iff } z_1 \leq \frac{(1-\nu s) R + s(1-\nu)\gamma z_0}{(1-\nu s) R - (1-s)\Delta_1} \end{aligned}$$

The necessary and sufficient for indeterminacy (sink) (22) follows. The system undergoes, generically, a transcritical bifurcation at $P(1) = 0$, a flip bifurcation at $P(-1) = 0$, and, if $P(1), P(-1) > 0$, a Hopf at $D = 1$. ■

Proof of Corollary 8. We notice that

$$D = \frac{\Delta_2}{\gamma}\left[\frac{\Delta_1}{\gamma} + \frac{1-s\nu}{1-\nu}\frac{R-\Delta_1}{\gamma}\left(1 - \frac{\Delta_1}{\Delta_2}\frac{1-s}{s}\frac{1-s}{1-s\nu}\frac{\gamma-\Delta_2}{R-\gamma}\right)\right] < 1$$

iff

$$\frac{1-s}{s}\frac{1-s}{1-s\nu}\left(\frac{\Delta_1}{\Delta_2}\frac{\gamma-\Delta_2}{R-\gamma}\right) + \frac{1-\nu}{1-s\nu}\left(\frac{\gamma}{\Delta_2}\frac{\gamma-\Delta_2}{R-\Delta_1} + \frac{\gamma-\Delta_1}{R-\Delta_1}\right) > 1$$

Since inequalities (26) imply the terms into brackets to be greater than one, a mild sufficient condition to obtain $D < 1$ becomes

$$\frac{1-s}{s}\frac{1-s}{1-s\nu} + \frac{1-\nu}{1-s\nu} > 1$$

or, equivalently, (27). ■

Proof of Proposition 9. From (13), (R^*, x^*) is a steady state, iff there exists a value A^* for the parameter A, such that $e(\nu(1/\beta - 1)/(1-\Delta_2)) A^* f'(1) = 1/\beta - \Delta_1$. ■

Proof of Proposition 10. The finding is obtained from Lemma 5, by substituting $\gamma = 1$ and, then, $R = 1/\beta$. ■

Proof of Lemma 11. We note that: $\partial D/\partial\Delta_2 = \omega + (1-\omega)/\beta + \varkappa[\omega + (1-\omega)(1/\beta - \Delta_1)]$. Two alternative cases matter. First, if $\omega < 1$, then $\partial D/\partial\Delta_2 > 0$. Second, assume $\omega > 1$. We observe that, according to definition of ω, we have $\omega + (1-\omega)/\beta = s/\beta + (1-s)\Delta_1 > 0$ and, eventually,

$\partial D/\partial\Delta_2 > 0$. ■

Proof of Lemma 12. (i) $\mathcal{D}_0 = -\omega\Delta_1 (1-s) / [s(1-\nu)]$ is the determinant for $\Delta_2 = 0$. We observe that $\mathcal{D}_0 < -1$, iff $\nu > \nu_3 \equiv 1 - \omega\Delta_1 (1-s)/s$. (ii) The Δ_0-half-line will cross the AC line, iff its origin, that is $(\mathcal{T}_0(\sigma), \mathcal{D}_0)$ with $\sigma = 0$, lies above the line AC. But, $\mathcal{D}_0 > \mathcal{T}_0(0) - 1$, iff $(\omega - 1)[1/\beta - 1 + (1/\beta - \Delta_1)\varkappa] > 0$. This inequality always holds, since, according to Assumption 4, $s < 1 - (1-\beta)/(1-\beta\Delta_1)$ and, therefore, $\omega - 1 > 0$. (iii) The Δ_0-half-line will cross the AB line, iff its origin $(\mathcal{T}_0(\sigma), \mathcal{D}_0)$ stands below the line AB. But $\mathcal{D}_0 < -\mathcal{T}_0(0) - 1$, iff $\varkappa > [1 + \omega + (1-\omega)/\beta] / [(1+\omega)\Delta_1 - (1-\omega)/\beta]$ or, equivalently, $\nu > \nu_1$, where $\varkappa \equiv (1-s)/[s(1-\nu)]$ and υ_2 is given by (29). (iv) We notice that $\nu_3 > \nu_1$, iff $\Delta_1\omega < [(1+\omega)\Delta_1 - (1-\omega)/\beta] / [1 + \omega + (1-\omega)/\beta]$. This inequality is equivalent to $1 - \beta\Delta_1 + \omega(1-\beta)\Delta_1 > 0$, since, from Assumption 4, $\beta > (\omega - 1)/(\omega + 1)$ and, then, $1 + \omega + (1-\omega)/\beta > 0$. But, still from Assumption 4, $\omega - 1 > 0$ and, therefore, the last inequality is satisfied. ■

Proof of Proposition 13. The determinant attains the highest value with the Δ_1-half-line, since, given ν, it is an increasing function of Δ_2. If this value is lower than -1, the eigenvalues can never be both in the unit circle, and indeterminacy is no longer possible. For $\Delta_2 = \Delta_1$, we have $\mathcal{D}_1 = (\omega + (1-\omega)/\beta + (1-\omega)(1/\beta - \Delta_1)\varkappa)\Delta_1$. Then $\mathcal{D}_1 < -1$, iff

$$\varkappa > (1 + \Delta_1[\omega + (1-\omega)/\beta]) / [\Delta_1(\omega - 1)(1/\beta - \Delta_1)]$$

or, equivalently, $\nu > \nu_5$, where υ_5 is given by (32). ■

Proof of Lemma 14. (i) The half-line Δ_1 lies below the line BC, iff $\mathcal{D}_1 < 1$, which is entailed by $1 - \omega < 0$ and $\Delta_1 < \beta$ (Assumption 4). In order to understand whether the Δ_1-half-line crosses the lines AB and AC, following the same line of the Lemma 12, we just look for the starting point of the horizontal half-line. We compute the trace for $\Delta_2 = \Delta_1$ and $\sigma = 0$: $\mathcal{T}_1(0) = \Delta_1 + \omega + (1-\omega)/\beta + (1-\omega)(1/\beta - \Delta_1)\varkappa$. (ii) The Δ_1-half-line crosses the line AC, iff the origin of the half-line, corresponding to $\sigma = 0$ is above the line AC, that is iff $\mathcal{D}_1 > \mathcal{T}_1(0) - 1$. More explicitly: $(\omega - 1)(1/\beta - 1) + \varkappa(\omega - 1)(1/\beta - \Delta_1) > 0$. Assumption 4 ensures that this inequality always holds. (iii) The Δ_1-half-line crosses the line AB, iff the origin of the half-line $(\sigma = 0)$ is below the line AB, that is iff $\mathcal{D}_1 < -\mathcal{T}_1(0) - 1$. This inequality rewrites: $\varkappa > [1 + \omega + (1-\omega)/\beta] / [(\omega - 1)(1/\beta - \Delta_1)]$ or, equivalently, $\nu > \nu_4$, where ν_4 is given by (31). ■

Proof of Lemma 15. (i) We compute the values σ_0 and σ_1 at which, respectively, the Δ_0 and the Δ_1-half-line cross the line AC. When $\Delta_2 = 0$, $\sigma_0 = (\omega - 1)/\omega$ solves the equation $\mathcal{T}_0(\sigma_0) = 1 + \mathcal{D}_0$. When $\Delta_2 = \Delta_1$, $\sigma_1 = (1 - 1/\omega)(1 - \Delta_1)/(1 - \beta\Delta_1)$ solves the equation $\mathcal{T}(\sigma_1) = 1 + \mathcal{D}$. Moreover, since Assumption 4 ensures $\omega - 1$ to be positive, the inequality $\sigma_1 < \sigma_0$ becomes equivalent to $(1/\beta - 1)\Delta_1 > 0$, which is always satisfied. (ii) Since the determinant of the characteristic polynomial is a decreasing function of ν, when ν increases, the Δ_0-half-line shifts down. Thus, $\sigma_2 < \sigma_1$, when the nationalization degree is low; $\sigma_2 = \sigma_1$, when ν is such that the Δ_0-half-line goes through A; and, eventually, $\sigma_2 > \sigma_1$, when the nationalization degree is high. Since we know that $\sigma_2 < \sigma_1$ if $\nu < \nu_1$ and that $\sigma_2 > \sigma_1$ if $\nu > \nu_3$, we conclude that there exists $\nu_2 \in (\nu_1, \nu_3)$, such that $\sigma_2 > \sigma_1$ iff $\nu > \nu_2$. (iii) We notice that $\nu_4 > \nu_3$ iff $\omega\Delta_1 > [(\omega - 1)(1/\beta - \Delta_1)] / [1 + \omega + (1 - \omega)/\beta]$. Since the denominator of the LHS is positive from Assumption 4, after substituting ω by the expression provided in Proposition 10, we obtain an equivalent inequality:

$$1 + \Delta_1 [s + (1 - s)\beta\Delta_1 + 2\beta - 1/\Delta_1](1 - s)/(1 - \beta) > 0 \qquad (41)$$

From Assumption 4, $\beta < \Delta_1$ and $\Delta_1 > 2/3$ imply $2\Delta_1 - 1/\beta > 0$ and $s + (1 - s)\beta\Delta_1 + 2\beta - 1/\Delta_1 > 0$. The latter inequality entails (41). ■

References

[1] Barro R. J. 1990. Government Spending in a Simple Model of Endogenous Growth. *Journal of Political Economy* **98**, 103-125.

[2] Baxter M. and R. King. 1991. Productive Externalities and Business Cycles. Discussion Paper 53, Institute for Empirical Macroeconomics, Federal Reserve Bank of Minneapolis.

[3] Campbell J. 1999. Asset Prices, Consumption and the Business Cycle. In Taylor J.B. and M. Woodford eds., *Handbook of Macroeconomics*, North-Holland, Amsterdam.

[4] Cazzavillan G. 1996. Public Spending, Endogenous Growth, and Endogenous Fluctuations. *Journal of Economic Theory* **71**, 394-415.

[5] Cazzavillan G., T. Lloyd-Braga and P. Pintus. 1998. Multiple Steady States and Competitive Endogenous Fluctuations with Productive Externalities, *Journal of Economic Theory* **80**, 60-107.

[6] Gibson B. and A. K. Dutt. 1993. Privatization and Accumulation in Mixed Economies. *Journal of Comparative Economics* **17**, 1-22.

[7] Grandmont, J.-M., P. Pintus and R. de Vilder. 1998. Capital-Labor Substitution and Competitive Nonlinear Endogenous Business Cycles. *Journal of Economic Theory* **80**, 14-59.

[8] Hansen G. 1985. Indivisible Labor and the Business Cycle. *Journal of Monetary Economics* **16**, 309-327.

[9] Jones C. I. 1998. *Introduction to Economic Growth.* Norton, New York.

[10] King R., C. Plosser and S. Rebelo. 1988. Production, Growth and Business Cycles: II. New Directions. *Journal of Monetary Economics* **21**, 309-341.

[11] Kocherlakota N. 1996. The Equity Premium: It's Still a Puzzle. *Journal of Economic Literature* **36**, 42-71.

[12] Maddison A. 1987. Growth and Slowdown in Advanced Capitalist Economies: Techniques of Quantitative Assessment. *American Economic Review* **25**, 649-698.

[13] Mankiw N. G., D. Romer and D. N. Weil. 1992. A Contribution to the Empirics of Economic Growth. *Quarterly Journal of Economics* **107**, 407-437.

[14] Obstfeld M. and K. S. Rogoff. 1999. *Elements of Differentiable Dynamics and Bifurcation Theory.* MIT Press, Cambridge.

[15] Ruelle D. 1989. *Foundations of International Macroeconomics.* Academic Press, San Diego.

[16] Summers R. and A. Heston. 1988. A New Set of International Comparisons of Real Product and Price Levels: Estimates of 130 Countries. *Review of Income and Wealth* **34**, 1-25.

[17] Zou H.-F. 1994. On the Dynamics of Privatization. *China Economic Review* **5**, 221-233.

2005

0501 Animal Spirits in Woodford and Reichlin Economies: The Representative Agent Does Matter
Stefano BOSI & Thomas SEEGMULLER

0502 Fiscal Policy and Fluctuations in a Monetary Model of Growth
Stefano BOSI & Francesco MAGRIS

0503 Is Training More Frequent When the Wage Premium Is Smaller? Evidence from the European Community Household Panel
Andrea BASSANINI & Giorgio BRUNELLO

0504 Training, Wages and Employment Security: An Empirical Analysis on European Data
Andrea BASSANINI

0505 Financial Development, Labor and Market Regulations and Growth
Raquel FONSECA & Natalia UTRERO

0506 Testing Heterogeneity within the Euro Area Using a Structural Multi-Country Model
Eric JONDEAU & Jean-Guillaume SAHUC

0507 On Outward-Looking Comparison Utility, Heterogeneous Preferences & the Third Dimension: A Geometric Perspective
Jean-Paul BARINCI & Jean-Pierre DRUGEON

0508 Welfare Effects of Social Security Reforms across Europe: the Case of France and Italy
Raquel FONSECA & Theptida SOPRASEUTH

0509 Can Heterogeneous Preferences Stabilize Endogenous Fluctuations?
Stefano BOSI & Thomas SEEGMULLER

0510 Default Recovery Rates and Implied Default Probability Estimations: Evidence from the Argentinean Crisis
Ramiro SOSA NAVARRO

0511 Selective Immigration Policies, Human Capital Accumulation and Migration Duration in Infinite Horizon
Francesco MAGRIS & Giuseppe RUSSO

0512 A Necessary and Sufficient Condition for Weak Exogeneity in Vector Error Correction Models
Christophe RAULT

0513 La PPA est-elle vérifiée pour les pays développés et en développement ? Un ré-examen par l'économétrie des panels non-stationnaires
Imed DRINE & Christophe RAULT

0514 The Influences Affecting French Assets Abroad Prior 1914
Antoine PARENT & Christophe RAULT

0515 The Balassa-Samuelson Effect in Central and Eastern Europe: Myth or Reality?
Balázs EGERT, Imed DRINE, Kirsten LOMMATZSCH & Christophe RAULT

0516	**Animal Spirits and Public Production in Slow Growth Economies**
	Stefano BOSI & Carine NOURRY

2004

0401	**Instabilité de l'emploi : quelles ruptures de tendance?**
	Yannick L'HORTY
0402	**Vingt ans d'évolution de l'emploi peu qualifié et du coût du travail : des ruptures qui coïncident?**
	Islem GAFSI, Yannick L'HORTY & Ferhat MIHOUBI
0403	**Allègement du coût du travail et emploi peu qualifié : une réévaluation**
	Islem GAFSI, Yannick L'HORTY & Ferhat MIHOUBI
0404	**Revenu minimum et retour à l'emploi : une perspective européenne**
	Yannick L'HORTY
0405	**Partial Indexation, Trend Inflation, and the Hybrid Phillips Curve**
	Jean-Guillaume SAHUC
0406	**Partial Indexation and Inflation Dynamics: What Do the Data Say?**
	Jean-Guillaume SAHUC
0407	**Why Do Firms Evaluate Individually Their Employees: The Team Work Case**
	Patricia CRIFO, Marc-Arthur DIAYE & Nathalie GREENAN
0408	**La politique environnementale française : une analyse économique de la répartition de ses instruments du niveau global au niveau local**
	Jean DE BEIR, Elisabeth DESCHANET & Mouez FODHA
0409	**Incentives in Agency Relationships: To Be Monetary or Non-Monetary?**
	Patricia CRIFO & Marc-Arthur DIAYE
0410	**Mathematics for Economics**
	Stefano BOSI
0411	**Statistics for Economics**
	Stefano BOSI
0412	**Does Patenting Increase the Private Incentives to Innovate? A Microeconometric Analysis**
	Emmanuel DUGUET & Claire LELARGE
0413	**Should the ECB Be Concerned about Heterogeneity? An Estimated Multi-Country Model Analysis**
	Eric JONDEAU & Jean-Guillaume SAHUC
0414	**Does Training Increase Outflows from Unemployment? Evidence from Latvian Regions**
	Jekaterina DMITRIJEVA & Michails HAZANS
0415	**A Quantitative Investigation of the Laffer Curve on the Continued Work Tax: The French Case**
	Jean-Olivier HAIRAULT, François LANGOT & Thepthida SOPRASEUTH

0416	**Intergenerational Conflicts and the Resource Policy Formation of a Short-Lived Government**
	Uk HWANG & Francesco MAGRIS
0417	**Voting on Mass Immigration Restriction**
	Francesco MAGRIS & Giuseppe RUSSO
0418	**Capital Taxation and Electoral Accountability**
	Toke AIDT & Francesco MAGRIS
0419	**An Attempt to Evaluate the Impact of Reorganization on the Way Working Time Reduction Has Been Implemented by French Firms since 1996**
	Fabrice GILLES
0420	**Dette souveraine: crise et restructuration**
	Facundo ALVAREDO & Carlos WINOGRAD
0421	**Renouvellement des générations, asymétrie de position et dynamique technologique des entreprises**
	Marc-Arthur DIAYE, Nathalie GREENAN, Claude MINNI & Sonia ROSA MARQUES