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Black Market, Labor Demand and Tax Evasion

Marc-Arthur DIAYE[*], Gleb KOSHEVOY[†]

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Abstract

According to many empirical studies, the size of the black market is growing in the OECD countries. Among the reasons for such a phenomena, is the labor market structure (mainly high total labor costs and the reduction in working hours in the official economy). The supply side of the labor market has been widely studied in the literature (see for instance Lemieux et al. 1994, Frederiksen et al. 2005). However there exist few analyses of the demand side (see for instance Fugazza and Jacques 2003) and in general they consider that the firms operate either on the official market or on the underground one. To the best of our knowledge there exists no *formal* analysis of the demand side of the labor market in which the firms can operate both in the official and (directly or indirectly) in the underground markets and this is very surprising given the (estimated) high number of illegal workers in most OECD countries. On the contrary, this paper focuses on the demand side and analyses the main driving forces behind the demand by (legal) firms for labor force in the black market. We show that the firms' technological characteristics matter. Moreover we construct a Principal-Multiagents model with an endogeneous probability for the Agents (the firms) to be detected by the Principal (the government) when using a black labor force. We assume the agents to compete in a Cournot Oligopoly structure and we show that at the Cournot equilibrium, the more the Principal controls the production reported by the agents, the more the probability of detection decreases.

[**RESUME**]

Selon plusieurs études, la taille de l'économie parallèle (non criminel) est non négligeable dans les pays de l'OCDE. Par exemple, cette taille est estimée pour le Royaume-Unie sur données microéconomiques (Family Expenditure Survey 1993) à au moins 9.4% du PIB par Lyssiotou, Pashardes et Stengos (1999). Par ailleurs selon Schneider et Enste (2000), il y a eu un accroissement continue de la taille du marché noir

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dans la plupart des pays de l'OCDE (y compris les Etats-Unis) de 1960 à 1999. Il semble donc que la structure du marché du travail ait une grande influence sur la taille du marché noir. Si le côté offre de travail a été abondamment analysé dans la literature (Frederiksen et al. 2005, Lemieux et al. 1994), le côté demande l'a moins été (voir par exemple Fugazza et Jacques 2003). Peut-être parce que l'idée qu'une entreprise légale puisse demander du travail illégal choque à première vue. Nous analysons dans cet article les déterminants de la demande de travail illégal par les firmes légales. Nous montrons que les caractéristiques technologiques des firmes jouent un rôle important. Par ailleurs nous développons un modèle original Principal-MultiAgents interagissant dans une structure oligopolistique (sans coopération) à la Cournot et où une entreprise légale (l'Agent) peut demander à la fois du travail sur le marché du travail officiel et sur le marché noir. Le Principal (le gouvernement) procède à des contrôles et l'entreprise paye une taxe proportionnelle au montant de la fraude si elle est détectée. La probabilité de détection est cependant endogène c'est-à-dire que plus l'entreprise fraude et plus la probabilité de se faire détecter augmente. Nous montrons qu'à l'équilibre de Cournot plus le Principal contrôle la production déclarée par les entreprises et plus la probabilité de détection d'équilibre baisse. Ce résultat paradoxal s'explique par le fait que la probabilité de détection étant endogène, ce sont en réalité les entreprises qui déterminent cette probabilité. Plus le Principal contrôle la production que les entreprises déclarent et plus ces dernières vont baisser la part de la production effectuée sur le marché noir de telle sorte que la probabilité de détection baisse et mais que la fraude leur soit toujours profitable.

Keywords : non reported activities, black market.

JEL Codes : J23, H26.

1 Introduction

According to many empirical studies, the size of the black market in the OECD countries is not negligible. For instance using a microeconomic data (1993 Family Expenditure Survey), Lyssiotou, Pashardes and Stengos (1999) find out that the size of the black market in the UK amounts to 9.24% of the GDP. Moreover according to Schneider and Enste (2000), there is a sizeable increase of the black market in many OECD countries (including the US) from 1960 to 1999. It seems therefore that the importance of the black market relative to the official one is a robust fact. The main reasons of the increase of the black market' size according to various authors are : *the increase of the tax and social security contribution burdens*, *the intensity of regulations* (i.e. countries with more general regulation -for instance labor market regulations, trade barriers, labor restrictions for foreigners- have a high size of the black market : see Johnson et al. 1997, Friedman et al. 1999), *the social transfers* (the social welfare system can incite the beneficiaries to work in the black economy, indeed their overall income is therefore much higher), *the labor market* (high total labor costs and the reduction in working hours in the official economy) and finally *the public sector services*[1]. Hence the analysis of labor market is of great importance in order to understand the rationale of the black market in a market economy. The supply side (of the labor market) has been widely studied in the literature (Cowell 1985, Clotfelter 1983, Lacroix and Fortin 1992, Lemieux et al. 1994, Frederiksen et al. 2005). However as pointed out by Sandmo (2004), there exist few analyses of the demand side (see for instance Fugazza and Jacques 2003) and in general they consider that the firms operate either on the official market or on the underground one. To the best of our knowledge there exists no *formal* analysis of the demand side of the labor market in which the firms can operate both in the official and (directly or indirectly -through subcontracting for instance-) in the underground markets. Two empirical facts seem to stress the importance of analysing this case. The first one is the (estimated) high number of illegal workers in several OECD countries. The second empirical fact is that in most OECD countries, some data sets concerning the use by legal firms of black labor force are now available. In France for instance such data sets are available since 1998. According to Cornu-Pauchet (2003) from the Agence Centrale des Organismes de Sécurité Sociale (Acoss)[2], the use of black labor force by legal firms represents in France a monetary loss for the state (in terms of tax evasion) of at least 4 billion Euro a year. Given the topic of our paper, by black market we simply mean the legal non reported activities. It is the purpose of our third and fourth sections to focus on the demand side and to analyse some driving forces behind the demand by legal firms for labor force in the black market ? In section 3, we show that the firms' technological characteristics matter. In section 4, we analyse this issue in a Principal-Multiagents model with an endogeneous probability for the Agents to be detected when using a black labor force. Moreover we assume the Agents (the firms) to compete in a Cournot Oligoply structure. We show that at the Cournot equilibrium, the more the Principal (who is here the Government) controls the production reported by the Agents, the more the probability of detection decreases. In section 2, we start our paper with the general question of the existence of an equilibrium in an economy in which there is a (formal or informal) cooperation between an official market and an underground one. From our point of view, the best way to make such an analysis is in the framework of a centrally planned economy because of the clear cut relationship between the issue of a mixed *centrally planned/free* economy and the one of a black

[1] If a firm invests in the black economy, it will not beneficiate from the public sector economy. Thus the more the quality and quantity of the public sector service, the more firms will have incentives to produce in the official market.

[2] The Acoss is especially in charge in fighting against the use by *legal* firms of black labor force (in french "travail dissimulé").

market in market economy. Finally section 5 concludes.

2 Equilibrium in a mixed economy with cooperation

In the model there are two markets, the state market and a black (free) market[3]. Prices at the state market are fixed and this is the characteristic feature of a centrally planned economy. Prices at the free market are flexible and vary depending on the difference between supply and demand. The behavior of the production sector is specified by profit maximization in free market prices. Another characteristic feature of the centrally planned economy is the presence of quotas, within which the agents may buy and sell goods at fixed prices. We consider two ways of cooperation between the state and free markets, the strong one and the flexible. According to the strong way of cooperation it is allowed to use the rests of the quotas in the free market, and according to the flexible one there are non restrictions on the free market. For both ways of cooperations we prove existence of equilibria and show that the strong way may cause big difference between the fixed and equilibrium prices (at the free market), while due to the flexible way of cooperation, the prices might be normalized.

2.1 Model

An economy $\mathcal{E}^{I,II}$ is specified by the list $\{(X_i^I, X_i^{II}, Y_i, \alpha_i, \beta_i, u_i, \omega_i), i \in [n], q, \mathcal{P}\}$, where $[n] = \{1, \ldots, n\}$ we let to denote a finite set of agents; $q \in \mathbb{R}^l$ denotes the vector of fixed prices (l denotes the set of available goods); $\mathcal{P} \subset \mathbb{R}^l$ denotes the set of free market prices; Y_i denotes the production set of the i-th agent; ω_i the endowments of the i-th agent. The aggregate production set is $Y := \sum_i Y_i + \sum_i \omega_i$. The quota functions $\beta_i : Y \to \mathbb{R}^l_+$ specify the consumption correspondence $X_i^I : Y \to 2^{\mathbb{R}^l_+}$, $X_i^I(y) := \{x_i \in \mathbb{R}^l_+ \,|\, x_i \leq \beta_i(y)\}$, that is the set of goods available at fixed prices q for the i-th agent. The wages are specified by the functions $\alpha_i : Y \times \{q\} \times \mathcal{P}$.

To define the free market consumption correspondence X_i^{II}, we specify the following two sets. Given q and $p \in \mathcal{P}$, we let to denote $K_p := \{k \in [l] \,|\, p_k < q_k\}$ and $\overline{K}_p := [l] \setminus K_p$.

Definition 1 *The strong way of cooperation is specified by the following*

[3] See for instance Stahl and Alexeev (1985) or Makarov et al.(1995).

choice of $X_i^{II} : Y \times \{q\} \times \mathcal{P} \to 2^{\mathbb{R}^l_+}$,

$$X_i^{II,s}(y,q,p) \\ = \\ \left\{ \begin{array}{c} x \in \mathbb{R}^l_+ \,|\, x_k \leq \sum_i (y_{ik} + \omega_{ik} - \beta_{ik}(y)),\ \textit{if } k \in K_p, \\ \textit{and } x_k \in proj_k(co(Y)) \ \textit{if } k \in \overline{K}_p \end{array} \right\},$$

where $proj_k(T)$ *denotes the projection of a set* $T \subset \mathbb{R}^l$ *on the* k*-th coordinate axe.*

This specification of the consumption sets corresponds to the strong way of cooperation between the state and free economies, under which the rests of quotas are available at the free market.

Definition 2 *The flexible way of cooperation is specified by the following consumption correspondence:*

$$X_i^{II,f} := co(Y) \cap \mathbb{R}^l_+.$$

This definition specifies a kind of "open" free market.

Given a pair of prices q and $p \in \mathcal{P}$, the set of allocations in the economy $\mathcal{E}^{I,II}_{s,f}$ is specified as

$$Z^{s(f)}(p,q) \\ := \\ \left\{ \begin{array}{c} z = (\{x_i^I\}_{i\in[n]}, \{x_i^{II}\}_{i\in[n]}, \{y_i\}_{i\in[n]}, p, q \,|\, (y_i \in Y_i)_{i\in[n]}, \\ (x_i^I \in X_i^I(y,p,q))_{i\in[n]}, (x_i^{II} \in X_i^{II,s(f)})_{i\in[n]} \end{array} \right\}$$

Obviously holds $Z^{s(f)}(p,q) \subset \prod_{i=1}^n \mathbb{R}^{3l}_+$.

The main point of the market in such mixed economies is the rule of construction of the budget mapping $B_i : Z^{s(f)}(p,q) \to 2^{Z_i}$. Specifically,

$$B_i^{s(f)}(z) \\ := \\ \left\{ \begin{array}{c} \tilde{z}_i = (\tilde{x}_i^I, \tilde{x}_i^{II}, \tilde{y}_i), x_i^I \in X_i^I(y|\tilde{y}_i), \tilde{x}_i^{II} \in X_i^{II,s(f)}, \\ \tilde{y}_i \in Y_i \,|\, q(\tilde{x}_i^I) + p(\tilde{x}_i^{II}) \leq \alpha_i(y|\tilde{y}_i,p,q) + \sum_{k\in\overline{K}_p} (\beta_{ik}(y|\tilde{y}_i) - \tilde{x}^I_{ik})(p_k - q_k) \end{array} \right\}$$

Here we assumed that the i-th agent, in addition to his wage $\alpha_i(y,p,q)$ in the official sector, gets the below surplus amount (due the difference between the official and free prices):

$$\sum_{k\in\overline{K}_p} (\beta_{ik}(y|\tilde{y}_i) - \tilde{x}^I_{ik})(p_k - q_k),$$

The preference mapping $P_i : Z^{s(f)}(p,q) \to 2^{Z_i}$ is specified by the rule

$$P_i(z) \\ := \\ \left\{ \begin{array}{l} \tilde{z}_i = (\tilde{x}_i^I, \tilde{x}_i^{II}, \tilde{y}_i), x_i^I \in X_i^I(y|\tilde{y}_i), \tilde{x}_i^{II} \in X_i^{II,s(f)}, \\ \tilde{y}_i \in Y_i \,|\, p(\tilde{y}_i) \geq p(y_i), u_i(\tilde{x}_i^I, \tilde{x}_i^{II}) > u_i(x_i^I, x_i^{II}) \end{array} \right\}$$

Recall that $u_i : \mathbb{R}^l \times \mathbb{R}^l \to \mathbb{R}$ denotes the utility function of the i-th agent.

Definition 3 *An equilibrium in the economy $\mathcal{E}_{s,f}^{I,II}$ is an allocation $z^* \in Z^{s(f)}(p^*, q)$ such that $B_i(z^*) \cap P_i(z^*) = \emptyset$ and all markets are clear $\sum_i (x_i^{*I} + x_i^{*II}) = y$.*

In order to ensure existence of an equilibrium in the mixed economies, we assume the following :

- The sets Y_i and X_i^{II} are non-empty convex compacts;
- The functions $\alpha_i(y, p, q)$, $\beta_i(y)$ are continuous and concave by y;
- For any $i \in [n]$ and $z \in Z^{s(f)}(p,q)$ there exists $(x_i^{*I}, x_i^{*II}, y_i^*) \in Z_i$ such that

$$\max(p,q)x_i^{*I} + px_i^{*II} < \alpha_i(y|y_i^*, p, q) + \sum_{k \in \overline{K}_p} \beta_{ik}(y|y_i^*)(p_k - q_k).$$

From these assumptions, it follows that $X_i^I(y)$ is a non-empty convex compact, the mappings B_i are continuous and the sets $B_i(z)$ are non-empty convex compacts for any $z \in Z(p,q)$.

We need a kind of the Walras law: for any $z \in Z^{s(f)}(p,q)$ there holds

$$\sum_{i \in [n]} \alpha_i(y, p, q) = \sum_{i \in [n]} \sum_{k \in \overline{K}_p} \beta_{ik}(y)(q_k - p_k) + \sum_k (\sum_{i \in [n]} (y_{ik} + \omega_{ik}))p_k.$$

Finally, we assume that goods from the state and free markets are substitutable, specifically, for any i, there holds

$$u_i(x_i^{*I}, x_i^{*II}) = w_i(x_i^{*I} + x_i^{*II}),$$

where $w_i : \mathbb{R}_+^l \to \mathbb{R}$ is a concave strictly monotone function. Then we get the following results on the existence of equilibria in the mixed economies.

Proposition 1 *Under the above assumptions, for any $a \geq 1$, there exists an equilibrium in the economy $\mathcal{E}_f^{I,II}$ with $\mathcal{P} := \{p : \sum_k p_k = a\}$.*

For the strong way of co-joint behavior between the state and free markets, we can not ensure any bound on the absolute value of the equilibrium free market prices.

Specifically, we get

Proposition 2 *Under the above assumptions, and homogeneous assumption* $\alpha_i(y, tp, tq) = t\alpha_i(y, p, q)$*, for any state price* q*, there exists an equilibrium in the economy* $\mathcal{E}_s^{I,II}$ *with* $\mathcal{P} := \mathbb{R}_+^l$.

To resume, three messages can be derived from our analysis. The first is that an equilibrium does exist in both cases of strong and flexible cooperation between state and free markets, the second is that the equilibrium prices depend on the kind of cooperation between the official and the free markets, and the third message is that since a mixed economy is a degenerate case of a market economy including a black market, then it must be the case that there exists also an equilibrium in such a market economy including a black market. Hence the subsequent analysis in which we analyse the determinants of the demand of black labor force by legal firms has a meaning.

3 Labor demand on the black market by legal firms

Let us consider the legal (official) labor market. That means that $y = (y_1, \dots, y_n) \in \mathbb{R}^n$ indicates the vector of n-types of labor. We consider that the labor with index 1 is the top qualified and we rank the other types in a descending order. Likewise, we consider also the black labor market with bundles $z = (z_1, \dots, z_n) \in \mathbb{R}^n$.

We assume a growth of an economy with a rate δ and all markets, except the labor market, are clear. Let $F : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ denotes the production function, θ be the price of the goods.

The goods at black labor economy have the same differentiating scale and we assume a kind of substitutability between goods in the legal and black markets. Namely, if $v = \theta F(y, z)$, then

$$\begin{gathered} v(y_1, \dots, y_n; z_1, \dots, z_n) = \\ v(y_1, \dots, y_i - \kappa, \dots, y_n; z_1, \dots, z_{i-2}, z_{i-1} + \kappa, z_i, \dots, z_n), \end{gathered} \tag{SR1}$$

and

$$\begin{gathered} v(y_1, \dots, y_n; z_1, \dots, z_n) = \\ v(y_1, \dots, y_i - \kappa, , \dots, y_n; z_1, \dots, z_{i-1}, \pi_i\kappa + z_i, z_{i+1}, \dots, z_n), \end{gathered} \tag{SR2}$$

and

$$\begin{gathered} v(y_1, \dots, y_n; z_1, \dots, z_n) = \\ v(y_1, \dots, y_i - \kappa, \lambda_i\kappa + y_{i+1}, y_{i+2}, \dots, y_n; z_1, \dots, z_n), \end{gathered} \tag{SR3}$$

with :

$$\kappa \geq 0, \pi_i > 1, \lambda_i > 1$$

The meaning of these substitution rules SR1, SR2 and SR3 is as follows. Labor forces from the black market are a kind of spoiling goods, they loose skills for the time of no use. Therefore, we assume that each producer expects that if he substitutes one unit of the i-th level labor in the legal sector by a unit of the $i-1$-th level labor from the black market, it will be no loss in production; and if he substitutes by the i-th level labor from the black market, the producer has to use π_i such units. For substitution from the legal label, a producer can use $i+1$ level with the transactions proportional to λ_i. These transactions might be a kind of payments for improving labor skills of $i+1$-th level workers.

Let $q = (q_1, \ldots, q_n) \in \mathbb{R}^{n+}$ be a vector of legal prices of the different types of labor and $p = (p_1, \ldots, p_n) \in \mathbb{R}^{n+}$ be a vector of prices of the different types of labor in the black economy. Let A be a random variable equal to 1 if the firm has used a labor force from the black labor market and 0 otherwise. Let X be a random variable equal to 1 if the firm has been detected (by the government) using a labor force from the black labor market and 0 otherwise. Of course, we assume that $\Pr(X = 1|A = 0) = 0$. Let $\eta = \Pr(X = 1)$. If a firm is detected using a labor force from the black labor market then it pays an amount $T(z)$ to the government. This amount depends on the total amount and the distribution of the labor forces used by the firm in the black market. $S(z)$ is a social cost (see for instance Allingham and Sandmo 1972) of being detected. It includes the financial consequences (except the previous T) when all the society knows that the firm has cheating it: for instance, the image of this firm can be damaged so that the customers boycott its products. However $S = 0$ if the firm has been detected cheating but there is no public information about that. Finally, let us denote

$$\begin{aligned} q(y) &= q.y' \\ p(z) &= p.z' \end{aligned}$$

Then each producer solves the problem

$$\begin{aligned} &\max_{y,z} \theta.f - q(y) - p(z) - \eta\,(T(z) + S(z)) \\ s.t. \quad : \quad &F(y,z) \geq f \end{aligned}$$

Let us now take a linear T and a linear S. That is :

$$T(z) = t.\sum_{i=1}^{n} z_i$$

$$S(z) = s.\sum_{i=1}^{n} z_i$$

where s and t are positive scalars.

Proposition 3 *Assuming the substitution rules, in the optimal solution, we have the following bounds for prices :*

$$q_i \leq \lambda_{i+1} q_{i+1}, \quad i = 1, \ldots, n-1 \tag{1}$$

and

$$p_i \geq \frac{q_i - \eta(t+s)}{\pi_i}, \quad i = 1, \ldots, n \tag{2}$$

and

$$p_{i-1} \geq q_i - \eta(t+s), \quad i = 2, \ldots, n. \tag{3}$$

Therefore equilibrium prices are:

$$p_{i-1} = Max\left(\; q_i - \eta(t+s), \quad \tfrac{q_{i-1} - \eta(t+s)}{\pi_{i-1}} \; \right) \tag{4}$$

Recall that $\eta(t+s)$ is the expected cost for the firm when using one unit of black labor force. That is, the equilibrium prices p_{i-1} are either the legal price of the lower labor minus the expected cost for using one unit of black labor force or the state price of the same level minus the expected cost for using one unit of black labor force divided by π_{i-1} (which is the required amount of units for substitution). From our point of view, two messages can be drawn from equation (4). The first one is that the expected cost for the firm for using one unit of black labor force $\eta(t+s)$ does not play the main role concerning the black labor force demand by firms. Indeed, according to equation (4) the firms transfer this cost to the black market workers[4]. The influence of the expected cost for using one unit of black labor force is indirect. For instance if the Government designs t and s in such a way that for some i and $i-1$, $q_i - \eta(t+s) = 0$ and $q_{i-1} - \eta(t+s) = 0$, then it must be the case that $p_{i-1} = 0$. The meaning is that in such a case, firms will ask for black labor force of level $i-1$ iff they can pay the workers a null wage. Since this is impossible then the demand for black labor force of level $i-1$ will be equal to zero. The second message is that it is π_i which plays the major role on the demand for black labor force of level i. For instance if π_i is not too high in such a way that $\pi_i < \frac{q_{i+1} - \eta(t+s)}{q_i - \eta(t+s)}, \forall i$ then any firm which uses a legal labor force of level i will also use a black labor force of level i.

From equation (4) we get the following result :

Proposition 4 *The following consistency condition is true :*

$$p_i \leq p_{i-1}$$

[4]Loosely speaking, their net wages are equal to their gross wages minus $\eta(t+s)$.

Then we can describe possible structures of interrelations between legal and black markets. Namely, consider bipartite graph with the vertices $V_s \cup V_b$, $V_* \cong \{1, \ldots, n\}$ and the set of edges E, joining pairs of vertices of the form $(i, i-1)$, (i, i), $i = 1, \ldots, n$, where the first entry in the pairs denotes a vertex in V_s and the second one in V_b (of course, we have to omit the pair $(1, 0)$). Then due to Propositions 3 and 4, we get that the bundle of equilibrium prices $p_1, \ldots, p_n$ defines a matching in this bipartite graph[5]. The meaning is that as we said above, it is π_i which plays the major role on the demand for black labor force of level i. Let us illustrate with the case $n = 3$ in which there are three levels of technology : the top labelled 1, the medium labelled 2 and the low labelled 3. Then depending on the value of $q_i - \eta(t+s)$ and $\frac{q_{i-1} - \eta(t+s)}{\pi_{i-1}}$, $i = 1, 2, 3$, we have four possible bipartite graphs (see Appendix B).

The first graph corresponds to the case where the three technological sectors use black labor force because π_i is not high whatever i. The second graph corresponds to the case where high tech sector 1 does not use a black labor force and where black labor force of level 2 is not used by any firm. The reason is that π_1 and π_2 are high while π_3 is weak. The third graph corresponds to the case where the medium sector 2 does not use a black labor force and where black labor force of level 3 is not used by any firm. The reason is that π_2 and π_3 are high while π_1 is weak. Finally the fourth graph corresponds to the case where high tech sector 1 does not use a black labor force and where black labor force of level 3 is not used by any firm. The reason is that π_1 , π_2 and π_3 are high. Let us also remark that all these four configurations can theoretically happen. However if we assume that π_i is a decreasing function of label i (that is, the more the job requires a qualified workforce, the higher is π_i), then the bipartite graph 2 is the more likely to happen. In this graph as we said above, high tech firms (label 1) do not use black labor force, medium tech (label 2) firms use high tech black labor force, medium tech (label 2) black labor forces are not used by any firm, and low tech (label 3) firms use low tech black (label 3) labor forces.

4 Detection mechanism and Labor demand on the black market: analysis in a Cournot oligopoly framework

The previous section has stressed out the crucial role played by π_i comparing to the one played by the expected cost for the firm when using one unit of black labor force: $\eta(t+s)$. In this section, we will analyse more precisely the role of this expected cost. *More generally, we will analyse the influence*

[5] Recall that a matching in a graph is defined by a collection of edges such that each vertex is an end point of at most one edge.

of a black labor force detection mechanism on the labor demand by firms. In order to do that we consider an oligoply structure with $m \geq 2$ firms with only one type of labor[6]. Moreover y_j is the production made by firm j in the official market and z_j is the production made by firm j in the black market. Let us take the following inverse linear[7] demand function :

$$\begin{aligned} \theta &= \theta\left(\sum_{j=1}^{m}(y_j+z_j)\right) = a - b\sum_{j=1}^{m}(y_j+z_j) \\ a,b &> 0 \end{aligned}$$

and the following production costs[8] in (respectively) the legal and black markets :

$$LC\left(y_j\right) = c_1 \times y_j$$

$$BC\left(z_j\right) = c_2 \times z_j$$

where : $c_1 > c_2 > 0$ and $a > c_1$. We assume that c_1 and c_2 are unknown by the Principal.

On the contrary to the previous model, the probability of being detected is not constant but it depends on the total amount of production :

$$\eta = \eta\left(\sum_{j=1}^{m}(y_j+z_j)\right)$$

Actually we have a Principal - Multiagents model in which the Agents do not cooperate each other. The government is the Principal and the firms are the Agents. *We assume that Principal does not observe nor z_j nor y_j.* However he observes the equilibrium market price and he knows the inverse demand function. Therefore an Agent j will not necessarily report y_j as his production. Indeed since the market equilibrium price depends on the total amount of production (legal + black markets) then the Principal can detect some black market production by comparing the price $\theta_0 = \theta\left(\sum\limits_{j=1}^{m} y_j\right)$ associated to the reported production $(\sum\limits_{j=1}^{m} y_j)$ to the observed market equilibrium price

[6]There are n types of labor in the previous section.

[7]Taking a more general inverse demand function will not change qualitatively the results. More fundamentally, the inverse demand function should be random (for the firms) if consumers can boycott their products when they are detected being cheating.

[8]The cost functions include here all the taxes and the workers training costs.

$\theta\left(\sum_{j=1}^{m}(y_j+z_j)\right)$. The difference between these two prices is:

$$\theta\left(\sum_{j=1}^{m} y_j\right)-\theta\left(\sum_{j=1}^{m}(y_j+z_j)\right)=b\sum_{j=1}^{m} z_j=mbz_j$$

Thus Principal can easily calculate firm j black market production :

$$z_j=\frac{\theta\left(\sum_{j=1}^{m} y_j\right)-\theta\left(\sum_{j=1}^{m}(y_j+z_j)\right)}{mb}$$

Hence it must be the case that Agent j will report y_j+z_j as his production in the legal sector. Since the Principal does not know the marginal costs over the legal and the black markets, there is a priori no way for him to know if the reported production y_j+z_j includes a black market production. Check and detect a black market production by a firm imply a cost for the Principal. Of course, there exist several detection mechanisms denoted $D=\{d,\eta\}$ where d is the unit cost when controlling the reported quantity y_j+z_j and η is the probability of being detected. If θ_0 is weaker than the market equilibrium price then the Principal can implement some additional controls. And this increases the probability of being detected for the firms. The choice of a such a mechanism is endogeneous, however (in order to simplify the analysis), we will assume that this choice is here exogeneous to the model. Finally here by choosing y_j and z_j , the firm j actually chooses the probability at which it wants to be detected. Let $\Delta=\rho\sum_{j=1}^{m} z_j$ where $\rho\in[0,1]$ is the part of the reported production (y_j+z_j) which is controlled by the Principal. We assume that the probability of being detected is the following :

$$\eta\left(\sum_{j=1}^{m}(y_j+z_j)\right)=\begin{cases} 0 & if\ \ \Delta<\Delta_1 \\ \frac{\Delta-\Delta_1}{\Delta_2-\Delta_1} & if\ \ \Delta_1\leq\Delta\leq\Delta_2 \\ 1 & if\ \ \Delta>\Delta_2 \end{cases}$$

with[9] $\Delta_2>\Delta_1\geq 0$.

The detection mechanism is therefore $D=\left\{d,\eta\left(\rho,\sum_{j=1}^{m} z_j,\Delta_1,\Delta_2\right)\right\}$ where η is (ceteris paribus) an increasing function of ρ and $\sum_{j=1}^{m} z_j$ and a

[9] This means that the Principal can control without detecting a black market production even if there is a black market production. The reason is that firms can use some dissimulation' strategies. If we had allowed $\Delta_1=\Delta_2=0$ then any black market production would have been detected with probability 1.

decreasing function of Δ_1 and Δ_2. Let us remark that ρ is choosen by the Principal, $\sum_{j=1}^{m} z_j$ is choosen by the Agents and Δ_1 and Δ_2 are choosen by the Principal provided they are technically available. Let us assume that the parameters of the detection mechanism D are known by the Agents[10].

Like in the previous section, let us take :

$$\begin{array}{ll} T(z_j) = t.z_j \ , & \textit{with } t \geq 0 \\ S(z_j) = s.z_j \ , & \textit{with } s \geq 0 \end{array}$$

The profit function of firm j is the following :

$$\left(a - b\sum_{j=1}^{m}(y_j + z_j)\right).(y_j + z_j) - c_1 y_j - c_2 z_j - \frac{\Delta - \Delta_1}{\Delta_2 - \Delta_1}.z_j.(t+s)$$

for $\Delta_1 \leq \Delta \leq \Delta_2$.

The first question we address is the following : does this Cournot Oligopoly game with endogeneous probability of detection admit an equilibrium ? For the sake of clarity, let us take $m = 2$. From a mathematical standpoint, the results presented here can easily be generalized to the case $m > 2$ using the concept of inclusive reaction function in the sense of Novshek (1985).

Let us set the following condition :

$$\frac{1}{2b}(a - c_1) > \frac{1}{2\rho^2}\left[\rho\Delta_1 + \frac{(c_1 - c_2)(\Delta_2 - \Delta_1)}{t+s}\right] \qquad \text{(DK)}$$

Proposition 5 *The Cournot duopoly game has two equilibria :*

1. $((0, z^*); (0, z^*))$ *with* $z^* > 0$, *if Condition (DK) is not fulfilled.*
2. $((y^*, z^*); (y^*, z^*))$ *with* $y^*, z^* > 0$, *if Condition (DK) is fulfilled.*

   where :

$$z^* = \tfrac{2}{3}\Sigma$$

$$y^* = \tfrac{2}{3}\Sigma'$$

$$\Sigma = \tfrac{1}{2\rho^2}\left[\rho\Delta_1 + \tfrac{(c_1-c_2)(\Delta_2-\Delta_1)}{t+s}\right]$$

$$\Sigma' = -\Sigma + \tfrac{1}{2b}(a - c_1)$$

[10] This hypothesis is without loss of generality.

Remark 1 *For any finite t and s, $z^* > 0$. Moreover z^* does not directly depend (as it could be expected) to the demand variables (a and b). z^* only depends on the difference of marginal costs between the official and black markets ($c_1 - c_2$) and to the variables (Δ_1, Δ_2 and ρ) of the control mechanism D.*

Remark 2 *Let us denoted by y^c the Cournot duopoly production of each firm when there is no black market (or when $c_1 = c_2$): $y^c = \frac{a-c_1}{3b}$. It is easy to see from proposition 5 that if condition (DK) is fulfilled then the production of each firm $y^* + z^* = \frac{a-c_1}{3b} = y^c$. However when condition (DK) is not fulfilled then the production of each firm $y^* + z^* \geq y^c$ (with $y^* = 0$).*

Proposition 6 *z^* is*

1. *an increasing function of $c_1 - c_2$ and Δ_2 ,*
2. *a decreasing function of ρ, t and s.*

Proposition 7 *The following two conditions are equivalent:*

1. $\frac{\partial z^*}{\partial \Delta_1} \geq 0$
2. $c_1 - c_2 \leq \rho(t+s)$

Propositions 6 and 7 are interesting, especially concerning the role of Δ_1 and Δ_2. According to proposition 6, z^* decreases when Δ_2 decreases. Let us recall that Δ_2 is the point at which η^* equal 1. So when Δ_2 decreases, if z^* remains the same, the likelihood to be detected with probability equal to 1 and the expected cost $\eta^* \rho z^* (t+s)$ will increase. In order to avoid that, Agent will decrease z^* in such a way that the expected cost decreases. Concerning proposition 7, Δ_1 is simply the point from which the probability to be detected is strictly positive. Therefore when Δ_1 decreases it is simply the likelyhood to be detected with a (strictly) positive probability which will increase. Hence Agent will not systematically decrease his black market production. His decision will depend on the comparison of the marginal costs of producing in the black market ($c_2 + \rho(t+s)$) and in the official market (c_1). If $c_1 > c_2 + \rho(t+s)$ then Agent will increase his black market production when Δ_1 decreases.

Proposition 8 *The equilibrium probability of being detected is :*

$$\eta^* = \frac{2}{3} . \frac{c_1 - c_2}{\rho(t+s)} - \frac{1}{3} . \frac{\Delta_1}{\Delta_2 - \Delta_1}$$

Moreover, we have :

1. $\frac{\partial \eta^*}{\partial \rho} < 0$

2. $\frac{\partial \eta^*}{\partial (c_1 - c_2)} > 0$

3. $\frac{\partial \eta^*}{\partial \Delta_1} < 0$

4. $\frac{\partial \eta^*}{\partial \Delta_2} > 0$

5. $\frac{\partial \eta^*}{\partial t} = \frac{\partial \eta^*}{\partial s} < 0$

At first glance, the point (1) of proposition 8 looks contradictory. Indeed we know by $\eta = \frac{\rho \sum z_j - \Delta_1}{\Delta_2 - \Delta_1}$ that η increases with ρ ceteris paribus. However at equilibrium, firms chooses z^* in such a way that when ρ increases, z^* decreases such that $2\rho z^*$ decreases. Hence η^* decreases and *black market production still remains profitable for the firms*. Likewise, point (4) of the same proposition looks contradictory. Indeed when Δ_2 decreases, the control mechanism D is more efficient in the sense that it detects more often with probability equal to 1. Therefore we expect the probability of detection to increase. However recall again that the firms actually chooses the probability of detection. Hence when Δ_2 decreases, the firms will decrease their black market production z^* in such a way that $2\rho z^* - \Delta_1$ decreases more than $\Delta_2 - \Delta_1$. Leading to the decreasing of η^*.

Proposition 8 leads also to some interesting remarks from a policy-making point of view. If the probability of detection is endogeneous then an economic policy whose goal is to reach *black-market proofness* by increasing the number of controls (ρ) or/and by increasing $t + s$, will be a failure. Indeed, firms will answer to such a policy by decreasing the equilibrium probability of detection η^* in such a way that black market production remains profitable. For instance when $t + s \longrightarrow +\infty$ then $\eta^* \longrightarrow 0$ and $z^* \longrightarrow \frac{\Delta_1}{3\rho} \geq 0$ (this ratio is strictly positive if $\Delta_1 > 0$). Likewise if $\rho = 1$ then $\eta^* = \frac{2}{3} . \frac{c_1 - c_2}{(t+s)} - \frac{1}{3} . \frac{\Delta_1}{\Delta_2 - \Delta_1}$ and $z^* = \frac{1}{3}\left[\Delta_1 + \frac{(c_1 - c_2)(\Delta_2 - \Delta_1)}{t+s}\right] > 0$. This means that when the probability of detection is endogeneous the purpose of an economic policy should not be to have a black-market proof economy but merely to have (through the design of ρ and $t + s$) $\eta^* = 0$.

5 Conclusion

We have provided in this paper a formal study of the demand by firms for labor force in the black market. According to our analysis the technological characteristics of the firms matter. Broadly speaking high tech firms will not use a black labor force. Moreover when analysing a Principal-Multiagents model with an endogeneous probability for the Agents (which are the firms) to be detected by the Principal (the government) when using a black labor force, we found out that at the Cournot equilibrium, the more the Principal controls the production reported by the Agents, the more the probability of detection decreases.

Appendix A.

Let us first set the following lemmata.

Lemma 1 *Let $p_k < q_k$ for some k. Then, in the flexible economy $E_f^{I,II}$, for each $z \in Z(p,q)$ and $\overline{z_i} \in Argmax_{\tilde{z}_i \in B(z)} u_i(x_i^{*I}, x_i^{*II})$, there holds $\overline{x}_{ik}^I$.*

Proof. From the contrary, suppose $\overline{x}_{ik}^I \neq 0$. Consider another allocation $\hat{z}_i$ which differs from $\overline{z}_i$ at two places: $\hat{x}_{ik}^I = 0$ and $\hat{x}_{ik}^{II} = \frac{q_k}{p_k}\overline{x}_{ik}^I + \overline{x}_{ik}^{II}$. Obviously this new allocation belongs to the budget set, and

$$u_i(\bar{z}_i) = w_i(\overline{z}_i + (\frac{q_k}{p_k} - 1)\overline{x}_{ik}^I) > w_i(\overline{z}_i) = u_i(\overline{z}_i),$$

that contradicts to $\overline{z_i} \in Argmax_{\tilde{z}_i \in B(z)} u_i(x_i^{*I}, x_i^{*II})$. ■

Lemma 2 *Suppose an allocation $\overline{z} = ((\overline{x}_i^I)_{i\in N}, (\overline{x}_i^{II})_{i\in N}, (\overline{y}_i)_{i\in N}, \overline{p}, q)$ is an equilibrium in the strong way of cooperation economy $\mathcal{E}_s^{I,II}$. Then, for any k such that $\overline{p}_k < q_k$, there holds $\sum_i \overline{x}_{ik}^I = 0$.*

Proof. We exploit the Walras law and the balance (clearness of all markets) $\sum_i(\overline{y}_{ik} + \omega_{ik}) = \sum_i(\overline{x}_{ik}^I + \overline{x}_{ik}^{II})$ for every k. Summing up over i the budget constrains, with the help of the Walras law, and due to the balance, one can easily get

$$\sum_{k\in K_{\overline{p}}} \sum_{i\in[n]} \overline{x}_{ik}^I (q_k - p_k) \leq 0.$$

Since $\overline{x}_{ik}^I \geq 0$, we get $\sum_i \overline{x}_{ik}^I = 0$ for every $k \in K_{\overline{p}}$. ■

Proof of proposition 1. Now we will prove existence of the equilibrium in the flexible economy. We follow the standard way and exploit the Gale lemma. The supply function of the ith agent is

$$\psi^i(p) = Argmax_{y\in Y_i} py.$$

Continuity and non-emptiness of the image immediately follows from the fact that we have assumed the sets Y_i and X_i^{II} to be non-empty convex compacts. Let $\Psi(p) = \sum_i(\psi^i(p) + \omega_i)$ denote the aggregate supply.

To define demand functions we use Lemma 1. Specifically, we set :

$$\phi^i(p) = \left\{ x_i^I + x_i^{II} \;\middle|\; \begin{array}{c} x_i^I \in X^I(\Psi(p), p, q), x_i^{II} \in X^{II}, \\ \text{and } u_i(x_i^I, x_i^{II}) \geq u_i(\tilde{x}_i^I, \tilde{x}_i^{II}) \\ \text{for any } (\tilde{x}_i^I, \tilde{x}_i^{II}) \textit{ such that}: \\ \sum_{k\in\overline{K}_p} p_k\tilde{x}_{ik}^I + \sum_k \tilde{x}_{ik}^{II} \leq \\ \alpha_i(\Psi(p), p, q) + \sum_{k\in\overline{K}_p} \beta_{ik}(\Psi(p))(p_k - q_k) \end{array} \right\}$$

From our assumptions, it follows that the functions $\phi^i(p)$, $i \in [n]$, are continuous. Let $\Phi(p) = \sum_i \phi^i(p)$ denote the aggregate demand.

Then one can check that the Walras law takes the standard form

$$p(\Psi(p) - \Phi(p)) \geq 0.$$

From the compactness of $B_i(p)$, we have that there exists a convex compact T such that

$$\Phi(p) \subset \cup_i \cup_{p \in \mathcal{P}} B_i(p) \subset T,$$

where here $\mathcal{P}$ is the standard unit simplex in $\mathbb{R}^l_+$.

Thus by the Gale lemma there exists p^*, $y^* \in \Psi(p^*)$, $x^* \in \Phi(p^*)$ such that $y^* - x^* \geq 0$. The equality follows from strict monotonicity of the functions w_i. Thus, we established existence of an equilibrium in the flexible way of cooperation economy. ■

Proof of proposition 2. To establish existence of equilibrium in the strong way of cooperation economy, we have to use several modifications comparing to the previous proof.

Firstly, we assume the functions α_i are homogeneous, $\alpha_i(y, tp, tq) = t\alpha_i(y, p, q)$. Then any pair of prices (p, q), $\sum_k p_k = M$, is equivalent to the pair $(\tilde{p}, q/M)$, $\sum_k \tilde{p}_k = 1$. Thus getting $M \to \infty$, we may, equivalently, consider $p \in \mathcal{P}$ and $q \to 0$. Thus, we consider the case $p \in \mathcal{P}$ and $q \leq (\epsilon, \dots, \epsilon)$.

Now, we define the supply functions as in the above case. For $p \in \mathcal{P}_\epsilon := \{p \geq q\} \cap \mathcal{P}$, we set the demand function by the same formula as in the above case, and extend these functions by continuity for the complement set of prices $p \in \mathcal{P} \setminus \mathcal{P}_\epsilon$, by the same rule, but taking into account the definition of the set X_i^I. One can check that due to Lemma 2, such an extension does not change equilibrium if such exists. Now at $\epsilon = 0$, we are in the case of the flexible economy and an equilibrium exists. Strict monotonicity assumption ensures $\overline{p}(0) > (0, \dots, 0)$. One can check that assuming all involving functions twice differentiable, we get, for small ϵ and $q \leq \epsilon\mathbf{1}$ and the equilibrium prices $\overline{p}$ in the flexible economy with such q, that there holds

$$|\alpha_i(\Psi(\overline{p}), \overline{p}, q) + \sum_{k \in \overline{K}_p} \beta_{ik}(\Psi(\overline{p})(\overline{p}_k - q_k) - \alpha_i(\Psi(\overline{p}(0)), \overline{p}(0), 0) + \sum_k \beta_{ik}(\Psi(\overline{p}(0))p_k(0)| \leq C\epsilon,$$

with a constant C does not depending on ϵ. From this follows that there exists $\epsilon > 0$ such that for any $q \leq \epsilon\mathbf{1}$, there holds $\overline{p} \geq \epsilon\mathbf{1}$. ■

Proof of proposition 3. Obviously the set of feasible (y, z) is a polytope because of our subsitution rules.

- Let us first show bound (3). Let (y^*, z^*) be a point at the Argmax, then if $y_i \neq 0$, $(y^*, z^*) + \gamma(-e_i, e_{i-1})$ is feasible with some $\gamma > 0$ and the additional cost of this shift is :

$$-q_i + p_{i-1} + \eta(t+s) \geq 0$$

Indeed (y^*, z^*) is not optimal otherwise. And this leads to bound (3).

- To show bound (2), let (y^*, z^*) be a point at the Argmax, then if $y_i \neq 0$, $(y^*, z^*) + \gamma(-e_i, e_i)$ is feasible with some $\gamma > 0$ and the additional cost of this shift is :

$$-q_i + \pi_i p_i + \eta(t+s) \geq 0$$

- Finally, let (y^*, z^*) be a point at the Argmax, then if $y_i \neq 0$, $(y^*, z^*) + \gamma(-e_i + e_{i+1}, 0)$ is feasible with some $\gamma > 0$ and the additional cost of this shift is :

$$-q_i + \lambda_{i+1} q_{i+1} \geq 0$$

And this leads to bound (1).

■

Proof of proposition 4. It follows directly from the fact that :

$$\begin{array}{c} Max\left(\; q_i - \eta(t+s), \quad \frac{q_{i-1}-\eta(t+s)}{\pi_{i-1}} \; \right) \geq \\ Max\left(\; q_{i+1} - \eta(t+s), \quad \frac{q_i-\eta(t+s)}{\pi_i} \; \right) \end{array}$$

■

Proof of proposition 5. Let us first calculate the reaction functions :

$$-b(y_j + z_j) + a - b\sum_j (y_j + z_j) - c_1 = 0 \qquad (1)$$

$$\left\{ \begin{array}{l} -b(y_j + z_j) + a - b\sum\limits_j (y_j + z_j) - c_2 - \\ \frac{\rho \sum\limits_j z_j - \Delta_1}{\Delta_2 - \Delta_1}.\rho.(t+s) - \frac{\rho^2}{\Delta_2 - \Delta_1}.z_j.(t+s) = 0 \end{array} \right. \qquad (2)$$

(1) implies :

$$y_j = \frac{1}{2b}\left[a - b\left(y_{j'} + z_{j'}\right) - 2bz_j - c_1\right] \quad , \; j \neq j' \qquad (3)$$

(2) implies :

$$c_1 - c_2 - \frac{\rho \sum z_j - \Delta_1}{\Delta_2 - \Delta_1}.\rho.(t+s) - \frac{\rho^2}{\Delta_2 - \Delta_1}.z_j.(t+s) = 0$$

Hence :

$$\begin{aligned} z_j &= -\frac{1}{2} z_{j'} + \Sigma \quad , \; j \neq j' \\ &\quad with \\ \Sigma &= \frac{1}{2\rho^2}\left[\rho\Delta_1 + \frac{(c_1 - c_2)(\Delta_2 - \Delta_1)}{t+s}\right] \end{aligned} \tag{4}$$

It is easy to see that under the above assumptions, Σ is strictly positive. If we replace z_j in equation (3), we get :

$$\begin{aligned} y_j &= -\frac{1}{2} y_{j'} + \Sigma' \quad , \; j \neq j' \\ &\quad with \\ \Sigma' &= -\Sigma + \frac{1}{2b}(a - c_1) \end{aligned} \tag{5}$$

We derive the two Cournot equilibria by the following way.

$z^* = -\frac{1}{2} z^* + \Sigma$ implies :

$$z^* = \frac{2}{3}\Sigma \tag{6}$$

However the value of y^* depends on the value of Σ'. If $\Sigma' > 0$ then :

$$y^* = \frac{2}{3}\Sigma' \tag{7}$$

Otherwise :

$$y^* = 0 \tag{8}$$

■

Proof of proposition 6. Straightforward. ■

Proof of proposition 7. $z = \frac{2}{3}\Sigma$ where :

$$\Sigma = \frac{1}{2\rho^2}\left[\rho\Delta_1 + \frac{(c_1 - c_2)(\Delta_2 - \Delta_1)}{t+s}\right]$$

Therefore :

$$\frac{\partial z^*}{\partial \Delta_1} = \frac{1}{3\rho^2}\left[\rho - \frac{(c_1 - c_2)}{t+s}\right]$$

■

Proof of proposition 8. It follows directly from proposition 5. ■

Appendix B.

Figure 1: **Bipartite Graph 1.**

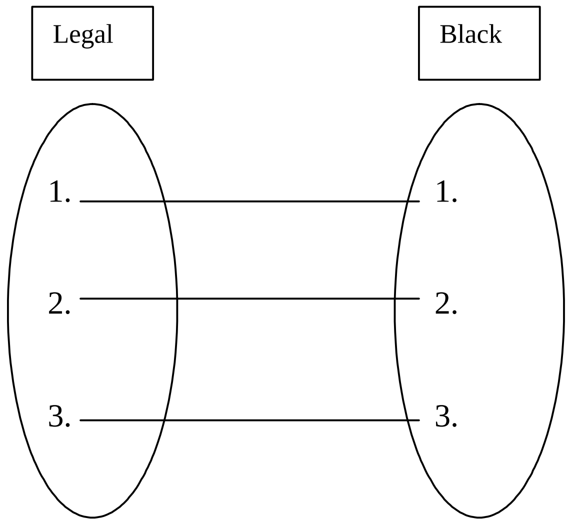


Figure 2: **Bipartite Graph 2.**

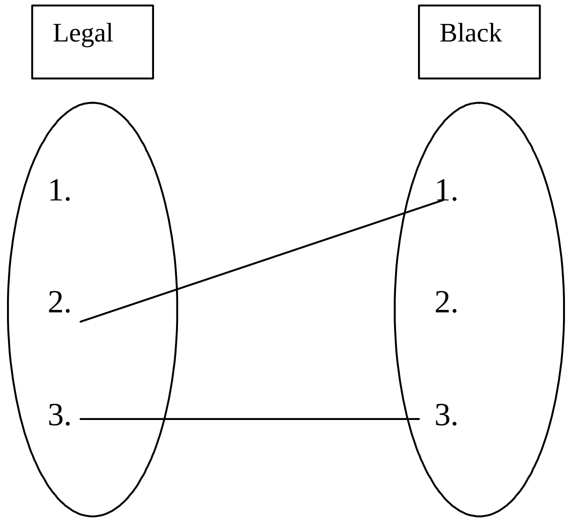


Figure 3: **Bipartite Graph 3.**

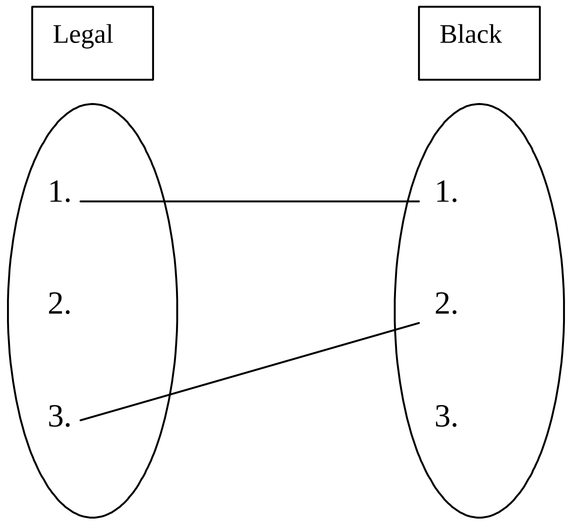
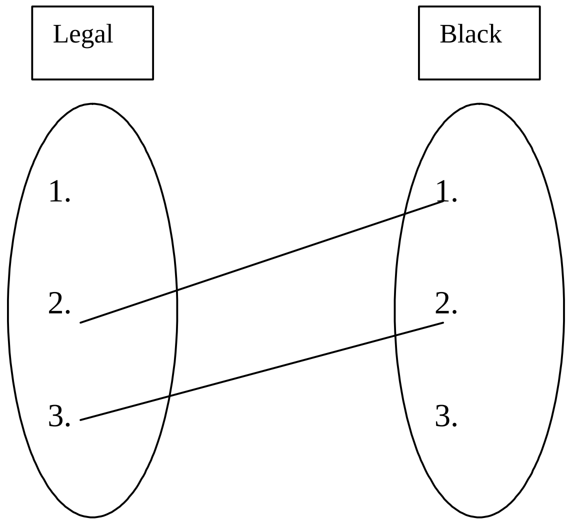


Figure 4: **Bipartite Graph 4.**



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