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Military R&D, Growth and the Optimal Allocation of Government Spending

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Abstract

The purpose of the model developed in the paper is to provide a simple economic framework to address an economic policy question, namely the optimal size of military R&D investment within total public expenditures. To capture the long-run impact of military R&D on the growth rate of the economy, one develops an endogenous growth model in line with Barro [1990] and Shieh & *alii* [2002]; the model focuses on the optimal sharing of public resources between civil investments, public consumption, military R&D investment and "standard" military spending. It emphasizes the key role played by public military R&D investments in determining the long-run levels of economic growth and welfare. According to our numerical simulations – based on a very prudential set of assumptions concerning the economic impact of military R&D – a 3.65 billions euros permanent reallocation of public spending from civilian unproductive public consumption towards military R&D investment, would induce a 380 billions euros GDP discounted benefit over a decade. In such a framework, characterized by productive externalities originating in military R&D, the government optimal policy should be to massively invest in military R&D: a global tax rate below 12% would drive to a 5.6% GDP long-run yearly growth rate.

JEL: D58, H23, O31, O32.

Keywords: endogenous growth, R&D, military expenditures, public spending.

1 Introduction

Is the military spending a useless spending? A large number of empirical and theoretical papers investigate the economic effects of military spending on eco-

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nomic growth with no clear and definitive answer to this question. The so-called theory of *peace dividends* (McNamara [1991]), suggesting that during peace times milex wastes and diverts away important resources from the civil sector, is broadly put forward to explain why the milex impact on economic growth is frequently found to be non-significant or even negative (see for example Barro & Sala-i-Martin [1995]), or less efficient than civilian public spending (Macnair & *alii* [1995]).

The early cross-country analysis by Benoit [1973, 1978], concluding to a positive relationship between external threats and economic growth, opened the way to generations of empirical models reflecting different theoretical framework, econometric procedures or sampling (time series, cross-section *etc.*) and leading to widely various results. Lipow [1990], Macnair & *alii* [1995], Brumm [1997] and Murdoch & *alii* [1997] confirm Benoit [1973] conclusions while, for instance, Biswas & Ram [1986] as well as Huand & Mintz [1991] find no significant correlation between military public spending and growth. Deger & Smith [1983], Faini & *alii* [1984] and Deger [1986] even show the existence of a negative relationship between military expenditures and growth.[1]

Despite contrasted empirical results, the theoretical economic literature devoted to the analysis of the effects of military spending upon growth and welfare, has suggested three main channels through which milex can influence GDP. *Demand effects* originate in public spending multiplier consequences of milex: from a Keynesian point of view military spending promotes activity by stimulating the aggregate demand, and eventually, employment, growth and welfare through a standard multiplier mechanism.[2] *Supply effects* describe the impact of military spending on the efficiency of the production process: in the long run, positive externalities associated to military innovations, spill over the entire productive system, affecting both the quantity and productivity of inputs, which together determine potential output. *Safety effects* highlight the crucial role played by the national defense in protecting persons and properties from domestic or foreign threats: safety conditions being an essential component of the incentives to invest and innovate, military expenditures, to the extent they increase national security, contribute to increase GDP (*cf.* for example Aizenman & Glick [2003]).

The purpose of this article is to study the long-run impact, on economic growth and social welfare of a very specific component of milex, namely military R&D expenditures, and to define the optimal economic policy concerning the optimal share of R&D military spending into total public spending. Our strong feeling and starting point, is that both the economic literature and almost all the governments have surprisingly underestimate the crucial role played by military R&D investments and their global effects on input productivity through

[1] See Sandler & Hartley [1995], Ram [1995] or Dunne & *alii* [2004] for some surveys on the relationship between milex and growth.

[2] Of course, the extent to which military expenditures can crowd out other forms of expenditure, such as civilian investment, depends on how the former is financed (Dakurah & *alii* [2001]). From a classical point of view, any spending is necessarily financed through current taxes, inflationary taxes or future taxes (Ricardian equivalence) and substitutes private investments through a crowding-out effect.

knowledge externalities. To give some insights on this question, we build a general equilibrium endogenous growth model which allow us to distinguish between productive and unproductive defense expenditures – whose effects transit through very different channels – and which can be used to compute the benefits of a permanent reallocation of public spending from civilian unproductive public consumption toward military R&D investments.

To avoid any further misunderstandings let us now define what one denotes by productive *vs* unproductive expenditures. Expenditures, eventually leading to a production costs cut, through a classical *supply side* effect, are called *productive*; this definition is voluntarily wide to include public substructures investments (airports, roads, communication networks *etc.*), public R&D investments and education spending as well as public subsidies to private R&D *etc.* An *unproductive* spending does not denote expenditures with no effects on the economy, but rather expenditures generating *demand side* effects through a Keynesian multiplier mechanism. Following this distinction, military R&D is a productive expenditure, because it contributes to decrease both the production cost of the defense service and – through an innovation diffusion process (positive externalities) – the production cost of the civilian good; on the other hand, military wages constitute an unproductive spending, because raising the wage bill positively affects aggregate demand in a Keynesian way.

On a theoretical ground, one knows that a government intervention is necessary to implement the first or the second best, when the economic activity is characterized by the existence of externalities, defined as economic phenomena having welfare effects not fully accounted for in the price and market system. A major class of externalities is constituted by public goods, such as transportation or communication networks, which are useful to all firms but whose corresponding investments cannot be realized by any single firm; in such a situation, without a public intervention, these goods are under-provided or not provided at all. Knowledge constitutes another case of externality, which stands at the heart of the present paper; as it cannot be the subject of property rights, one can consider it as a public good. Innovations, originating in R&D investments of one particular company, benefit, sooner or later, to other firms of the same sector and, step by step, to the whole economy; such transmission mechanisms allow us to understand how scientific and technical externalities bypass the market, by switching from one firm to another without any priced transaction. This "non-priced" diffusion process particularly characterizes military R&D activities, which generate almost immediate effects in the military sector and next in the civilian sector (Benoit [1973], [1978]).

The first attempts to achieve an economic modeling of the dynamic economic effects of military R&D, follow the developments, at the end of the 80s, of the so-called *New Growth Theories*, which consider the economic growth rate as an endogenous variable depending on the fundamentals of the economy. As sketched above, at a very rough level, two main effects of military research on economic activity can be highlighted:

(i) On the one hand, military R&D and its applications improve the performance of military equipment, *i.e.*, the quality of national defense services

provided by the army and, eventually, the global welfare.

(ii) On the other hand, military R&D increases the total stock of available scientific and technical knowledge, diffusing sooner or later, to the overall economy and, eventually, contributing to a more efficient production process which results in a higher growth rate.

The first mechanism directly affects the consumers' utility function and the second the aggregate production function: military R&D spending, through innovations diffusion, generates spillovers effects from the military sector towards the whole productive system. These positive externalities constitute a fundamental non-priced productive factor generating increasing returns and endogenous growth. Because a particular firm does not take into account, when making its R&D investments decisions, the positive impact of such investments on other firms and the overall economy, total R&D spending stands far below its socially optimal level; the role of the government is thus to design the appropriate incentive schemes to encourage firms of the military sector to sufficiently invest into R&D.

The introduction of national defense services as a component of the utility function is an old idea – Brito, [1972], Deger & Sen [1984], Van der Ploeg & Zeeuw, [1990], Zou, [1995], Chang & *alii* [1996] – providing an economic modeling of the *Demand Side* impact of military expenditures. The long-run *Supply Side* effects of military R&D have not been widely analyzed in economic literature even if one can find few theoretical papers devoted to formalize the effects of military R&D operating as an external production factor. In line with Barro [1990] seminal paper – considering public spending as an external growth factor and evaluating its impact on GDP growth – Shieh & *alii* [2002] develop a dynamic general equilibrium model where the military spending – research and substructures – is analyzed as an external effect "doping" the production function and generating a self-maintained growth. The model developed in this paper, in line with Shieh & *alii* [2002], differs from the latter in four main points:

(i) To compute the optimal share of military R&D investments into public spending and military spending, we distinguish four main public spending components: civilian consumption, civilian investment, ordinary milex and military R&D.[3]

(ii) To investigate the double nature of military R&D, we carefully distinguish the direct effect of military R&D on the utility function – through the quality of defense services – from its *Supply Side* impact on the aggregate production function

(iii) Depreciation of capital and public spending is allowed.

(iv) A numerical simulation of the model on French data allows us to give an evaluation of the impact of an increase in military R&D investments on GDP and to compute the optimal relative size of military R&D.

After the presentation of the model (Section 2), and the equilibrium (Section 3), Section 4 is devoted to the dynamic analysis. Eventually, Section 5 addresses policy issues.

[3] Shieh & *alii* [2002] only deal with civilian *vs* military public spending.

2 The model

The main purpose of the model developed in this section is to provide a simple economic framework to address a policy question, namely the optimal relative size of military R&D investment within total public expenditures. In order to be able to provide such an estimation, one needs first to understand how military R&D investments affect the global welfare; two main effects can be distinguished. On the one hand, military R&D and its applications raise the army productivity, *i.e.*, the quality of national defense services provided by the army: this first R&D transmission channel from the defense sector to the civil one directly affects the welfare through the utility function of economic agents. On the other hand, military R&D investments increase the total stock of scientific and technical knowledge available in the economy and, consequently, positively affect inputs productivity (externalities) and eventually the growth rate of the economy and the welfare: this indirect effect constitute a second R&D transmission channel from the defense sector to the civil one.

To capture the long-run impact of military R&D on the growth rate of the economy, we develop an endogenous growth model in line with Barro [1990] and Shieh & *alii* [2002]. The following subsections present the behavior of the three agents of the model.

2.1 Households

Households are supposed to live an infinite number of periods during which they consume a private consumption good c, a public consumption good b and a national defense public service denoted by e. The overall level of utility reached by the representative household during his life is given by the intertemporal utility function:

$$\sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right) + v\left(b_t\right) + w\left(e_t\right)\right] \tag{1}$$

where $0 < \beta \equiv 1/\left(1+i\right) < 1$ denotes the discount factor and $i > 0$ the time-preference rate.

We split the total military expenditures into two components – standard military expenditures n and R&D military expenditures m – and we assume that each of them is used to produce the national defense public good $e_t \equiv e\left(m_t, n_t\right)$; the function $e\left(.\right)$ is supposed to have constant returns to scale: $e\left(\mu m, \mu n\right) = \mu e\left(m, n\right)$.

Such a distinction allows us to further analyze the specific role played by military R&D investments and their effects on the global economy and the welfare.

At each period of time the representative household faces the following budget constraint:

$$c_t + \left(k_{t+1} - \Delta_k k_t\right) \leq \left(1-\tau\right)\left(r_t k_t + \omega_t l_t\right) \tag{2}$$

where $\Delta_k \equiv 1 - \delta_k$ denotes the capital depreciation rate between two periods.

Consumption and investment net expenditures stand on the left side of equation (2) while on the right side figures the disposable income with r the real return on capital, ω the real wage rate and τ the tax rate.

Labor supply is assumed to be inelastic and normalized to one:[4]

$$l_t = 1 \tag{3}$$

In such a framework the consumer's problem is to maximize the intertemporal utility function (1) with respect to k_t, and c_t. The infinite horizon Lagrangian function can thus be written:

$$\sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right) + v\left(b_t\right) + w\left(e_t\right)\right] + \sum_{t=0}^{\infty} \lambda_t \left[\left(1-\tau\right)\left(r_t k_t + \omega_t\right) - c_t - k_{t+1} + \Delta_k k_t\right]$$

After elimination of Lagrange multipliers, first order conditions lead directly to the Euler equation,

$$\frac{u'\left(c_t\right)}{u'\left(c_{t+1}\right)} = \beta\left[\Delta_k + \left(1-\tau\right) r_{t+1}\right] \tag{4}$$

and the budget constraint (2), now with equality.

Moreover, the optimal solution must respect the following transversality condition:

$$\lim_{t\to\infty} \lambda_t k_{t+1} = 0 \tag{5}$$

Assumption 1. *The utility function is characterized by a constant ε_u elasticity of intertemporal substitution in consumption:*

$$u\left(c\right) \equiv c_u \frac{c^{1-1/\varepsilon_u} - 1}{1 - 1/\varepsilon_u} \tag{6}$$

where c_u is a constant.

2.2 Firms

The state of technology is represented by a production function including four inputs: capital (k), labor (l), public investment (a) and military R&D expenditures (m). The public investment good is summarized by the amount a of public expenditures devoted to raise the quantity and/or quality of education and public substructures as roads, airports, cable networks *etc.* (productive externalities). On their side, military R&D expenditures affect the global productivity through a standard R&D externality (spillovers effects from the defense sector to the civil sector).

Assumption 2. *The production function $F\left(k,l,a,m\right)$ exhibits constant returns to scale in capital and labor:*

$$F\left(\mu k, \mu l, a, m\right) = \mu F\left(k,l,a,m\right)$$

[4] A more general approach would, of course, introduce the labor disutility into the utility function, but such a change would not affect the main results of the model.

The intensive production function $f(\kappa, a, m) \equiv F(k/l, 1, a, m)$ is supposed to be homogeneous of degree one with respect to its arguments: $f(\mu\kappa, \mu a, \mu m) = \mu f(\kappa, a, m)$.

The producer problem is to maximize the profit with respect to capital stock k_t and labor force l_t, considering all public goods externalities – *i.e.*, a and m - as constants.

$$\max_{k_t, l_t} F(k_t, l_t, a_t, m_t) - r_t k_t - \omega_t l_t$$

The firm equilibrium is thus defined by the equality between the real cost of each input and its productivity:

$$\begin{aligned} r &= F_k(k_t, l_t, a_t, m_t) \\ \omega &= F_l(k_t, l_t, a_t, m_t) \end{aligned}$$

These equalities can be rewritten in terms of an intensive production function:

$$\begin{aligned} r_t &= f_\kappa(\kappa_t, a_t, m_t) \\ \omega_t &= f(\kappa_t, a_t, m_t) - \kappa f_\kappa(\kappa_t, a_t, m_t) \end{aligned} \tag{7}$$

2.3 Government

As already mentioned, one assumes that the total amount of public expenditures is constituted of civil investment a (public networks infrastructures, education), public consumption b (health, justice, employment and social policies *etc.*), military R&D investment m and standard military spending n (arms, troops, buildings *etc.*):

$$g_t \equiv a_t + b_t + m_t + n_t$$

The government budget constraint at time t is thus given by:

$$a_{t+1} - \Delta_a a_t + b_{t+1} - \Delta_b b_t + m_{t+1} - \Delta_m m_t + n_{t+1} - \Delta_n n_t \leq \tau(r_t k_t + \omega_t l_t) \tag{8}$$

where $\Delta_i \equiv 1 - \delta_i$ and δ_i is the depreciation rate of the public expenditure of type i; the right-hand side of (8) represents the total amount of taxes.[5]

In such an economy the economic policy is simply described by the overall tax rate τ and the breakdown of fiscal revenues into the four components of public spending:

$$(\sigma_a, \sigma_b, \sigma_m, \sigma_n) \equiv (a_t/g_t, b_t/g_t, m_t/g_t, n_t/g_t) \tag{9}$$

with, of course,

$$\sigma_a + \sigma_b + \sigma_m + \sigma_n = 1 \tag{10}$$

Using the key (9) and the budget constraint, equation (8) can be rewritten:

$$\begin{aligned} & \sigma_a[g_{t+1} - \Delta_a g_t] + \sigma_b[g_{t+1} - \Delta_b g_t] + \sigma_m[g_{t+1} - \Delta_m g_t] + \sigma_n[g_{t+1} - \Delta_n g_t] \\ = \; & g_{t+1} - [\sigma_a\Delta_a + \sigma_b\Delta_b + \sigma_m\Delta_m + \sigma_n\Delta_n] g_t \\ \leq \; & \tau(r_t k_t + \omega_t l_t) \end{aligned}$$

[5] A lag could be introduced between fiscal revenues and public expenditures, but this should not change the long term analysis and the stationary state of the model.

or, equivalently:

$$g_{t+1} - \Delta g_t \leq \tau \left(r_t k_t + \omega_t l_t \right) \tag{11}$$

where the depreciation factor of public expenditure can be viewed as a weighted average of specific depreciation factors:

$$\Delta \equiv \sigma_a \Delta_a + \sigma_b \Delta_b + \sigma_m \Delta_m + \sigma_n \Delta_n \tag{12}$$

3 Equilibrium

Equilibrium in the labor market is characterized by an inelastic labor supply (*cf.* (3)). The general equilibrium of the model requires equilibrium on both goods and inputs markets. Noticing that $r_t k_t + \omega_t l_t = r_t \kappa_t + \omega_t = f(\kappa, a, m)$, that $a_t = \sigma_a g_t$ and $m_t = \sigma_m g_t$, one easily rewrites the representative agent budget constraint (2) as an aggregate resources constraint:

$$c_t + \kappa_{t+1} - \Delta_k \kappa_t = (1 - \tau) f \left(\kappa_t, \sigma_a g_t, \sigma_m g_t \right) \tag{13}$$

while the government budget constraint (11) becomes:

$$g_{t+1} - \Delta g_t = \tau f \left(\kappa_t, \sigma_a g_t, \sigma_m g_t \right) \tag{14}$$

Substituting (7) in the Euler equation (4), one gets:

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta \left[\Delta_k + (1 - \tau) f_\kappa \left(\kappa_{t+1}, \sigma_a g_{t+1}, \sigma_m g_{t+1} \right) \right] \tag{15}$$

Observing that the homogeneity property of the intensive production function implies that its derivatives are homogeneous of degree zero,

$$f_\kappa (\mu \kappa, \mu a, \mu m) = f_\kappa (\kappa, a, m)$$

it follows immediately $f_\kappa (\kappa_t, \sigma_a g_t, \sigma_m g_t) = f_\kappa (\kappa_t / g_t, \sigma_a, \sigma_m)$. Defining

$$\begin{aligned} x_t &\equiv \kappa_t / g_t \\ \varphi(x_t) &\equiv f(\kappa_t / g_t, \sigma_a, \sigma_m) \end{aligned}$$

one easily shows that $\varphi'(x_t) = f_\kappa (\kappa_t, \sigma_a g_t, \sigma_m g_t)$ which implies: $r_t = \varphi'(x_t) = f_\kappa (x_t, \sigma_a, \sigma_m)$. Equations (13), (14) and (15) can be now rewritten:

$$\begin{aligned} c_t + \kappa_{t+1} - \Delta_k \kappa_t &= (1 - \tau) g_t \varphi(x_t) \qquad (16) \\ g_{t+1} - \Delta g_t &= \tau g_t \varphi(x_t) \qquad (17) \\ \frac{u'(c_t)}{u'(c_{t+1})} &= \beta \left[\Delta_k + (1 - \tau) \varphi'(x_{t+1}) \right] \end{aligned}$$

Setting

$$\begin{aligned} y_t &\equiv c_t / g_t \qquad (18) \\ \gamma_t &\equiv g_{t+1} / g_t \end{aligned}$$

and dividing both sides of (16) and (17) by g_t, one eventually gets:

$$y_t + \gamma_t x_{t+1} - \Delta_k x_t = (1-\tau)\varphi(x_t) \quad (19)$$
$$\gamma_t = \Delta + \tau\varphi(x_t) \quad (20)$$

On the other hand, the Euler equation can be revisited under Assumption 1:

$$c_{t+1}/c_t = (\beta[\Delta_k + (1-\tau)\varphi'(x_{t+1})])^{\varepsilon_u}$$

So, we have:

$$\frac{y_{t+1}}{y_t}\gamma_t = (\beta[\Delta_k + (1-\tau)\varphi'(x_{t+1})])^{\varepsilon_u} \quad (21)$$

4 Dynamic system

Substituting (20) into (21) and (19), one straightforwardly obtains:

$$[\Delta + \tau\varphi(x_t)]\, y_{t+1}/y_t = (\beta[\Delta_k + (1-\tau)\varphi'(x_{t+1})])^{\varepsilon_u} \quad (22)$$
$$y_t - \Delta_k x_t + \Delta x_{t+1} = (1-\tau-\tau x_{t+1})\varphi(x_t) \quad (23)$$

These equations constitute a two-dimensional dynamic system in (x_t, y_t) where x_t – but not y_t – is a predetermined variable.

4.1 Stationary state

In order to compute the steady state, omit the time subscripts in (22-23) and solve the system:

$$\gamma = \Delta + \tau\varphi(x) = (\beta[\Delta_k + (1-\tau)\varphi'(x)])^{\varepsilon_u} \quad (24)$$
$$y = (\Delta_k - \Delta)x + (1-\tau-\tau x)\varphi(x) \quad (25)$$

Growth is balanced (usual arguments of the endogenous growth literature apply): $\gamma \equiv g_{t+1}/g_t = c_{t+1}/c_t = k_{t+1}/k_t$. Noticing that $\lambda_t = \beta^t u'(c_t)$ and using (6), the transversality condition (5) becomes:

$$\lim_{t\to\infty} c_u c_0^{-1/\varepsilon_u} k_0 \gamma \left(\beta\gamma^{1-1/\varepsilon_u}\right)^t = 0$$

i.e., $\beta\gamma^{1-1/\varepsilon_u} < 1$. Thus we get $\gamma < \Delta_k + \rho$ from (24), where $\rho \equiv (1-\tau)r$ is the after-tax return on capital.

4.2 Local dynamics

The question of equilibrium uniqueness under rational expectations is a crucial theoretical issue. This section is devoted to prove that the equilibrium is unique and is a saddle path converging to the stationary state. The starting point of such equilibrium trajectory is defined by the predetermined variable x_0: the adjustment of the jump-variable y_0 ensures that the starting point belongs to the converging saddle path, which is the only solution sustainable in the long-run (variables remain non-negative in the transition and the transversality condition is satisfied). In order to study the local dynamics and to show the saddle-point stability, we will linearize the dynamic system around the steady state.

Differentiating equation (22) w.r.t. the dynamic variables $(x_{t+1}, y_{t+1}, x_t, y_t)$ and using (24-25), one gets,

$$\gamma\varepsilon_u \frac{\varphi'' x}{\varphi'} \frac{(1-\tau)\,\varphi'}{\Delta_k + (1-\tau)\,\varphi'} \frac{dx_{t+1}}{x} - \gamma \frac{dy_{t+1}}{y} = \tau\varphi' x \frac{dx_t}{x} - \gamma \frac{dy_t}{y} \tag{26}$$

where the differentials are relative to the stationary state.

Linearizing now equation (23) around the steady state, one has:

$$\gamma \frac{dx_{t+1}}{x} = \left[\Delta_k + (1-\tau-\tau x)\,\varphi'\right] \frac{dx_t}{x} - \frac{y}{x}\frac{dy_t}{y} \tag{27}$$

Let $\varepsilon_2 \equiv x\varphi''/\varphi' < 0$ denote the elasticity of the interest rate with respect to the ratio κ/g (capital per capita over public spending); the linear system (26-27) is equivalently written as follows:

$$\begin{bmatrix} \frac{dx_{t+1}}{x} \\ \frac{dy_{t+1}}{y} \end{bmatrix} = \begin{bmatrix} \gamma\varepsilon_u\varepsilon_2 \frac{\rho}{\Delta_k+\rho} & -\gamma \\ \gamma & 0 \end{bmatrix}^{-1} \begin{bmatrix} x\rho\frac{\tau}{1-\tau} & -\gamma \\ \Delta_k + \rho\frac{1-(1+x)\tau}{1-\tau} & -\frac{y}{x} \end{bmatrix} \begin{bmatrix} \frac{dx_t}{x} \\ \frac{dy_t}{y} \end{bmatrix}$$

The determinant and the trace of the Jacobian matrix are respectively:

$$D = \frac{\Delta_k + \rho}{\gamma} - \frac{\rho}{\gamma}\frac{\tau}{1-\tau}\left(\frac{y}{\gamma} + x\right) \tag{28}$$

$$T = 1 + D + \frac{\rho}{\gamma}\left(\frac{\tau}{1-\tau}\frac{y}{\gamma} - \frac{\varepsilon_u\varepsilon_2}{\Delta_k+\rho}\frac{y}{x}\right) \tag{29}$$

The following proposition and the relevant proof states the uniqueness of the equilibrium transition.

Proposition 1 *The equilibrium is unique.*

Proof. See the Appendix.

The next proposition allow us to make a step forward under a mild additional hypothesis and to characterize explicitly the stability: in order to demonstrate the saddle-path stability, we assume that the weighted average of the public expenditures depreciation rates turns out to be equal to the capital depreciation rate.

Assumption 3. $\Delta = \Delta_k$.

Proposition 2 *The stationary state is a saddle point.*

Proof. See the Appendix.

Proposition 2 confirms Barro [1990] conjecture. The core of his seminal paper is a dynamic equation with one non-predetermined variable; the model exhibits one unstable and so determined stationary state which is the only possible equilibrium. In our model, as in Barro [1990], the equilibrium is still determined (that is unique), but an equilibrium transition is now possible.

5 Economic policy

Shieh & *alii* [2002] analyze how growth and welfare depend on the relative weights of civilian and military expenditures into public spending. They not only show that there exists an optimal ratio milex/GDP, which maximizes the economic growth, but also highlight that this ratio stands below the ratio maximizing the social welfare. Their finding contributes to explain why in some countries, military expenditures cuts associated with disarmament policies reduce the welfare level.

Our model focuses on the optimal (welfare-maximizing) sharing of public resources between civilian investments (a), public consumption (b), military investment in R&D (m) and standard military spending (n). In the previous sections economic agents were supposed to solve their programs, considering the economic policy $(\tau, \sigma_a, \sigma_b, \sigma_m, \sigma_n)$ as given, *i.e.*, the tax rate and the breakdown of fiscal revenues into the four components we have defined above.

Given such an agents' best response, the problem the government faces now becomes the optimal economic policy to implement, say $(\tau^*, \sigma_a^*, \sigma_b^*, \sigma_m^*, \sigma_n^*)$. Of course, the representative agent's shortcut, makes equivalent to maximize with respect to these five policy tools any social welfare function (but strictly increasing in the individual utilities) or the representative agent's utility function (1).

To keep things as simple as possible, let us focus directly on the case of regular growth (in the long-run the equilibrium will be sufficiently close to the steady state).[6] Clearly, as in Shieh & *alii* [2002], maximizing the economic growth rate is not equivalent to maximize the social welfare.

We will choose a Cobb-Douglas production function not only to satisfy the homogeneity requirement (see Assumption 2), but also to get straightforward numerical simulations:

$$\begin{aligned} F(\mu k, \mu l, a, m) &= \mu F(k, l, a, m) \\ f(\mu \kappa, \mu a, \mu m) &= \mu f(\kappa, a, m) \end{aligned}$$

For similar reasons, we assume a Cobb-Douglas as defense good production function.

Assumption 4. *The production function $F(.)$ and the defense good production function $e(.)$ are specified as follows:*

$$\begin{aligned} F(k, l, a, m) &= \theta k^{\alpha} l^{1-\alpha} a^{\alpha_a} m^{\alpha_m} \\ e(\sigma_m, \sigma_n) &\equiv B \sigma_m^{\beta_m} \sigma_n^{\beta_n} \end{aligned}$$

with $\alpha + \alpha_a + \alpha_m = 1$ and $\beta_m + \beta_n = 1$.

[6] In fact, because of the uniqueness of the equilibrium, we could compute utility along a transitional path, whenever the starting point stands off the steady state, and maximize its value with respect to the policy parameters, but it would drive us to hard analytical computations. As the main goal of the paper is to analyze long-run policy effects, we can focus on the steady state or transitional equilibria close enough (by continuity, the optimal policy rule does not change too much, if the equilibrium path remains in a neighborhood of the stationary state).

Assumption 3 is now replaced by the following assumption, which is more restrictive in terms of public expenditure depreciation factors.

Assumption 5. $\Delta_a = \Delta_b = \Delta_m = \Delta_n = \Delta_k$.

Eventually, we restrict ourselves to the case of a logarithmic utility functions, easier to handle and more widely accepted by RBC scholars.

Assumption 6. $u(c) \equiv c_u \ln c$, $v(b) \equiv c_v \ln b$, $w(e) \equiv c_w \ln e$.

A logarithmic utility function corresponds to the case of a unit elasticity of intertemporal substitution. The social welfare function becomes

$$W = \sum_{t=0}^{\infty} \beta^t c_u \ln c_t + \sum_{t=0}^{\infty} \beta^t c_v \ln b_t + \sum_{t=0}^{\infty} \beta^t c_w \ln e_t$$

where, without loss of generality:

$$c_u + c_v + c_w = 1 \tag{30}$$

Proposition 3 *Under Assumptions 4, 5 and 6, the optimal economic policy is obtained in two steps.*

(i) First, solve for x the following implicit equation:

$$\theta \sigma_a^{\alpha_a} \sigma_m^{\alpha_m} x^{\alpha} = \frac{x\Delta(1-\beta)}{\beta\varepsilon_1(1-\tau) - \tau x} \tag{31}$$

where τ is defined by

$$\begin{aligned}
\tau(x) &= \left[1 + A - B \pm \sqrt{(1+A-B)^2 - 4A}\right]/2 \qquad (32)\\
A &\equiv \frac{\varepsilon_1(\varepsilon_1 c_u(\beta+x) + (1-\varepsilon_1)(\varepsilon_1 c_u - 1)\beta x)}{c_u(\varepsilon_1+x)(\beta\varepsilon_1 + 2\beta\varepsilon_1 x + x^2) - \beta\varepsilon_1 x(1+x)}\\
B &\equiv \frac{x^2([c_u(1+\varepsilon_1) - \varepsilon_1]\beta\varepsilon_1 + c_u(x-\varepsilon_1))}{c_u(\varepsilon_1+x)(\beta\varepsilon_1 + 2\beta\varepsilon_1 x + x^2) - \beta\varepsilon_1 x(1+x)}
\end{aligned}$$

and σ_a, σ_m, σ_n by:

$$\sigma_a = \left(1 + \frac{\alpha_m}{\alpha_a} + \frac{\varepsilon_1}{\alpha_a} \frac{[c_v + (\beta_n + \beta_m) c_w][\beta(1-\varepsilon_1)(1-\tau) + \tau x]}{c_u + c_u \frac{\tau x}{\tau x - (1-\tau)} + \frac{\beta}{1-\beta}\frac{\tau x}{\tau x - (1-\tau)\varepsilon_1} - \left(c_u + \frac{\beta}{1-\beta}\right)\frac{\tau x}{\tau x - (1-\tau)\beta\varepsilon_1}}\right)^{-1} \tag{33}$$

$$\sigma_m = \frac{\alpha_m(c_v + \beta_n c_w) - \alpha_a \beta_m c_w}{\alpha_a(c_v + \beta_n c_w + \beta_m c_w)}\sigma_a + \frac{\alpha_a \beta_m c_w}{\alpha_a(c_v + \beta_n c_w + \beta_m c_w)} \tag{34}$$

$$\sigma_n = -\frac{(\alpha_a + \alpha_m)\beta_n c_w}{\alpha_a(c_v + \beta_n c_w + \beta_m c_w)}\sigma_a + \frac{\beta_n c_w}{c_v + \beta_n c_w + \beta_m c_w} \tag{35}$$

(ii) Then, after getting x^*, compute τ^* from (32), σ_a^* from (33) and, eventually, σ_m^*, σ_n^* and σ_b^* from (34), (35) and (10), respectively.

Proof. See the Appendix.

Let us notice that (31) is an implicit equation easy to solve numerically, taking as constant the fundamental parameters of the economy.

6 Numerical simulations

The purpose of this subsection is to calibrate the parameter values we use for simulating the model, to evaluate the impact on the GDP of an increasing public investment in military R&D and, eventually, to determine the optimal economic policy.

6.1 Parametrization and calibration

The relative weights σ_a, σ_b, σ_m, σ_n of public spending components simply correspond to the values observed in the French economy during the year 2005 as they are summarized in Table 2 (*cf.* Appendix 2). According to this table, the share σ_a of civil investment (a) into overall fiscal revenues is equal to 21.12%;[7] similar computations give the values of the relevant shares of public consumption (b), military R&D (m) and standard military spending (n), into the total amount of taxes: $\sigma_b = 73.12\%$, $\sigma_m = 0.5\%$ and $\sigma_n = 5.26\%$. The overall tax rate τ, measuring the global fiscal pressure, is given by the ratio of the total amount of taxes, 737 billions euros, over GDP (1691 billions euros for year 2005), *i.e.*, 43.59%.

Available estimations for capital depreciation rate show an important difference between human and physical capital: human capital is characterized by a lower depreciation rate, often below 2%,[8] to be compared to 8% for physical capital. As our model does not allow us to distinguish between the two types of capital, we assume an average 5% annual depreciation rate corresponding, more or less, to a 50% depreciation in 13 years. The rate of time preference is set equal to 5%.[9]

The share α of capital remuneration in GDP is set to 72.5% according to the results of Mankiw & *alii* [1992] and most of the empirical estimations; α is a measure of both human and physical capital share in total income, while $1-\alpha$ is the weight of productive externalities.

One goal of the paper is to analyze the impact of military R&D on economic growth and global welfare; as noticed in the previous section, such an analysis highlights the crucial role played by productive externalities associated to R&D public investment. In order to get a prudential evaluation of the economic impact of an increase in public military R&D expenditures and to avoid any overestimation of this impact, we minimize the size of military R&D externalities by setting $\alpha_m = 2.5\%$.

For similar reasons we decided:

(i) To limit the relative weight of defense services in the household's utility function – *i.e.*, the impact of military R&D on global welfare – by considering that households strongly prefer non-military goods to military ones: $c_u = c_v = 49\%$, $c_w = 2\%$.

[7] Corresponding to $9.84\% + 1.98\% + 8.59\% + 0.72\%$ (see Table 2 in the Appendix).

[8] See, for instance, Arrazola & de Hevia [2004].

[9] Corresponding to a yearly discount factor $\beta = 0.9524$.

(ii) To limit the role played by military R&D in the defense good production function: $\beta_m = 10\%$.

This set of very cautious and, in a way, pessimistic assumptions, concerning the role played by military R&D in the global economy, should protect us against any overestimation of its impact on GDP and the results obtained can probably be considered as lower bounds.

Eventually the yearly real growth rate of the economy has been set equal to 1.5%, corresponding to the observed values over the last few years in the French economy.

The table below summarizes the parameters values we used for simulating the theoretical model introduced in Section 2.

Parameter	**Definition**	**Observed (%)**
σ_a	Share of civil investment into overall fiscal revenues	21.12
σ_b	Share of public consumption into overall fiscal revenues	73.12
σ_m	Share of military R&D into overall fiscal revenues	0.50
σ_n	Share of non-R&D military spending into overall fiscal revenues	5.26
τ	Overall tax rate (global fiscal pressure)	43.59
$\gamma - 1$	GDP growth rate	1.50
	GDP (billions euros)	1691
		Calibrated (%)
i	Rate of time preference	5.00
Δ_k	Average depreciation rate of human and physical capital	5.00
α	(Human and physical) capital share in GDP	72.50
α_m	Elasticity of production to military R&D expenditures	2.50
α_a	Elasticity of production to public investment spending	25.00
β_m	Elasticity of the defense good production to military R&D	10.00
c_u	Private consumption utility share	49.00
c_v	Public consumption utility share	49.00
c_w	Defense good utility share	2.00

Table 1. Model parametrization and calibration (France, 2005)

The previous set of assumptions concerning the growth rate of the economy, the fiscal pressure and the relative weights of public spending components, eventually leads to set the value of the global productivity parameter θ. Considering that:

$$\gamma = \Delta + \tau\theta\sigma_a^{\alpha_a}\sigma_m^{\alpha_m}x^{\alpha} \tag{36}$$

and using (31), one gets:

$$\theta = \frac{\gamma - \Delta}{\tau\sigma_a^{\alpha_a}\sigma_m^{\alpha_m}}\left[\varepsilon_1\frac{\beta(\gamma - \Delta)}{\gamma - \beta\Delta}\frac{1-\tau}{\tau}\right]^{-\alpha} = 0.3997$$

6.2 Increasing military R&D under constant global public spending

In this section one proceeds to numerical simulations of the model to address the following question: what is the impact of doubling the military R&D share into public spending σ_m,[10] on the growth rate of the economy and the GDP, while decreasing at the same time the public consumption share σ_b, in order to keep constant global public expenditures? In other words: would it be optimal, from an economic policy point of view, to switch some fiscal resources from civilian unproductive spending to military R&D investments? One must notice that such a switch represents a decrease of σ_b from 73.12% to 72.62%, *i.e.*, a slight variation by -0.68% in the relative weight of public consumption; taking in account that the total amount of public consumption spending is equal to 538.98 billions euros, this means a 3.65 billions euros decrease of this amount (corresponding to the actual size of R&D military spending).[11]

Formally the problem consists in providing an evaluation of the GDP increase involved by a $d\sigma_m$ increase of σ_m coming with a joint decrease $d\sigma_b = -d\sigma_m$ of public unproductive consumption. Setting $d\sigma_m = -d\sigma_b$ and $d\tau = d\sigma_a = d\sigma_b = 0$, differentiating (36) and noticing that $\varphi(x, \sigma_a, \sigma_m) = \theta \sigma_a^{\alpha_a} \sigma_m^{\alpha_m} x^{\alpha}$, one gets in the long run (after a transitional adjustment):

$$d\gamma = \tau \varphi_m d\sigma_m + \tau \varphi_x dx \tag{37}$$

Totally differentiating

$$\varphi(x, \sigma_a, \sigma_m) = \frac{(1-\beta)\Delta x}{\beta \varepsilon_1 (1-\tau) - \tau x} \tag{38}$$

we have $dx = x_\tau d\tau + x_a d\sigma_a + x_m d\sigma_m$, where $d\tau = d\sigma_a = 0$; using equation (60) given in the Appendix, dx can then be written:

$$dx = \frac{\alpha_m}{\sigma_m} \frac{x}{\varepsilon_1} \frac{1}{\beta(1-\varepsilon_1)(1-\tau) + \tau x} d\sigma_m \tag{39}$$

Substituting (39) into (37), one gets the impact of the military R&D increase on the yearly growth factor of the economy:

$$\frac{d\gamma}{d\sigma_m} = (\gamma - \Delta) \frac{\alpha_m}{\sigma_m} \left[1 + \frac{1}{\beta(1-\varepsilon_1)(1-\tau) + \tau x} \right] > 0$$

Using the parameterization presented in Section 6.1 and noticing that the following equation stands at the stationary state:

$$x = \left(\frac{\gamma - \Delta}{\tau \theta \sigma_a^{\alpha_a} \sigma_m^{\alpha_m}} \right)^{1/\alpha}$$

[10] *i.e.*, a shift of σ_m from 0.5% to 1%.

[11] See Table 2 in the Appendix.

we eventually get:

$$\gamma' \approx \gamma + \frac{d\gamma}{d\sigma_m}\left(\sigma'_m - \sigma_m\right) = 1.0209$$

where γ' represents the yearly stationary growth factor, corresponding to the new share of military R&D into global public spending: $\sigma'_m = 0.01$. To summarize: if the government would decide to double the share of military R&D into public spending, while keeping the latter constant, through a decrease of the unproductive public consumption share, the growth rate of the economy would shift from 1.5% to 2.09% corresponding to a 9.98 billions euros GDP benefit for one full year (for a slight decrease by -0.68% in public consumption).[12]

Noticing that a permanent increase in the relative weight of military R&D, affects not only the current but the permanent growth rate of the economy, one needs to compute the discounted value of all future GDP increases associated with the initial change.

Let us denote with z_0 the GDP of the current year, with z_t the GDP of year t and with z'_t the GDP after the increase in the share of military R&D: $\sigma'_m = 2\sigma_m$; the GDP benefit associated to the economic policy change is equal, at time t, to: $z'_t - z_t$. Denoting γ' the annual growth rate corresponding to σ'_m, we have $\gamma' = \beta R'$ (Euler equation) along the new regular growth path, where $R' \equiv \Delta + (1-\tau)\,r'$; at the stationary state, we just need to discount future GDP benefits, to get the intertemporal benefit Γ_T over T years:

$$\Gamma_T \equiv \sum_{t=0}^{T} \frac{z'_t - z_t}{R'^t} = z_0 \sum_{t=0}^{T} \frac{\gamma'^t - \gamma^t}{\left(\gamma'/\beta\right)^t} \tag{40}$$

since $z_t = z_0\gamma^t$ and $z'_t = z_0\gamma'^t$ along the regular growth path. Considering that $z_0 = 1691$ billions euros (French 2005 GDP) and applying the formula (40), we find that a limited 3.65 billions euros permanent reallocation of public spending, $\left(\sigma'_m - \sigma_m\right)\tau z_0 = 3.65$, from civilian unproductive public consumption toward military R&D investment, induces a 380 billions euros discounted benefit over a decade ($T = 10$).[13]

It is worth noting that this result would remain the same with a budget reallocation going from military non-R&D expenditures toward military R&D; from this point of view a decrease in σ_n is strictly equivalent to a decrease in σ_b as far as it is used to double the relative weight σ_m of military R&D inside global spending. However the decrease in σ_n, from 0.0526 to 0.0476 (-9.5%), which would be necessary to keep constant the global public spending despite the increase of military R&D investment, is far higher and more perceptible than the corresponding 0.7312 to 0.7262 decrease of σ_b (-0.68%). In other words,

[12] *i.e.*, a raise of the growth rate close to 40%.

[13] In the case $T = +\infty$, one gets:

$$\Gamma_{+\infty} = z_0 \frac{\beta}{1-\beta} \frac{\gamma' - \gamma}{\gamma' - \beta\gamma}$$

it is always possible, through an internal reallocation of the defense budget, to reach the same level of benefits than one could get with a budget reallocation going from civilian expenditures toward military R&D, but this implies a higher level of "effort" for the military sector than it would cost to the civilian sector.

6.3 Optimal policy

The purpose of this subsection is to compute the optimal economic policy, *i.e.*, the tax rate and the public spending shares – $(\tau^*, \sigma_a^*, \sigma_b^*, \sigma_m^*, \sigma_n^*)$ – maximizing the stationary state global welfare, considering as given all the exogenous parameters i, Δk, α, α_a α_m, β_m, c_u, c_v, c_w and θ. Applying the method described in Proposition 3, one gets

$$x^* = 3.8192$$

and eventually:

$$(\tau^*, \sigma_a^*, \sigma_b^*, \sigma_m^*, \sigma_n^*) = (11.25\%, 81.70\%, 9.73\%, 8.21\%, 0.36\%)$$

With these optimal values for the economic policy variables, a tax rate under 12%, would lead to an optimal growth factor γ equal to 1.0561, corresponding to an annual growth rate around 5.6%.

Despite a very prudential set of assumptions (see Section 6.1) concerning the productive impacts of military R&D, a small effect of the defense good into the utility function and equal weights for private and public consumption goods ($c_u = c_v$), one gets a large gap between "productive" expenditures on the one hand ($\sigma_a = 81.70\%$, $\sigma_m = 8.21\%$) and non-productive public expenditures on the other hand ($\sigma_b = 9.73\%$, $\sigma_n = 0.36\%$). This result is a well-known characteristic of endogenous growth models where productive externalities play a central role: in such models, investments in R&D, education or public substructures are associated with high long-run returns because they induce productive externalities which durably improve the efficiency of the production process and, consequently, the social welfare. In such an economic word, if policy makers do not care with electoral cycles, but only with long-run growth and welfare, they must minimize the level of unproductive investments,[14] to strongly invest in R&D, networks substructures and education.

7 Conclusion

The general equilibrium endogenous growth model presented in this paper emphasizes the key role played by public military R&D investments in determining the long-run levels of economic growth and welfare. While inspired by Shieh & *alii* [2002], it departs from the latter by using more general specifications (CES functions), distinguishing unproductive defense services from military R&D, taking into account the impact – through the quality of national defense – of military research on the household utility function, allowing us to compute the

[14] As, for instance, civilian or military bureaucracies.

optimal degree of military R&D and to proceed to a numerical simulation on French data.

From a theoretical point of view we obtain four main results: *(i)* the equilibrium path is unique, *(ii)* market imperfections (externalities + taxes) make the equilibrium inefficient under an arbitrary policy, *(iii)* an opportune fiscal policy (tax rate + public spending shares) can restore the second best, *(iv)* military research matters more than the ordinary milex in order to achieve the social welfare target in the long run. This last point is crucial as it stresses how the military research, as a productive externality, is a powerful engine for growth, compared to alternative policies, such as, public consumption or ordinary milex. This depends on the fact that in an endogenous growth framework, knowledge accumulation has a dramatic and unbounded impact on factors productivity.

A numerical simulation, based on a prudential set of assumptions concerning the global impact of military R&D, shows that a slight 3.65 billions euros permanent reallocation of public spending from civilian unproductive public consumption toward military R&D investment, induces an immediate about 10 billions euros GDP benefit for the first year and a 380 billions euros discounted benefit over a decade. In such a framework, characterized by productive externalities originating in military R&D, the government optimal economic policy should be to massively invest in military R&D.

Even if the previous numerical simulation can be seen as a fruitful illustration of what could be the benefits associated to a moderate public spending reallocation in favor of military R&D, a more general and reliable model should include a full description of the strategic interactions between countries as well as an internalization of the negative long-run externalities potentially associated with high-tech arms production.

On the one hand, one needs to analyze how technology and industrial leaks, through innovations diffusion, can generate international impacts of local military R&D. For example, military R&D investment and endogenous growth could be combined in a dynamic model of capital and arms accumulation, *à la* Zou [1995], using dynamic game theory to formalize arms racing and strategic interactions between countries.

On the other hand, military R&D contributes to the development of high-tech arms – biological or nanotechnological weapons, robotic arms etc. – which can eventually be exported towards under-developed countries characterized by imperfect arms controls; in a new mass terrorism environment this long-run negative externality, must certainly be encompassed in the previous analysis.

8 Appendix

8.1 Appendix 1: Proofs of propositions

Proof of Proposition 1. We need to prove that the equilibrium is always determined, *i.e.*, that the dimension of the stable manifold is less or equal to the number of predetermined variables (here, namely, one). In other words, we rule out any sink configuration (characterized by two eigenvalues inside the unit circle). As $\varepsilon_2 < 0$, (29) implies:

$$D = T - 1 - \frac{\rho y}{\gamma}\left(\frac{\tau}{1-\tau}\frac{1}{\gamma} - \frac{1}{x}\frac{\varepsilon_u \varepsilon_2}{\Delta_k + \rho}\right) < T - 1 \tag{41}$$

It is easy to show that the two eigenvalues lie inside the unit circle if and only if $D < 1$, $D > T - 1$ and $D < -T - 1$. As inequality (41) violates the second condition, any sink configuration turns out to be impossible: at least one eigenvalue lies outside the unit circle and the uniqueness of the equilibrium, if any, is guaranteed. ■

Proof of Proposition 2. In the (T, D)-plane, the saddle points match with the two areas:

$$\begin{aligned} -T - 1 &< D < T - 1 \\ T - 1 &< D < -T - 1 \end{aligned}$$

We knows, from Proposition 1, that $D < T - 1$. To show that the stationary state is a saddle point, one needs only to prove that $D > -T - 1$. Substituting formulas (28) and (29) into $D > -T - 1$, one gets the following condition:

$$2 + 2\left[\frac{\Delta_k + \rho}{\gamma} - \frac{\rho}{\gamma}\frac{\tau}{1-\tau}\left(\frac{y}{\gamma} + x\right)\right] + \frac{\rho}{\gamma}\left(\frac{\tau}{1-\tau}\frac{y}{\gamma} - \frac{\varepsilon_u \varepsilon_2}{\Delta_k + \rho}\frac{y}{x}\right) > 0$$

or, equivalently:

$$\gamma(1-\tau)\left[2x(\Delta_k + \rho)(\gamma + \Delta_k + \rho) - \rho y \varepsilon_u \varepsilon_2\right] - \rho \tau x (y + 2x\gamma)(\Delta_k + \rho) > 0 \tag{42}$$

Noticing that $\rho = (1-\tau)\varphi'$ and $\tau\varphi = \gamma - \Delta$, and dividing (42) by $1 - \tau$, yields the equivalent condition:

$$2x\gamma(\Delta_k + \rho)(\gamma + \Delta_k + \rho) - \rho y \gamma \varepsilon_u \varepsilon_2 - \varepsilon_1(\gamma - \Delta)(\Delta_k + \rho)(y + 2x\gamma) > 0 \tag{43}$$

where $\varepsilon_1 \equiv x\varphi'/\varphi \in (0, 1)$. In order to show that (43) is satisfied, we observe that $\Delta < \gamma$. We have immediately:

$$2x\gamma\left[(1+\varepsilon_1)\Delta + (1-\varepsilon_1)\gamma\right]/\varphi + \varepsilon_1\left[\tau x(\gamma - \Delta) + (1-\tau)(\gamma + \Delta)\right] > 0 \tag{44}$$

Multiplying both sides by φ, applying the definition of ε_1 and rearranging, yields the following inequality:

$$2x\gamma\left[\gamma + \Delta + (1-\tau)\varphi'\right] - \varepsilon_1(\gamma - \Delta)\left[(1 - \tau - \tau x)\varphi + 2x\gamma\right] > 0 \tag{45}$$

Using (25), and Assumption 3, we get $y = (1 - \tau - \tau x)\varphi$, while noticing that $\rho = (1-\tau)\varphi'$, we rewrite (45):

$$2x\gamma(\gamma + \Delta + \rho) - \varepsilon_1(\gamma - \Delta)(y + 2x\gamma) > 0 \tag{46}$$

Multiplying (46) by $\Delta_k + \rho$ and adding the positive term $-\rho y \gamma \varepsilon_u \varepsilon_2$, one eventually gets (43) under Assumption 3. ■

Proof of Proposition 3. Before maximizing, we need to compute the welfare function (utility function) along the balanced growth path: $(c_t, b_t, m_t, n_t) = (c_0, b_0, m_0, n_0)\,\gamma^t$, where γ is the common (regular) growth factor:

$$e_t \equiv e\,(m_t, n_t) = e\left(m_0\gamma^t, n_0\gamma^t\right) = e\,(m_0, n_0)\,\gamma^t = e_0\gamma^t$$

(notice that the defence good production function is supposed to be homogeneous of degree one). Denoting $e_0 \equiv e\,(m_0, n_0)$, one gets under restriction (30):

$$\begin{aligned}
W &= c_u \sum_{t=0}^{\infty} \beta^t \ln\left(c_0\gamma^t\right) + c_v \sum_{t=0}^{\infty} \beta^t \ln\left(b_0\gamma^t\right) + c_w \sum_{t=0}^{\infty} \beta^t \ln\left(e_0\gamma^t\right) \\
&= (c_u \ln c_0 + c_v \ln b_0 + c_w \ln e_0) \sum_{t=0}^{\infty} \beta^t + (c_u + c_v + c_w) \ln\gamma \sum_{t=0}^{\infty} \beta^t t \\
&= \frac{1}{1-\beta}\left(c_u \ln c_0 + c_v \ln b_0 + c_w \ln e_0 + \frac{\beta}{1-\beta}\ln\gamma\right)
\end{aligned}$$

Equilibrium uniqueness under rational expectations (Proposition 1) requires c_0, b_0, e_0 to be compatible with the stationary state γ characterizing the regular growth. Definition (9) concerning the economic policy implies at the beginning: $(a_0, b_0, m_0, n_0) = (\sigma_a, \sigma_b, \sigma_m, \sigma_n)\,g_0$ and $e_0 = e\,(m_0, n_0) = e\,(\sigma_m g_0, \sigma_n g_0) = e\,(\sigma_m, \sigma_n)\,g_0$. From definition (18) one gets $c_0 = y g_0$. The endogenous growth of steady state implies a regular growth path; under restriction (30) we obtain:

$$\begin{aligned}
W &= \frac{c_u \ln(y g_0) + c_v \ln(\sigma_b g_0) + c_w \ln\left(e\,(\sigma_m, \sigma_n)\,g_0\right)}{1-\beta} + \frac{\beta}{(1-\beta)^2}\ln\gamma \\
&= \frac{1}{1-\beta}\left(c_u \ln(y g_0) + c_v \ln(\sigma_b g_0) + c_w \ln\left[e\,(\sigma_m, \sigma_n)\,g_0\right] + \frac{\beta}{1-\beta}\ln\gamma\right) \\
&= \frac{1}{1-\beta}\left[c_u \ln y + c_v \ln\sigma_b + c_w \ln e\,(\sigma_m, \sigma_n) + \ln g_0 + \frac{\beta}{1-\beta}\ln\gamma\right]
\end{aligned}$$

where $g_0 \equiv a_0 + b_0 + m_0 + n_0$ is an initial condition.

As β and g_0 aren't in the set of choice variables, the problem of maximizing W turns out to be equivalent to the following:

$$\max\left[c_u \ln y + c_v \ln\sigma_b + c_w \ln e\,(\sigma_m, \sigma_n) + \frac{\beta}{1-\beta}\ln\gamma\right] \tag{47}$$

Under Assumption 4, the policy of public spending (9) entails:

$$\begin{aligned}
f\,(\kappa, a, m) &\equiv F\,(\kappa, 1, a, m) = \theta\kappa^{\alpha} a^{\alpha_a} m^{\alpha_m} \\
\varphi\,(x_t) &= f\,(\kappa_t, a_t, m_t)\,/g_t = A\,(\kappa_t/g_t)^{\alpha}\,(a_t/g_t)^{\alpha_a}\,(m_t/g_t)^{\alpha_m} = \theta\sigma_a^{\alpha_a}\sigma_m^{\alpha_m}x_t^{\alpha} \\
\varphi'\,(x_t) &= \alpha\theta\sigma_a^{\alpha_a}\sigma_m^{\alpha_m}x_t^{\alpha-1}
\end{aligned}$$

Still under Assumption 4 we have: $\varepsilon_1 \equiv x\varphi'/\varphi = \alpha$.

Since $\varepsilon_u = 1$, one gets from (24) an implicit equation defining the stationary state x:

$$\Delta + \tau\theta\sigma_a^{\alpha_a}\sigma_m^{\alpha_m}x^{\alpha} = \beta\left[\Delta_k + (1-\tau)\,\alpha\theta\sigma_a^{\alpha_a}\sigma_m^{\alpha_m}x^{\alpha-1}\right]$$

Taking into account that $\tau\varphi = \gamma - \Delta$, equation (24) becomes

$$\gamma = \beta \left[\Delta_k + \frac{1-\tau}{\tau} \frac{1}{x} \frac{x\varphi'}{\varphi} (\gamma - \Delta) \right] \tag{48}$$

Substituting ε_1 into equation (48) and solving for γ, the growth factor is now explicitly computed:

$$\gamma = \beta \frac{\Delta (1-\tau) \varepsilon_1 - \Delta_k \tau x}{(1-\tau) \beta \varepsilon_1 - \tau x} \tag{49}$$

From (25) and (49), problem (47) is restated as follows:

$$\begin{aligned} & \max c_v \ln \sigma_b + c_w \ln e(\sigma_m, \sigma_n) + c_u \ln [(1 - \tau - \tau x) \varphi(x) + (\Delta_k - \Delta) x] \\ & + \frac{\beta}{1-\beta} \ln \left[\beta \frac{\Delta (1-\tau) \varepsilon_1 - \Delta_k \tau x}{(1-\tau) \beta \varepsilon_1 - \tau x} \right] \end{aligned} \tag{50}$$

with

$$\frac{\Delta \varepsilon_1 (1-\tau) - \Delta_k \tau x}{\beta \varepsilon_1 (1-\tau) - \tau x} = \Delta_k + (1-\tau) \varphi'(x)$$

Noticing that $\tau\varphi = \gamma - \Delta$ implies

$$\varphi(x) = \frac{(\Delta - \beta \Delta_k) x}{\beta \varepsilon_1 (1-\tau) - \tau x} \tag{51}$$

and substituting $\varphi(x)$ into (50) yields:

$$\begin{aligned} & \max c_u \ln x + c_v \ln \sigma_b + c_w \ln e(\sigma_m, \sigma_n) \\ & + c_u \ln \left[(\Delta - \beta \Delta_k) \frac{1 - \tau - \tau x}{\beta \varepsilon_1 (1-\tau) - \tau x} + \Delta_k - \Delta \right] \\ & + \frac{\beta}{1-\beta} \ln \left[\beta \frac{\Delta \varepsilon_1 (1-\tau) - \Delta_k \tau x}{\beta \varepsilon_1 (1-\tau) - \tau x} \right] \end{aligned}$$

or, equivalently under Assumption 5,

$$\begin{aligned} & \max c_u \ln x + c_v \ln \sigma_b + c_w \ln e(\sigma_m, \sigma_n) + c_u \ln [(1-\beta) \Delta] \\ & + \frac{\beta}{1-\beta} \ln (\beta \Delta) + c_u \ln \frac{1 - \tau - \tau x}{\beta \varepsilon_1 (1-\tau) - \tau x} + \frac{\beta}{1-\beta} \ln \frac{\varepsilon_1 (1-\tau) - \tau x}{\beta \varepsilon_1 (1-\tau) - \tau x} \end{aligned}$$

Observing that $\sigma_b = 1 - \sigma_a - \sigma_m - \sigma_n$ (see restriction (10)) and that β and Δ are not maximization arguments, we restate the program:

$$\begin{aligned} & \max c_u \ln x + c_w \ln e(\sigma_m, \sigma_n) + c_v \ln (1 - \sigma_a - \sigma_m - \sigma_n) + c_u \ln (1 - \tau - \tau x) \\ & + \frac{\beta}{1-\beta} \ln [\varepsilon_1 (1-\tau) - \tau x] - \left(c_u + \frac{\beta}{1-\beta} \right) \ln [\beta \varepsilon_1 (1-\tau) - \tau x] \end{aligned}$$

Under Assumption 4, the implicit equation (51) becomes,

$$\theta \sigma_a^{\alpha_a} \sigma_m^{\alpha_m} x^{\alpha} = \frac{(1-\beta) \Delta x}{\beta \varepsilon_1 (1-\tau) - \tau x}$$

This equation defines locally a function $x = x(\tau, \sigma_a, \sigma_m)$. We thus need to maximize an increasing function $\tilde{W}$ of the initial welfare function W with respect to variables $(\tau, \sigma_a, \sigma_m, \sigma_n)$:

$$\begin{aligned}
\tilde{W} \equiv\ & c_u \ln x(\tau, \sigma_a, \sigma_m) + c_w \ln e(\sigma_m, \sigma_n) + c_v \ln(1 - \sigma_a - \sigma_m - \sigma_n) \\
& + c_u \ln(1 - \tau - \tau x(\tau, \sigma_a, \sigma_m)) + \frac{\beta}{1-\beta} \ln[(1-\tau)\varepsilon_1 - \tau x(\tau, \sigma_a, \sigma_m)] \\
& - \left(c_u + \frac{\beta}{1-\beta}\right) \ln[(1-\tau)\beta\varepsilon_1 - \tau x(\tau, \sigma_a, \sigma_m)]
\end{aligned}$$

To simplify the writing, we will denote the partial derivatives as follows:

$$x_\tau \equiv \frac{\partial x}{\partial \tau}, x_a \equiv \frac{\partial x}{\partial \sigma_a}, x_m \equiv \frac{\partial x}{\partial \sigma_m}, e_m \equiv \frac{\partial e}{\partial \sigma_m}, e_n \equiv \frac{\partial e}{\partial \sigma_n}$$

The program encompasses now all the restrictions and we have to solve an unconstrained optimization problem; setting the gradient equal to zero:[15]

$$\frac{\partial \tilde{W}}{\partial \tau} = \frac{\partial \tilde{W}}{\partial \sigma_a} = \frac{\partial \tilde{W}}{\partial \sigma_m} = \frac{\partial \tilde{W}}{\partial \sigma_n} = 0$$

we get, respectively:

$$\begin{aligned}
& c_u \frac{x_\tau}{x} - c_u \frac{1 + x + \tau x_\tau}{1 - \tau - \tau x} - \frac{\beta}{1-\beta} \frac{x + \tau x_\tau + \varepsilon_1}{\varepsilon_1(1-\tau) - \tau x} + \left(c_u + \frac{\beta}{1-\beta}\right) \frac{x + \tau x_\tau + \beta\varepsilon_1}{\beta\varepsilon_1(1-\tau) - \tau x} \\
=\ & 0 \qquad (52) \\
& c_u \frac{x_a}{x} - c_u \frac{\tau x_a}{1 - \tau - \tau x} - \frac{\beta}{1-\beta} \frac{\tau x_a}{\varepsilon_1(1-\tau) - \tau x} + \left(c_u + \frac{\beta}{1-\beta}\right) \frac{\tau x_a}{\beta\varepsilon_1(1-\tau) - \tau x} \\
=\ & \frac{c_v}{1 - \sigma_a - \sigma_m - \sigma_n} \qquad (53) \\
& c_u \frac{x_m}{x} - c_u \frac{\tau x_m}{1 - \tau - \tau x} - \frac{\beta}{1-\beta} \frac{\tau x_m}{\varepsilon_1(1-\tau) - \tau x} + \left(c_u + \frac{\beta}{1-\beta}\right) \frac{\tau x_m}{\beta\varepsilon_1(1-\tau) - \tau x} \\
=\ & \frac{c_v}{1 - \sigma_a - \sigma_m - \sigma_n} - c_w \frac{e_m}{e} \qquad (54)
\end{aligned}$$

and

$$c_w \frac{e_n}{e} = \frac{c_v}{1 - \sigma_a - \sigma_m - \sigma_n} \qquad (55)$$

Using (53), (54) and (55), we have

$$\begin{aligned}
& c_u \frac{1}{x} - c_u \frac{\tau}{1 - \tau - \tau x} - \frac{\beta}{1-\beta} \frac{\tau}{\varepsilon_1(1-\tau) - \tau x} + \left(c_u + \frac{\beta}{1-\beta}\right) \frac{\tau}{\beta\varepsilon_1(1-\tau) - \tau x} \\
=\ & \frac{c_w}{x_m} \frac{e_n}{e} - \frac{c_w}{x_m} \frac{e_m}{e} = \frac{c_w}{x_a} \frac{e_n}{e} \qquad (56)
\end{aligned}$$

[15] Concerning the second order conditions, the concavity of the initial problem guarantees that the Hessian matrix of $\tilde{W}$ with respect to $(\tau, \sigma_a, \sigma_m, \sigma_n)$ is negative semi-definite.

and

$$\frac{x_m}{x_a} = 1 - \frac{e_m}{e_n} \tag{57}$$

The partial derivatives x_τ, x_a, x_m are straightforwardly computed by totally differentiating

$$\varphi(x, \sigma_a, \sigma_m) = \frac{(1-\beta)\,\Delta x}{\beta\varepsilon_1(1-\tau) - \tau x}$$

and recalling that $\varphi(x, \sigma_a, \sigma_m) = \theta\sigma_a^{\alpha_a}\sigma_m^{\alpha_m}x^{\alpha}$. Noticing that $\varepsilon_1 = \alpha$, $\sigma_a\varphi_a/\varphi = \alpha_a$, $\sigma_m\varphi_m/\varphi = \alpha_m$, we obtain:

$$\begin{aligned}
x_\tau &\equiv \frac{\partial x}{\partial \tau} = -\frac{x}{\varepsilon_1}\frac{\beta\varepsilon_1 + x}{\beta(1-\varepsilon_1)(1-\tau) + \tau x} && (58)\\
x_a &\equiv \frac{\partial x}{\partial \sigma_a} = \frac{\alpha_a}{\sigma_a}\frac{x}{\varepsilon_1}\frac{1}{\beta(1-\varepsilon_1)(1-\tau) + \tau x} && (59)\\
x_m &\equiv \frac{\partial x}{\partial \sigma_m} = \frac{\alpha_m}{\sigma_m}\frac{x}{\varepsilon_1}\frac{1}{\beta(1-\varepsilon_1)(1-\tau) + \tau x} && (60)
\end{aligned}$$

Equation (57) gives

$$\frac{\alpha_m\sigma_a}{\alpha_a\sigma_m} = 1 - \frac{e_m}{e_n} \tag{61}$$

According to Assumption 4, we observe that:

$$\begin{aligned}
e_m/e &= \beta_m/\sigma_m \\
e_n/e &= \beta_n/\sigma_n && (62)\\
e_m/e_n &= \beta_m\sigma_n/(\beta_n\sigma_m) && (63)
\end{aligned}$$

Using (62) and (63), we compute σ_m and σ_n from system (55-61) as linear functions of σ_a (equations (34) and (35)). Substituting (58) into (52) and solving for τ, we get the latter as function of x (equation (32)); clearly we keep the meaningful solution $\tau(x) \in [0,1]$. Replacing (59), (34) and (35) into (53), we find σ_a as function of x and τ (equation (33)). Eventually, we obtain the optimal policy solving equation (31) for x, where τ is given by (32), whereas σ_m and σ_a are provided by (34) and (33), respectively. Once x^* has been determined, it remains to compute τ^* from (32), σ_a^* from (33) and, finally, σ_m^*, σ_n^* and σ_b^* from (34), (35) and (10), respectively. ■

8.2 Appendix 2: French tax structure

France, 2005		**Billions euros**	**% of overall taxes**	**Type of expenditure**
National budget		**288.2**	**39.10**	
including	Ministry of Education and Research	72.5	9.84	Civilian investment
	Ministry of Employment. Health and Social Cohesion	50.3	6.82	Public consumption
	Ministry of Defense	42.4	5.75	
	including Military R&D	3.65	0.49	Military R&D expenditures
	Other military expenditures	38.75	5.26	Other military expenditures
	Ministry of Finance and Industry	14.9	2.02	Public consumption
	Ministry of Transport and Infrastructure	14.6	1.98	Civilian investment
	Ministry of the Interior	15.2	2.06	Public consumption
	Ministry of Justice	5.5	0.75	Public consumption
	Ministry of Agriculture	4.9	0.66	Public consumption
	Ministry of Foreign Affairs	4.4	0.60	Public consumption
	Ministry for Ex-Servicemen	3.4	0.46	Public consumption
	Ministry of Arts and Communication	2.8	0.38	Public consumption
	Prime Minister	0.9	0.12	Public consumption
	Ministry of the Environment and Sustainable Development	0.8	0.11	Public consumption
	Ministry of Sports	0.5	0.07	Public consumption
	Common expenditures	55.1	7.48	Public consumption
Local public administrations		**90.41**	**12.27**	
including	Investments	63.29	8.59	Civilian investment
	Consumption	27.12	3.68	Public consumption
Social security administrations		**352.82**	**47.87**	Public consumption
European Union		**5.66**	**0.77**	
including	Investments	5.32	0.72	Civilian investment
	Consumption	0.34	0.05	Public consumption
Overall taxes		**737.09**	**100**	

Table 2. Breakdown of all taxes paid by French citizens by type of expenditure (2005)

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