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Can Using Interest Rates to Check Domestic Demand Raise the Strength of the Sterling in the Long Run?

Eleni ILIOPOULOS & Marcus MILLER

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Can using interest rates to check domestic demand raise the strength of the sterling in the long run?*

Eleni Iliopulos[†]

University of Pavia and EPEE, University of Evry

Marcus Miller

University of Warwick and CEPR

March 14, 2006

Abstract

An updated version of Krugman's (1993) MMF framework is used to consider the implications of buoyant domestic demand for the real exchange rate and debt dynamics. The updating includes a Taylor rule for monetary policy and explicit treatment of external assets and liabilities.

In response to an exogenous rise in the aggregate demand, short-run appreciation of the real exchange rate is followed by a prolonged decline as external debt accumulates and net wealth deteriorates. Whether in equilibrium the real exchange rate is stronger or weaker depends crucially on a comparison of real interest rates and the growth rate. If the domestic growth rate is higher than

[†]Corresponding author: e_iliopulos@eco.unipv.it

global real interest rates, the currency may strengthen in the long run despite the deterioration of net external assets.

To see whether the strength of sterling is sustainable, the analysis is briefly calibrated to UK data over the last decade. Blanchard, Giavazzi and Sa (2005) suggest that international liabilities to be treated as imperfect substitutes: so we check to see how this would affect our results.

JEL numbers: F31, F32, F41.

Keywords: international macroeconomics, debt dynamics, open economy, real exchange rate dynamics.

1 Introduction

In his Frank Graham's lecture of 1993 Paul Krugman recommended a MMF or "modified Mundell-Fleming model" as a useful workhorse for policy issues in international macroeconomics. In this essay we update this approach by including three key features which have bulked large in the intervening decade: the use of a Taylor rule instead of a monetary target to guide monetary policy (Taylor, 1993); the presence of significant external borrowing financing accumulated current account deficits[1]; valuation effects (Gourchinas and Rey, 2004; WEO, 2005). This updated framework, where the Central Bank manipulates real interest rates to stabilise the economy, is used to study the impact of demand shocks on the real exchange rate in the short and long run and to study the strength of the sterling.

The importance of tracking the course of indebtedness has been emphasised in two papers by Leith, and Wren-Lewis (2000, 2002). In the former, where a Taylor rule is used to raise real interest rates when inflation is above target in a closed economy, their principal conclusion is that fiscal policy needs to be tightened as and when Government debt increases – otherwise rising real interest rates destabilise debt dynamics. In the latter the analysis is carried over to a two open economies model with floating exchange rates. Their elegant analysis involves two special assumptions; that agents have access to perfect capital markets so consumption depends on wealth and not current income; that price setting is as described in Calvo (1983) where the price level is sluggish but not inflation itself.

As with Leith and Wren-Lewis (LWL), we find that the response to a domestic demand shock is a rise in the real interest rates and real exchange rate, which is reversed in the long run– but there has to be a sufficiently strong wealth effect to ensure macroeconomic stability.

We provide an application, namely the UK under current policies. We show how the current imbalances do not necessarily lead to a long term weakening of the currency. In fact, the strength of the sterling depends crucially on the interaction of UK real growth rate and real interest rates; if UK real

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[1] WEO 2005 (chapter 3) discusses how countries have been more prompt to accumulate external liabilities in times of globalization.

interest rates are smaller than its growth rate, long-term equilibrium is consistent with a strong currency. In fact, the accumulation of trade deficits due to the strong currency is offset by the strong growth of the economy. The result is reversed when real interest rates are larger than the growth rate.

Blanchard, Giavazzi and Sa (2005) stress that international liabilities are not necessarily perfect substitutes. In order to take into account for their considerations, we incorporate in our model the effect of home bias. We find that introducing home bias in our model implies smaller imbalances in the long run.

2 Stabilising feedback

Consider a small open economy with no inflation and which does not therefore suffer from Original Sin i.e., it can borrow internationally in its own domestic currency (Eichengreen, Hausmann and Panizza, 2003a). Assume that the real interest rate is used to manage aggregate demand, according to a type of Taylor rule; and that the latter depends on the real exchange rate. Then the dynamics of asset accumulation and the real exchange rate can be analyzed in a two-dimensional dynamic system, as follows.

2.1 Asset dynamics and the real exchange rate

Consider now a country's net international investment position. Changes in net foreign asset positiions are given by net exports of goods and services, current transfers, the investment income balance[2], capital transfers and capital gains (see also WEO (2006), Chapter 3). If we ignore current and capital transfers on account of their relatively small size and we assume that real interest rates are equal to real returns, the accumulation of external indebtedness can be represented by the following equation:

$$\dot{d}(t) = (r(t) - g(t))\left(d(t) + \frac{A(t)}{Q(t)}\right) + \beta q(t) - (r^*(t) - g(t))\frac{A(t)}{Q(t)} + \frac{A(t)}{Q(t)}\dot{q}(t) \tag{1}$$

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[2] Where the sum of the three is the current account balance.

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where d indicates net external indebtedness (ratio to GDP), q is the log difference between the value of the real exchange rate level that takes trade into balance and the current level (relative price of traded goods – high q means uncompetitive exchange rate), A represents gross foreign currency external assets (ratio to GDP), $\frac{A}{Q}$ indicates gross external assets in domestic currency[3], r is the domestic real interest rate, r^* is the rest of the world real interest rate, g is the long run real growth rate and β is the elasticity of trade with respect to the real exchange rate.

Notice in particular that the last term on the right-hand side of equation (1) represents the valuation effects stressed by Gourchinas and Rey (2004). For simplicity we have assumed that all gross external assets are denominated in foreign currency and all gross external liabilities are denominated in domestic currency. This hypothesis fits realistically U.S. data: its liabilities are almost entirely denominated in dollars and most of its assets are in foreign currencies. We consider this semplification a resonable aproximation also for other industrial countries' external positions; industrial countries are in fact generally characterizad by a relarively large share of domestic currency liabilities and large share of foreign currency assets.

Let us now assume $\frac{A(t)}{Q(t)}$ fixed and define for simplicity $c \equiv \frac{A(t)}{Q(t)}$. Then, assuming r^* and g fixed, equation (1) can be rewritten in the following form:

$$\dot{d}(t) = (r(t) - g)d(t) + c(r(t) - r^*) + \beta q(t) + c\dot{q}(t) \tag{2}$$

Let now interest rates be used to stabilise output, i.e. a sort of Taylor rule, so,

$$r(t) - r^* = \alpha(y(t) - \bar{y}) \tag{3}$$

where y indicates real output (in log) and $\bar{y}$ is the output target for monetary policy. Finally, the determination of output depend on the following IS curve, i.e.:

$$y(t) - \bar{y} = -\delta q(t) - \gamma r(t) - \eta d(t) + x(t) \tag{4}$$

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[3]Where Q is the real exchange rate level

Note that in addition to high interest rates and a high exchange rate, external debt exerts a dampening effect on aggregate demand. In addition we have included the variable x to represent an exogenous component of aggregate demand for domestic goods.

Substituting into equation (2) using the Taylor rule (3) for real interest rates and aggregate demand (4) for real output, we obtain the following equation for debt accumulation:

$$\dot{d}(t) = \left(\frac{r^* + \alpha x(t)}{1 + \alpha\gamma} - g - \theta c \right) d(t) - \theta d^2(t) + q(t) (\beta - \varepsilon d(t) - \varepsilon c) + c \left(\frac{+\alpha (x(t) - r^*\gamma)}{1 + \alpha\gamma} \right) + c\dot{q}(t) \quad (5)$$

where $\theta \equiv \alpha\eta/(1 + \alpha\gamma)$, $\varepsilon \equiv \alpha\delta/(1 + \alpha\gamma)$. We also observe that

$$r(t) = \frac{r^* + \alpha x(t) - \alpha\delta q(t) - \alpha\eta d(t)}{1 + \alpha\gamma}$$

is UK real interest rate.

2.2 International financial arbitrage

The second dynamic equation involves the arbitrage condition. Converting the usual uncovered interest parity condition into real terms implies an arbitrage condition:

$$\dot{q}(t) = r^* - r(t) \quad (6)$$

On substituting for r and y, we find:

$$\dot{q}(t) = r^* - r(t) = -\alpha (y(t) - \bar{y}) = -\alpha (-\delta q(t) - \gamma r(t) - \eta d(t) + x(t))$$

So, dropping constant terms and collecting terms in r, we can write

$$\dot{q}(t) = \theta d(t) + \varepsilon q(t) - \left(\frac{\alpha (x(t) - \gamma r^*)}{1 + \alpha\gamma} \right) \quad (7)$$

2.3 External indebtedness and real exchange rate: the dynamics of the system

On combining equation (5) with equation (7) we now derive the dynamics that link external indebtedness to the evolution of the real exchange rate. On substituting equation (7) for valuation effects in equation

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(5), we obtain the following system of differential equations:

$$\dot{d}(t) = \left(\frac{r^* + \alpha x(t)}{1 + \alpha\gamma} - g\right) d(t) - \varepsilon d(t) q(t) - \theta d^2(t) + \beta q(t) \tag{8}$$

$$\dot{q}(t) = \theta d(t) + \varepsilon q(t) - \left(\frac{\alpha(x(t) - \gamma r^*)}{1 + \alpha\gamma}\right) \tag{9}$$

Choosing units so that $\bar{y} = 0$ and and $q(t) = 0$ in equilibrium, and assuming for convenience that $d(t) = 0$, then $\bar{y} = y$ requires that $x(t) = \gamma r^*$. In this case, the constant terms appearing in equation (9) disappears and that in equation (8) becomes simply $r^* - g$. The stationarity schedules for external indebtedness and the real exchange rate are sketched in Figure 1. Notice that at the right of point A on the non-linear locus of stationarity for debt, real interest rates fall below the growth rate. Note also that whenever $\varepsilon d(t) - \beta$ approaches zero, q tends to infinity (i.e. external liabilities follow an explosive path; $\varepsilon d(t) - \beta$ represents the asymptote of the curve). We restrict our analysis by assuming $\varepsilon d(t) - \beta < 0$, because for reasonable parameters[4], $\varepsilon d(t) - \beta = 0$ implies net external liabilities are larger than 200% of GDP. After linearising equation (8) around equilibrium (in which $q(t) = d(t) = 0$), the dynamic system can then be written as:

$$\begin{bmatrix} \dot{d}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} r^* - g & \beta \\ \theta & \varepsilon \end{bmatrix} \begin{bmatrix} d(t) \\ q(t) \end{bmatrix} \tag{10}$$

where the determinant of the Jacobian matrix of (10) is $(r^* - g)\varepsilon - \theta\beta$. While the system will be unstable with no feedback $(\theta = 0)$, there will have a saddle point structure as long as $\theta > (r^* - g)\varepsilon/\beta$. Assuming this condition is satisfied, the dynamics are shown in Figure 2 - where it ensures that the line of stationarity for q is steeper than that for d. Given that in steady state $r^* = r$, the stability condition implies that the extra cost of servicing one unit of external debt, $r - g$, must generate an offsetting improvement in the trade balance in the long term (due to depreciation of the currency induced by the negative wealth effect). Effectively, the negative wealth effect depresses aggregate demand and allows the

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[4] For parameter estimates we have drawn eclectically on Krugman (1988), Leith and Wren-Lewis (2000, 2002), Obstfeld and Rogoff (2000) and Smith and Wickens (1990).

monetary authority to cut interest rates and so the exchange rate.

Figure 2 is constructed on the assumption that $x = \gamma r^*$ so equilibrium is at the origin. Figure 2 also shows the effects of an increase in x both on impact and in equilibrium. We will discuss the effect of an exogenous demand shock in the next section.

3   Impact of demand shocks

Let the economy be affected by a positive demand shock and consider for simplicity the effect of an increase in $x(t)$ such that $x(t) + \Delta x(t) \equiv x > \gamma r^*$. It is easy to show that the steady state of the economy is no longer at the origin. The stationarity schedule for external liabilities is in fact characterized by the following equation:

$$q(t) = \frac{\left(\frac{(r^* + \alpha x)}{1 + \alpha \gamma} - g\right) d(t) - \theta d^2(t)}{\varepsilon d(t) - \beta} \tag{11}$$

where we assume $\theta d(t) - \beta < 0$. As a result of the exogenous shock, the curve (11) continues to pass through the origin as in Figure 2 and 3. Note that the minimum of the curve is characterized by a positive level of debt whenever: $\alpha x > g(1 + \gamma) - r^*$. We assume this condition to be satisfied (see the Mathematical Appendix, A1). Note also that the derivative of (11) is negative; therefore, an exogenous demand shock swivels the curve clockwise (see Figures 2-3). For what concerns the stationarity schedule for real exchange rate, q, the impact of the shock in the exogenous demand component implies a rightward shift. Its slope is not affected.

3.1   Comparative statics

In order to analyze the long-term effects of an exogenous demand shock, we now focus on the steady state. Given the system of differential equations (8-9), steady-state values are given by the following:

$$(d, q)^* = \left(\frac{\beta(x - \gamma r^*)}{\beta \eta + \delta(g - r^*)}, \frac{(x - \gamma r^*)(g - r^*)}{\beta \eta + \delta(g - r^*)} \right)$$

It is possible to show (see (12-13)) that while an increase in x always determines an increase in the steady-state value for debt, its effect on the steady-state value for real exchange rate is not uniquely signed. In fact, if for example $g - r^* > 0$, an increase in x leads to an appreciation of the currency. If instead $g - r^* < 0$, the real exchange rate needs to depreciate. These considerations imply that, as shown in Figure 2 and 3, a positive exogenous demand shock always shifts the saddle path rightward.

$$\left(\frac{\partial d^*}{\partial x(t)} \right) = \frac{\beta}{\beta\eta + \delta(g - r^*)} > 0 \tag{12}$$

$$\left(\frac{\partial q^*}{\partial x(t)} \right) = \frac{(g - r^*)}{\beta\eta + \delta(g - r^*)} \tag{13}$$

Note finally that an increase in η implies a decrease in steady state values for debt and an appreciation of the currency if $g - r^* < 0$, a depreciation if $g - r^* > 0$.

3.2   Transition

We now proceed by focusing the attention on the path of the system towards the steady state. Given the linearized form of the debt schedule:

$$\dot{d} \approx \left(\frac{r^* + \alpha x}{1 + \alpha\gamma} - g - \varepsilon q^* - 2\theta d^* \right) d + q(\beta - \varepsilon d^*)$$

it is then possible to substitute for the steady-state value of the real exchange rate and write the Jacobian matrix of system (8-9) in the following form:

$$\begin{bmatrix} r^* - g - \theta d^* & \beta - \varepsilon d^* \\ \theta & \varepsilon \end{bmatrix}$$

As in Section 2, saddle-path stability requires $\beta\theta > \varepsilon(r^* - g)$. In order to analyze the transition of the system toward steady state, we now focus on the saddle path. Let the general equation for the saddle path (linearized around steady state) be represented by:

$$q\left(t\right) = \frac{v_{21}}{v_{11}} d\left(t\right) + q^* - \frac{v_{21}}{v_{11}} d^* \tag{14}$$

where v_{11} and v_{11} are respectively the first and the second component of the stable eigenvector. It is easy to see that the slope of (14) is always negative and can be represented by the following:

$$\frac{v_{21}}{v_{11}} = \frac{\theta}{\lambda_s - \varepsilon} = \frac{2\theta}{-\varepsilon + r^* - g - \theta d^* - \sqrt{\left(\varepsilon + r^* - g - \theta d^*\right)^2 + 4\left(\beta\theta - \varepsilon(r^* - g)\right)}} < 0 \tag{15}$$

where λ_s is the stable eigenvalue. It is possible to show that an exogenous demand shock (such that $x\left(t\right) > \gamma r^*$) has the effect of swivelling the saddle path anticlockwise; in fact, under very plausable conditions (see Mathematical appendix, A2), the derivative of its slope with respect to x is positive.

The above considerations suggest that an exogenous demand shock shifts the saddle-path rightwards and turns it anti-clockwise; during the adjustment process, external liabilities accumulate. Whether the real exchange rate depreciates in the long run crucially depends upon $(g - r^*)$. If $g - r^* > 0$, the steady state level for the real exchange rate will be more appreciated than before the shock. If $g - r^* < 0$, steady state will be characterized by a weaker currency.

3.3 Trajectory

As a consequence of an exogenous demand shock (such that $x\left(t\right) > \gamma r^*$), the real exchange rate immediately accomodates x and external liabilities in order to keep the economy on a path towards steady state. External liabilities only adjust in the long run. The real exchange rate initially jumps to a level $q(0)$ – which depends on the predetermined value of $d\left(t\right)$ that we indicate as $d(0)$. Eventually, both net external liabilities and the real exchange rate move towards steady state values. It possible to show that the long term adjustment process from $q(0)$ towards steady state always implies a depreciation. In fact:

$$q(0) - q^* = \frac{\theta}{\lambda_s - \varepsilon} \left(d\left(0\right) - d^*\right) > 0$$

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4   Demand shocks, Taylor rules and real exchange rates

An interesting feature of the current situation is that growth rates exceed real interest rates in both US and UK. This is illustrated in the table below where the first line shows yields on indexed debt in the medium and long run. In the US for example these are around 1% in the medium term, slightly less than 2% in the long run i.e. substantially less than current estimates of US growth rate (for which Godley et al. 2004 use a figure of 3.2% for example). For the UK, real interest rates of around one and an half per cent (both medium and long term) are less than the growth of potential GDP, currently estimated at 2.5%. (As a check on these real rates, for the US and the UK we compare them with yields on benchmark bonds to provide the implicit inflation estimates shown in parenthesis in the last row.)

Table 1: **Real and nominal Government Bond Yields, Medium and Long term - Source: FT (04/10/05)**

	US		UK		Germany	
Real	(2008) 1.03	(2028) 1.93	(2009) 1.44	(2024) 1.43	(2009) "0.77"	(2037) "1.63"
Nominal	(2010) 4.25	(2031) 4.62	(2010) 4.23	(2036) 4.27	(2010) 2.77	(2037) 3.63
Inflation(Estimated)	(3.22)	(2.69)	(2.79)	(2.84)	"2.0"	"2.0"

As there is no quoted indexed debt on issue in Germany, real rates are crudely estimated by subtracting an inflation forecast from the nominal yields on benchmark government bonds shown in the table. If German inflation was constant at 2%, for example, this would imply real yields arising from something under 1% in the medium term to about 1.5% in the long term.

4.1 Buoyant exogenous demand, monetary policy and external debt

The strength of sterling over the last few years has been widely attributed to buoyant domestic demand and whose inflationary consequences have been checked by tight monetary policy. What are the implications for external indebtedness and exchange rate in the long run?

Starting from the origin let there be a sustained increase in exogenous demand (which can be attributed both to the rise in public expenditure and to the recent real estate bubble), which triggers a rise of real interest rates as the monetary authorities implement a Taylor rule. This has the effect of pushing up the value of the currency so that in the short run it jumps from the equilibrium, 0, to A, the intersection between the vertical axis and the new stable manifold, S'S' (see Figure 3). The currency appreciation and the associated trade deficit lead to a rise in external liabilities; and, given a sufficiently large wealth effect, demand will subside allowing real interest rates to be reduced, i.e. the economy moves down the stable manifold along which external liabilities accumulate and the real exchange rate slowly depreciates. In equilibrium, the arbitrage condition requires domestic and foreign real interest rates to be equalised, say at 2%. For convenience assume that this is also the growth rate for UK in the long run. The final equilibrium E must then be at a point of trade balance i.e. on the horizontal axis. So the real exchange rate will first rise and then fall to its initial level, despite the increased external indebtedness.

What if UK growth rate lies above the world real interest rate over the long run – as now appears to be the case? In the presence of relatively low real interest rates, countries can sustain a stronger currency than otherwise. Therefore, if present conditions reflect the long term, strong sterling does not imply an explosive growth of UK external imbalances.

To the extent that buoyant domestic demand has been driven by public sector investment, an interesting issue is whether such an equilibrium will occur with government debt peaking at less than 40% of GDP. If not, then it will trigger the Treasury's "investment sustainability rule" designed to restrain demand directly.

5 Effects of an increase in the cost of borrowing

Blanchard et al. (2005) stress home bias, i.e. that more international borrowing in home currency involves paying higher interest rates. Allowing for home bias - in their case issuing more dollars - leads to a fall in the dollar keeping the domestic real interest rate constant. In this application, however, we treat the domestic real rate, r, as endogenous and explore how home bias alters the impact of an exogenous demand shock on indebtedness and competitiveness.

Introducing the hypothesis of imperfect substitutability implies modifying the arbitrage condition such that:

$$\dot{q}(t) = r^* - r(t) + \varphi d(t) \tag{16}$$

where φ represents the effect of home bias on investors' preferences and is related to the net external indebtedness. Following the same logic of Section 3, we now let the economy be affected by a positive exogenous demand shock, such that $x(t) + \Delta x(t) \equiv x > \gamma r^*$. The dynamics of the system are defined by:

$$\dot{d}(t) = \left(\frac{r^* + \alpha x}{1 + \alpha \gamma} - g\right) d(t) - \varepsilon d(t) q(t) - \theta d^2(t) + \beta q(t) \tag{17}$$

$$\dot{q}(t) = (\theta + \varphi) d(t) + \varepsilon q(t) - \frac{\alpha(x - \gamma r^*)}{1 + \alpha \gamma} \tag{18}$$

By comparing system (8-9) - i.e. relative to an exogenous demand shock in absence of home bias - with system (17-18), one difference is clearly evident. Home bias implies a more negatively sloped stationarity schedule for q (sketched in Figure 4 with bold lines, both before and after the exogenous demand shock; stationarity schedules in absence of home bias are represented with the dotted lines).

5.1 Steady state and comparative statics

In order to analyze the long-term effects of an exogenous demand shock in presence of home bias in investors' preferences, we now focus on steady-state values. Given the stationarity schedules for external

liabilities and for the real exchange rate (17-18), the existence of real solutions for steady-state values requires the following condition to hold:

$$\frac{\beta}{\varepsilon}(\theta + \varphi) + g - r^* > \sqrt{4\varphi \frac{\beta}{\varepsilon} \frac{\alpha (x - \gamma r^*)}{1 + \alpha\gamma}} \tag{19}$$

which is verified for φ sufficiently small.

Note that the steady-state value for debt is defined by two roots, d_1^* and d_2^*, suggesting the existence of multiple equilibria; however, one of them can be excluded from the analysis. Under (19) d_1^* is real and therefore,

$$0 < d_1^* < d_2^*$$

It is possible to show that for $\varepsilon d(t) - \beta < 0$, d_1^* is positive and consistent with our analysis, while d_2^* implies $\varepsilon d(t) - \beta > 0$. The steady state of the system is defined by:

$$d_1^* = \frac{\frac{\beta}{\varepsilon}(\theta + \varphi) + g - r^* - \sqrt{\left(\frac{\beta}{\varepsilon}(\theta + \varphi) + g - r^*\right)^2 - 4\varphi\frac{\beta}{\varepsilon}\frac{\alpha(x-\gamma r^*)}{1+\alpha\gamma}}}{2\varphi} \tag{20}$$

$$q_1^* = \frac{\alpha(x - \gamma r^*)}{\varepsilon(1 + \alpha\gamma)} - \frac{\theta + \varphi}{\varepsilon}d_1^* \tag{21}$$

An exogenous demand shock always leads to an increase in external indebtedness also in presence of home bias (the derivative of (20) with respect to x is in fact always positive). As can be seen in Figure 4, the shock shifts the steady state from the origin, O, to a point such as $E\prime$ characterized by a positive amount of net external liabilities. In Figure 4 we consider an increase in aggregate demand which strengthens or does not weaken the exchange rate in the absence of home bias (see the movement from the origin O to E); in the presence of home bias, however, the extent of external debt will be less but the exchange rate will be weaker (see the movement from O to $E\prime$). The basic reason for this is that although there is less debt in equilibrium, the cost of servicing this debt is higher, requiring a larger trade surplus and a lower exchange rate.

If instead we consider an increase in aggregate demand which weakens the exchange rate in the absence of home bias ($r^* > g$), introducing home bias will also lead to a depreciated currency.in the long run. Note finally that whenever home bias is sufficiently large ($\varphi > \frac{\alpha}{d_1^*} \frac{(x-\gamma r^*)}{(1+\alpha\gamma)} - \theta$), an exogenous demand shock always implies a depreciated currency in the long run.

5.2 Transition towards steady state

In order to analyze the transition of the economy toward steady state we now proceed computing the Jackobian matrix of system (17-18) (linearized around steady state). Thus:

$$
J = \begin{bmatrix} r^* - g - (\theta - \varphi) d_1^* & \beta - \varepsilon d_1^* \\ \theta + \varphi & \varepsilon \end{bmatrix}
$$

where saddle-path stability requires:

$$
\frac{\beta}{\varepsilon} (\theta + \varphi) + g - r^* > 2\varphi d_1^* \tag{22}
$$

Note however that condition(19) implies that (22) is always satisfied (see Mathematical appendix, A3). Therefore, if steady state exists, then it is a saddle point. The saddle path towards steady state is qualitatively analogous to the saddle path in absence of home bias (compare in fact equation (23) with (15) letting $\varphi = 0$). Its slope is always negative and it is given by:

$$
\frac{v_{21}}{v_{11}} = \frac{\theta + \varphi}{\lambda_s - \varepsilon} = \frac{2(\theta + \varphi)}{r^* - g - (\theta - \varphi) d_1^* - \varepsilon - \sqrt{(r^* - g - (\theta - \varphi) d_1^* - \varepsilon)^2 + 4(\varphi + \theta)(\beta - d_1^*\varepsilon)}} < 0 \tag{23}
$$

where v_{11} and v_{11} are respectively the first and the second component of the stable eigenvector and λ_s is the stable eigenvalue. In response to a positive exogenous demand shock the real exchange rate adjusts immediately and jumps to a level $q(0)$. Then, external liabilities accumulate during the transition towards steady state. This adjustment process always implies a depreciation of the currency. In fact:

$$q(0) - q_1^* = \left(\frac{\theta + \varphi}{\lambda_s - \varepsilon} \right) (d(0) - d_1^*) > 0$$

5.3 The effects of home bias on the transition process

We now shift the attention towards the impact of different degrees of home bias on the adjustment process. It is possible to show that for reasonable values of the parameters, the introduction of imperfect substitutability always implies a more negatively sloped saddle path: the higher the degree of home bias, the more negatively sloped the curve (for some baseline cases, see tables 2-4). The basic reason is that the higher cost of servicing the debt implies a stronger currency during the adjustment (see arbitrage equation, (16)).(for further discussion on steady-state values see also the Mathematical appendix A4).

We also provide some baseline cases that show the speed of adjustment towards steady state for different degrees of home bias (see Figure 4). Note that for φ significant[5], higher degrees of home bias imply a faster adjustment toward steady state. The reason is that the higher cost of servicing debt implied by home bias acts as a constraint for debt accumulation.

6   Conclusions

We have looked at Taylor rules and debt dynamics in an open economy, emphasising the role played by the wealth effect in the standard case where the real interest rate is greater than the growth rate. In response to an exogenous rise in the aggregate demand, the dynamics indicate that the short run appreciation followed by a depreciation as external debt accumulates and net wealth deteriorates. But when the domestic growth rate exceeds the world real interest rate, the net effect of increased external borrowing is to raise the exchange rate.

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[5]If φ that tends to zero and both x and r^* are relatively large, a marginal increase in home bias makes convergence slower. The reason is that home bias' contstraining effect on external liabilities is more than offset by its effect in keeping interest rates high (and therefore the real exchange rate strong) slowing therefore the adjustment process.

The effect of rising debt on sovereign spreads emphasized by Blanchard et al. can change this result. With sufficiently strong home bias, an exogenous demand will lead to a depreciation of the currency even when the domestic growth rate exceeds the world real interest rate.

A final word of warning: if real interest rates were to return to more normal levels, where the cost of borrowing exceeds the rate of interest for the UK, then extra spending will of course weaken the currency. The UK – and the US – could be in a honeymoon period when more external borrowing is relatively painless; but this honeymoon could end soon.

7   Mathematical appendix

7.1   A1

Consider now the equation defining the stationarity schedule for external liabilities accumulation (see equation 11). We now compute its derivative with respect to $d\left(t\right)$. We obtain:

$$\left(\frac{\partial q\left(t\right)}{\partial d\left(t\right)}\right) = \frac{-\theta\varepsilon\left(1+\gamma\alpha\right)d^{2}\left(t\right)+2\beta\theta\left(1+\gamma\alpha\right)d\left(t\right)+\beta\left(g\left(1+\gamma\alpha\right)-\left(r^{*}+\alpha x\left(t\right)\right)\right)}{\left(1+\gamma\alpha\right)\left(\beta-\varepsilon d\left(t\right)\right)^{2}} \tag{A1}$$

While its denominator is always positive, the sign of A1 depends on the numerator. The optima of the function are given by the following (where we exclude the larger root because it implies $\beta-\varepsilon d\left(t\right)<0$ – we restrict the analysis to $\beta-\varepsilon d\left(t\right)>0$):

$$d_{1,2} = \frac{\beta}{\varepsilon} \pm \sqrt{\left(\frac{\beta}{\varepsilon}\right)^{2}+\frac{\beta}{\varepsilon}\frac{1}{\theta}\left(g-z\right)}$$

where we let $z = \left(r^{*}+\alpha x\right)/\left(1+\alpha\gamma\right)$.

The system of equations (8-9) can present the following features:

1: $z > g + \beta\theta/\varepsilon$, two positive eigenvalues in the dynamic system of equations.

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2: $z < g + \beta\theta/\varepsilon$ one positive and one negative eigenvalue in the dynamic system of equations (saddle path).

2.1 $z < g$ one positive and one negative optimum for d

2.2 $z > g$ two positive optima for d

We focus our attention on case 2.2.

7.2   A2

In order to analyze the effect of an increase in $x(t)$ on the slope of the saddle path (where we let $x(t) + \Delta x(t) \equiv x > \gamma r^*$), we proceed by calculating the derivative of its slope with respect to x.

$$\left(\frac{\partial \frac{v_{21}}{v_{11}}}{\partial x}\right) = \left(\frac{\partial\left(\frac{v_{21}}{v_{11}}\right)}{\partial \lambda_s}\right)\left(\frac{\partial \lambda_s}{\partial d^*}\right)\left(\frac{\partial d^*}{\partial x}\right)$$

$$= \frac{\left(\frac{v_{21}}{v_{11}}\right)^2}{2}\left(1 + \frac{1}{1 - \frac{2\lambda s}{\varepsilon + r^* - g - \theta d^*}}\frac{1}{(\varepsilon + r^* - g - \theta d^* - 2\lambda_s)^2}\right)\frac{1}{\eta + (g - r^*)\,\delta/\beta}$$

It is possible to show that an increase in x has the effect of swivelling the saddle path anti-clockwise whenever at least one of these conditions is satisfied:

$$\varepsilon + r^* - g - \theta d^* \quad > \quad 0$$

$$\frac{1}{2\lambda_s - (\varepsilon + r^* - g - \theta d)} \quad > \quad \frac{(\varepsilon + r^* - g - \theta d^* - 2\lambda_s)^2}{\varepsilon + r^* - g - \theta d^*}$$

$$-(\varepsilon + r^* - g - \theta d^*) \quad > \quad -2\lambda_s$$

$$(\varepsilon + r^* - g - \theta d^* - 2\lambda_s)^3 \quad > \quad -(\varepsilon + r^* - g - \theta d^*)$$

Since they are very general conditions, we can reasonably assume they are satisfied. Thus, the above considerations suggest that an exogenous demand shock implies a rightward shift in the saddle path and an increase in its slope.

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7.3   A3

If a steady state exists, then it is a saddle point.

Proof. The condition of existence of the steady state is

$$\frac{\beta}{\varepsilon}\left(\theta + \varphi\right) + g - r^* > \sqrt{4\varphi\frac{\beta}{\varepsilon}\frac{\alpha\left(x - \gamma r^*\right)}{1 + \alpha\gamma}} \tag{A3}$$

It is easy to show that condition (A3) entails

$$\sqrt{4\varphi\frac{\beta}{\varepsilon}\frac{\alpha\left(x - \gamma r^*\right)}{1 + \alpha\gamma}} > 2\varphi d\left(t\right)$$

and, therefore, $\left(\theta + \varphi\right)\beta/\varepsilon + g - r^* > 2\varphi d\left(t\right)$. Since determinant $D = \varepsilon\left(r^* - g + 2\varphi d\left(t\right)\right) - \beta\left(\theta + \varphi\right)$ the inequality $D < 0$ is satisfied. Then $\lambda_1\lambda_2 < 0$ and the steady state is a saddle. ■

7.4   A4

The sign of the derivative of the steady-state level of the real exchange rate with respect to φ crucially depends upon a critical value for d. If the external liabilities are less than the minimum of the stationarity schedule for external liabilities, d_m, a marginal increase in φ leads to an appreciation. (however, the currency remains weaker than the level that takes trade into balance). If instead $d\left(t\right) > d_m$, a marginal increase in φ always leads to a long run currency depreciation. Given the minimum of the stationarity schedule for external liabilities,

$$d_m = \frac{\beta}{\varepsilon} - \sqrt{\left(\frac{\beta}{\varepsilon}\right)^2 + \frac{\beta}{\varepsilon\theta}\left(g - \frac{r^* + \alpha x}{1 + \alpha\gamma}\right)}$$

it is possible to express the same condition by finding a critical value for φ below which a marginal increase in home bias always lead to a currency depreciation, i.e.:

$$\varphi^c = \frac{1}{d_m}\left(-\varepsilon q\left(t\right) - \theta d_m + \frac{\alpha\left(x - \gamma r^*\right)}{1 + \alpha\gamma}\right)$$

Note finally that if $x\left(t\right) = \gamma r^*$, the steady state is in the origin and is not affected by $\Delta\varphi$.
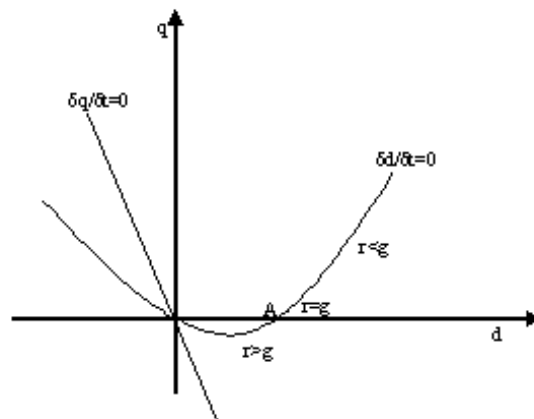
8   Figures and tables

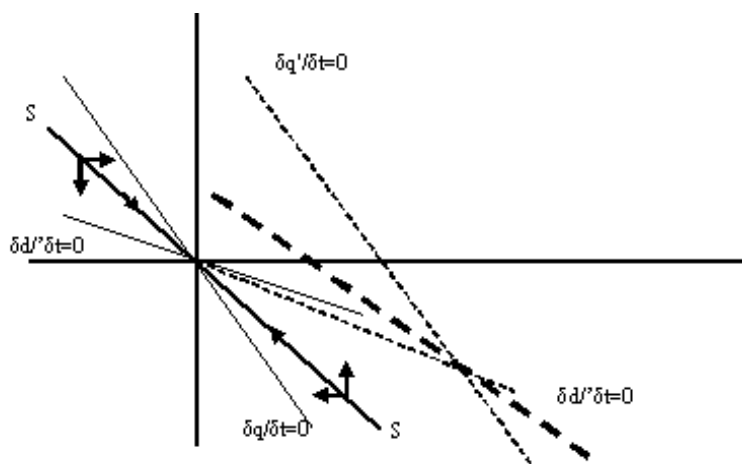Figure 1. Allowing for reversal in $r - g$

Figure 2. Saddlepoint stability when feedback is sufficient
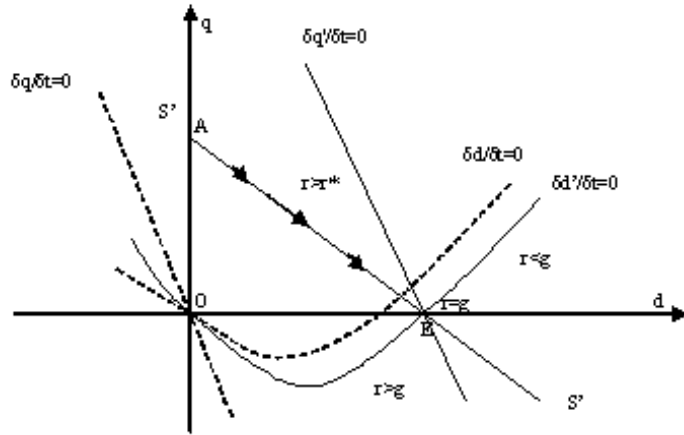
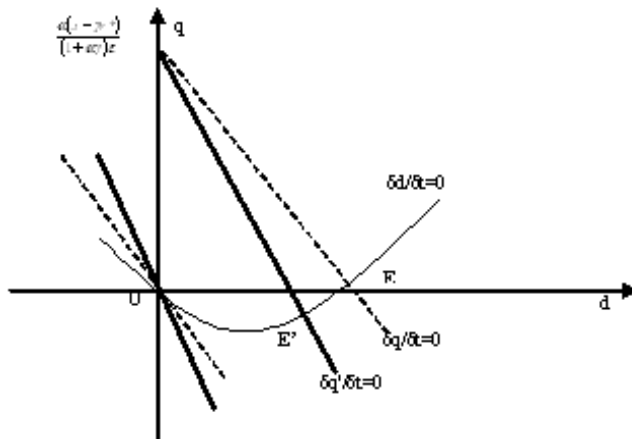Figure 3. UK: an exogenous demand shock in the short and long run

Figure 4. Exogenous demand shock with home bias.

8.1 Simulation results[6]

$$\varphi = 0, r^* = 0.025$$

$$g = 0.025$$

$$x = \gamma r^* + 0.01$$

$$\eta = 0.025$$

$$\varphi = 0.01, r^* = 0.025,$$

$$g = 0.025$$

$$x = \gamma r^* + 0.01$$

$$\eta = 0.025$$

$$\varphi = 0.1, r^* = 0.025,$$

$$g = 0.025$$

$$x = \gamma r^* + 0.01$$

$$\eta = 0.025$$

$$\varphi = 0, r^* = 0.02,$$

$$g = 0.025,$$

$$x = \gamma r^* + 0.01$$

$$\eta = 0.025$$

$$\varphi = 0.01, r^* = 0.02$$

$$g = 0.025$$

$$x = \gamma r^* + 0.01$$

$$\eta = 0.025$$

$$\varphi = 0.1, r^* = 0.02$$

$$g = 0.025$$

$$x = \gamma r^* + 0.01$$

$$\eta = 0.025$$

$$\varphi = 0, r^* = 0.03$$

$$g = 0.025$$

$$x = \gamma r^* + 0.01$$

$$\eta = 0.025$$

$$\varphi = 0.01, r^* = 0.03$$

$$g = 0.025$$

$$x = \gamma r^* + 0.01$$

$$\eta = 0.025$$

$$\varphi = 0.1, r^* = 0.03$$

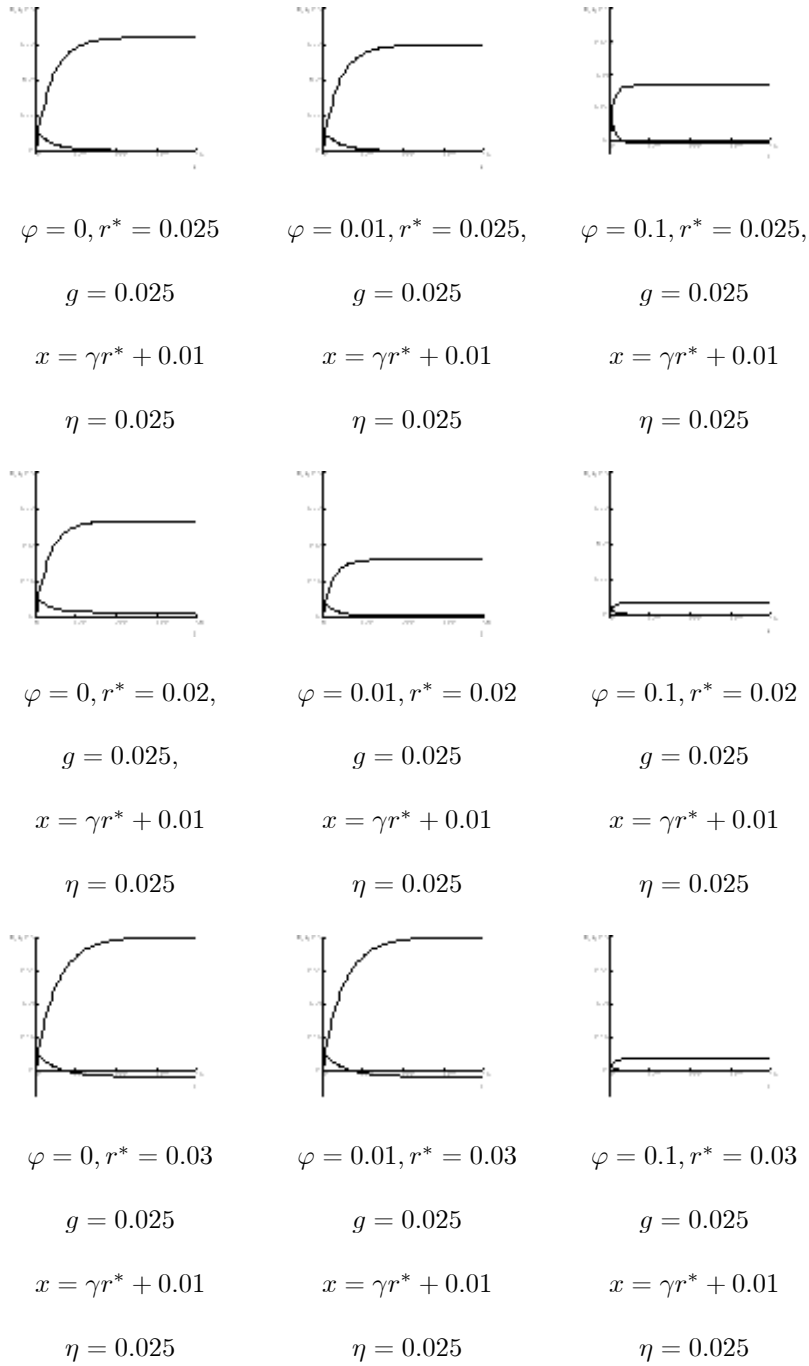$$g = 0.025$$

$$x = \gamma r^* + 0.01$$

$$\eta = 0.025$$

Figure 4. Change in φ for plausible values of $(r^* - g)$ and saddle path.

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[6] We keep $\alpha = 0.5 \beta = 0.1, \gamma = 0.2, \delta = 0.1, (\theta = \frac{\alpha\eta}{1+\alpha\gamma}, \varepsilon = \frac{\alpha\delta}{1+\alpha\gamma})$

Table 2: **change in $(r^* - g)$ and saddle path**

r^*	g	slope for $\varphi = 0$	slope for $\varphi = 0.001$	slope for $\varphi = 0.01$	slope for $\varphi = 0.02$	slope for $\varphi = 0.1$
$r^* = 0.035$	$g = 0.035$	-0.17637	-0.18993	-0.29546	-0.38987	-0.87356
$r^* = 0.03$	$g = 0.03$	-0.17637	-0.18993	-0.29546	-0.38987	-0.87356
$r^* = 0.025$	$g = 0.025$	-0.17637	-0.18993	-0.29546	-0.38987	-0.87356
$r^* = 0.03$	$g = 0.035$	-0.16625	-0.17903	-0.28032	-0.37276	-0.85292
$r^* = 0.025$	$g = 0.03$	-0.16625	-0.17903	-0.28032	-0.37276	-0.852925
$r^* = 0.02$	$g = 0.025$	-0.16625	-0.17903	-0.28032	-0.37276	-0.85292
$r^* = 0.035$	$g = 0.03$	-0.18745	-0.20202	-0.31221	-0.40833	-0.89482
$r^* = 0.03$	$g = 0.025$	-0.18745	-0.20202	-0.31221	-0.40833	-0.89482

Table 3: **Change in exogenous demand shock and the saddle path**

r^*	g	x	slope for $\varphi = 0$	slope for $\varphi = 0.001$	slope for $\varphi = 0.01$	slope for $\varphi = 0.02$	slope for $\varphi = 0.1$
$r^* = 0.03$	$g = 0.025$	$x = \gamma r^* + 0.02$	-0.18375	-0.20045	-0.32544	-0.42754	-1.34112
$r^* = 0.03$	$g = 0.025$	$x = \gamma r^* + 0.001$	-0.19052	-0.20324	-0.30314	-0.39454	-0.87440
$r^* = 0.03$	$g = 0.025$	$x = \gamma r^* + 0.0005$	-0.19069	-0.20331	-0.30270	-0.39385	-0.87331

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Table 4: **Change in wealth effect and the saddle path**

r^*	g	x	η	slope for $\varphi = 0$	slope for $\varphi = 0.001$	slope for $\varphi = 0.01$	slope for $\varphi = 0.02$	slope for $\varphi = 0.1$
$r^* = 0.03$	$g = 0.025$	$x = \gamma r^* + 0.001$	$\eta = 0.025$	-0.18745	-0.20324	-0.30314	-0.39454	-0.87440
$r^* = 0.03$	$g = 0.025$	$x = \gamma r^* + 0.001$	$\eta = 0.027$	-0.20190	-0.21426	-0.31201	-0.40207	-0.87859
$r^* = 0.03$	$g = 0.025$	$x = \gamma r^* + 0.001$	$\eta = 0.03$	-0.21838	-0.23027	-0.32503	-0.41318	-0.88485

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