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A Nonlinear Panel Unit Root Test under Cross Section Dependence

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Abstract

We propose a nonlinear heterogeneous unit root test for testing the null hypothesis of unit-roots processes against the alternative that allows a proportion of units to be generated by globally stationary ESTAR processes and a remaining non-zero proportion to be generated by unit root processes. The proposed test is simple to apply and accommodates cross sectional dependence. Monte Carlo simulation shows that our test holds correct size and under the hypothesis that data are generated by globally stationary ESTAR processes has a better power than the recent test proposed in Pesaran (2005).An application to a panel of bilateral real exchange rate series with the US Dollar from the 11 major OECD countries is provided.

JEL Classification: C12, C15, C22
Key Words: Non-linear panel unit root tests, cross sectional dependence, Purchasing Power Parity.

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1. Introduction

There is now a large literature on testing for a unit root in time series and panel data. The growth in that area is mainly due to empirical applications on, for example, Purchasing Power Parity (PPP) and Growth (see Cerrato and Sarantis, 2007 and Emerson and Kao, 2006 amongst the others).

Standard univariate and multivariate unit root tests can be expected to have low power if the time series contain a nonlinear type of dynamics (e.g. structural breaks). If in the univariate time series literature this problem has been extensively investigated, recently researchers have started to address this problem in the context of multivariate unit root tests (see amongst the others Bai and Silvestre, 2005; Tzavalis, 2002).

However, in many practical applications a smooth change in the level and/or time trend might be preferable to a structural break and as a consequence they suggest a Panel Logistic Smooth Transition Autoregressive Test.

Recently two novel unit root tests have been developed by Kapetanios et al. (2003) and Pesaran (2005) respectively. The former test is a univariate non-linear unit root test while the latter is a panel unit root test. Kapetanios *et al.*. (2003) develop a formal unit root test against the alternative of nonlinear mean reversion by considering an ESTAR model and they show that, under the null hypothesis, the distribution of the test is not normal and therefore provide critical values.

Pesaran (2005), on the other hand, extends the Im *et al.* (2003) panel unit root test to account for heterogeneous cross-section dependence. The individual CADF (Cross Augmented Dickey Fuller) and the panel statistic (CIPS) have non-normal distributions, so their critical values (for different N and T) are obtained by Monte Carlo simulations. The panel unit root test proposed by Pesaran (2005) differs from other tests such as Choi (2001) in that while the latter all assume that individual time series are independent, the former proposes a novel method to account for cross sectional dependence. In fact, Pesaran (2005) shows that cross sectional dependence can be accounted for by augmenting the standard DF regression with the cross section averages of lagged levels and first differences of the individual series. In this paper we propose a novel nonlinear panel unit root test. The proposed test is a direct extension of the ESTAR test proposed in Kapetanios et al. (2003) to a panel setting.

2. A Nonlinear Dynamic Panel with Cross-Section Dependence

Suppose the observation y_{it} on the i^{th} cross-section unit at time t is generated according to the dynamic nonlinear heterogeneous panel ESTAR model below:

$$y_{it} = \beta_i y_{i,t-1} + \upsilon_i y_{i,t-1} Z(\theta_i; y_{i,t-d}) + u_{it}, \; t = 1, \ldots, T, \; i = 1, \ldots, N, \tag{1a}$$

where initial value, y_{i0}, is given, and the error term, u_{it}, has the one-factor structure:

$$u_{it} = \gamma_i f_t + \varepsilon_{it} \qquad \varepsilon_{it} \approx \text{i.i.d.}(0, \sigma^2_{\varepsilon i}) \tag{1b}$$

in which f_t is the unobserved common effect, and ε_{it} is the individual-specific (idiosyncratic) error. Following the literature on STAR models, the transition function adopted here is of the exponential form, i.e.,

$$Z(\theta_i; y_{i,t-d}) = 1 - \exp(-\theta_i y^2_{i,t-d}) \tag{1c}$$

where we assume that $\theta_i \geq 0$, and $d \geq 1$ is the delay parameter. To begin with we assume that y_{it} is a mean zero stochastic process. We discuss processes with nonzero mean and later. To simplify the model and following the existing literature, the delay parameter d is set to be equal to one and (1a)-(1c) are re-written in first difference form as:

$$\Delta y_{it} = \phi_i y_{i,t-1} + \upsilon_i y_{i,t-1}[1 - \exp(-\theta_i y^2_{i,t-1})] + \gamma_i f_t + \varepsilon_{it}, \tag{2}$$

where $\phi_i = -(1 - \beta_i)$. If y_{it} is assumed to follow a unit root process in the middle regime, then $\phi_i = 0$,[1] and equation (2) can be re-written as:

$$\Delta y_{it} = \upsilon_i y_{i,t-1}[1 - \exp(-\theta_i y^2_{i,t-1})] + \gamma_i f_t + \varepsilon_{it} \tag{3}$$

[1] It follows the practice in the literature (e.g. Balke and Fomby, 1997, in the context of TAR models and Michael et al., 1997 in the context of ESTAR models).

Using (3), we are interested in testing the hypotheses:

$$Ho: \theta_i = 0 \quad \text{for all } i \tag{3a}$$

$$Ha: \theta_i > 0 \ \text{ for } i = 1,2...,N_1;\ \theta_i = 0 \ \text{ for i} = \mathrm{N}_1 + 1, \mathrm{N}_1 + 2, ..., \mathrm{N}$$

Remark 1: The alternative hypothesis above implies that some units are generated by a stationary ESTAR model but it also allows a proportion of units being a unit root process.

The following assumptions are introduced:

Assumption 1: $N_1 / N \to q$ as $N \to \infty$, with $0 < q \leq 1$ under the alternative hypothesis.[2]

Assumption 2: ε_{it} are independently distributed for all $i = 1,...,N$ and $t = 1,...,T$, with zero mean, constant variance $\sigma^2_{\varepsilon i}$, and finite fourth order moment.

Assumption 3: f_t is serially uncorrelated with zero mean, constant variance σ^2_f, and finite fourth moment. (Without loss of generality σ^2_f will be set equal to unity.)

Assumption 4: ε_{it}, f_t, and γ_i are independently distributed for all i.

Assumption 5: Following Pesaran (2004a), we define the weights $\{\varphi_i\}$ having the following properties: $\varphi_i = \mathrm{O}(\frac{1}{N})$; $\sum_{i=1}^{N} \varphi_i = 1$; $\sum_{i=1}^{N} |\varphi_i| < K$ for $K < \infty$; $\sum_{i=1}^{N} \varphi_i^2 = \mathrm{O}(\frac{1}{N})$.

Assumption 6: Let $\bar{\gamma} = \frac{1}{\mathrm{N}} \sum_{\mathrm{j}=1}^{\mathrm{N}} \gamma_{\mathrm{j}}$. We suppose $\bar{\gamma} \neq 0$ for a fixed N and for $\mathrm{N} \to \infty$.

[2] As noted in Im, Pesaran and Shin (2003) this condition is necessary for the consistency of the panel unit root tests.

3. Nonlinear Unit Root Tests with Serially Uncorrelated Errors

Assumptions 1 and 2 together imply that the composite error, u_{it}, is serially uncorrelated. This restriction will be relaxed in Section 3.3.

3.1 Individual NCADF Test

Testing the null hypothesis (3a) directly is not feasible, since υ_i is not identified under the null.[3] To overcome this problem, we follow Luukkonen et al. (1988), and derive below a *t*-type test statistic. Using Taylor expansion on (3), under the null hypothesis, the following auxiliary regression is obtained:

$$\Delta y_{it} = a_i + b_i y_{i,t-1}^3 + \gamma_i f_t + e_{it}\,. \qquad (4)$$

Lemma 1: If Assumptions (2)-(6) are satisfied, then the common factor f_t can be approximated by:

$$f_t \approx \frac{1}{\bar{\gamma}} \Delta \bar{y}_t - \frac{\bar{w}_i}{\bar{\gamma}} \bar{y}_{t-1}^3 \qquad (4a)$$

Proof: see Appendix 1.

Therefore, it follows that Equation (4) can be written as the following nonlinear cross-sectionally augmented DF (NCADF) regression:

$$\Delta y_{it} = a_i + b_i y_{i,t-1}^3 + c_i \Delta \bar{y}_t + d_i \bar{y}_{t-1}^3 + e_{it} \qquad (6)$$

[3] See for example Davies (1987).

The idea is, given the framework above, to develop a unit root test in heterogeneous panel model based on Equation (6). Extending the idea in Kapetanios *et al.* (2003), we suggest using model (6) and *t*-statistic on b_i, that is denoted by

$$t_{iNL}(N,T) = \frac{\hat{b}_i}{s.e.(\hat{b}_i)},$$

where $\hat{b}_i$ is the OLS estimate of b_i, and s.e.($\hat{b}_i$) its associated standard error. Denote the student statistic on the ratio of b_i in Equation (6) as:

$$t_{iNL}(N,T) = \frac{y_{i,-1}^{'} M \, \Delta y_i}{(\Delta y_i^{'} M \, \Delta y_i)^{1/2} (y_{i,-1}^{'} M \, y_{i,-1})^{1/2}} \qquad (7)$$

where $\Delta y_i = (\Delta y_{i1}, \Delta y_{i2}, ..., \Delta y_{i,T})'$, $y_{i,-1}^3 = (y_{i,0}^3, y_{i,1}^3, ..., y_{i,T-1}^3)'$, $X = (\tau, \Delta y_i, y_{i,-1})'$, M the projection matrix onto $\delta(X)$, the orthogonal complement of the span of X, $\tau' = (1,1,...,1)$ and $\Delta \bar{y} = N^{-1} \sum_{j=1}^{N} \Delta y_j$, $\bar{y}_{i,-1} = N^{-1} \sum_{j=1}^{N} y_{j,-1}$. The critical values of the NCADF test can be computed by stochastic simulation for any fixed T > 3, and for given distributional assumptions for the random variables (ε_i,f).

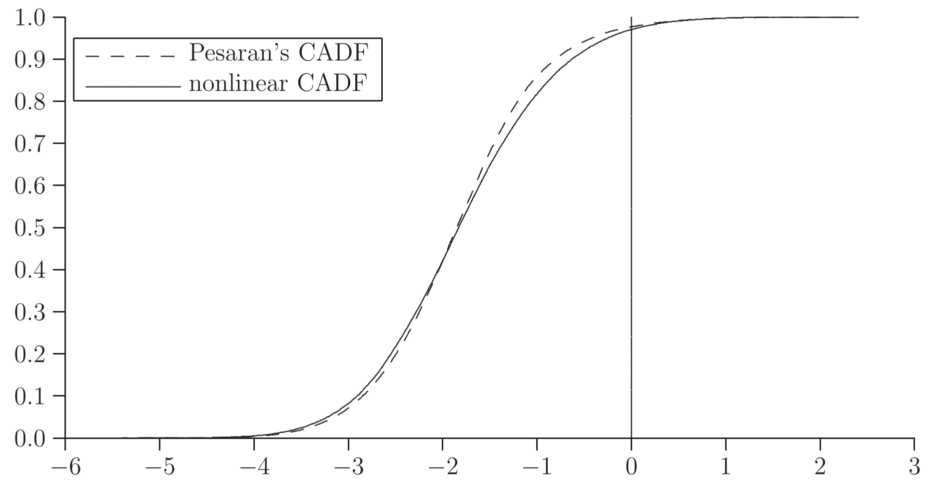
To accommodate stochastic processes with nonzero means, we need the following modifications. In the case where the data has nonzero mean, i.e., where $x_t = \mu + y_t$, we use the de-meaned data $y_t = x_t - \bar{x}$, where $\bar{x}$ is the sample mean. In this case the asymptotic distribution of the t_{NL} statistic is basically the same as (7), except that data are replaced by the de-meaned data.[4]

Figure 1 displays the simulated cumulative distribution function of the individual NCADF statistic under the null hypothesis using 50,000 replications for N = 100 and T = 500. For comparison the simulated cumulative distribution function of Pesaran CADF statistic is

[4] Similarly, for the case with nonzero mean and nonzero linear trend, i.e., where $x_t = \mu + \delta t + y_t$, we use the de-meaned and de-trended data $y_t = xt - \hat{\mu} - \hat{\delta}$ t, where $\hat{\mu}$ and $\hat{\delta}$ are the OLS estimators of μ and δ. Now the associated asymptotic distributions are such that W(r) is replaced by the de-meaned and de-trended standard Brownian motion W (r).

also provided. The series $y_{i,t}=y_{i,t-1}+f_t+u_{i,t}$, for i = 1, 2, …,100, and t = -50,-49,…, 1, 2, …, 500 were first generated from $y_{i,-50}$ = 0, with f_t and $u_{i,t}$ as i.i.d. N(0,1). Then 50,000 NCADF regressions of $\Delta y_{i,t}$ on $y_{i,t-1}^3$, $\Delta \bar{y}_t$, and $\bar{y}_{t-1}^3$. $\Delta \bar{y}_t$ and $\bar{y}_{t-1}^3$ were computed over the sample t = 1, 2, …,500. Figure 1 plots the ordered values of the OLS t-ratios of $y_{i,t-1}^3$ in these regressions.

Figure 1: Cumulative Distribution Function of Pesaran's Cross-Sectionally Augmented DF, and nonlinear Cross-Sectionally Augmented DF Statistics



Not surprisingly the nonlinear CADF distribution, as the Pesaran's CADF distribution, is more skewed to the left as compared to the standard DF distribution. This is clearly reflected in the critical values of the distributions summarized in Table 1.

Table 1: Critical Values of the DF, Pesaran's CADF, and nonlinear CADF Distributions (N=100,T=500, 50,000 replications)

Level	DF	CADF	NCADF
1 %	-2.60	-3.80	-3.72
2.5%	-2.23	-3.49	-3.41
5 %	-1.94	-3.22	-3.15
10 %	-1.61	-2.91	-2.85

Critical values of the individual nonlinear CADF distribution for values of T and N in the range of 10 to 200 are given in Appendix 2.

The nonlinear CADF distribution, like the Pesaran's CADF distribution and the standard DF distribution, departs from normality in two important respects: it has a substantially negative mean and its standard deviation is less than unity, although not by a large amount. The simulated density functions of the standardized NCADF, computed with N = 100, T = 500, and 50,000 replications are displayed in Figure 2. The mean, standard deviation, skewness and Kurtosis -3 coefficients of the NCADF and the Pesaran's CADF distributions are reported in Table 3. They are quite small, although statistically highly significant.

Figure 2: Simulated Density Function of the Standardized NCADFi and the Standardized Pesaran's CADFi Distributions as Compared to the Normal Density

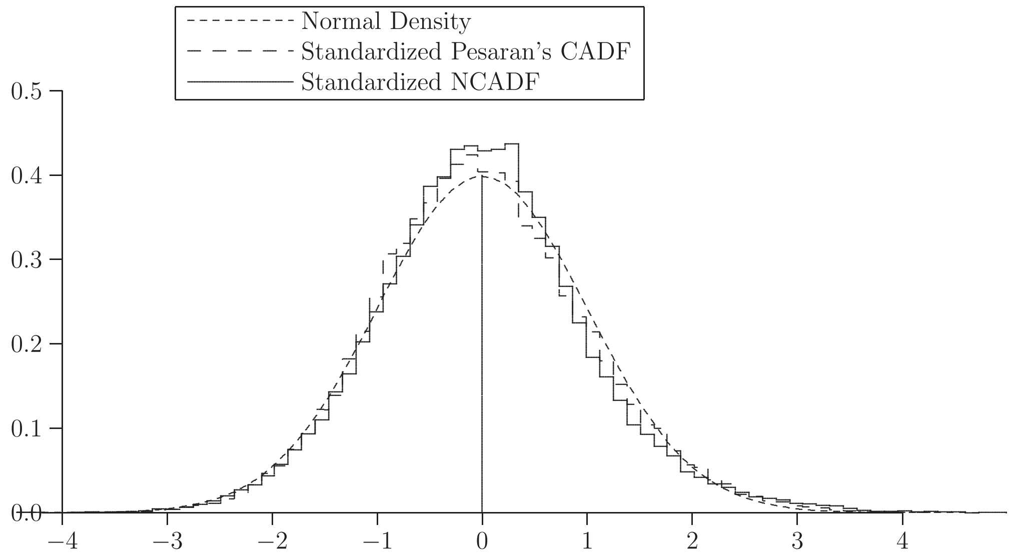


Table 3: moments of the CADF distributions

	Pesaran's CADF	NCADF
Mean	-1.80	-1.83
Standard deviation	0.90	0.83
Skewness	0.20	0.28
Kurtosis -3	0.19	0.77

Since cross-sectional dependence in panel data is widely known now to be a serious problem, in the next sections we shall be using model (6) to develop a unit root test to test for the null hypothesis of unit root against an ESTAR stationary alternative.

3.2 Panel nonlinear CADF Test

Following Pesaran (2005), we suggest using the *t*-statistic in Equation (7) to construct a panel unit root test by averaging the individual test statistics:

$$\bar{t}_{NL}(N,T) = N^{-1}\sum_{i=1}^{N} t_{iNL}(N,T) \tag{8}$$

This is a nonlinear cross-sectionally augmented version of the IPS test based (NCIPS). The test statistic defined in Equation (8) can also be extended to the case where serial correlation is present in the data. In this particular case, one may include, in the model, lags of the left hand side variable after using an information criteria to select the lag order.

We simulated the distribution of NCIPS setting N = 100, T = 500, and using 50,000 replications. The simulated density functions of the NCIPS and the Pesaran's CIPS Statistics are displayed in Figure 3. Both the densities show marked departures from normality. The density shows a great degree of departure from normality. The skewness and Kurtosis -3 coefficients of the NCIPS and the Pesaran's CIPS distributions are reported in Table 4.

Figure 3: Simulated Density Function of the NCIPS Statistic and the Pesaran's CIPS Distributions

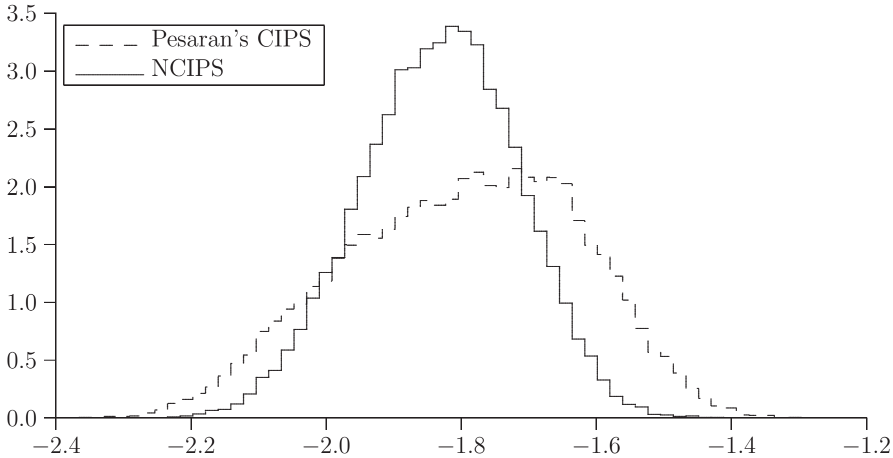


Table 4: moments of the CIPS distributions

	Pesaran's CIPS	NCIPS
Mean	-1.80	-1.83
Standard deviation	0.17	0.12
Skewness	-0.10	-0.068
Kurtosis -3	-1.67	-1.45

The critical values of the nonlinear CIPS test are given in Table 5 in Appendix 2.

3.3 The Serially Correlated Errors Case

Serial correlation can be incorporated in the model in a variety of different ways. In what follows, we use the model in Equation (4) and specify the serial correlation structure as:

$$u_{it} = \rho_i u_{i,t-1} + \eta_{it} \tag{9}$$

We first model serial correlation as above and thereafter cross section dependence as

$$\eta_{it} = \gamma_i f_t + \varepsilon_{it} \tag{10}$$

Using Equation (4) jointly with (9) above we obtain:

$$\Delta y_{it} = a_i(1-\rho_i) + b_i(1-\rho_i)y^3_{i,t-1} + \rho_i \Delta y_{i,t-1} + \eta_{it} \quad (11)$$

And substituting (10) into (11)

$$\Delta y_{it} = a_i(1-\rho_i) + b_i(1-\rho_i)y^3_{i,t-1} + \rho_i \Delta y_{i,t-1} + \gamma_i f_t + \varepsilon_{it} \quad (12)$$

Finally by imposing the unit root null on Equation (12):

$$\Delta y_{it} = a_i(1-\rho_i) + \rho_i \Delta y_{i,t-1} + \gamma_i f_t + \varepsilon_{it} \quad (13)$$

Using Equation (13) and the same approach as in Appendix 1, one can obtain proxies for f_t. We suggest in this case using the following non-linear CADF regression:

$$\Delta y_{it} = a_i + b_i y^3_{i,t-1} + c_i \bar{y}^3_{i,t-1} + \sum_{j=0}^{p} d_{ij} \Delta \bar{y}_{t-j} + \sum_{j=1}^{p} \delta_{ij} \Delta y_{i,t-j} + e_{it} \quad (14)$$

Information criteria can be used to choose the length of p.

4. Small Sample Analysis

In this section we assess the size of the nonlinear panel test defined in Equation (8) under different scenarios. Firstly, we look at power of the test in the case of weak and strong cross sectional dependence but not moving average structure for the error term. In the next section, we generalise this scenario by allowing a moving average specification for the error term and weak-strong cross sectional dependence. For comparison, in all the above experiment we also report the size of the Pesaran (2005) test when a nonlinear DGP is considered.

The data generating process (DGP) considered is the following Panel ESTAR:

$$\Delta y_{it} = \upsilon_i y_{i,t-1}[1-\exp(-\theta_i y^2_{i,t-1})] + \gamma_i f_t + \varepsilon_{it}$$

with i=1, 2, …, N; t=-51, -50, …, 1, 2, …, T; $f_t \sim i.i.d.N(0,1)$; $\varepsilon_{it} \sim i.i.d.N(0,\sigma_i^2)$; $\sigma^2_i \sim i.i.d.U[0.5,1.5]$. We consider two scenarios for cross sectional dependence, namely low cross sectional dependence $\gamma_i \sim i.i.d.U[0,0.20]$, and high cross sectional dependence $\gamma_i \sim i.i.d.U[-1,3]$.

4.1 Size Distortion Analysis

In our size analysis below, we generate data by setting $\theta_i = 0$ for all *i*. Size is computed at the 5% nominal significance level. The number of replications is set to 5,000. The standard error of the computed size is 0.0031. Results for the size are reported in Table 6 below.

Table 6: Size of Nonlinear Cross-Sectionally Augmented Panel Unit Root Tests
No Serial Correlation, Low and High Cross Section Dependence Case

Low Cross Section Dependence						
N/T	Test	10	20	30	50	100
10	CIPS	0.0498	0.0492	0.0540	0.0506	0.0496
	NCIPS	0.0468	0.0474	0.0582	0.0438	0.0494
20	CIPS	0.0538	0.0508	0.0464	0.0520	0.0520
	NCIPS	0.0532	0.0484	0.0444	0.0556	0.0488
30	CIPS	0.0554	0.0560	0.0426	0.0498	0.0490
	NCIPS	0.0516	0.0456	0.0490	0.0478	0.0448
50	CIPS	0.0516	0.0564	0.0508	0.0432	0.0496
	NCIPS	0.0474	0.0520	0.0486	0.0474	0.0512
100	CIPS	0.0526	0.0454	0.0490	0.0468	0.0488
	NCIPS	0.0470	0.0458	0.0434	0.0452	0.0478
High Cross Section Dependence						
N/T	Test	10	20	30	50	100
10	CIPS	0.0550	0.0492	0.0594	0.0520	0.0616
	NCIPS	0.0508	0.0432	0.0456	0.0414	0.0474
20	CIPS	0.0488	0.0492	0.0566	0.0568	0.0614
	NCIPS	0.0468	0.0432	0.0448	0.0402	0.0432
30	CIPS	0.0568	0.0518	0.0568	0.0466	0.0504
	NCIPS	0.0470	0.0394	0.0438	0.0332	0.0354
50	CIPS	0.0616	0.0566	0.0422	0.0448	0.0458
	NCIPS	0.0408	0.0410	0.0386	0.0328	0.0348
100	CIPS	0.0522	0.0518	0.0496	0.0530	0.0500
	NCIPS	0.0436	0.0414	0.0348	0.0354	0.0350

The test seems to have an acceptable size for large cross section dimension and somehow slightly undersized with respect to the Pesaran (2004) test.

4.2 Power Analysis

In this section we assess the power of the test defined in Equation (8) under the same DGP as above but we consider the cases of weak and strong alternatives, namely we assume for the weak alternative:

$$\theta_i = 0 \text{ for } i = 1,2,...,N/2 \qquad \theta_i = 0.01 \text{ for } i = N/2+1,...,N,$$

while for the strong alternative:

$$\theta_i = 0 \text{ for } i = 1,2,...,N/2 \qquad \theta_i = 0.05 \text{ for } i = N/2+1,...,N.$$

The power is computed at the 5% nominal significance level, and results are reported in Table 7 and 8.

Table 7: Power of Cross-Sectionally Augmented Nonlinear Panel Unit Root Tests

No Serial Correlation, Low and High Cross Section Dependence Case

Weak Alternative

Low Cross Section Dependence						
N/T	Test	10	20	30	50	100
10	CIPS	0.0584	0.0884	0.1356	0.2398	0.7280
	NCIPS	0.0598	0.1098	0.1934	0.3620	0.8942
20	CIPS	0.0590	0.1134	0.1594	0.3960	0.9480
	NCIPS	0.0818	0.1724	0.2970	0.6516	0.9954
30	CIPS	0.0712	0.1116	0.1818	0.4696	0.9864
	NCIPS	0.0906	0.2002	0.3806	0.7988	0.9998
50	CIPS	0.0660	0.1246	0.2088	0.5280	0.9990
	NCIPS	0.0978	0.2678	0.5032	0.9334	1.00
100	CIPS	0.0744	0.1264	0.2398	0.6428	1.00
	NCIPS	0.1006	0.3346	0.6682	0.9912	1.00
High Cross Section Dependence						
N/T	Test	10	20	30	50	100
10	CIPS	0.0632	0.1144	0.2294	0.4952	0.9180
	NCIPS	0.0680	0.1590	0.3314	0.6666	0.9722
20	CIPS	0.0554	0.1278	0.2678	0.7026	0.9918
	NCIPS	0.0820	0.2240	0.4618	0.8780	0.9986
30	CIPS	0.0486	0.1302	0.3234	0.8080	0.9964
	NCIPS	0.0842	0.2508	0.5628	0.9536	0.9998
50	CIPS	0.0516	0.1466	0.3638	0.8916	1.00
	NCIPS	0.0846	0.3134	0.6700	0.9900	1.00
100	CIPS	0.0478	0.1476	0.4122	0.9592	1.00
	NCIPS	0.0986	0.3598	0.7776	0.9976	1.00

Table 8: Power of Cross-Sectionally Augmented Nonlinear Panel Unit Root Tests
No Serial Correlation, Low and High Cross Section Dependence Case
Strong Alternative

N/T	Test	10	20	30	50	100
Low Cross Section Dependence						
10	CIPS	0.0862	0.1928	0.4066	0.8394	1.00
	NCIPS	0.0994	0.3190	0.6336	0.9616	1.00
20	CIPS	0.0890	0.2692	0.5978	0.9870	1.00
	NCIPS	0.1582	0.5706	0.8988	0.9998	1.00
30	CIPS	0.1010	0.3014	0.6978	0.9974	1.00
	NCIPS	0.1722	0.6862	0.9716	1.00	1.00
50	CIPS	0.1016	0.4010	0.8064	1.00	1.00
	NCIPS	0.2158	0.8592	0.9984	1.00	1.00
100	CIPS	0.1124	0.3994	0.9210	1.00	1.00
	NCIPS	0.2596	0.9700	1.00	1.00	1.00
High Cross Section Dependence						
N/T	Test	10	20	30	50	100
10	CIPS	0.0886	0.3448	0.7300	0.9516	0.9914
	NCIPS	0.1378	0.5020	0.8490	0.9846	0.9986
20	CIPS	0.0898	0.4930	0.8976	0.9956	1.00
	NCIPS	0.1958	0.7346	0.9656	0.9982	1.00
30	CIPS	0.0978	0.5900	0.9654	0.9996	1.00
	NCIPS	0.2268	0.8488	0.9948	1.00	1.00
50	CIPS	0.1048	0.7074	0.9908	1.00	1.00
	NCIPS	0.2752	0.9250	0.9990	1.00	1.00
100	CIPS	0.1004	0.7626	0.9996	1.00	1.00
	NCIPS	0.3150	0.9720	1.00	1.00	1.00

The test we propose seems to have stronger power than the Pesaran (2005) test when the true DGP is nonlinear.

4.3 Serial Correlated Errors Case

In this section we analyze size and power of the proposed test when serial correlation is incorporated into the DGP. We consider positive serial correlation. The errors ε_{it} were generated as:

$$\varepsilon_{it} = \rho_i \varepsilon_{i,t-1} + \zeta_{it},$$
$$\zeta_{it} \sim i.i.d.N(0,\sigma_i^2),$$
$$\sigma_i^2 \sim i.i.d.U[0.5; 1.5],$$

$\rho_i \sim$ i.i.d.U[0.2; 0.4] in the case of positive correlation,

$\rho_i \sim$ i.i.d.U[-0.4; -0.2] in the case of negative correlation.

We only consider here for the power analysis the case where

$$\theta_i = 0 \text{ for } i = 1,2,...,N/2, \qquad \theta_i = 0.05 \text{ for } i = N/2+1,...,N,$$

and high cross-sectional dependence:

$$\gamma_i \sim \text{i.i.d.U}[-1,3].$$

The size and power are computed at 5% nominal significance level and it are based on the following non-linear CADF regression:

$$\Delta y_{it} = a_i + b_i y_{i,t-1}^3 + c_i \bar{y}_{t-1}^3 + d_{i,0}\Delta \bar{y}_t + d_{i,1}\Delta \bar{y}_{t-1} + \delta_{i,1}\Delta y_{i,t-1} + e_{it}$$

$$i = 1,2,...,N;\ t = 1,2,...,T,\ \bar{y}_t = \frac{1}{N}\sum_{i=1}^{N} y_{it}.$$

The test is computed as:

$$\bar{t}_{NL}(N,T) = N^{-1}\sum_{i=1}^{N} t_{iNL}(N,T)$$

where $\bar{t}_{NL}(N,T)$ is the OLS t-ratio of b_i in the above non-linear ADF regression. The number of simulation is set equal to 5,000. Table 9 below shows the results.

Table 9: Size of Cross-Sectionally Augmented Nonlinear Panel Unit Root Tests
Strong alternave, High Cross Section Dependence Case

		Positive Serial Correlation				
N/T		10	20	30	50	100
10	CIPS	0.5504	0.1402	0.1066	0.0764	0.0668
	NCIPS	0.2928	0.06	0.0646	0.0514	0.0534
20	CIPS	0.6520	0.1492	0.1034	0.0796	0.0800
	NCIPS	0.3534	0.061	0.0558	0.0542	0.0526
30	CIPS	0.6990	0.1504	0.0976	0.0768	0.0664
	NCIPS	0.3774	0.0534	0.0478	0.0528	0.0484
50	CIPS	0.7700	0.1666	0.0932	0.0734	0.0618
	NCIPS	0.4174	0.0482	0.0458	0.0538	0.0374
100	CIPS	0.8292	0.1502	0.1012	0.0726	0.0648
	NCIPS	0.4672	0.0476	0.0464	0.0448	0.0432

		Negative Serial Correlation				
N/T		10	20	30	50	100
10	CIPS	0.5688	0.1606	0.1162	0.0756	0.0694
	NCIPS	0.2544	0.0396	0.0326	0.0294	0.0384
20	CIPS	0.6960	0.1722	0.1124	0.0886	0.0744
	NCIPS	0.3038	0.0262	0.0228	0.0306	0.0306
30	CIPS	0.7598	0.1888	0.1160	0.0866	0.0606
	NCIPS	0.3194	0.0244	0.0238	0.0244	0.0306
50	CIPS	0.8132	0.2000	0.1172	0.0794	0.0680
	NCIPS	0.3620	0.0216	0.0146	0.0228	0.0300
100	CIPS	0.8758	0.2100	0.1284	0.0898	0.0742
	NCIPS	0.3896	0.0164	0.0142	0.0208	0.0240

Both tests have a good size with the Pesaran (2005) being consistently oversized. In Table 10 we show results on the power of the test in the case when positive as well as negative serial correlation is present in the DGP.

Table 10: Power of Cross-Sectionally Augmented Nonlinear Panel Unit Root Tests
Strong alternave, High Cross Section Dependence Case

		Positive Serial Correlation				
N/T		10	20	30	50	100
10	CIPS	0.5836	0.2762	0.4712	0.8246	0.9808
	NCIPS	0.3840	0.3684	0.6998	0.9592	0.9978
20	CIPS	0.6922	0.3472	0.6214	0.9576	0.9984
	NCIPS	0.4968	0.5556	0.8986	0.9966	0.9998
30	CIPS	0.7464	0.3858	0.7222	0.9884	0.9998
	NCIPS	0.5470	0.6476	0.9580	0.9994	1
50	CIPS	0.8016	0.4738	0.8238	0.9996	1
	NCIPS	0.6320	0.767	0.9916	1	1
100	CIPS	0.8734	0.485	0.9234	1	1
	NCIPS	0.7290	0.8896	0.9984	1	1

		Negative Serial Correlation				
N/T		10	20	30	50	100
10	CIPS	0.5746	0.3378	0.5846	0.8714	0.9756
	NCIPS	0.3004	0.2394	0.5534	0.8966	0.9898
20	CIPS	0.6870	0.4642	0.7680	0.9802	0.9952
	NCIPS	0.3814	0.3466	0.7468	0.9846	0.9990
30	CIPS	0.7676	0.5350	0.8618	0.9966	0.9996
	NCIPS	0.4356	0.4034	0.8466	0.9962	1
50	CIPS	0.8202	0.6054	0.9288	0.9998	1
	NCIPS	0.4936	0.5024	0.9230	0.9996	1
100	CIPS	0.8810	0.6742	0.9806	1	1
	NCIPS	0.5552	0.6038	0.9688	1	1

For panels of a moderate size, the gain in power from using the non-linear panel unit root test with respect to the Pesaran (2004) test is evident.

5. Empirical Applications: Real Exchange Rates

In this section we apply our test to real exchange rates against the US dollar for twenty OECD countries over the period 1973Q1-1998Q2. The data set is the same used by Murray and Papell (2002, 2005). Since the long-run Purchasing Power Parity (PPP) relationship is one of the main components of theoretical international macroeconomic models, a large number of studies have tested this relationship by applying unit root tests to real exchange rates. Most of these studies show evidence of unit root behaviour in real exchange rates, which has become a puzzle in international finance. The growing literature on nonlinear exchange rates argues that transaction costs and frictions in financial markets may lead to nonlinear convergence in real exchange rates. Consequently, the non-mean reversion reported by linear unit root tests may be due to the fact these tests are based on a mis-specified stochastic process. The individual statistics for our unit root test are shown in Table 11. For comparison purposes, we also report the statistics for the Pesaran (2005) test which accounts for cross section dependence but not for nonlinearity.

Table 11: Individual Unit Root Tests for Real Dollar Exchange

Country	Lag	Cerrato et al (NCADF)	Pesaran (CADF)
Australia	3	-2.1765	-1.6501
Austria	4	-2.2085	-2.1432
Belgium	4	-2.4220	-1.2380
Canada	6	-1.1528	-1.3575
Denmark	3	-3.3390	-2.8699
Finland	7	-1.7015	-2.4148
France	4	-0.9386	-2.1170
Germany	4	-3.3166	-2.6044
Greece	4	-0.1449	-2.1730
Ireland	6	-0.1855	-1.0970
Italy	4	-2.6717	-2.0218
Japan	3	-2.5943	-1.9477
Netherlands	4	-2.7076	-1.9930
N Zealand	3	-3.7296	-3.8758
Norway	7	-2.2595	-1.8869
Portugal	8	-1.9120	-0.6359
Spain	8	-1.6911	-2.1622
Sweden	8	-3.8830	-1.5888
Switzerland	4	-5.1263	-2.7768
UK	7	-2.5354	-2.0689
Critical Values (N=20, T=100):			
1%		-3.74	-3.87
5%		-3.09	-3.24
10%		-2.80	-2.92

Rejection Rates of the Panel Unit Root Tests

	Cerrato et al(2007)		Pesaran (2005)	
	H0	H1	H0	H1
1%	90%	10%	95%	5%
5%	75%	25%	95%	5%
10%	75%	25%	95%	5%

The Pesaran (2005) test rejects the unit root null hypothesis in only 1 out of 20 cases at all levels of significance. By contrast, the nonlinear test rejects the null in 2 cases at the 1% significance level, and in 5 cases at the 5% and 10% level. Hence our test rejects the unit root null more frequently and therefore yields stronger support for the long-run PPP.

As we argued above, univariate tests have low power and this problem is overcome by employing panel unit root tests. The results for our panel unit root test and the Pesaran panel unit root test are shown in Table 12.

Table 12: Panel Unit Root Tests

	Cerrato et al (NCIPS)	Pesaran (CIPS)
	-2.3348	-2.0311
Critical Values (N=20, T=100):		
1%	-2.24	-2.36
5%	-2.11	-2.20
10%	-2.03	-2.11

The contrast between the two panel statistics is rather strong. The Pesaran (2005) test fails to reject the unit root null at all levels of significance, thus implying non-mean reversion in the whole panel of real exchange rates. On the other hand, our nonlinear panel test rejects the unit root null for the panel of real exchange rates at all levels of significance, giving support to the long-run PPP for the whole panel of OECD countries. This evidence of nonlinear mean reversion in the OECD real exchange rates may suggest that previous evidence of non-mean reversion in real exchange rates is due to using linear unit root tests.

6. Conclusion

A number of panel unit root tests allowing for cross section dependence have been proposed in the literature. In this paper we propose a nonlinear heterogeneous panel unit root test for testing the null hypothesis of unit-root processes against the alternative that allows a proportion of units to be generated by globally stationary ESTAR processes and a remaining non-zero proportion to be generated by unit root processes. The proposed test is simple to apply and accommodates cross sectional dependence. Our test is compared to Pesaran's (2005) linear test via Monte Carlo simulation exercises, and it is found that our test holds correct size and under the hypothesis that data are generated by globally stationary ESTAR processes has a better power than the Pesaran test.

We provide an application to a panel of bilateral real exchange rate series with the US dollar from the 20 major OECD countries. In contrast to the evidence obtained by linear tests, we find evidence of nonlinear mean-reversion in the real exchange rates for the whole OECD panel that gives support to the long-run PPP hypothesis. Given the importance of the PPP in international macroeconomic models, our evidence suggests that the employment of nonlinear panel unit root tests may provide a solution to the PPP puzzle.

Given the growing literature of nonlinear models, we believe that the development of panel nonlinear unit root tests has large potential in empirical macroeconomic and financial applications. Evidence indicates that different time series may follow different nonlinear specifications. Consequently, one could consider unit root tests with different types of transition functions that allow for asymmetric dynamic adjustment. Another extension would be to allow for different transition variables. Further applications of our tests and theoretical extensions are left for future work.

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Appendix 1

Proof of Lemma 1

We assume that the error term u_{it} in (6) follows a stationary process, for all i, with summable auto-covariance given by $u_{it} = \sum_{l=0}^{\infty} a_{il} \varsigma_{i,t-l}$, with $\varsigma_{i,t}$ being a zero mean random variable with variance matrix defined by I_{l+1} and finite fourth order moment.

The variance of u_{it} is finite and given by:

$$Var(u_{it}) = \sum_{l=0}^{\infty} a_{il}^2 = \sigma_i^2 \leq \bar{\sigma}^2 < \infty .$$

First note that, after using cross sectional averages, Equation (6) can be written as:

$$\Delta \bar{y}_{\varphi t} = \bar{\omega}_{\varphi} \bar{y}^3_{\varphi t-1} + \bar{\gamma}_{\varphi} f_t + \bar{u}_{\varphi t}$$

With $\Delta \bar{y}_{\varphi t} = \sum_{i=1}^{N} \varphi_i \Delta y_{it}$; $\bar{y}^3_{\varphi t-1} = \sum_{i=1}^{N} \varphi_i y^3_{i,t-1}$; $\bar{\omega}_{\varphi} = \sum_{i=1}^{N} \varphi_i \omega_i$; $\bar{\gamma}_{\varphi} = \sum_{i=1}^{N} \varphi_i \gamma_i$ and $\bar{u}_{\varphi t} = \sum_{i=1}^{N} \varphi_i a_{it}$.

Assuming that $\gamma > 0$, then f_t can be approximated as follows:

$$f_t \approx \frac{1}{\bar{\gamma}_{\varphi}} \Delta \bar{y}_{\varphi t} - \frac{1}{\bar{\gamma}_{\varphi}} \bar{\omega}_{\varphi} \bar{y}^3_{i,t-1} - \frac{1}{\bar{\gamma}_{\varphi}} \bar{u}_{\varphi t}$$

And since $Var(\bar{u}_{\varphi t}) = \sum_{i=1}^{N} \varphi_i^2 (\sum_{i=0}^{\infty} a_i^2)$ and $Var(\bar{u}_{\varphi t}) \leq \bar{\sigma}^2 (\sum_{i=1}^{N} \varphi_i^2) = O(\frac{1}{N})$, It follows that as $N \to \infty$, $E(\bar{u}_t) = 0$. consequently the factor f_t can now be approximated by:

$$f_t \approx \frac{1}{\bar{\gamma}_{\varphi}} \Delta \bar{y}_{\varphi t} - \frac{\bar{\omega}_{\varphi}}{\bar{\gamma}_{\varphi}} \bar{y}^3_{i,t-1}$$

Appendix 2:

Critical values

Table 2: Critical Values of Individual NCADF Distribution

N	T	1 %	2.5 %	5 %	10 %
10	10	-5.18	-4.17	-3.50	-2.87
	15	-4.19	-3.60	-3.16	-2.69
	20	-3.93	-3.44	-3.07	-2.67
	30	-3.79	-3.38	-3.05	-2.70
	50	-3.81	-3.41	-3.11	-2.78
	70	-3.67	-3.39	-3.12	-2.80
	100	-3.71	-3.39	-3.12	-2.80
	200	-3.73	-3.40	-3.12	-2.82
15	10	-5.35	-4.22	-3.52	-2.92
	15	-4.21	-3.64	-3.15	-2.67
	20	-3.96	-3.42	-3.06	-2.68
	30	-3.81	-3.36	-3.06	-2.69
	50	-3.69	-3.32	-3.06	-2.75
	70	-3.75	-3.41	-3.11	-2.78
	100	-3.70	-3.38	-3.13	-2.76
	200	-3.67	-3.37	-3.09	-2.78
20	10	-5.05	-4.20	-3.47	-2.89
	15	-4.27	-3.63	-3.13	-2.73
	20	-3.94	-3.39	-3.04	-2.67
	30	-3.71	-3.39	-3.09	-2.74
	50	-3.70	-3.28	-3.04	-2.73
	70	-3.66	-3.35	-3.07	-2.75
	100	-3.74	-3.38	-3.09	-2.80
	200	-3.77	-3.40	-3.14	-2.84
30	10	-5.62	-4.37	-3.55	-2.95
	15	-4.22	-3.62	-3.14	-2.68
	20	-3.87	-3.42	-3.09	-2.70
	30	-3.86	-3.42	-3.14	-2.73
	50	-3.69	-3.37	-3.06	-2.75
	70	-3.71	-3.32	-3.07	-2.75
	100	-3.77	-3.32	-3.10	-2.79
	200	-3.68	-3.37	-3.11	-2.84

N	T	1 %	2.5 %	5 %	10 %
50	10	-5.16	-4.17	-3.52	-2.91
	15	-4.21	-3.57	-3.15	-2.68
	20	-4.10	-3.47	-3.11	-2.69
	30	-3.75	-3.33	-3.00	-2.69
	50	-3.68	-3.35	-3.04	-2.76
	70	-3.70	-3.36	-3.07	-2.75
	100	-3.59	-3.31	-3.09	-2.79
	200	-3.72	-3.36	-3.10	-2.81
70	10	-5.17	-4.23	-3.52	-2.92
	15	-4.32	-3.64	-3.22	-2.74
	20	-3.97	-3.47	-3.10	-2.65
	30	-3.79	-3.41	-3.06	-2.71
	50	-3.73	-3.41	-3.11	-2.76
	70	-3.68	-3.37	-3.05	-2.76
	100	-3.71	-3.40	-3.10	-2.81
	200	-3.62	-3.34	-3.11	-2.83
100	10	-4.89	-3.99	-3.39	-2.81
	15	-4.04	-3.53	-3.16	-2.75
	20	-3.91	-3.45	-3.05	-2.66
	30	-3.76	-3.36	-3.06	-2.70
	50	-3.63	-3.33	-3.04	-2.75
	70	-3.64	-3.31	-3.01	-2.74
	100	-3.74	-3.35	-3.10	-2.79
	200	-3.69	-3.40	-3.11	-2.82
200	10	-5.21	-4.17	-3.42	-2.84
	15	-4.30	-3.67	-3.21	-2.78
	20	-3.91	-3.44	-3.11	-2.70
	30	-3.69	-3.34	-3.04	-2.73
	50	-3.77	-3.40	-3.10	-2.77
	70	-3.66	-3.28	-3.08	-2.75
	100	-3.70	-3.38	-3.11	-2.79
	200	-3.64	-3.38	-3.14	-2.81

Table 2.1: Critical Values of Average of Individual Nonlinear Cross-Sectionally Augmented Dickey-Fuller Distribution

N	T	1%	2.5%	5%	10%	N	T	1%	2.5%	5%	10%
10	200	-2.50	-2.40	-2.33	-2.25	50	200	-2.14	-2.09	-2.04	-1.99
	100	-2.42	-2.31	-2.22	-2.11		100	-2.10	-2.05	-2.01	-1.96
	70	-2.39	-2.27	-2.19	-2.10		70	-2.08	-2.03	-1.99	-1.94
	50	-2.36	-2.26	-2.16	-2.05		50	-2.05	-2.00	-1.96	-1.91
	30	-2.31	-2.20	-2.12	-2.01		30	-2.00	-1.95	-1.90	-1.84
	20	-2.32	-2.20	-2.09	-1.97		20	-1.96	-1.90	-1.85	-1.79
	15	-2.34	-2.19	-2.08	-1.94		15	-1.95	-1.88	-1.82	-1.75
	10	-2.53	-2.34	-2.17	-1.98		10	-2.01	-1.91	-1.83	-1.75
15	200	-2.33	-2.25	-2.18	-2.09	70	200	-2.11	-2.06	-2.02	-1.98
	100	-2.30	-2.22	-2.14	-2.06		100	-2.07	-2.03	-1.99	-1.95
	70	-2.26	-2.19	-2.13	-2.04		70	-2.05	-2.00	-1.97	-1.92
	50	-2.24	-2.16	-2.08	-2.00		50	-2.02	-1.98	-1.94	-1.89
	30	-2.20	-2.11	-2.03	-1.95		30	-1.96	-1.91	-1.87	-1.83
	20	-2.17	-2.09	-2.00	-1.90		20	-1.92	-1.87	-1.83	-1.77
	15	-2.19	-2.08	-1.98	-1.88		15	-1.91	-1.84	-1.80	-1.73
	10	-2.34	-2.18	-2.04	-1.90		10	-1.95	-1.88	-1.80	-1.72
20	200	-2.26	-2.19	-2.13	-2.06	100	200	-2.08	-2.04	-2.01	-1.97
	100	-2.24	-2.16	-2.11	-2.03		100	-2.05	-2.01	-1.97	-1.93
	70	-2.20	-2.13	-2.08	-2.00		70	-2.02	-1.99	-1.95	-1.91
	50	-2.18	-2.11	-2.05	-1.98		50	-1.99	-1.95	-1.92	-1.88
	30	-2.14	-2.07	-2.00	-1.92		30	-1.94	-1.89	-1.86	-1.81
	20	-2.11	-2.03	-1.95	-1.86		20	-1.89	-1.84	-1.81	-1.76
	15	-2.10	-2.00	-1.93	-1.84		15	-1.87	-1.82	-1.77	-1.72
	10	-2.22	-2.09	-1.97	-1.84		10	-1.92	-1.85	-1.78	-1.70
30	200	-2.20	-2.14	-2.09	-2.02	200	200	-2.05	-2.01	-1.99	-1.95
	100	-2.18	-2.11	-2.06	-2.00		100	-2.01	-1.98	-1.96	-1.92
	70	-2.15	-2.09	-2.03	-1.97		70	-2.00	-1.96	-1.93	-1.89
	50	-2.11	-2.05	-2.00	-1.94		50	-1.96	-1.93	-1.90	-1.86
	30	-2.07	-2.00	-1.95	-1.88		30	-1.90	-1.87	-1.84	-1.80
	20	-2.02	-1.95	-1.90	-1.83		20	-1.86	-1.81	-1.78	-1.73
	15	-2.02	-1.94	-1.87	-1.79		15	-1.82	-1.78	-1.74	-1.69
	10	-2.13	-2.00	-1.90	-1.80		10	-1.87	-1.80	-1.75	-1.68