Collaterals and Macroeconomic Volatility

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Abstract

In this paper, we study the effects of collaterals on business cycles and growth in monetary economies with credit market imperfections. We consider an endogenous growth model with a partial cash-in-advance constraint and inelastic labor supply. We assume that the share of consumption purchases paid with credit depends positively on the collateral available to the agent. In this framework, we find that money is no longer superneutral in the long run and short-run fluctuations, either deterministic or stochastic, can arise. On the one side, the monetary policy can enhance the growth rate and welfare, on the other side, reduce the macroeconomic volatility. Second, the sensitivity to collaterals alters the effectiveness of monetary policy in terms of welfare and stability. Finally, indeterminacy becomes more likely as long as the credit market is less sensitive to collaterals.

Keywords: liquidity constraint, superneutrality, endogenous fluctuations.

JEL classification: D90, E32, E41.

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1   Introduction

The US subprime crisis has become the focus of economists' attention in the last few years because of its dramatic effects on the global economy. As well known, subprime loans are offered to consumers with a poor credit history or insufficient collateral at high interest rates. Whenever the subprime credit market has expanded rapidly, the rate of defaults has increased, resulting in sharp decline in house prices and the crisis happened.

In the view of the subprime crisis, we address the stability issue, stressing the role of collaterals, in a monetary economy with credit market imperfections. The role of collaterals is usually introduced through a credit constraint imposed on agents; an individual can borrow as long as the repayment does not exceed the market value of his collateral (Kiyotaki and Moore (1997), Kiyotaki (1998), and Cordoba and Ripoll (2003)). We depart from these models by considering a framework in which the collateral is introduced through a partial cash-in-advance constraint in the spirit of Grandmont and Younès (1972). Namely, we assume that agents pay a part of their consumption purchases in cash while the remainder is financed with credit. Further, the amount of credit depends on the collateral available to the agent.

Within this framework, our objective is to provide the conditions that make the equilibrium indeterminate and allow for self-fulfilling revisions in expectations to be consistent with rational expectations.

In this line of research, several papers showed that a partial cash-in-advance constraint promotes the occurrence of multiple equilibria. In one-sector monetary models, Carlstrom and Fuerst (2003) considered partial cash-in-advance with transaction cost. They assume that the fraction of consumption purchases paid cash is endogenously chosen by agents. Subsequently, this fraction is assumed to be constant in Bosi and Magris (2003). In the latter, a small departure from the traditional Ramsey-Cass-Koopmans model (that is, the share of consumption purchases requiring CIA is below certain threshold which is very small) is sufficient to make the equilibrium indeterminate, for any value of the elasticity of intertemporal substitution. In addition, when the share is above this threshold, indeterminacy arises only when the elasticity of intertemporal substitution is sufficiently low. However, the inclusion of investment in the liquidity constraint modifies the range of parameters values giving rise to indeterminacy (Bosi and Dufourt, 2008).[1]

Moreover, we are close to cash-credit models pioneered by Lucas and Stokey (1987). Following this literature, we refer to goods purchased with money as "cash goods" and goods purchased with credit as "credit goods".

In precise, this paper is an extension of Bosi and Magris (2003) model. We assume that the fraction of consumption goods paid with credit (the credit share) depends positively on the capital-consumption ratio. That is, if the value of agent's capital is high relative to his current consumption, he can easily

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[1] Abel (1985) showed that, whenever both the consumption and investment expenditures are subject to the CIA constraint, the equilibrium is always locally unique.

support the repayments of the credit. So, this agent has incentive to pay more with credit and less in cash and the credit share increases. We show that this specification has important implications on the effect of money on growth, the choice of an optimal monetary policy that maximizes the welfare and the stability properties of the economic system.

First, money is not neutral in our model. An increase in the monetary growth lowers the return of money, leading agents to substitute cash by capital. Hence, higher rate of money growth is associated with a larger capital stock and a higher growth rate. In addition, agents are willing to pay more with credit due to the increase in the capital-consumption ratio.

Our first result says that the choice for the means of payment depends on the opportunity cost of holding money. Such a result is similar to the results obtained previously. Lucas and Stokey (1987) showed that an increase in the nominal interest rate leads agents to substitute against cash goods. After that, for an economy with capital formation and credit-goods production, Aiyagari and Eckstein (1995) and Aiyagari, Braun and Eckstein (1998) showed that the nominal interest rate determines the price of credit good relative to the price of cash good. Subsequently, in Ireland (1994) and Hromcova (2003), agents can use the costly financial intermediary as an alternative to cash. These papers showed that, as the economy grows, it is cheaper to buy via intermediaries and so money is relatively less used.

Second, when the credit sensitivity to collaterals increases, the positive impact of monetary policy on the growth rate becomes stronger only when the elasticity of intertemporal substitution in consumption is sufficiently weak. In this case, under a high sensitivity to collaterals, agents can avoid the opportunity cost of holding money by accumulating more capital and then paying their consumption with credit.

Third, when the sensitivity to collateral is low, indeterminacy arises for low values of the elasticity of intertemporal substitution in consumption. This case appears to be consistent with the results found in Bosi and Magris (2003).

Finally, indeterminacy becomes more and more likely to emerge as long as the sensitivity of credit market to collaterals is lower: agents are willing to pay more with credit to avoid the opportunity cost of holding money and, thus, the share of cash good decreases. In Bosi and Magris (2003), the indeterminacy region widens continuously as the share of consumption purchases paid cash lowers.

This paper is organised as follows. In section 2, we present the model and derive the intertemporal equilibrium. Section 3 is devoted to the analysis of the steady state and the comparative statics. Section 4 studies the local stability and the occurrence of local bifurcations. In section 5, we provide a numerical example. Finally, section 6 concludes.

2 The model

This paper considers an endogenous growth model with partial cash-in-advance constraint and inelastic labor supply. The economy is populated by a large number of identical infinitely-lived agents acting under perfect foresight. There is also a representative firm. The main feature of this model is that consumers are allowed to make their purchases of consumption goods by using money or credit. Therefore, only a part of the consumption goods is subject to the cash-in-advance requirement. This framework is an extension of Bosi and Magris (2003) model. While they assume that a *constant* share of the consumption goods are paid in cash previously accumulated, we assume that this share is endogenously determined.

2.1 Consumers

The lifetime utility function of a representative agent is given by

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \tag{1}$$

where $\beta \in (0,1)$ stands for the discount factor, c_t is the consumption demand and the instantaneous utility function u verifies the following basic assumption.

Assumption 1. *The single-period utility function $u\left(c\right)$ is twice continuously differentiable for all positive values of c and satisfies, for any $c > 0$, $u'\left(c\right) > 0$, $u''\left(c\right) < 0$, $\lim_{c \to 0} u'\left(c\right) = +\infty$ and $\lim_{c \to +\infty} u'\left(c\right) = 0$.*

In each period t, the household's portfolio is constituted by the stock of capital k_t and the amount of money balances M_t. S/he is subject to a usual budget constraint

$$p_t\left(c_t + k_{t+1} - \Delta k_t\right) + M_{t+1} \leq M_t + p_t r_t k_t + p_t w_t l_t + \tau_t \tag{2}$$

for $t = 0, 1, \ldots$, where $\Delta \equiv 1 - \delta \in [0,1]$ and δ is the depreciation rate of capital, $\tau_t \equiv M_{t+1} - M_t$ are nominal lump-sum transfers "helicoptered" by the monetary authority, r is the real interest rate on capital and w is the real wage. For simplicity, labor supply is assumed to be inelastic, namely, $l_t \equiv 1$ for every $t \geq 0$.

In addition, the representative household faces a partial cash-in-advance constraint.

In the spirit of Grandmont and Younès (1972), we assume that a share of purchases is paid cash, while the rest on credit. In a way, we are also close to Lucas and Stokey (1987), an economy with a cash and a credit good.

We define credit share the fraction of consumption good paid on credit. We further assume that the amount of disposable credit depends on the collaterals the household is endowed with. More precisely, the individual's capital stock relative to her/his consumption habits matters in order to open a credit line.

Formally, the credit share is given by

$$\gamma\left(\frac{k}{c}\right) \equiv \frac{\text{credit good}}{\text{credit good} + \text{cash good}}$$

An augmented version of the partial cash-in-advance considered by Bosi and Magris (2003) and Carlstrom and Fuerst (2003), is now provided in order to take into account the role of collaterals:

$$[1 - \gamma\left(k_t/c_t\right)] p_t c_t \leq M_t \tag{3}$$

for $t = 0, 1, \dots$

Introducing an endogenous credit share in a model of capital accumulation is the added value of our paper. We notice that the velocity of money becomes endogenous as well and, therefore, variable: $v\left(k/c\right) \equiv [1 - \gamma\left(k/c\right)]^{-1}$. In this sense, our formulation overcomes one of the main criticisms addressed to the cash-in-advance models: the implausibility of a constant velocity of money.[2]

The credit function verifies some reasonable restrictions.

Assumption 2. *The credit share $\gamma\left(k/c\right)$ is twice continuously differentiable and satisfies $\gamma\left(k/c\right) \in [0,1]$, $\gamma'\left(k/c\right) \geq 0$, $\gamma''\left(k/c\right) \leq 0$ for every $k/c \geq 0$.*

In Assumption 2, $\gamma \in [0,1]$ simply means that credit and money velocity are non-negative; $\gamma' \geq 0$ captures the role of collaterals relative to consumption habits: the higher the ratio, the larger the credit; and $\gamma'' \leq 0$ reconciles smoothness and the existence of an upper bound ($\gamma \leq 1$).

The representative agent maximizes (1) subject to (2) and (3), that is the infinite-horizon Lagrangian function

$$\sum_{t=0}^{\infty} \left[\beta^t u\left(c_t\right) + \lambda_t B_t + \mu_t C_t\right] \tag{4}$$

with respect to the consumption path $((c_t)_{t=0}^{\infty})$ and the saving path (capital and balances: $(k_t, M_t)_{t=1}^{\infty}$), where

$$\begin{aligned} B_t &\equiv M_t + p_t r_t k_t + p_t w_t l_t + \tau_t - p_t\left(c_t + k_{t+1} - \Delta k_t\right) - M_{t+1} \\ C_t &\equiv M_t - [1 - \gamma\left(k_t/c_t\right)] p_t c_t \end{aligned}$$

are non-negative implicit constraints.

Deriving (4) with respect the demand for balances and capital gives the portfolio arbitrage, while deriving with respect to the demand for capital and consumption the intertemporal arbitrage, the consumption smoothing over time.

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[2]MIU models are immunized against this criticism: the functional equivalence highlighted by Feenstra (1986) between CIA and MIU, no longer holds with a Cash-When-I'm-Done (CWID) timing or a CIA timing and strictly positive elasticity of substitution between consumption and real balances (see Carlstrom and Fuerst, 2003).

Deriving (4) with respect to the costate variables λ_t, μ_t, we recover the constraints, now binding.

More precisely, after eliminating the multipliers, we get a sequence of Euler equations

$$\frac{u'(c_t)}{u'(c_{t+1})} = \frac{\beta}{\pi_{t+1}} \frac{I_{t+1} - \gamma'(x_{t+1})\pi_{t+1}}{1 - \gamma'(x_t)\pi_t}$$
$$\frac{1 - \gamma'(x_t)\pi_t + (I_t - 1)[1 - \gamma(x_t) + \gamma'(x_t)x_t]}{1 - \gamma'(x_{t+1})\pi_{t+1} + (I_{t+1} - 1)[1 - \gamma(x_{t+1}) + \gamma'(x_{t+1})x_{t+1}]}$$

$$(5)$$

for $t = 0, 1, \ldots$, where $x_t \equiv k_t/c_t$ is the key argument of the credit function, while $\pi_{t+1} \equiv p_{t+1}/p_t$ and $I_t \equiv (\Delta + r_t)\pi_t$ denote the gross rates of inflation and nominal interest, respectively.

A positive marginal utility implies binding budget constraints: $B_t = 0$ for $t = 0, 1, \ldots$, while the augmented CIA constraints are also binding ($C_t = 0$ for $t = 0, 1, \ldots$) under a sequence of strictly positive interest rates: $I_t > 1$, $t = 0, 1, \ldots$

We observe that positive interest rates are necessary to the existence of a monetary equilibrium. Under no uncertainty, when balances are dominated by a capital asset, the opportunity cost of holding money induces agents to keep in the pockets the minimal amount of money for transaction purposes. As a consequence, the liquidity constraint becomes binding.

Eventually, a rational household takes into account the initial endowments $M_0, k_0 \geq 0$ and the transversality condition:

$$\lim_{t \to +\infty} \beta^t u'(c_t)(k_{t+1} + M_{t+1}/p_t) = 0 \tag{6}$$

2.2 Firms

On the production side, there is a continuum of identical firms participating to competitive markets. The representative firm rents capital and labor in order to produce the good under constant (private) returns to scale. External effects of capital intensity spill over the other firms. Technology is rationalized by a Romer-type (1986) production function.

Assumption 3. $F(K, L) \equiv A\bar{k}^{1-s}K^s L^{1-s}$, $s \in (0, 1)$

The TFP $(A\bar{k}^{1-s})$ is affected by productive externalities of average capital intensity $\bar{k}$.

Each producer is price-taker and determines the demand for inputs to maximize the profit.

The assumption of a representative household and a representative firm implies at equilibrium $\bar{k} = k = K/L$ and first order conditions for profit maximization reduce to

$$r_t = sA \tag{7}$$
$$w_t = (1-s)Ak_t \tag{8}$$

We notice that the equilibrium interest rate is constant over time as usual in the endogenous growth literature *à la* Romer.

2.3 Monetary authority

Money supply follows a simple rule. Lump-sum money is "helicoptered" to consumers:

$$\tau_t = M_{t+1}^s - M_t^s \tag{9}$$

and the rate of money growth is kept constant over time by the monetary authority: $M_{t+1}^s = \mu M_t^s$ for $t = 0, 1, \ldots$

2.4 General equilibrium

The economy under study is a system of three markets: money, labor and goods.

The money market clears when the demand for real balances $m_t \equiv M_t/p_t$, supported by the liquidity constraint, equals the amount supplied by the monetary authority:

$$m_t = [1 - \gamma(x_t)] c_t = M_t^s/p_t \tag{10}$$

The dynamic version of (10):

$$\mu = \pi_{t+1} \frac{c_{t+1}}{c_t} \frac{1 - \gamma(x_{t+1})}{1 - \gamma(x_t)} \tag{11}$$

highlights the decomposition of the nominal growth in inflation and real growth.

In the labor market, the demand l_t is determined by profit maximization (8), while, for simplicity, the supply is supposed to be inelastic: $l_t = 1$.

By the Walras' law the good market clears too. The equilibrium is obtained, by replacing (9), (7) and (8) in the representative agent's budget constraint (2).

$$c_t + k_{t+1} - \Delta k_t = r_t k_t + w_t l_t = A k_t$$

Dividing by k_t both the sides, we obtain the growth rate:

$$g_{t+1} \equiv \frac{k_{t+1}}{k_t} = \Delta + A - \frac{1}{x_t} \tag{12}$$

In order to compute the intertemporal equilibrium, let us introduce an explicit utility function, very usual in the endogenous growth literature.

Assumption 4. *The instantaneous utility function is given by:*

$$
\begin{aligned}
u(c) &\equiv \ln c \text{ iff } \sigma = 1 \\
u(c) &\equiv c^{1-1/\sigma}/(1-1/\sigma) \text{ iff } \sigma \neq 1
\end{aligned} \tag{13}
$$

where $\sigma > 0$ is the constant elasticity of intertemporal substitution.[3]

Replacing (12) in (5) and (11), we obtain the dynamic system of a CIA economy with collaterals. Formally:

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[3] The CES preferences rationalized by (13), satisfy the Assumption 1.

Definition 1 *An intertemporal equilibrium with perfect foresight is a sequence* $(x_t, \pi_t)_{t=0}^{\infty}$ *that satisfies (i) the initial conditions* (M_0, k_0), *(ii) the transitional dynamics:*

$$
\begin{aligned}
\left(\Delta + A - \frac{1}{x_t}\right)\frac{x_t}{x_{t+1}} &= \left(\frac{\beta}{\pi_{t+1}}\frac{I_{t+1}-\gamma'(x_{t+1})\pi_{t+1}}{1-\gamma'(x_t)\pi_t}\frac{1-\gamma'(x_t)\pi_t+(I_t-1)\left[1-\gamma(x_t)+\gamma'(x_t)x_t\right]}{1-\gamma'(x_{t+1})\pi_{t+1}+(I_{t+1}-1)\left[1-\gamma(x_{t+1})+\gamma'(x_{t+1})x_{t+1}\right]}\right)^{\sigma} \\
\left(\Delta + A - \frac{1}{x_t}\right)\frac{x_t}{x_{t+1}} &= \frac{\mu}{\pi_{t+1}}\frac{1-\gamma(x_t)}{1-\gamma(x_{t+1})}
\end{aligned}
$$

(14)

for $t = 0, 1, \ldots$, *where* $I_t = (\Delta + r)\pi_t$ *and* $r = sA$, *and (iii) the transversality condition (6).*

3   Steady state

Growth is regular at the steady state:[4]

$$
g \equiv \frac{k_{t+1}}{k_t} = \frac{c_{t+1}}{c_t} = \frac{m_{t+1}}{m_t}
$$

Dropping the time index in (14) and solving the system gives the stationary state (x, π). The balanced growth factor is obtained from (12):

$$
g = \Delta + A - \frac{1}{x}
$$

(15)

Replacing

$$
(x, \pi) = \left(\frac{1}{\Delta + A - g}, \frac{\mu}{g}\right)
$$

in (14) gives the Euler equation of steady state, that is, a sort of modified golden rule where money is no longer superneutral because of the market imperfection $\gamma' > 0$:

$$
g = \left[\beta\frac{g}{\mu}\frac{\Delta + r - \gamma'\left(\frac{1}{\Delta+A-g}\right)}{\frac{g}{\mu} - \gamma'\left(\frac{1}{\Delta+A-g}\right)}\right]^{\sigma}
$$

(16)

Introducing the elasticities of credit share is an appropriate way of taking into account the role of collaterals in the effectiveness of monetary policy. More precisely, we define the first and second-order elasticities of credit share: $\varepsilon_1(x) \equiv x\gamma'(x)/\gamma(x) \in [0,1)$ and $\varepsilon_2(x) \equiv x\gamma''(x)/\gamma'(x) < 0$. Equation (16) becomes:

$$
g = \left[\beta\frac{g}{\mu}\frac{\Delta + sA - \gamma\varepsilon_1(\Delta + A - g)}{\frac{g}{\mu} - \gamma\varepsilon_1(\Delta + A - g)}\right]^{\sigma}
$$

(17)

The equilibrium existence requires the term into the brackets to be positive: $\gamma' < \min\{g/\mu, \Delta + r\}$ or $\gamma' > \max\{g/\mu, \Delta + r\}$. The existence of a monetary

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[4] In the long run, an unbalanced growth should violate either the positivity of variables or the transversality condition. Notice, however, that growth rates may be unbalanced along the transition path.

equilibrium requires also $I > 1$ and, therefore, $g/\mu = 1/\pi < \Delta + r$. Then, the positivity of g becomes equivalent to $\gamma' < 1/\pi$ or $\gamma' > \Delta + r$. For simplicity, we assume the first inequality as sufficient condition in a neighborhood of the steady state.

Assumption 5. $\pi\gamma'(x) < 1 < I$.

Eventually, the balanced growth path $(m_t, k_t, c_t) = (m_0, k_0, c_0) g^t$ must satisfy the transversality condition: $\lim_{t \to +\infty} \beta^t u'(c_t)(k_{t+1} + \pi m_{t+1}) = 0$. Substituting the path and solving the limit, we obtain a parametric restriction: $\beta < g^{1/\sigma - 1}$, or, equivalently:

$$\sigma \ln \beta < (1 - \sigma) \ln g \tag{18}$$

We notice that (18) is verified in the case of a logarithmic utility ($\sigma = 1$).

3.1 Comparative statics

Since money is no longer superneutral, we want to understand the mechanism of monetary policy and the long-run impact on growth and welfare. In the next section, we will analyze also the short-run effects of the rule on the business cycles and the sense of a stabilization policy.

Differentiating equation (16) w.r.t. μ and g gives the impact of the monetary rule on the growth rate:

$$\varepsilon_{g\mu} \equiv \frac{dg}{d\mu}\frac{\mu}{g} = \left(1 + \frac{1}{\sigma}\frac{1 - \pi\gamma'}{\pi\gamma'} - xg\varepsilon_2\frac{I - 1}{I - \pi\gamma'}\right)^{-1} > 0 \tag{19}$$

where $\varepsilon_{g\mu}$ is the elasticity of the growth factor with respect to the monetary policy. This elasticity is positive under Assumption 5.

The higher the monetary growth (μ), the higher the inflation (π) and the opportunity cost of holding money (I). Savers will keep more capital and less money in the portfolio. The capital growth rate will increase. In addition, when the capital/consumption ratio goes up, consumption purchases are more collateralized, agents are willing to substitute the cash by credit, the credit share increases and the need for money lowers.

Elasticity (19) can be rewritten as follows:

$$\varepsilon_{g\mu} = \left(1 + \frac{1}{\sigma}\frac{1 - \gamma\varepsilon_1\pi/x}{\gamma\varepsilon_1\pi/x} - xg\varepsilon_2\frac{I - 1}{I - \gamma\varepsilon_1\pi/x}\right)^{-1} > 0 \tag{20}$$

and, *ceteris paribus*, the impact of collaterals on the effectiveness of monetary policy is captured by the following derivatives:

$$\frac{\partial \varepsilon_{g\mu}}{\partial \varepsilon_1} \quad > \quad 0 \text{ iff } \sigma < -\frac{1}{xg\varepsilon_2}\frac{1}{I - 1}\left(\frac{I - \gamma\varepsilon_1\pi/x}{\gamma\varepsilon_1\pi/x}\right)^2$$

$$\frac{\partial \varepsilon_{g\mu}}{\partial \varepsilon_2} \quad > \quad 0$$

The first derivative can be interpreted as follows. Given the second-order moment (ε_2), the higher the sensitivity to collaterals (ε_1), the larger the impact of money growth on the growth rate ($\varepsilon_{g\mu}$), when households find hard smoothing consumption over time (sufficiently low σ). A weaker elasticity of intertemporal substitution makes more difficult to elude the CIA constraint through higher savings: in this case, a higher sensitivity to collaterals can free individuals from the burden of the constraint and promotes capital accumulation.

The second derivative is interpreted as follows: households prefer a higher capital/consumption ratio under a faster monetary growth, and the flatter the credit share (the higher and closer to zero ε_2), the sharper their response. To understand the point, fix a value x of the ratio and the slope γ of the credit share in x. The flatter the credit share around x, the steeper the credit share on the right of x. Rational households moves on the right and accumulate more capital to enjoy the easier credit.

3.2 Optimal monetary growth

Maximizing a welfare function W is equivalent to maximizing the utility function under the assumption of representative agent. Along the balanced growth path:

$$W \equiv \sum_{t=0}^{\infty} \beta^t u(c_t) = \frac{u(c_0)}{1 - \beta g^{1-1/\sigma}}$$

provided that $\beta < g^{1/\sigma - 1}$, that is, the transversality condition (18) holds.

Proposition 2 *The optimal (welfare-maximizing) monetary policy is given by*

$$\mu^* = \frac{[\beta(\Delta + A)]^\sigma}{\Delta + A} \left(1 + \frac{1-s}{\gamma \varepsilon_1} \frac{A}{\Delta + A - [\beta(\Delta + A)]^\sigma}\right) \tag{21}$$

The higher the sensitivity to collaterals, the lower the optimal monetary growth rate:

$$\frac{\partial \mu^*}{\partial \varepsilon_1} = -\frac{1-s}{\gamma \varepsilon_1^2} \frac{A}{\Delta + A} \frac{[\beta(\Delta + A)]^\sigma}{\Delta + A - [\beta(\Delta + A)]^\sigma} < 0 \tag{22}$$

Proof. We notice that, at the steady state, $c_0 = (\Delta + A - g)k_0$. Then

$$W = V(g) \equiv \frac{u((\Delta + A - g)k_0)}{1 - \beta g^{1-1/\sigma}}$$

Deriving w.r.t. g gives

$$V'(g) = \left[\left(1 - \frac{1}{\sigma}\right) \frac{\beta g^{-1/\sigma}}{1 - \beta g^{1-1/\sigma}} - \frac{1}{\Delta + A - g} \frac{c_0 u'(c_0)}{u(c_0)}\right] V(g)$$

Using (13), we obtain

$$V'(g) = \left(\frac{\beta g^{-1/\sigma}}{1 - \beta g^{1-1/\sigma}} - \frac{1}{\Delta + A - g}\right) \left(1 - \frac{1}{\sigma}\right) V(g)$$

10

and, therefore, $V'(g) > 0$ iff[5]

$$g < [\beta(\Delta + A)]^{\sigma} \equiv g^* \tag{23}$$

Defining the welfare as a function of the monetary policy:

$$W(\mu) \equiv V(g(\mu))$$

we obtain, according to (19), $W'(\mu) > 0$ iff $V'(g) > 0$, that is, iff (23) holds. Welfare is maximized by $g^{-1}(g^*)$, where g^{-1} is a well-defined function from the monotonicity of g (see (19)). More explicitly:

$$\mu^* = \frac{g^* - \beta g^{*1-1/\sigma}(\Delta + sA - \gamma\varepsilon_1(\Delta + A - g^*))}{\gamma\varepsilon_1(\Delta + A - g^*)}$$

and, replacing (23), we obtain (21). Deriving (21) w.r.t. ε_1 gives (22). ∎

The existence of an interior solution results from the trade-off between the initial consumption c_0 and the growth rate g.

Renouncing to consume today, raises individual saving, capital accumulation and, finally, the growth rate.

On the one side, too low growth rates supported by a slow monetary growth, are inefficient. But, on the other side, a fast monetary expansion can imply capital overaccumulation, which requires an excessively low level of initial consumption.

4   Local dynamics

In order to characterize the stability properties of the steady state and the occurrence of local bifurcations, we proceed by linearizing the dynamic system (23)-(15) around the steady state (x, π) defined by (16)-(**??**) and computing the Jacobian matrix J, evaluated at this steady state. Local dynamics are represented by a linear system $(dx_{t+1}/x, d\pi_{t+1}/\pi)^T = J(dx_t/x, d\pi_t/\pi)^T$. In the following, we exploit the fact that the trace T and the determinant D of J are the sum and the product of the eigenvalues, respectively. As highlighted by Grandmont, Pintus and de Vilder (1998), the stability properties of the system, that is, the location of the eigenvalues with respect to the unit circle, can be better characterized in the (T, D)-plane.

More explicitly, we evaluate the characteristic polynomial $P(z) \equiv z^2 - Tz + D$ at -1 and 1. Along the line AC, one eigenvalue is equal to 1, *i.e.* $P(1) = 1 - T + D = 0$. Along the line AB, one eigenvalue is equal to -1, *i.e.* $P(-1) = 1 + T + D = 0$. On the segment $[BC]$, the two eigenvalues are complex and conjugate with unit modulus, *i.e.* $D = 1$ and $|T| < 2$. Therefore, inside the triangle ABC, the steady state is a sink, *i.e.* locally indeterminate

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[5]Notice that the transversality condition ensures the convergence of the welfare series and is equivalent to $(\beta g)^{\sigma} < g$. Therefore, (23) is equivalent to $(\beta g)^{\sigma} < g < [\beta(\Delta + A)]^{\sigma}$. The existence of a nonempty range for g requires $g < \Delta + A$, which is satisfied by (15).

($D < 1$ and $|T| < 1 + D$). It is a saddle point if (T, D) lies on the right sides of both AB and AC or on the left sides of both of them ($|1 + D| < |T|$). It is a source otherwise. A (local) bifurcation arises when an eigenvalue crosses the unit circle, that is, when the pair (T, D) crosses one of the loci AB, AC or $[BC]$. (T, D) depends on the structural parameters. We choose and vary a parameter of interest and we observe how (T, D) moves in the (T, D)-plane. More precisely, according to the changes in the bifurcation parameter, a transcritical bifurcation (generically) occurs when (T, D) goes through AC, a flip bifurcation (generically) arises when (T, D) crosses AB, whereas a Hopf bifurcation (generically) happens when (T, D) goes through the segment $[BC]$.

Linearizing system (23)-(15) and taking in account the elasticities of credit share, gives

$$[\sigma\varepsilon_2 (A_1 + B_1) - 1] \frac{dx_{t+1}}{x} + \sigma B_2 \frac{d\pi_{t+1}}{\pi} = [\sigma\varepsilon_2 (A_2 + B_1) - C_1] \frac{dx_t}{x} + \sigma (A_2 + B_2) \frac{d\pi_t}{\pi}$$

$$(1 + C_2) \frac{dx_{t+1}}{x} - \frac{d\pi_{t+1}}{\pi} = (C_1 + C_2) \frac{dx_t}{x}$$

where, under Assumption 5:[6]

$$A_1 \equiv \frac{\gamma\varepsilon_1 \frac{\pi}{x}}{I - \gamma\varepsilon_1 \frac{\pi}{x}} > 0$$

$$A_2 \equiv \frac{\gamma\varepsilon_1 \frac{\pi}{x}}{1 - \gamma\varepsilon_1 \frac{\pi}{x}} > 0$$

$$B_1 \equiv \frac{\gamma\varepsilon_1 (I - 1) - \gamma\varepsilon_1 \frac{\pi}{x}}{[1 - \gamma (1 - \varepsilon_1)] (I - 1) + 1 - \gamma\varepsilon_1 \frac{\pi}{x}}$$

$$B_2 \equiv \frac{[1 - \gamma (1 - \varepsilon_1)] I - \gamma\varepsilon_1 \frac{\pi}{x}}{[1 - \gamma (1 - \varepsilon_1)] (I - 1) + 1 - \gamma\varepsilon_1 \frac{\pi}{x}}$$

$$C_1 \equiv \frac{\Delta + A}{g} > 1$$

$$C_2 \equiv \frac{\gamma\varepsilon_1}{1 - \gamma} > 0 \tag{26}$$

Since x_t and π_t are variables independently non-predetermined, the equilibrium is locally determinate if and only if the steady state is a source, while local indeterminacy requires a saddle point or a sink.

The Jacobian matrix is given by:

$$J = \begin{bmatrix} \sigma\varepsilon_2 (A_1 + B_1) - 1 & \sigma B_2 \\ 1 + C_2 & -1 \end{bmatrix}^{-1} \begin{bmatrix} \sigma\varepsilon_2 (A_2 + B_1) - C_1 & \sigma (A_2 + B_2) \\ C_1 + C_2 & 0 \end{bmatrix}$$

---

[6] Assumption 5 can be equivalently written:

$$0 < 1 - \gamma\varepsilon_1 \frac{\pi}{x} < I - \gamma\varepsilon_1 \frac{\pi}{x} \tag{24}$$

and implies also

$$[1 - \gamma (1 - \varepsilon_1)] (I - 1) + 1 - \gamma\varepsilon_1 \frac{\pi}{x} > 0 \tag{25}$$

while the trace and determinant by:

$$T(\sigma) = 1 + D(\sigma) - \frac{\left(\frac{1}{\sigma} + A_2\right)(1 - C_1) - \varepsilon_2(A_1 - A_2)}{\frac{1}{\sigma} - B_2(1 + C_2) - \varepsilon_2(A_1 + B_1)} \tag{27}$$

$$D(\sigma) = -\frac{(A_2 + B_2)(C_1 + C_2)}{\frac{1}{\sigma} - B_2(1 + C_2) - \varepsilon_2(A_1 + B_1)} \tag{28}$$

In the spirit of Grandmont, Pintus and de Vilder (1998), we apply the geometrical method and we characterize the locus $\Sigma \equiv \{(T(\sigma), D(\sigma)) : \sigma \geq 0\}$ obtained by varying the intertemporal substitution in consumption in the (T, D)-plane.

The following lemma provides technical results to prove the main proposition.

Lemma 3 *(i) Σ is linear with origin $(T(0), D(0)) = (C_1, 0)$, endpoint*

$$(T(\infty), D(\infty)) = \left(1 + D(\infty) + \frac{\varepsilon_2(A_2 - A_1) - A_2(C_1 - 1)}{\varepsilon_2(A_1 + B_1) + B_2(C_2 + 1)}, \frac{(A_2 + B_2)(C_1 + C_2)}{\varepsilon_2(A_1 + B_1) + B_2(C_2 + 1)}\right) \tag{29}$$

and slope

$$\begin{aligned}
S &= \left[1 + \frac{[A_2 - A_1 - (C_1 - 1)(A_1 + B_1)]\varepsilon_2 - (C_1 - 1)(A_2 + B_2 + B_2 C_2)}{(A_2 + B_2)(C_1 + C_2)}\right]^{-1} \\
&= \left[2 - C_1 + \frac{A_2 - A_1 - (C_1 - 1)(A_1 + B_1)}{(A_2 + B_2)(C_1 + C_2)}\left(\varepsilon_2 - \varepsilon_2^C\right)\right]^{-1}
\end{aligned}$$

where

$$\begin{aligned}
\varepsilon_2^B &\equiv \varepsilon_2^C - \frac{4(A_2 + B_2)(C_1 + C_2)}{A_2 - A_1 - (A_1 + B_1)(C_1 - 1)} \\
\varepsilon_2^C &\equiv -(C_1 - 1)\frac{A_2 C_2 + (A_2 + B_2)(C_1 - 1)}{A_2 - A_1 - (A_1 + B_1)(C_1 - 1)}
\end{aligned}$$

are the critical points such that the line Σ goes through the vertices B and C, respectively.

(ii) Under Assumption 5, $D'(\sigma) < 0$, that is, the point $(T(\sigma), D(\sigma))$ moves downwards in the (T, D)-plane when σ goes up.

Proof. The origin and the endpoint are obtained taking the limit of (27) and (28) as σ approaches 0 from above and $+\infty$ from below, respectively. The slope is obtained computing $T'(\sigma)$, $D'(\sigma)$ and the ratio $S = D'(\sigma)/T'(\sigma)$.

Moreover

$$D'(\sigma) = -\frac{[D(\sigma)/\sigma]^2}{(A_2 + B_2)(C_1 + C_2)} < 0$$

because $C_1 + C_2 > 0$ and, under Assumption 5,

$$A_2 + B_2 = \frac{I - \gamma\varepsilon_1\frac{\pi}{x}}{1 - \gamma\varepsilon_1\frac{\pi}{x}}\frac{1 - \gamma(1 - \varepsilon_1)}{[1 - \gamma(1 - \varepsilon_1)](I - 1) + 1 - \gamma\varepsilon_1\frac{\pi}{x}} > 0 \tag{30}$$

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(see equations (24) and (25) in the previous footnote). ∎

As in the case of comparative statics, Assumption 5 plays a key role in order to characterize the local dynamics.

Proposition 4 *Let Assumption 5 holds and*

$$\sigma_F \equiv \frac{1 + C_1}{(A_2 + 2B_2)(1 + C_1 + 2C_2) + (A_1 + A_2 + 2B_1)\varepsilon_2} \tag{31}$$

$$\sigma_H \equiv \frac{1}{B_2(1 - C_1) - A_2(C_1 + C_2) + (A_1 + B_1)\varepsilon_2}$$

where σ_F and σ_H are solutions of $D(\sigma) = -T(\sigma) - 1$ and $D(\sigma) = 1$, respectively.

(i) If $A_2 - A_1 - (C_1 - 1)(A_1 + B_1) < 0$ or $(A_2 - A_1 - (C_1 - 1)(A_1 + B_1) > 0$ and $\varepsilon_2^C < \varepsilon_2 < 0)$, then the steady state is a saddle point for $0 < \sigma < \sigma_F$ and a source for $\sigma_F < \sigma$. The system generically undergoes a flip bifurcation at $\sigma = \sigma_F$.

(ii) If $A_2 - A_1 - (C_1 - 1)(A_1 + B_1) > 0$ and $\varepsilon_2 < \varepsilon_2^B$, then the steady state is a saddle point for $\sigma < \sigma_F$ and a sink for $\sigma_F < \sigma$. The system undergoes a flip bifurcation at $\sigma = \sigma_F$.

(iii) If $A_2 - A_1 - (C_1 - 1)(A_1 + B_1) > 0$ and $\varepsilon_2^B < \varepsilon_2 < \varepsilon_2^C$, then the steady state is a saddle point for $\sigma < \sigma_F$, a source for $\sigma_F < \sigma < \sigma_H$ and a sink for $\sigma_H < \sigma$. The system generically undergoes a flip bifurcation and a Hopf bifurcation at $\sigma = \sigma_F$ and $\sigma = \sigma_H$, respectively.

Proof. $(C_1, 0)$, the origin of Σ, lies on the T-axis, on the right of the line AC.

Consider the endpoint (29). Since Assumption 5 holds, (24) holds as well and

$$A_2 - A_1 = \frac{(I - 1)\gamma\varepsilon_1\frac{\pi}{x}}{(I - \gamma\varepsilon_1\frac{\pi}{x})(1 - \gamma\varepsilon_1\frac{\pi}{x})} > 0$$

Then, according to (30), $D(\infty) > 0 \Leftrightarrow \varepsilon_2(A_1 + B_1) + B_2(C_2 + 1) > 0 \Leftrightarrow D(\infty) > T(\infty) - 1$. This implies that Σ never crosses the line AC (the eigenvalues never cross $+1$ and there is no room for saddle node, transcritical or pitchfork bifurcations).

Focus now on the impact of ε_2 on the location of Σ. When $\varepsilon_2 (< 0)$ increases, Σ rotates clockwise around the origin $(C_1, 0)$ iff $\partial S/\partial\varepsilon_2 < 0$, where

$$\frac{\partial S}{\partial\varepsilon_2} = -\frac{A_2 - A_1 - (C_1 - 1)(A_1 + B_1)}{(A_2 + B_2)(C_1 + C_2)}S^2 \tag{32}$$

(1) If $A_2 - A_1 - (C_1 - 1)(A_1 + B_1) < 0$, then $\partial S/\partial\varepsilon_2 > 0$ and Σ rotates counterclockwise around $(C_1, 0)$. We notice that $S(-\infty) = 0^+$ and $0 < \varepsilon_2^C < \varepsilon_2^B$. If $0 < S(\varepsilon_2^C)$, then $S(\varepsilon_2) \in (0, S(\varepsilon_2^C))$ for every $\varepsilon_2 < 0$. If $S(\varepsilon_2^C) < 0$, then $S(\varepsilon_2) \notin (S(\varepsilon_2^C), 0)$ for every $\varepsilon_2 < 0$. In both the cases, Σ never crosses the triangle. Indeed:

(1.1) if $S \in (0, 1)$, then $D'(\sigma) < 0$ and $\Sigma \cap AC = \varnothing$ prevent Σ from entering ABC (Σ is a segment included in the cone $\{(T, D) : D \leq \min\{0, T - 1\}\}$);

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(1.2) if $S \notin (0,1)$, since the line including Σ intersects $D = 1$ on the right of C, it can not cross the triangle.

In both the cases (1.1) and (1.2), there is room for a flip bifurcation when Σ crosses the line AB below A. More precisely, the steady state is a saddle point for $\sigma < \sigma_F$ and a source for $\sigma_F < \sigma$. The system undergoes a flip bifurcation at $\sigma = \sigma_F$.

(2) If $A_2 - A_1 - (C_1 - 1)(A_1 + B_1) > 0$, then $\partial S / \partial \varepsilon_2 < 0$ and Σ rotates clockwise around $(C_1, 0)$. We notice that $S(-\infty) = 0^-$ and $\varepsilon_2^B < \varepsilon_2^C < 0$.

(2.1) When $\varepsilon_2 < \varepsilon_2^B$, the steady state is a saddle point for $\sigma < \sigma_F$ and a sink for $\sigma_F < \sigma$. The system generically undergoes a flip bifurcation at $\sigma = \sigma_F$.

(2.2) When $\varepsilon_2^B < \varepsilon_2 < \varepsilon_2^C$, the steady state is a saddle point for $\sigma < \sigma_F$, a source for $\sigma_F < \sigma < \sigma_H$ and a sink for $\sigma_H < \sigma$. The system generically undergoes a flip bifurcation and a Hopf bifurcation at $\sigma = \sigma_F$ and $\sigma = \sigma_H$, respectively.

(2.3) When $\varepsilon_2^C < \varepsilon_2 < 0$, then the steady state is a saddle point for $\sigma < \sigma_F$ and a source for $\sigma_F < \sigma$. The system undergoes a flip bifurcation at $\sigma = \sigma_F$.

In all these cases, the existence of bifurcations requires positive critical values: $\sigma_F, \sigma_H > 0$. Cases (1) and (2.3) correspond to case (i) in the proposition. Cases (2.1) and (2.2) correspond to cases (ii) and (iii), respectively. ∎
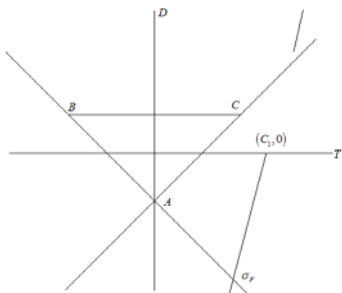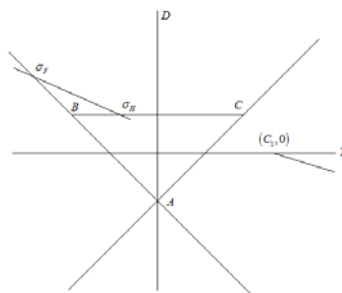

Figure 1: case (i)   Figure 2: case (iii)

Case (i) of Proposition 4 is closer to Bosi and Magris (2003) (set $\varepsilon_2 = 0$ with a constant credit share). This case shows that, when collaterals matter, cycles of period two appear for sufficiently low elasticity of intertemporal substitution in consumption. The intuition is given as follows.

Assume that k_t increases from the value of steady state. Then, the income Ak_t increases as well. If the intertemporal substitution σ is weak, the income effect prevails and raises the current consumption c_t. If the intertemporal substitution is sufficiently weak, the response in terms of c_t exceeds the increase of $(\Delta + A)k_t$ and, according to the budget constraint, $k_{t+1} = (\Delta + A)k_t - c_t$ decreases. Thus, an increase of k_t is followed by a decreases of k_{t+1} and two-period cycles arise.

This mechanism can be reinforced by a sensitivity to collaterals: when k_t goes up, x_t and the credit share also move up. The positive effect on the current

consumption adds to the income effect entailing a deeper fall of k_{t+1}. In this case (i), cycles of period two are compatible with lower values of intertemporal substitution and, in a way, more plausible. Equation (31) captures a sort of trade-off between the critical value of intertemporal substitution in consumption (σ_F) and the sensitivity of credit share to collaterals (ε_1). The impact of the second-order elasticity is characterized in the following corollary.

Corollary 5 *In case (i) of Proposition (4), the indeterminacy range $(0, \sigma_F)$ shrinks with $\varepsilon_2 \, (< 0)$ iff $A_1 + A_2 + 2B_1 > 0$.*

Proof. Simply, notice that

$$\frac{\partial \sigma_F}{\partial \varepsilon_2} = -\frac{A_1 + A_2 + 2B_1}{1 + C_1} \sigma_F^2$$

and $\partial \sigma_F / \partial \varepsilon_2 < 0$ iff $A_1 + A_2 + 2B_1 > 0$. ∎

Corollary 5 states that the range where supercritical cycles occur $(\sigma_F, +\infty)$ widens, if, plausibly, $A_1 + A_2 + 2B_1 > 0$. In this case, given ε_1, increasing ε_2 makes the credit share flatter and, thus, steeper on the right of x. Increasing k_t, moves x_t on the right of x. On the right, the credit share is now higher and households prefers to consume more. The mechanism described above takes place and k_{t+1} decreases more. In other terms, given ε_1, the closer ε_2 to zero, the lower the critical value of flip bifurcation σ_F: because of the additional effect of ε_2, cycles becomes compatible with lower values and, so, more plausible values of intertemporal substitution.

In Bosi and Magris (2003), the additional effects due to the elasticities of credit share were not taken in account. These effects amplify the traditional bifurcation mechanism which generates period-doubling cycles through a flip bifurcation when the intertemporal substitution is sufficiently weak.

We observe that, in case (i), when the flip bifurcation is subcritical, the cycle is unstable and there is room for indeterminacy around the saddle point which is now stable.

Assume that households anticipate an increase in k_{t+1} from the steady state k. Because of the income effect (Ak_{t+1}), they expect also an increase in their future consumption c_{t+1}. Under a large substitution effect, the consumption smoothing pushes up also c_t (see the Euler equation in (5)). But, since k_t is a predetermined variable, this violates the current budget constraint: $k_{t+1} = (\Delta + A) k_t - c_t$. Conversely, a self-fulfilling increase of k_{t+1} becomes possible only if c_t decreases, which happens only if a sufficiently weak intertemporal substitution makes the consumption smoothing difficult.

If the flip bifurcation is supercritical, the cycle arises around a source and is stable. A new kind of indeterminacy occurs: multiple equilibria converge to the cycle and there is room for sunspot equilibria around this attractor.

5 A numerical example

Let us consider an explicit formulation of credit share: $\gamma(x) \equiv \eta_0 + \eta_1 x^\varepsilon$, with $\varepsilon \in [0, 1]$, $\eta_0, \eta_1 \geq 0$. We obtain as required: $\varepsilon_1 = \varepsilon \eta_1 x^\varepsilon / (\eta_0 + \eta_1 x^\varepsilon) \in [0, 1]$

and $\varepsilon_2 = \varepsilon - 1 \leq 0$.

The steady state is defined by (17). We calibrate the TFP parameter A in order to get a plausible growth rate, that is we express A as an inverse function of g:

$$A = \frac{\left[\frac{g}{\mu} - \varepsilon_1\gamma\left(\Delta - g\right)\right]g^{1/\sigma} - \left[\Delta - \varepsilon_1\gamma\left(\Delta - g\right)\right]\beta\frac{g}{\mu}}{\gamma\varepsilon_1 g^{1/\sigma} + \left(s - \varepsilon_1\gamma\right)\beta\frac{g}{\mu}} \tag{33}$$

In addition, we require the model restrictions to be satisfied.

(i) Essential restrictions:

$x = 1/\left(\Delta + A - g\right) > 0$ (positivity of quantities),

$\pi = \mu/g > 0$ (positivity of prices),

$0 < \gamma < 1$ (credit share),

$\beta < g^{1/\sigma - 1}$ (transversality condition),

$I = \left(\Delta + sA\right)\mu/g > 1$ (monetary equilibrium).

(ii) Optional restrictions:

$\pi \geq 1$ (positive inflation),

$g > 1$ (growth),

$0 < 1 - \left(\Delta + A - g\right)\gamma\varepsilon_1\mu/g$ (Assumption 5).

In the following, for simplicity, we focus on the (locally) isoelastic case: $\eta_0 = 0$ implies $\varepsilon_1 = \varepsilon$.

We set some preliminary parameters according to quarterly data: $\beta = 0.99$, $\Delta = 0.94574$. In the spirit of Mankiw, Romer and Weil (1992), capital imbeds the human capital and we fix $s = 2/3$. For simplicity, we set also $\gamma = 1/2$: cash and credit good weight the same.

5.1 Comparative statics

Fix now: $\varepsilon = 0.5$ (intermediate sensitivity to collaterals), $\mu = g = 1.01$ (monetary growth accompanies growth), $\sigma = 1$ (logarithmic utility). Applying formula (33), we calibrate $A = 0.11134$, to implement a quarterly growth rate of 1%.

We notice that the essential and optional restrictions are satisfied: $x = 21.24 > 0$, $\pi = 1 \geq 1$, $0 < \gamma = 0.5 < 1$, $\beta = 0.99 < g^{1/\sigma - 1} = 1$, $I = 1.0200 > 1$, $g = 1.01 > 1$, $0 < 1 - \left(\Delta + A - g\right)\gamma\varepsilon\mu/g = 0.98823$.

To compute the effectiveness of monetary policy, we apply (20),

$$\frac{dg}{d\mu}\frac{\mu}{g} = \left[1 + \frac{1}{\sigma}\frac{1 - \gamma\varepsilon\left(\Delta + A - g\right)\mu/g}{\gamma\varepsilon\left(\Delta + A - g\right)\mu/g} + \frac{\left(1 - \varepsilon\right)g}{\Delta + A - g}\frac{I - 1}{I - \gamma\varepsilon\left(\Delta + A - g\right)\mu/g}\right]^{-1}$$

and we obtain:

$$\frac{dg}{d\mu}\frac{\mu}{g} = 0.011741 > 0 \tag{34}$$

The impact of a monetary acceleration is positive (money is non-superneutral), but extremely weak: increasing by 1% the monetary growth factor, raises by about 0.01% the growth factor.

An interesting question is whether a higher sensitivity to collaterals lowers the effectiveness of monetary policy. The answer is positive.

Indeed, if we increase ε from 0.5 to 0.9, living the other parameters unchanged, and we recompute the TFP parameter that ensures 1% of growth: $A = 0.11105$, we get

$$\frac{dg}{d\mu}\frac{\mu}{g} = 0.0046039 > 0$$

that is, less than half the impact in (34).

Therefore, there is a very little role to play for monetary policy when collaterals and social inequalities matters in the credit market.

In general, the impact of a monetary growth on economic growth remains extremely weak and one may wonder if the gain in terms of growth could justify high inflation rates.

5.2 Optimal monetary policy

In the two scenarios $\varepsilon = 0.5$ and $\varepsilon = 0.9$, the optimal growth factors $g^* \equiv (\beta (\Delta + A))^\sigma$ are respectively given by $g^* = 1.0465$ and $g^* = 1.0462$ and the corresponding optimal monetary rules by $\mu^* = 14.878$ and $\mu^* = 8.6753$ (see (21) and (22)).

These unrealistic values depend on the very small impacts of μ on the growth factor: passing from $g = 1.01$ to g^* requires a large injection of balances.

5.3 Local dynamics

Let us show the impact of intertemporal substitution on local dynamics. We provide two parametrizations corresponding to the source and the saddle case. In the following, fix $\mu = g = 1.01$ and $\varepsilon = 0.5$.

Before computing the Jacobian matrix and the eigenvalues, we need to find blocks (26). Under the assumption of isoelastic credit share, blocks and the Jacobian matrix simplify to:

$$
\begin{aligned}
A_1 &= \frac{\gamma\varepsilon (\Delta + A - g)\,\mu/g}{I - \gamma\varepsilon (\Delta + A - g)\,\mu/g} \\[2mm]
A_2 &= \frac{\gamma\varepsilon (\Delta + A - g)\,\mu/g}{1 - \gamma\varepsilon (\Delta + A - g)\,\mu/g} \\[2mm]
B_1 &= \frac{\gamma\varepsilon \left[I - 1 - (\Delta + A - g)\,\mu/g\right]}{1 + (1 - \gamma)(I - 1) + \gamma\varepsilon \left[I - 1 - (\Delta + A - g)\,\mu/g\right]} \\[2mm]
B_2 &= \frac{(1 - \gamma)I + \gamma\varepsilon \left[I - (\Delta + A - g)\,\mu/g\right]}{1 + (1 - \gamma)(I - 1) + \gamma\varepsilon \left[I - 1 - (\Delta + A - g)\,\mu/g\right]} \\[2mm]
C_1 &= \frac{\Delta + A}{g} \\[2mm]
C_2 &= \frac{\gamma\varepsilon}{1 - \gamma}
\end{aligned}
\tag{35}
$$

(where $I = (\Delta + sA)\,\mu/g$) and

$$J = \begin{bmatrix} \sigma(\varepsilon - 1)(A_1 + B_1) - 1 & \sigma B_2 \\ 1 + C_2 & -1 \end{bmatrix}^{-1} \begin{bmatrix} \sigma(\varepsilon - 1)(A_2 + B_1) - C_1 & \sigma(A_2 + B_2) \\ C_1 + C_2 & 0 \end{bmatrix} \tag{36}$$

respectively.

5.3.1 Equilibrium uniqueness (source)

In order to find equilibrium determinacy, we require sufficiently high intertemporal substitution effects. In this respect, we set $\sigma = 1$.

A calibrated TFP parameter implements the 1% growth rate (quarterly): $A = 0.11134$ according to formula (33).[7]

Using (35) and (36), we get

$$J = \begin{bmatrix} 0.90517 & 6.1638 \\ -0.18886 & 9.2458 \end{bmatrix}$$

with eigenvalues $\lambda_1 = 1.0472$, $\lambda_2 = 9.1038$. Both the eigenvalues are explosive, the steady state is a source and the jump variables adjust: $(x_t, \pi_t) = (x, \pi)$ for $t = 0, 1, \ldots$ Shocks on the fundamentals are neutralized by the rational expectations.

5.3.2 Equilibrium multiplicity (saddle)

It is known that small elasticities of intertemporal substitution promote indeterminacy in CIA models with capital accumulation (Cooley and Hansen (1989)).

Fixing $\sigma = 1/3$, we calibrate the TFP parameter to maintain a quarterly growth rate of 1%: $A = 0.14128$.[8]

Still using (35) and (36), we obtain

$$J = \begin{bmatrix} 1.0916 & -0.41169 \\ 0.061087 & -0.61753 \end{bmatrix}$$

with eigenvalues $\lambda_1 = -0.60269$ and $\lambda_2 = 1.0767$.

We observe that convergence to the steady state is non-monotonic ($-1 < \lambda_1 < 0$). When λ_1 approaches -1, a flip bifurcation arises ($\sigma = \sigma_F$).

5.3.3 Collaterals and macroeconomic volatility

Let us focus on the case (i) of Proposition (4) and assume the credit share to be isoelastic. We fix $\mu = g = 1.01$ and we calibrate the TFP parameter as a

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[7] The essential and optional restrictions hold because the parametrization is the same than in the first case of the comparative statics.

[8] Even in this case, the essential and optional restricitons hold: $x = 12.984$, $\pi = 1 \geq 1$, $0 < \gamma = 0.5 < 1$, $\beta = 0.99 < g^{1/\sigma - 1} = 1.0201$, $I = 1.0399 > 1$, $g = 1.01 > 1$, $0 < 1 - (\Delta + A - g)\gamma\varepsilon\mu/g = 0.98075$.

function of (ε, σ_F):

$$A(\varepsilon, \sigma_F) \equiv \frac{\left[\frac{g}{\mu} - \varepsilon\gamma(\Delta - g)\right] g^{1/\sigma_F} - [\Delta - \varepsilon\gamma(\Delta - g)]\beta\frac{g}{\mu}}{\gamma\varepsilon g^{1/\sigma_F} + (s - \varepsilon\gamma)\beta\frac{g}{\mu}} \qquad (37)$$

that is, given the degree of sensitivity to collaterals and the critical value of intertemporal substitution, we set A to implement $g = 1.01$.

Using (35) and (37), we represent the implicit equation (31), with $\varepsilon_2 = \varepsilon - 1$, in the (ε, σ_F)-plane. In other terms, we plot the behavior of the critical value σ_F as a function of the degree of sensitivity to collaterals ε.
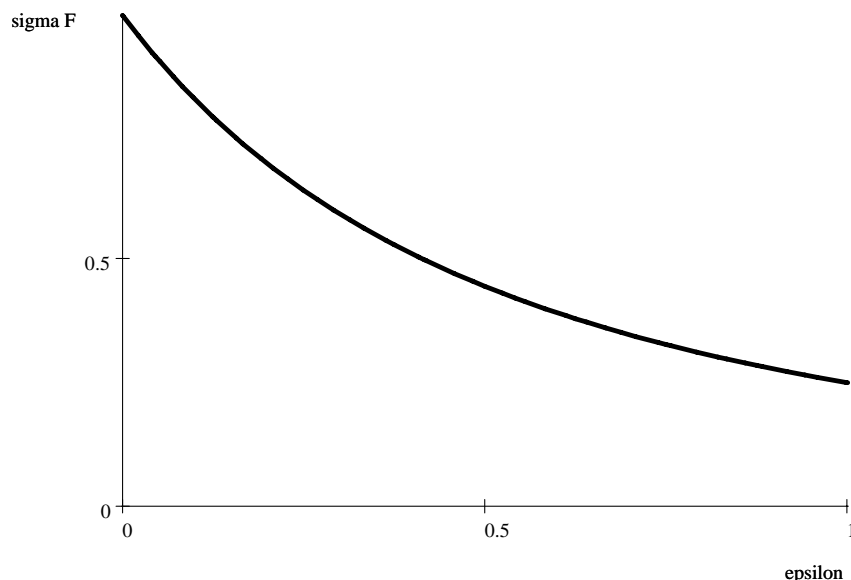

Figure 3: Sensitivity to collaterals and indeterminacy range

For a given ε, the vertical segment between the axis of abscissas and the downward-sloped curve represent the indeterminacy range $(0, \sigma_F)$ of Proposition (4). According to our parametrization, the indeterminacy range shrinks with the degree of sensitivity to collaterals of the credit market. In other terms, the greater the role of collaterals, the less likely the macroeconomic volatility.

Our paper can contribute to shed a light on the underlying mechanism of the subprime crisis. Indeed, concerning this sort of financial earthquake, the credit in the U.S. was not sufficiently collateralized in the past decade (low ε) and the risk of macroeconomic volatility (endogenous business cycles due to self-fulfilling prophecies) is now significant (high σ_F).

6 Conclusion

In the view of the US subprime crisis that hit the global economy in the last few years, we studied the role of collaterals in the emergence of endogenous fluc-

tuations. We considered a monetary one-sector model with endogenous growth where agents are subject to a partial cash-in-advance constraint. In this framework, we assume that the share of consumption purchases paid with credit depends positively on the collateral available to the individual. Our main results are as follows. Money is no longer neutral; an increase in the monetary growth leads to a higher growth in the economy, and more willingness to make the consumption purchases with credit rather than cash. Second, the sensitivity of credit market to collaterals enhances the effectiveness of monetary policy. Third, the endogenous fluctuations become more and more likely to occur as long as the sensitivity of credit market with respect to collateral is lower, that is, agents can more easily get credit and so pay more in credit and less in cash. The later result is consistent with the findings of Bosi and Magris (2003) which is considered as a particular case of our framework.

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