EPEE
UNIVERSITE D'EVRY
VAL D'ESSONNE

DOCUMENT DE RECHERCHE

EPEE

CENTRE D'ETUDE DES POLITIQUES ECONOMIQUES DE L'UNIVERSITE D'EVRY

Health, Growth and Welfare: Why Put Public Money on Medical R&D?

Stefano BOSI & Thierry LAURENT

08 – 18

www.univ-evry.fr/EPEE

UNIVERSITE D'EVRY – VAL D'ESSONNE, 4 BD. FRANÇOIS MITTERRAND, 91025 EVRY CEDEX

Health, Growth and Welfare: Why Put Public Money on Medical R&D?*

Stefano BOSI† and Thierry LAURENT‡
Center for Economic Policy Studies (EPEE)
Department of Economics, University of Paris-Evry

November 26, 2008

Abstract

This paper aims at providing a simple economic framework to address a somewhat neglected question of economic policy, namely the optimal share of investments in medical R&D in total public spending. In order to capture the long-run impact of tax-financed medical R&D on the growth rate, we develop an endogenous growth model in the spirit of Barro [1990]. The model focuses on the optimal sharing of public resources between consumption and (non-health) investment, medical R&D and other health expenditures. It emphasizes the key role played by the public health-related R&D in enhancing economic growth and welfare in the long run. According to our numerical simulations – based on prudential assumptions about the economic impact of medical R&D – a one billion euros permanent reallocation of public spending in favor of medical R&D, would induce about €4 billions GDP increase the first year and a GDP discounted benefit of about €60 billions over a decade. Then, in economies characterized by productive externalities of R&D, the government is recommended to invest substantially more in medical R&D in order to implement an optimal policy.

JEL: H23, H51, I18, O31.

Keywords: public health, medical R&D, public spending, endogenous growth.

*The authors would like to thank Ferhat Mihoubi, Professor at the University of Paris-Evry, for his precious help to improve a previous version of this article ; his technical support allowed to substancially rise the quality of the numerical simulations presented here. Of course any remaining errors are our own.

†EQUIPPE (University of Lille 1) and EPEE (University of Paris-Evry). E-mail: stefano.bosi@orange.fr.

‡Corresponding author. EPEE (Center for Economic Policy Studies), Department of Economics, University of Paris-Evry, 4 bd. F. Mitterrand, 91025 Evry cedex, France.

1 Introduction

Why spend public money in medical R&D? Even if many empirical and theoretical articles have focused on the effects of health on economic growth, surprisingly, there is little room in the growth literature on the specific impact of tax-financed medical R&D.

Human capital results from investments in education and health. The baseline of models devoted to the role of health in economic growth is that a "good health" increases human capital, workers' productivity and, eventually, the growth rate[1] (see Bloom, Canning & Sevilla [2004], Sala-i-Martin, Doppelhofer & Miller [2004], Jamison, Lau & Wang [2004], Gyimah-Brempong & Wilson [2004], Weil [2005]). Clearly, as the provision of health services is financed with public resources, there is a trade-off between health and other services that a government provides, such as education, defence, justice, etc., raising the question of the optimal share of health investments in total public spending.

The issue of the optimal size of specific components (education, health, defence etc.) of public expenditures has been especially addressed in a circumscribed literature inspired by Barro [1990], the seminal endogenous growth model where the government spending plays the role of a productive externality and determines the growth rate of the economy in the long run.[2]

Nevertheless, as pointed out by Agenor [2005], *"most of this literature does not account in a satisfactory manner for the macroeconomic effects of health services. (...), these services are generally described as a government-provided consumption good; models of this type almost invariably introduce a dichotomy in the composition of public spending : expenditure on utility-enhancing services is generally assumed not to affect the production side, whereas production-related spending (such as infrastructure) is assumed to have no effect on utility-enhancing services – for the very reason that these services are usually directly related only to an exogenous component of government spending"*.[3]

In order to tackle these questions and avoid the so-called public-spending dichotomy between *utility-enhancing* and *production-related* expenditures, we introduce R&D investments and distinguish four main components of public spending: public consumption and (non-health) investment, tax-financed medical R&D and other health expenditures. In our model, the R&D externalities play a twofold role as utility-enhancing and production-related public expenditures: medical R&D *(i)* contributes to increase the social welfare through the quality of publicly provided health services; *(ii)* generates positive externalities affecting the TFP and promoting more efficient production processes.

[1] Even if some papers have found evidence of reverse causality: economic growth may enable individuals to spend more on health services. For example Zon & Muysken [2001], suggest that a slow down in growth may be explained by a preference for health that is positively influenced by a growing income per head.

[2] Close to ours, Hosoya [2003] is a two-sector endogenous growth model where the government undertakes productive health expenditures.

[3] From a broader perspective, Agenor [2005, 2006] studies the optimal sharing of government spending between health and infrastructures in an endogenous growth model where the public spending is an input to produce final goods and provide health services.

Despite the fact that a large stream of endogenous growth literature has put forward the technological change as the primary determinant of growth in R&D-based models[4], at the best of our knowledge, no papers have focused on the macroeconomic impact of a tax-financed medical R&D.

Our work aims also at providing a finer description of the different components of public spending from a theoretical point of view and disentangling the specific effect of health-related R&D public expenditures. Our initial suspicion was that scholars and policy makers tend to underestimate the long-run positive effects of public investments in medical R&D and their knowledge spillovers across the main sectors of the economy. Surprisingly, simulating the optimal share of health-related R&D in total public spending backs up our feeling.

Innovations, originating in public R&D investments, benefit, sooner or later, to other firms of the same sector and, step by step, to the whole economy. Such transmission mechanisms allow us to understand how scientific and technical externalities bypass the market, by switching from one firm to another without any priced transaction. This non-priced diffusion process particularly characterizes medical R&D activities, which generate almost immediate effects in the health sector and next in the other sectors.

In order to capture the twofold nature of health-related R&D, we distinguish between productive and unproductive public expenditures, by identifying the different channels through which externalities work. This approach allows us to evaluate the benefits of a permanent reallocation of public spending from, for instance, unproductive public consumption to health-related R&D.

To avoid any misunderstanding, we need to define what we call productive and unproductive spending. Expenditures, eventually leading to a production costs cut, through a classical supply-side effect, are called productive; this definition is voluntarily wide to include public substructures investments – airports, roads, communication networks, *etc.* –, public R&D investments and educational spending as well as public subsidies to private R&D. Unproductive spending does not mean expenditures with no effects on the economy, but rather those generating demand-side effects through a Keynesian multiplier mechanism.

As sketched above, at a very rough level, two main effects of publicly-funded medical research on economic activity can be highlighted:

(i) On the one hand, health-related R&D and its applications improve the performance of medical equipment, *i.e.* the quality of the health services provided by the public health sector and, eventually, the global welfare.

(ii) On the other hand, medical R&D increases the total stock of available scientific and technical knowledge, diffusing sooner or later, to the overall economy and, eventually, contributing to a more efficient production process which results in a higher growth rate.

The first mechanism directly affects the consumers' utility function, while the second the aggregate production function: tax-financed medical R&D, through

[4] The reader is referred to the seminal papers by Romer [1990], Grossman & Helpman [1991], Aghion & Howitt [1992].

innovations diffusion, generates spillovers effects from the health sector towards the whole productive system. These positive externalities constitute a fundamental non-priced productive factor generating increasing returns and contributing to endogenous growth.

The model developed in the paper not only considers public medical R&D, but also private non-health R&D. Because a particular firm does not take into account, when making its R&D investments decisions, the positive impact of such investments on other firms and the overall economy, total R&D spending stands far below its socially optimal level; the role of the government is thus to design the appropriate incentive schemes to encourage private firms to sufficiently invest into R&D. Taking into account the two types of R&D – tax-financed medical R&D *vs* private (non-medical) R&D – allows us to analyze how the government manages the optimal allocation of tax resources between public funding of health-related R&D on the one side, and subsidies to private R&D on the other side.

According to our numerical simulations – based on prudential assumptions about the economic impact of medical R&D – a one billion euros permanent reallocation of public spending in favor of medical R&D, would induce about four billions euros GDP increase the first year and a GDP discounted benefit of about 60 billions euros over a decade. Then, in a framework characterized by productive externalities due to R&D, the optimal policy of the government should be to invest substantially more in medical R&D.

To summarize, the endogenous growth model we propose, allows us:

(i) To clearly distinguish four main public spending components: consumption, health-unrelated public investment, medical R&D and other health expenditures.

(ii) To compute the optimal share of tax-financed medical R&D into public spending and into overall public health expenditures.

(iii) To investigate the twofold nature of medical R&D, by carefully distinguishing its direct effect on welfare – through the quality of health services – from its *supply-side* impact on the aggregate production function.

(iv) To consider different depreciation rates for private capital, private R&D and the different components of public spending.

(v) To determine the optimal level of subsidies (tax-cut) associated to private R&D.

(vi) To present numerical simulations, based on French data, in order to give an evaluation of the impact of an increase in tax-financed R&D investments on GDP and to compute the optimal relative size of medical R&D.

After the presentation of the model (section 2) and the equilibrium analysis (section3), section 4 is devoted to the dynamic analysis. Sections 5 and 6 address the policy issues, while proofs and technicalities are gathered in the appendixes.

2 The model

The purpose of this section is to develop a general equilibrium model to address a policy issue, namely the optimal share of health R&D investment in total public expenditure. Before deriving the equilibrium condition and simulating the French economy, we need to understand how health R&D investments affect the social welfare. Two distinct effects can be highlighted. On the one hand, health R&D and relevant applications raise the health sector productivity *i.e.* improves the quality of the services provided by the health sector: this first R&D transmission channel, from the health sector to the whole economy, directly affects the welfare through the utility function of a representative agent. On the other hand, health R&D investments increase the total stock of scientific and technical knowledge available in the economy. Thereby, as externalities, they affect positively inputs productivity and, eventually, the growth rate of the economy and the welfare ; this indirect but crucial effect of medical R&D identifies a second transmission channel from the health sector to other economic compartments.

An appropriate way to capture the impact of health-related R&D on the growth rate, is to develop an endogenous growth model in the spirit of Barro [1990]. The economy is populated by three types of agents: households, firms and the government. Their behavior is characterized in the following sections.

2.1 Households

Households are supposed to live an infinite number of periods during which they consume a private consumption good c, a public consumption good b and health public services denoted by e. The overall level of utility reached by the representative household during his life is given by the intertemporal utility function:

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(b_t) + w(e_t)\right] \tag{1}$$

where $0 < \beta \equiv 1/(1+i) < 1$ denotes the discount factor and $i > 0$ the time-preference rate.

The overall amount of public health expenditures is divided into two components: the medical R&D m and the other health expenditures n. Viewed as accumulable stocks these two components are employed to produce the health public good $e_t \equiv e(m_t, n_t)$ under constant returns to scale: $e(\mu m, \mu n) = \mu e(m, n)$. The breakdown of (public) health spending between medical R&D and other health expenditures, will enable us to disentangle the specific role played by health R&D investments on social welfare.

As the public consumption good (b) and the public health services (e) are supposed to be 100% publicly funded, households expenditures are simply constituted by private consumption good (c), private investment in capital (k), private investment in R&D (p) and different kinds of taxes: at each period of time the representative household faces the following budget constraint:

$$c_t+k_{t+1}-\Delta_k k_t+p_{t+1}-\Delta_p p_t \leq (1-\tau_k)\, r_{kt} k_t+(1-\tau_p)\, r_{pt} p_t+(1-\tau_l)\, \omega_t l_t \quad (2)$$

where $\delta_i \equiv 1-\Delta_i$, with $i=k,p$, denotes different depreciation rates for private capital and private R&D from a period to another.

Consumption and investment net expenditures stand on the left side of equation (2) while on the right side figures the disposable income with r_k and r_p the real returns on capital and on private R&D, ω the real wage and τ_k, τ_p, τ_l the tax rates on capital, private R&D and labor income, respectively.[5]

For simplicity, labor supply is assumed to be inelastic and normalized to one:

$$l_t = 1 \quad (3)$$

In such a framework the consumer's problem is maximizing the intertemporal utility function (1) with respect to k_t, p_t and c_t. The infinite horizon Lagrangian function can thus be written:

$$\sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right)+v\left(b_t\right)+w\left(e_t\right)\right]$$
$$+\sum_{t=0}^{\infty} \lambda_t \left[(1-\tau_k)\, r_{kt} k_t+(1-\tau_p)\, r_{pt} p_t+(1-\tau_l)\, \omega_t-c_t-k_{t+1}+\Delta_k k_t-p_{t+1}+\Delta_p p_t\right]$$

Rearranging the first-order conditions, after eliminating the Lagrange multipliers, leads directly to a No-Arbitrage Condition, which is, indeed, an equilibrium condition:

$$\Delta_k+(1-\tau_k)\, r_{kt}=\Delta_p+(1-\tau_p)\, r_{pt} \quad (4)$$

to an Euler equation:

$$\frac{u'\left(c_t\right)}{u'\left(c_{t+1}\right)}=\beta\left[\Delta_k+(1-\tau_k)\, r_{kt+1}\right] \quad (5)$$

and to the budget constraint (2), now with equality.

Eventually, the optimal solution must satisfy the transversality condition:

$$\lim_{t\rightarrow\infty} \lambda_t\left(k_{t+1}+p_{t+1}\right)=0 \quad (6)$$

In order to simplify future calculations, and to get tractable equations, we assume that the utility functions u, v and w are characterized by constant elasticities of intertemporal substitution in consumption.

Assumption 1 *CIES preferences:*

$$h(x) \equiv c_h \frac{x^{1-1/\varepsilon_h}-1}{1-1/\varepsilon_h} \text{ if } \varepsilon_h \neq 1;\ h(x) \equiv c_h \ln x \text{ if } \varepsilon_h=1 \quad (7)$$

where $h \equiv u,v,w$ *and, without loss of generality,* $c_u+c_v+c_w=1$.

[5] In this model, the policy maker can take into account the positive externalities associated with private R&D, by reducing the tax rate on the real returns of private R&D, in order to raise private R&D investments.

2.2 Firms

Technology is represented by a production function which processes six inputs: three choice variables for the firm – its stock of capital (k), its stock of knowledge resulting from its (past and current) investments in R&D (p), the labor demand (l) – and three externalities – the stock of (health-unrelated) public capital (a), the stock of knowledge resulting from public investments in medical R&D (m) and, eventually, the average stock of knowledge resulting from other firms' private investments in R&D ($\bar{p}$). The public capital a, viewed as a productive externality, is simply the result of the accumulation of all past and current public expenditures generating productive externalities (*i.e.* devoted, for example, to raise the quantity and/or quality of education and public substructures as roads, airports, cable networks, *etc.*). On their side, health R&D expenditures affect also the global productivity through a standard R&D externality (spillovers effects from the health sector to the other sectors).

Assumption 2 *(i) The production function* $F(k, p, l, a, m, \bar{p})$ *exhibits constant returns to scale in capital, private R&D and labor:*

$$F(\mu k, \mu p, \mu l, a, m, \bar{p}) = \mu F(k, p, l, a, m, \bar{p})$$

(ii) The intensive production function $\tilde{f}(\kappa, \pi, a, m, \bar{p}) \equiv F(\kappa, \pi, 1, a, m, \bar{p})$*, where* $\kappa \equiv k/l$ and $\pi \equiv p/l$ *is supposed to be homogeneous of degree one with respect to its arguments:*

$$\tilde{f}(\mu\kappa, \mu\pi, \mu a, \mu m, \mu\bar{p}) = \mu\tilde{f}(\kappa, \pi, a, m, \bar{p})$$

The producer problem is to maximize the profit with respect to capital stock k_t, R&D p_t and labor force l_t, considering all the externalities – *i.e.*, a, m and $\bar{p}$ – as constants.

$$\max_{k_t, p_t, l_t} F(k_t, p_t, l_t, a_t, m_t, \bar{p}_t) - r_{kt}k_t - r_{pt}p_t - \omega_t l_t$$

The firm equilibrium is thus defined by the equality between the real cost and the productivity of each input:

$$\begin{aligned} r_{kt} &= F_k(k_t, p_t, l_t, a_t, m_t, \bar{p}_t) \\ r_{pt} &= F_p(k_t, p_t, l_t, a_t, m_t, \bar{p}_t) \\ \omega_t &= F_l(k_t, p_t, l_t, a_t, m_t, \bar{p}_t) \end{aligned}$$

Of course, these equalities can be rewritten in terms of the intensive production function:

$$\begin{aligned} r_{kt} &= \tilde{f}_\kappa(\kappa_t, \pi_t, a_t, m_t, \bar{p}_t) \qquad (8) \\ r_{pt} &= \tilde{f}_\pi(\kappa_t, \pi_t, a_t, m_t, \bar{p}_t) \\ \omega_t &= \tilde{f}(\kappa_t, \pi_t, a_t, m_t, \bar{p}_t) - \kappa_t\tilde{f}_\kappa(\kappa_t, \pi_t, a_t, m_t, \bar{p}_t) - \pi_t\tilde{f}_\pi(\kappa_t, \pi_t, a_t, m_t, \bar{p}_t) \end{aligned}$$

2.3 Government

The overall stock of public capital g is the sum of the stocks of *(i)* (health-unrelated) public capital a (public networks substructures, education *etc.*), *(ii)* public consumption b, *(iii)* knowledge m resulting from public investments in health R&D and *(iv)* other (non-R&D) health spending n (medical equipment, current wages, hospital buildings *etc.*):

$$g_t \equiv a_t + b_t + m_t + n_t$$

All these stocks result from accumulation of flows and depreciation across time. For instance, public consumption includes durable consumption goods and can be considered partially accumulable.

The government budget constraint at time t is thus given by:

$$\begin{aligned} & a_{t+1} - \Delta_a a_t + b_{t+1} - \Delta_b b_t + m_{t+1} - \Delta_m m_t + n_{t+1} - \Delta_n n_t \\ \leq \; & \tau_k r_{kt} k_t + \tau_p r_{pt} p_t + \tau_l \omega_t l_t \end{aligned} \tag{9}$$

where $\delta_i \equiv 1 - \Delta_i$ is the depreciation rate of the public capital of type i, the right-hand side of (9) representing the total amount of tax receipt.[6]

In such an economy the economic policy of the government is simply described by the tax vector (τ_k, τ_p, τ_l) and the breakdown of the public "capital" g into its four components:

$$(\sigma_a, \sigma_b, \sigma_m, \sigma_n) \equiv (a_t/g_t, b_t/g_t, m_t/g_t, n_t/g_t) \tag{10}$$

with, of course,

$$\sigma_a + \sigma_b + \sigma_m + \sigma_n = 1 \tag{11}$$

Using the sharing (10) and the budget constraint, equation (9) can be rewritten:

$$\begin{aligned} & \sigma_a (g_{t+1} - \Delta_a g_t) + \sigma_b (g_{t+1} - \Delta_b g_t) + \sigma_m (g_{t+1} - \Delta_m g_t) + \sigma_n (g_{t+1} - \Delta_n g_t) \\ = \; & g_{t+1} - (\sigma_a \Delta_a + \sigma_b \Delta_b + \sigma_m \Delta_m + \sigma_n \Delta_n) g_t \\ \leq \; & \tau_k r_{kt} k_t + \tau_p r_{pt} p_t + \tau_l \omega_t l_t \end{aligned}$$

or, equivalently:

$$g_{t+1} - \Delta g_t \leq \tau_k r_{kt} k_t + \tau_p r_{pt} p_t + \tau_l \omega_t l_t \tag{12}$$

where the depreciation factor of public capital g can be viewed as a weighted average of specific depreciation factors:

$$\Delta \equiv \sigma_a \Delta_a + \sigma_b \Delta_b + \sigma_m \Delta_m + \sigma_n \Delta_n \tag{13}$$

[6] A lag could be introduced between fiscal revenues and public expenditures, but this would not change the long term analysis and the stationary state of the model.

Notice that the "usual" breakdown of the total amount of public spending into four flow components – investment (excluding health), consumption, medical R&D and other health expenditures – can be recovered as:[7]

$$(\tilde{\sigma}_a, \tilde{\sigma}_b, \tilde{\sigma}_m, \tilde{\sigma}_n) \equiv \left(\frac{a_{t+1} - \Delta_a a_t}{g_{t+1} - \Delta g_t}, \frac{b_{t+1} - \Delta_b b_t}{g_{t+1} - \Delta g_t}, \frac{m_{t+1} - \Delta_m m_t}{g_{t+1} - \Delta g_t}, \frac{n_{t+1} - \Delta_n n_t}{g_{t+1} - \Delta g_t} \right) \tag{14}$$

still with $\tilde{\sigma}_a + \tilde{\sigma}_b + \tilde{\sigma}_m + \tilde{\sigma}_n = 1$.

3 Equilibrium

The equilibrium in the labor market is characterized by an inelastic labor supply (*cf.* (3)): in order to compute the general equilibrium, we focus on the markets of inputs and goods. Since all (competitive) firms are identical, we have in equilibrium:

$$\bar{p}_t = p_t = \pi_t$$

and, since $l_t = 1$,

$$\begin{aligned} r_{kt} k_t + r_{pt} p_t + \omega_t l_t &= r_{kt} \kappa_t + r_{pt} \pi_t + \omega_t \\ &= \tilde{f}(\kappa_t, \pi_t, a_t, m_t, \pi_t) \equiv f(\kappa_t, \pi_t, a_t, m_t) \end{aligned}$$

We notice that $f(\kappa, \pi, a, m)$ is still homogeneous of degree one and

$$f_\kappa(\kappa, \pi, a, m) = \tilde{f}_\kappa(\kappa, \pi, a, m, \pi)$$

Let us now define five elasticities of interest:[8]

$$\begin{aligned} s_{kt} &\equiv \frac{f_\kappa(\kappa_t, \pi_t, a_t, m_t)\kappa_t}{f(\kappa_t, \pi_t, a_t, m_t)} = \frac{r_{kt}\kappa_t}{f(\kappa_t, \pi_t, a_t, m_t)} \\ s_{pt} &\equiv \frac{\tilde{f}_\pi(\kappa_t, \pi_t, a_t, m_t, \bar{p}_t)\pi_t}{\tilde{f}(\kappa_t, \pi_t, a_t, m_t, \bar{p}_t)} = \frac{r_{pt}\pi_t}{f(\kappa_t, \pi_t, a_t, m_t)} \\ s_{\pi t} &\equiv \frac{f_\pi(\kappa_t, \pi_t, a_t, m_t)\pi_t}{f(\kappa_t, \pi_t, a_t, m_t)} = 1 - s_{kt} - s_{at} - s_{mt} \\ s_{at} &\equiv \frac{f_a(\kappa_t, \pi_t, a_t, m_t)a_t}{f(\kappa_t, \pi_t, a_t, m_t)} \\ s_{mt} &\equiv \frac{f_m(\kappa_t, \pi_t, a_t, m_t)m_t}{f(\kappa_t, \pi_t, a_t, m_t)} \end{aligned}$$

The first two elasticities are the shares of capital and private R&D revenues in total income.

[7] The link between σ and $\tilde{\sigma}$ becomes an explicit function at the steady state (see formula (80) in the Appendix 2).

[8] These elasticities turns out to be constant under a Cobb-Douglas technology.

Since $a_t = \sigma_a g_t$ and $m_t = \sigma_m g_t$, the representative agent budget constraint (2) rewrites as an aggregate resources constraint:

$$\begin{aligned} & c_t + \kappa_{t+1} - \Delta_k \kappa_t + \pi_{t+1} - \Delta_p \pi_t \\ \leq \ & \left[(1-\tau_k) s_{kt} + (1-\tau_p) s_{pt} + (1-\tau_l)(1 - s_{kt} - s_{pt}) \right] f(\kappa_t, \pi_t, \sigma_a g_t, \sigma_m g_t) \end{aligned} \tag{15}$$

On its side, the government budget constraint (12) becomes:

$$g_{t+1} - \Delta g_t = \left[\tau_k s_{kt} + \tau_p s_{pt} + \tau_l (1 - s_{kt} - s_{pt}) \right] f(\kappa_t, \pi_t, \sigma_a g_t, \sigma_m g_t) \tag{16}$$

Substituting (8) in the Euler equation (5), one gets:

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta \left[\Delta_k + (1-\tau_k) f_\kappa (\kappa_{t+1}, \pi_{t+1}, a_{t+1}, m_{t+1}) \right] \tag{17}$$

Observing that the homogeneity property of the intensive production function implies that its derivatives are homogeneous of degree zero,

$$f_\kappa(\mu\kappa, \mu\pi, \mu a, \mu m) = f_\kappa(\kappa, \pi, a, m)$$

it follows immediately

$$f_\kappa(\kappa_t, \pi_t, \sigma_a g_t, \sigma_m g_t) = f_\kappa\left(\frac{\kappa_t}{g_t}, \frac{\pi_t}{g_t}, \sigma_a, \sigma_m \right)$$

Noticing that,

$$r_{pt} = \frac{\kappa_t}{\pi_t} \frac{s_{pt}}{s_{kt}} f_\kappa(\kappa_t, \pi_t, a_t, m_t)$$

NAC (4) becomes:

$$\Delta_k + (1-\tau_k) f_\kappa(\kappa_t, \pi_t, a_t, m_t) = \Delta_p + (1-\tau_p) \frac{\kappa_t}{\pi_t} \frac{s_{pt}}{s_{kt}} f_\kappa(\kappa_t, \pi_t, a_t, m_t)$$

In order to simplify the analytical results, but without a substantial loss of generality, we assume:

Assumption 3 $\Delta_p = \Delta_k$.

Under this assumption, we find**:**

$$\pi_t = \frac{1-\tau_p}{1-\tau_k} \frac{s_{pt}}{s_{kt}} \kappa_t \tag{18}$$

Then

$$r_{kt} = f_\kappa(\kappa_t, \pi_t, a_t, m_t) = f_\kappa\left(\frac{\kappa_t}{g_t}, \frac{\pi_t}{g_t}, \sigma_a, \sigma_m \right) = f_\kappa\left(\frac{\kappa_t}{g_t}, \frac{1-\tau_p}{1-\tau_k} \frac{s_{pt}}{s_{kt}} \frac{\kappa_t}{g_t}, \sigma_a, \sigma_m \right)$$

For simplicity, we assume that the shares of capital income and private R&D in total income are constant.

Assumption 4 *The elasticities vector* $(s_{kt}, s_{pt}, s_{at}, s_{mt}) = (s_k, s_p, s_a, s_m)$ *is constant.*

Let us define $x_t \equiv \kappa_t / g_t$ and

$$\varphi(x_t) \equiv f\left(x_t, \frac{1-\tau_p}{1-\tau_k}\frac{s_{pt}}{s_{kt}}x_t, \sigma_a, \sigma_m\right)$$

Under Assumption 4, we obtain:

$$\varphi'(x_t) = f_\kappa + \frac{1-\tau_p}{1-\tau_k}\frac{s_p}{s_k}f_\pi = \frac{s_k+s_\pi}{s_k}f_\kappa\left(x_t, \frac{1-\tau_p}{1-\tau_k}\frac{s_p}{s_k}x_t, \sigma_a, \sigma_m\right)$$

since (18) holds and

$$f_\pi = \frac{s_\pi}{s_k}\frac{\kappa_t}{\pi_t}f_\kappa = \frac{1-\tau_k}{1-\tau_p}\frac{s_\pi}{s_p}f_\kappa$$

where $s_\pi = 1 - s_k - s_a - s_m$. Therefore,

$$\begin{aligned} f_\kappa &= \frac{s_k}{s_k+s_\pi}\varphi'(x_t) \\ r_{kt} &= f_\kappa(\kappa_t, \pi_t, a_t, m_t) = \frac{s_k}{s_k+s_\pi}\varphi'(x_t) \end{aligned}$$

Defining the tax pressure as an average tax rate,

$$\tau \equiv s_k\tau_k + s_p\tau_p + (1-s_k-s_p)\tau_l \tag{19}$$

equations (15), (16) and (17) can be then rewritten under Assumptions 3 and 4:

$$c_t + \left(1+\frac{1-\tau_p}{1-\tau_k}\frac{s_p}{s_k}\right)(\kappa_{t+1}-\Delta_k\kappa_t) \leq (1-\tau)\, g_t\varphi(x_t) \tag{20}$$

$$g_{t+1} - \Delta g_t = \tau g_t \varphi(x_t) \tag{21}$$

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta\left[\Delta_k + (1-\tau_k)\frac{s_k}{s_k+s_\pi}\varphi'(x_{t+1})\right]$$

since

$$f(\kappa_t, \pi_t, \sigma_a g_t, \sigma_m g_t) = g_t f\left(\frac{\kappa_t}{g_t}, \frac{\pi_t}{g_t}, \sigma_a, \sigma_m\right) = g_t\varphi(x_t)$$

Setting,

$$\begin{aligned} y_t &\equiv c_t/g_t \\ \gamma_t &\equiv g_{t+1}/g_t \end{aligned} \tag{22}$$

and dividing both sides of (20) and (21) by g_t, one eventually gets:

$$y_t + \left(1+\frac{1-\tau_p}{1-\tau_k}\frac{s_p}{s_k}\right)(\gamma_t x_{t+1}-\Delta_k x_t) \leq (1-\tau)\,\varphi(x_t) \tag{23}$$

$$\gamma_t = \Delta + \tau\varphi(x_t) \tag{24}$$

On the other hand, the Euler equation can be revisited under Assumption 1:

$$\frac{c_{t+1}}{c_t} = \left(\beta\left[\Delta_k + (1-\tau_k)\frac{s_k}{s_k+s_\pi}\varphi'(x_{t+1})\right]\right)^{\varepsilon_u}$$

So, we have:

$$\frac{y_{t+1}}{y_t}\gamma_t = \left(\beta\left[\Delta_k + (1-\tau_k)\frac{s_k}{s_k+s_\pi}\varphi'(x_{t+1})\right]\right)^{\varepsilon_u} \tag{25}$$

4 Dynamic system

Substituting (24) into (25) and (23) **gives:**

$$[\Delta + \tau\varphi(x_t)]\frac{y_{t+1}}{y_t} = \left(\beta\left[\Delta_k + (1-\tau_k)\frac{s_k}{s_k+s_\pi}\varphi'(x_{t+1})\right]\right)^{\varepsilon_u} \tag{26}$$

$$y_t + \left(1+\frac{1-\tau_p}{1-\tau_k}\frac{s_p}{s_k}\right)(\Delta x_{t+1} - \Delta_k x_t) = \left[1-\tau-\left(1+\frac{1-\tau_p}{1-\tau_k}\frac{s_p}{s_k}\right)\tau x_{t+1}\right]\varphi(x_t) \tag{27}$$

These equations constitute a two-dimensional dynamic system in (x_t, y_t) where x_t – but not y_t – is a predetermined variable.

4.1 Stationary state

In order to compute the steady state, one omits the time subscripts in (26-27) and one solves the system:

$$\gamma = \Delta + \tau\varphi(x) = \left(\beta\left[\Delta_k + (1-\tau_k)\frac{s_k}{s_k+s_\pi}\varphi'(x)\right]\right)^{\varepsilon_u} \tag{28}$$

$$y = \left(1+\frac{1-\tau_p}{1-\tau_k}\frac{s_p}{s_k}\right)(\Delta_k - \Delta)x + \left[1-\tau-\left(1+\frac{1-\tau_p}{1-\tau_k}\frac{s_p}{s_k}\right)\tau x\right]\varphi(x) \tag{29}$$

Growth is balanced (usual arguments of endogenous growth theory apply): $\gamma \equiv g_{t+1}/g_t = c_{t+1}/c_t = k_{t+1}/k_t = p_{t+1}/p_t$. Noticing that $\lambda_t = \beta^t u'(c_t)$ and using (7), the transversality condition (6) becomes:

$$\lim_{t\to\infty} c_u c_0^{-1/\varepsilon_u}(k_0+p_0)\gamma\left(\beta\gamma^{1-1/\varepsilon_u}\right)^t = 0$$

i.e., $\beta\gamma^{1-1/\varepsilon_u} < 1$. Thus, we get $\gamma < \Delta_k + \rho$ from (28), where $\rho \equiv (1-\tau_k) r_k = (1-\tau_k)\varphi'(x) s_k/(s_k+s_\pi)$ is the after-tax return on capital.

4.2 Local dynamics

Raising the question of saddle-path stability is not a mere theoretical matter. Indeed, as shown by Blanchard and Quah [1989], saddle-path stability implies the uniqueness of equilibrium under rational expectations.

In this section, we show, without introducing additional restrictions on the fundamentals, that the equilibrium is a saddle path and converges to the stationary state. Our proof is sufficiently general to show the uniqueness of equilibrium as a robust feature of Barro-like models.

In the saddle case, the converging path is the unique solution of the dynamic system under rational expectations because the other trajectories either make some variable negative, soon or later, or violate the transversality condition. Since x_0 is a predeterminate variable, the control variable y_0 jumps to place the starting point (x_0, y_0) on the saddle path.

In order to study the local dynamics and prove the saddle-path stability, we linearize the dynamic system (26-27) around the steady state.

Differentiating (26) with respect to dynamic variables $(x_{t+1}, y_{t+1}, x_t, y_t)$ and using (28-29), one gets,

$$\gamma\varepsilon_u \frac{\rho}{\rho+\Delta_k}\frac{x\varphi''}{\varphi'}\frac{dx_{t+1}}{x} - \gamma\frac{dy_{t+1}}{y} = \tau\varphi' x\frac{dx_t}{x} - \gamma\frac{dy_t}{y} \tag{30}$$

where the differentials are relative to the stationary state.

Linearizing now equation (27) around the steady state, one has:

$$\eta\gamma\frac{dx_{t+1}}{x} = \left[\eta\left(\Delta_k - \tau x\varphi'\right) + \left(1-\tau\right)\varphi'\right]\frac{dx_t}{x} - \frac{y}{x}\frac{dy_t}{y} \tag{31}$$

where,

$$\eta \equiv 1 + \frac{s_p}{s_k}\frac{1-\tau_p}{1-\tau_k} \tag{32}$$

We observe that (32) implies:

$$y = \eta\left(\Delta_k - \Delta\right)x + \left(1 - \tau - \eta\tau x\right)\varphi\left(x\right) \tag{33}$$

Let $\varepsilon_2 \equiv x\varphi''/\varphi' < 0$ denote the elasticity of the interest rate with respect to the ratio κ/g (capital per head over public spending). The linear system (30-31) rewrites equivalently:

$$\begin{bmatrix} \frac{dx_{t+1}}{x} \\ \frac{dy_{t+1}}{y} \end{bmatrix} = \begin{bmatrix} \gamma\varepsilon_u\varepsilon_2\frac{\rho}{\rho+\Delta_k} & -\gamma \\ \gamma\eta & 0 \end{bmatrix}^{-1} \begin{bmatrix} \tau x\varphi' & -\gamma \\ \Delta_k\eta + \left(1-\tau-\tau x\eta\right)\varphi' & -\frac{y}{x} \end{bmatrix} \begin{bmatrix} \frac{dx_t}{x} \\ \frac{dy_t}{y} \end{bmatrix}$$

The determinant and the trace of the Jacobian matrix are respectively:

$$D = \frac{1}{\gamma}\left[\Delta_k + \frac{\varphi'}{\eta}\left(1 - \tau - \tau x\eta - \tau\frac{y}{\gamma}\right)\right] \tag{34}$$

$$T = 1 + D + \frac{1}{\gamma}\frac{y}{x\eta}\left(\frac{\varphi'}{\gamma}\tau x - \frac{\rho}{\rho+\Delta_k}\varepsilon_2\varepsilon_u\right) \tag{35}$$

The following proposition proves the uniqueness of equilibrium transition.

Proposition 1 *The equilibrium is unique (saddle-path stability).*

Proof. See the Appendix 1.

Proposition 1 recovers the equilibrium determinacy of Barro [1990] where, however, dynamics are poorer due to the lack of short-run transitions. The economy jumps from the very beginning on the steady state because dynamics are driven by a simple equation with one non-predetermined variable and an unstable eigenvalue. In our model, determinacy still prevails, but equilibrium transitions becomes possible.

5 Optimal policy

As seen above, a key issue of this model is to find the optimal (welfare-maximizing) breakdown of public capital (g) into four components: productive public capital (a), public consumption (b), stock of knowledge issued from public investments in medical R&D (m) and other public expenditures on health (n). Until now, economic agents (households and firms) were supposed to solve their programs, taking as given the economic policy, *i.e.* the tax rates and the breakdown of public capital into these four components. The government is supposed now to compute the optimal policy, that is the vector of optimal shares of public capital and tax rates $\left(\sigma_a^*, \sigma_b^*, \sigma_m^*, \sigma_n^*, \tau_k^*, \tau_p^*, \tau_l^*\right)$, given the private agents' best responses. As the different shares resulting from the breakdown of public capital add up to unity (*cf.* (11)): $\sigma_a + \sigma_b + \sigma_m + \sigma_n = 1$, the number of policy tools reduces to six endogenous variables and policy making sums up to computing and announcing an optimal vector $\left(\sigma_a^*, \sigma_m^*, \sigma_n^*, \tau_k^*, \tau_p^*, \tau_l^*\right)$.

Under an inelastic labor supply and no restrictions on the tax rates, the optimal policy should be a corner solution consisting in levying taxes on labor income at a full rate ($\tau_l^* = 1$) and subsidizing ($\tau_k^*, \tau_p^* < 0$) the inputs that generate positive externalities *i.e.* private capital and private R&D. A welfare maximization without any constraint on the tax rates should entail an electoral suicide.[9]

In order to rule out such nonsensical policy, we assume the same tax rate τ_q on capital and labor income: $\tau_q \equiv \tau_k = \tau_l$. This restriction is far from being unrealistic and is compatible with a balanced growth path.[10] A common tax rate on capital and labor implements an interior solution because capital supply is elastic and the capital is an essential input in the production function (see the Inada conditions).

This restriction brings back to five the number of policy variables, while leaving the tax rate τ_p on private R&D an independent tool ; we can freely play

[9] Of course, introducing an endogenous labor supply would allow to bypass this problem (a unit labor income tax rate would imply no labor supply and zero output, which is clearly inefficient).

[10] Leisure demand is bounded and can not grow as the other arguments in the utility function, namely private and public consumption. The King-Plosser-Rebelo utility function can not be considered because of separability.

with τ_p in order to evaluate the macroeconomic impact of subsidizing private investments in R&D.

5.1 Characterization

The shortcut of a representative agent, makes equivalent for the government to maximize, with respect to the five policy tools $(\sigma_a, \sigma_m, \sigma_n, \tau_p, \tau_q)$, any social welfare function – but strictly increasing in the individual utilities – or the representative agent's utility function (1).

To keep things as simple as possible, let us focus directly on the case of regular growth (in the long-run the equilibrium will be sufficiently close to the steady state).[11]

We will use a Cobb-Douglas production function not only to satisfy the homogeneity property (see Assumption 2),

$$\begin{aligned} F(\mu k, \mu p, \mu l, a, m, \bar{p}) &= \mu F(k, p, l, a, m, \bar{p}) \\ f(\mu \kappa, \mu \pi, \mu a, \mu m) &= \mu f(\kappa, \pi, a, m) \end{aligned}$$

but also to simplify numerical simulations. Similarly, we assume a Cobb-Douglas as production function of medical cares.

Assumption 5 *The production functions F and e are specified as follows:*

$$\begin{aligned} F(k, p, l, a, m, \bar{p}) &= \theta k^{s_k} p^{s_p} l^{1-s_k-s_p} a^{s_a} m^{s_m} \bar{p}^{1-s_k-s_p-s_a-s_m} \\ e(\sigma_m, \sigma_n) &\equiv B \sigma_m^{\beta_m} \sigma_n^{\beta_n} \end{aligned}$$

with $\beta_m + \beta_n = 1$.

Eventually, we restrict ourselves to the case of logarithmic utility functions, easier to handle and widespread in the RBC literature.

Assumption 6 $u(c) \equiv c_u \ln c$, $v(b) \equiv c_v \ln b$, $w(e) \equiv c_w \ln e$.

A logarithmic utility function corresponds to the case of a unit elasticity of intertemporal substitution. The social welfare function becomes,

$$W = \sum_{t=0}^{\infty} \beta^t c_u \ln c_t + \sum_{t=0}^{\infty} \beta^t c_v \ln b_t + \sum_{t=0}^{\infty} \beta^t c_w \ln e_t$$

where, without loss of generality:

$$c_u + c_v + c_w = 1 \tag{36}$$

[11] In fact, because of the uniqueness of the equilibrium, we could compute utility along a transitional path, whenever the starting point stands off the steady state, and maximize its value with respect to the policy parameters, but it would drive us to hard analytical computations. As the main goal of the paper is to analyze long-run policy effects, we can focus on the steady state or transitional equilibria close enough (by continuity, the optimal rule will change little along an equilibrium path in a neighborhood of the stationary state).

Instead of maximizing welfare with respect to policy tools $(\sigma_a, \sigma_m, \sigma_n, \tau_p, \tau_q)$, one maximizes it indirectly with respect to an auxiliary vector $(\sigma_a, \sigma_m, \sigma_n, \eta, h)$, where,

$$h \equiv \gamma - \Delta_k \tag{37}$$

and finally compute $(\tau_p, \tau_q)^*$ using $(\sigma_a, \sigma_m, \sigma_n, \eta, h)^*$.

Proposition 2 *The optimal policy is a vector* $(\sigma_a, \sigma_m, \sigma_n, \eta, z, \varphi)^*$ *solution of the following system:*

$$\begin{aligned}
\varphi &= \theta \sigma_a^{s_a} \sigma_m (\varphi, \sigma_a)^{s_m} (\eta (\varphi, \sigma_a) - 1)^{1-s_k-s_a-s_m} \\
&\quad \times \left[\frac{\varphi - z(\varphi, \sigma_a)}{z(\varphi, \sigma_a) + \Delta(\varphi, \sigma_a) - \beta \Delta_k} \frac{\beta s_k}{1 - s_p + (\eta(\varphi, \sigma_a) - 1) s_k} \right]^{1-s_a-s_m} \\
0 &= \frac{\varphi \frac{1-s_k-s_a-s_m}{\eta(\varphi,\sigma_a)-1} - (\varphi - z(\varphi, \sigma_a)) \frac{s_k}{1-s_p+(\eta(\varphi,\sigma_a)-1)s_k}}{\varphi (s_a + s_m) - z(\varphi, \sigma_a)} \\
&\quad + \frac{(1-\beta)(z(\varphi, \sigma_a) + \Delta(\varphi, \sigma_a))}{\frac{1-s_p-s_k}{s_k} (z(\varphi, \sigma_a) + \Delta(\varphi, \sigma_a) - \beta \Delta_k) + \eta(\varphi, \sigma_a)(1-\beta)(z(\varphi, \sigma_a) + \Delta(\varphi, \sigma_a))}
\end{aligned}$$

where

$$\begin{aligned}
z(\varphi, \sigma_a) &\equiv \varphi (s_a + s_m) - \frac{c_u \left(\Delta_a - \Delta_b + \varphi \frac{s_a}{\sigma_a} \right) \left(1 - \sigma_a - \frac{\varphi s_m}{\Delta_a - \Delta_m + \varphi \frac{s_a}{\sigma_a}} \right)}{c_v + c_w \left(\Delta_a - \Delta_b + \varphi \frac{s_a}{\sigma_a} \right) \left(\frac{\beta_m}{\Delta_a - \Delta_m + \varphi \frac{s_a}{\sigma_a}} + \frac{\beta_n}{\Delta_a - \Delta_n + \varphi \frac{s_a}{\sigma_a}} \right)} \\
\sigma_n(\varphi, \sigma_a) &= \frac{\beta_n}{\Delta_a - \Delta_n + \varphi \frac{s_a}{\sigma_a}} \frac{c_w \left(\Delta_a - \Delta_b + \varphi \frac{s_a}{\sigma_a} \right) \left(1 - \sigma_a - \frac{\varphi s_m}{\Delta_a - \Delta_m + \varphi \frac{s_a}{\sigma_a}} \right)}{c_v + c_w \left(\Delta_a - \Delta_b + \varphi \frac{s_a}{\sigma_a} \right) \left(\frac{\beta_m}{\Delta_a - \Delta_m + \varphi \frac{s_a}{\sigma_a}} + \frac{\beta_n}{\Delta_a - \Delta_n + \varphi \frac{s_a}{\sigma_a}} \right)} \\
\sigma_m(\varphi, \sigma_a) &\equiv \frac{\varphi s_m}{\Delta_a - \Delta_m + \varphi \frac{s_a}{\sigma_a}} + \sigma_n(\varphi, \sigma_a) \frac{\beta_m}{\beta_n} \frac{\Delta_a - \Delta_n + \varphi \frac{s_a}{\sigma_a}}{\Delta_a - \Delta_m + \varphi \frac{s_a}{\sigma_a}} \\
\eta(\varphi, \sigma_a) &\equiv \frac{1}{1-\beta} \frac{1 - s_p - s_k}{s_k} \frac{1 - \frac{\varphi + \Delta(\varphi,\sigma_a) - \beta \Delta_k}{\varphi(s_a+s_m) - z(\varphi,\sigma_a)} + \frac{1}{c_u} \frac{\beta}{1-\beta} \frac{z(\varphi,\sigma_a) + \Delta(\varphi,\sigma_a) - \beta \Delta_k}{z(\varphi,\sigma_a) + \Delta(\varphi,\sigma_a)}}{\frac{\varphi + \Delta(\varphi,\sigma_a) - \beta \Delta_k}{\varphi(s_a+s_m) - z(\varphi,\sigma_a)} \frac{z(\varphi,\sigma_a) + \Delta(\varphi,\sigma_a)}{z(\varphi,\sigma_a) + \Delta(\varphi,\sigma_a) - \beta \Delta_k} - \frac{1}{c_u} \frac{\beta}{1-\beta} - 1}
\end{aligned}$$

and

$$\Delta(\varphi, \sigma_a) \equiv \Delta_b + \sigma_a (\Delta_a - \Delta_b) + \sigma_m (\varphi, \sigma_a)(\Delta_m - \Delta_b) + \sigma_n(\varphi, \sigma_a)(\Delta_n - \Delta_b)$$

Proof. See the Appendix 1.

After this system has been solved, one gets $\sigma_a, \sigma_m, \sigma_n, \eta, z, \varphi$ and Δ; then one computes $\sigma_b = 1 - \sigma_a - \sigma_m - \sigma_n$, the growth factor $\gamma \equiv z + \Delta$ and (see Appendix 1)

$$x = \frac{\varphi - z}{z + \Delta - \beta \Delta_k} \frac{\beta s_k}{1 - s_p + (\eta - 1) s_k}$$

We thus have,

$$\tau = \left[1 + x \frac{\gamma - \beta \Delta_k}{\gamma - \Delta} \frac{1 - s_p + (\eta - 1) s_k}{\beta s_k} \right]^{-1}$$

and eventually:

$$\tau_p = 1 - \frac{1-\tau}{1-s_p+(\eta-1)\,s_k}\frac{s_k}{s_p}(\eta-1)$$
$$\tau_q = 1 - \frac{1-\tau}{1-s_p+(\eta-1)\,s_k}$$

5.2 Numerical computation

The purpose of this subsection is to compute the optimal economic policy $\left(\sigma_a^*, \sigma_b^*, \sigma_m^*, \sigma_n^*, \tau_p^*, \tau_q^*, \tau^*\right)$ characterized above (see Proposition 2). As the analytical resolution of the system is not possible, we fix plausible values for the structural parameters and we solve the resulting system.

5.2.1 Parametrization

Table 1 below summarizes the calibration of the 14 free parameters of the model.

The yearly rate of time preference is plausibly set equal to 4%. As our model does not allow us to distinguish between the depreciation rate of private capital δ_k and the depreciation rate of private R&D δ_p, we assume a common 8% annual depreciation rate for both types of capital, corresponding, more or less, to a half depreciation after 8.5 years. To avoid any bias in fanonsensicalvor of public medical R&D and to be consistent with the calibration of the other depreciation rates, we set to 8% the depreciation rate δ_m of the stock of knowledge issued from public investments in health-related R&D.[12]

The depreciation rate δ_a of productive public capital is set to 5% to take into account that public and private capital usually depreciate at different rates, reflecting *(i)* the casual observation that some types of governmentally supplied infrastructure (*e.g.* roads, port facilities, nuclear power stations, *etc.*) are typically more durable than those provided by private agents, *(ii)* the fact that a significant part of public investment is devoted to increase human capital which is characterized by a lower depreciation rate, often below 2%, than the physical one.[13] In this case, if we assume that *(i)* the investment in human capital represents 10% of the total amount of public (human+ physical) investment, *(ii)* the depreciation rate of public physical capital is equal to 6% and *(iii)* the depreciation rate of human capital is equal to 2%, then the average depreciation rate of total public capital is equal to 5%.

Finally, the depreciation rate δ_b of public consumption is set to 100% (full yearly depreciation) whereas the one of ordinary health expenditures (δ_n) – weighted average of a 100% depreciation rate associated to the public health

[12] The assumption of a common depreciation rate, equal to 8%, for private capital, private R&D and public medical-R&D is not immune from criticism. In a more specified setting, one would fix higher depreciation rates for private and public R&D (around 12%) and a lower depreciation rate for private capital (around 7%). On this particular point, one can see Nadiri & Prucha [1996], Hall [2006], [2007], Mead [2007].

[13] See, for instance, Arrazola & de Hevia [2004], Arrazola, de Hevia, Risueño & Sanz [2005].

consumption (wage bill of the public health sector, drugs/medical consumption refunded by social security administrations, *etc.*) and the lower depreciation rates associated to medical equipment, hospital buildings, *etc.* – is fixed to 61%.[14] The share s_k of capital remuneration in GDP is set to 75% according to the empirical estimates by Mankiw, Romer & Weil [1992], Aghion & Howitt [1997] and other empirical estimations;[15] s_k is a measure of both human and physical capital share in total income, while $1 - s_k = s_a + s_m + s_p$ represents the overall weight of the three productive externalities associated with public capital (a), private R&D (p) and medical R&D (m).

Our theoretical analysis has shed a light on the role of productive externalities (associated to public or private investments in R&D) as powerful growth engine. One of the goals of the paper is to evaluate the impact of R&D on economic growth and social welfare. In order to provide a cautious evaluation of the macroeconomic impact of R&D expenditures and to avoid any overestimation, we minimize the size of public and private R&D externalities by setting $s_p = s_m = 1\%$.

With the same attitude, we decide *(i)* to limit the relative weight of the health public good in the household's utility function – i.e. the indirect impact of health-related R&D on social welfare – by considering that households strongly prefer private consumption and public consumption: $c_u = c_v = 46\%$ *i.e.* $c_w = 8\%$,[16] *(ii)* to limit the direct role played by medical R&D in the production of health services: $\beta_m = 10\%$.

This set of prudential and, in a way, pessimistic assumptions, about the R&D mechanisms at work in the economy, could shelter us from criticisms about a possible overestimation of their effects on the equilibrium growth rate and welfare.

Eventually, the productivity parameter θ is set to 0.5631. More precisely, the TFP is revealed by the observed growth rate: we calibrate θ to replicate the average yearly value observed in the French economy during the last decade (2%), while setting the policy parameters $(\tilde{\sigma}_a, \tilde{\sigma}_b, \tilde{\sigma}_m, \tilde{\sigma}_n, \tau_p, \tau_q, \tau)$ to the values experienced in 2006.[17]

---

[14] In France, the amount of public health investment (investment in public hospitals, excluding consumption of intermediate goods) in 2006, stands around 5.2 billions €, representing more or less 4% of non-R&D public health expenditures and meaning that 96% of them are in fact pure consumption; in such a case, considering a 100% depreciation rate for the consumption part and a 6% depreciation rate for the investment part, one gets an average 61% depreciation rate for non-R&D public health expenditures.

[15] See, among the others, Jones [2007], Chari, Kehoe & McGrattan [1997], Howitt [2000], Klenow & Rodriguez-Clare [2005], Manuelli & Seshadri [2005] and Erosa, Koreshkova & Restuccia [2006].

[16] Notice that the share of the total amount of public health expenditures in GDP in France is more or less equal to 9%; setting $c_w = 8\%$ is a slightly prudential, but reasonable hypothesis.

[17] An extensive calibration of the model and the evaluation of $(\tilde{\sigma}_a, \tilde{\sigma}_b, \tilde{\sigma}_m, \tilde{\sigma}_n, \tau_p, \tau_q, \tau)$ are provided in section 6.1 below.

Parameter	Definition	Value (%)
i	Rate of time preference	4.00
c_u	Weight of the private consumption in the utility function	46.00
c_v	Weight of the public (non-health) consumption in the utility function	46.00
$c_w = 1 - c_u - c_v$	Weight of the health good in the utility function	8.00
s_k	Parameter of the productive private capital in the production function	75.00
s_a	Parameter of the productive public capital in the production function	23.00
s_m	Parameter of the public health-R&D in the production function	1.00
s_p	Parameter of the private R&D in the production function	1.00
θ	Scale parameter of the production function	56.31
β_m	Parameter of the health-R&D expenditures in the production function of the health good	10.00
$\beta_n = 1 - \beta_m$	Parameter of the "other" health expenditures in the production function of the health good	90.00
$\delta_k = \delta_p$	Depreciation rate of the productive private capital = Depreciation rate of the private R&D	8.00
δ_a	Depreciation rate of the productive public capital	5.00
δ_b	Depreciation rate of the public consumption	100.00
δ_m	Depreciation rate of the stock of knowledge issued from public investments in health-related R&D	8.00
δ_n	Depreciation rate of other health expenditures	61.00

Table 1. Optimal policy: calibration

5.2.2 Results

In order to compute the optimal policy corresponding to the calibration of the parameters listed in Table 1, we solve numerically the implicit system in Proposition 2. The optimal values we are looking for are:

(i) the breakdown of the public capital into the four components: σ_a, σ_b, σ_m and σ_n,

(ii) the breakdown of the total amount of public spending into the four components: $\tilde{\sigma}_a$, $\tilde{\sigma}_b$, $\tilde{\sigma}_m$ and $\tilde{\sigma}_n$,

(iii) the tax pressure (average tax rate) τ and its breakdown into the tax rate on labor and capital income τ_q and the specific tax rate on private R&D τ_p,

(iv) the growth rate of the economy: $\gamma - 1$,

(v) the social welfare: W.

Table 2 below summarizes the results under the benchmark parametrization of Table 1, but also shows the optimal policies corresponding to alternative parametrizations.

More precisely, in Table 2, we analyze the government response to a structural change involving one or more parameters. Each line considers the deviation of one or more parameters with respect to the benchmark, while the remaining parameters takes the values in Table 1: the vertical arrows indicate whether the parameter value increases (↑) or decreases (↓) with respect to the benchmark of

Table 1.

Parameters	σ_a	σ_b	σ_m	σ_n	$\tilde{\sigma}_a$	$\tilde{\sigma}_b$	$\tilde{\sigma}_m$	$\tilde{\sigma}_n$	τ_q	τ_p	τ	$\gamma - 1$	W
Basic parametrization (*cf.* Table 1)	**83.3**	**10.5**	**3.9**	**2.3**	**46.6**	**44.6**	**2.6**	**6.2**	**15.5**	**-13.1**	**15.2**	**9.3**	**0.62**
Alternative parametrizations													
$\uparrow i = 5\%$	76.7	16.1	3.8	3.3	**30.6**	**59.7**	**1.9**	**7.8**	**13.4**	**-20.1**	**13.1**	**6.5**	0.34
$\uparrow \beta_m = 20\%$	83.1	10.4	4.4	2.0	46.9	44.6	3.0	5.5	15.5	-13.0	15.2	9.4	0.63
$\uparrow c_u = 51.4\%$, $\downarrow c_v = 41.4\%$, $\downarrow c_w = 7.2\%$	84.3	9.6	3.9	2.1	48.9	42.5	2.7	5.9	14.5	-12.8	14.2	9.4	0.73
$\downarrow c_u = 41.4\%$, $\uparrow c_v = 51.4\%$, $\downarrow c_w = 7.2\%$	82.6	11.5	3.8	2.0	44.5	47.6	2.5	**5.3**	16.3	-13.3	16.1	9.2	0.56
$\downarrow c_u = 43.7\%$, $\downarrow c_v = 43.7\%$, $\uparrow c_w = 12.6\%$	82.6	9.7	4.2	3.5	46.1	41.5	2.8	**9.6**	15.9	-13.1	15.6	9.3	0.53
$\uparrow \delta_k = 10\%$	84.4	9.5	3.9	2.1	47.3	43.9	2.7	6.1	15.7	-12.3	15.4	8.1	0.24
$\uparrow \delta_a = 7\%$	82.3	11.2	4.1	2.4	47.1	44.3	2.5	6.1	15.4	-13.4	15.1	8.8	0.57
$\downarrow \delta_b = 90\%$	82.8	11.1	3.9	2.2	47.3	43.8	2.7	6.2	15.5	-13.1	15.2	9.4	0.66
$\uparrow \delta_m = 12\%$	83.6	10.5	3.6	2.3	46.2	44.6	3.0	6.2	15.5	-13.1	15.2	9.3	0.62
$\uparrow \delta_n = 80\%$	83.6	10.5	3.9	1.9	46.2	44.5	2.6	6.6	15.5	-13.1	15.2	9.3	0.61
$\uparrow s_a = 26\%$, $\downarrow s_k = 72.5\%$, $\downarrow s_m = 0.75\%$, $\downarrow s_p = 0.75\%$	86.6	8.7	2.8	1.9	**51.9**	**40.4**	**2.0**	5.7	**18.1**	-13.0	**17.9**	9.1	0.38
$\downarrow s_a = 20\%$, $\uparrow s_k = 78.5\%$, $\downarrow s_m = 0.75\%$, $\downarrow s_p = 0.75\%$	80.9	12.9	3.4	2.7	**43.1**	48.2	2.2	6.5	**12.6**	-12.3	12.4	**10.9**	1.21
$\downarrow s_a = 22.75\%$, $\downarrow s_k = 74.5\%$, $\uparrow s_m = 2\%$, $\downarrow s_p = 0,75\%$	80.5	10.1	7.1	2.2	44.5	44.5	**4.8**	6.2	16.0	-13.5	15.8	8.7	0.44
$\downarrow s_a = 22.75\%$, $\downarrow s_k = 74.5\%$, $\downarrow s_m = 0.75\%$, $\uparrow s_p = 2\%$	83.4	11.1	3.1	2.4	44.5	47.0	**2.0**	6.5	15.1	-13.7	14.5	8.7	0.53
Raising R&D externalities													
$\downarrow s_a = 21\%$, $\uparrow s_m = 2\%$, $\uparrow s_p = 2\%$	77.9	11.9	7.6	2.6	**38.8**	**49.8**	**4.6**	**6.8**	14.3	-14.5	13.8	7.9	0.43
$\downarrow s_a = 19\%$, $\uparrow s_m = 3\%$, $\uparrow s_p = 3\%$	71.9	13.8	11.4	2.9	**31.4**	**55.0**	**6.3**	**7.3**	**13.2**	**-15.9**	**12.3**	**6.7**	0.31

Table 2. Optimal policy: results

Concerning the growth rate and fiscal pressure, results are somewhat usual, in line with the endogenous growth literature *à la* Barro [1990]. The overall tax rate of the economy stands to 15.2% and generates an equilibrium growth rate of the economy equal to 9.3%; these findings are consistent with those generally found in the endogenous growth literature where an optimal tax rate under 20% can sustain a 10% growth rate of the economy.[18] The specific tax rate on labor and capital income stands at 15.5%, *i.e.* above the overall tax rate, allowing the government to save fiscal resources in order to subsidize private R&D through an appropriate transfer characterized by a negative -13.1% tax rate on private R&D income. The usual breakdown of the total amount of public spending into the four components – investment (46.6%), consumption (44.6%), health R&D (2.6%) and other health expenditures (6.2%) – highlights the central roles played by public medical research and development. Despite a pessimistic set of assumptions concerning the role played by the R&D in the global economy, 2.6% of the total amount of public spending should be devoted to medical R&D, in order for the government to implement an optimal fiscal policy. This result can be usefully compared to the real value observed in France during the year 2006:[19] public health R&D stands to 2.95 billions euros for a total amount of fiscal revenues equal to 792.49 billions euros, corresponding to

[18] In Barro [1990] the second best fiscal pressure has to be equal to the production elasticity w.r.t. the externality of public spending, that is, under a Cobb-Douglas technology, to $1-\alpha$, where α is the capital share in total income. $1-\alpha$ can be small under weak externalities (according to empirical estimates), consistently with the assumption (usually retained in the endogenous growth literature) that capital includes human capital (as in Mankiw, Romer and Weil [1992]).

[19] See Fenina & Geffroy [2007] and Appendix 3.

a share of medical R&D into public spending equal to 0.37%; then, according to our numerical simulation, the public investment in medical R&D, stands 17.6 billions euros under its socially optimal level.

As can be seen on Table 2, the results are pretty sensitive to parameters s_p and s_m describing the size of R&D externalities, *i.e.* the impact of private and public R&D on the whole economy; increasing these two parameters leads without any surprise to *(i)* a higher share of medical R&D into the total amount of public spending: 4.6% for $s_m = s_p = 2\%$ and 6.3% for $s_m = s_p = 3\%$, *(ii)* a higher subsides to private R&D: $\tau_p = -14.5\%$ for $s_m = s_p = 2\%$ and $\tau_p = -15.9\%$ for $s_m = s_p = 3\%$.

Increasing the time-preference rate (from 4% to 5%) tends to decrease all kinds of public investments – the public investment share shifts from 46.6% to 30.6%, while the medical R&D share shifts from 2.6% to 1.9% – but to increase the public consumption share from 44.6% to 59.7%.

Regard to the sensitivity of the optimal policy to the deep parameters, one can clearly distinguish two subsets of parameters:

(i) our main conclusions are not too sensitive to certain parameters in the households' utility function (weights c_u, c_v and c_w), to the parameters in the provision public health services (elasticities β_m and β_n) or to the depreciation rates (δ_k, δ_a, δ_b, δ_m and δ_n).

(ii) results are pretty sensitive to the assumptions made on the main production function (parameters s_k, s_a, s_m and s_p); an increase of the size of the externality associated to medical R&D (*resp.* public investment) leads the government to reallocate its fiscal receipts in favor of medical R&D (*resp.* public investment); symmetrically reducing externalities associated with public spending (medical R & D or public investment) leads the government to reallocate spending in favor of public consumption.

6 Raising public investment in medical R&D: an evaluation

The purpose of this section is to realize additional numerical simulations, in order to assess the macroeconomic impact on the GDP and the growth rate of the economy, of increasing public investment in medical R&D. The added value of this exercise for policy makers is to have an idea about the expected gains from a fiscal policy oriented to support R&D. The first part is devoted to the calibration of the model on French data including the current economic policy. The second part presents the main results of numerical simulations.

6.1 Calibration

The calibration process consists to set the values of two types of parameters we need to implement the numerical simulations:

(i) The structural parameters of the model: these deep parameters have been already defined in the previous section; in order to draw a coherent picture, we

use in this section the same values that the ones employed to compute the optimal policy (see Table 1). As stressed above, the parameters of households' preferences and R&D externalities are fixed according to a prudential view, somewhat pessimistic, on the role of R&D in the economy: such approach should prevent any criticism regarding a possible overestimation of the impact of R&D (including medical R&D) on the growth rate of the economy and welfare. As above, the scale parameter θ is calibrated in order to get, in our model, a growth rate for the French economy which fits the observed value of 2%.

(ii) Other parameters that were endogenous in the optimal policy section (see above), are now exogenous and fixed according to the observed policy practice in the French economy: namely, the proportion in the total amount of tax receipts of public investment, public consumption, health R&D and other health expenditures; the tax rate on labor and capital incomes and the tax rate on private R&D income; eventually, the GDP growth rate. The level of GDP, expressed in euros, is also used as a convenient basis for providing a monetary evaluation of the impact on the French economy of increasing public investments in medical R&D (rather than getting the impact on the growth rate only).

All theses parameters are simply derived from French national accounts for the year 2006. The shares into the total amount of fiscal revenues of public investment ($\tilde{\sigma}_a$), public consumption ($\tilde{\sigma}_b$), health-related R&D ($\tilde{\sigma}_m$) and other (unproductive) health expenditures ($\tilde{\sigma}_n$), are simply derived from Table 6 (see Appendix 3). For instance, the amount of health-related public R&D, standing to 2.95 billions euros for the year 2006, represents 0.37% of the total amount of fiscal receipts (792.49 billions euros): we thus obtain $\tilde{\sigma}_m = 0.37$. The same method has been used to compute the respective shares of public investment, public consumption and ordinary (unproductive) health expenditures into the overall public spending: $\tilde{\sigma}_a = 7.58\%$, $\tilde{\sigma}_b = 75.88\%$ and $\tilde{\sigma}_n = 16.17\%$.

The ratio of the total amount of taxes (792.49 billions euros in 2006) to the French 2006 GDP (1792 billions euros) gives immediately the French fiscal pressure in 2006: $\tau = 44,22\%$. Since the overall tax rate of the economy τ is defined in equation (19) as a weighted average of the specific tax rates applied to private capital incomes (τ_k) , private R&D incomes (τ_p), and labor incomes (τ_l), one gets immediately $\tau_q \equiv (\tau - s_p\tau_p) / (1 - s_p)$, where τ_q denotes the common tax rate on capital and labor income. This formula allow us to compute the tax rate applied to capital and labor incomes as a function *(i)* of the overall tax rate of the economy τ and *(ii)* of the specific tax rate applied to private R&D incomes.

In order to be consistent with the parametrization of the aggregate production, we assume that private R&D returns represents, more or less, 1% of the GDP, *i.e.* 17.92 billions euros for the year 2006. Such incomes would generate, if taxed at the average level τ, a total amount of taxes equal to 44.22% times 17.92 billions euros, that is 7.92 billions.

Considering that the so-called *Research Tax Credit (RTC)*, the main tax measure aimed at supporting the development of private R&D, represents an annual cost of about 1.1 billion euros for the government budget for year 2006, one can deduce that the total amount of taxes on private R&D incomes stands around $7.92 - 1.1 = 6.82$ billions euros. Dividing this latter amount of taxes by the corresponding amount of incomes (17.92 billions), one can approximate the value of the specific tax rate on private R&D incomes: $\tau_p = 38,09\%$ and, finally, compute $\tau_q = 44,29\%$.

Eventually, the yearly real growth rate of the economy has been set equal to 2%, corresponding to the average value observed in the French economy during the last decade; as explained above, in order to have a coherent representation of the economy, we need to calibrate the productivity parameter θ (TFP) which implements the observed growth rate. Using the observed policy values $(\tilde{\sigma}_a, \tilde{\sigma}_b, \tilde{\sigma}_m, \tilde{\sigma}_n, \tau_p, \tau_q, \tau)$ derived above from the national accounts, and the calibration of the structural parameters presented in Table 1, we obtain: $\theta = 0.5631$. The details of this procedure are provided in the Appendix 2.

Table 3 complements Table 1 to give a complete representation of the set of parameters that we use in the numerical simulations.

Parameter	**Definition**	**Observed** (%)
$\tilde{\sigma}_a$	Share of public (non-health) investment in overall tax revenues	7.58
$\tilde{\sigma}_b$	Share of public consumption in overall tax revenues	75.88
$\tilde{\sigma}_m$	Share of tax-financed medical R&D in overall tax revenues	0.37
$\tilde{\sigma}_n$	Share of other health expenditures in overall tax revenues	16.17
τ	Tax pressure (overall tax rate)	44.22
τ_q	Tax rate on labor and capital incomes	44,29
τ_p	Tax rate on private R&D incomes	38,09
θ	Scale parameter of the production function	56.31
$\gamma - 1$	GDP growth rate	2.00
	GDP (2006; billions €)	1792

Table 3. Economic policy simulations: calibration

6.2 Increasing public health R&D under constant global public spending

In this section, one proceeds to numerical simulations of the model to address the following question: what is the macroeconomic impact on the growth rate of the economy, the GDP and eventually the fiscal receipts of a one billion euros increase of public investment in medical R&D ? To do this, we distinguish two scenarios:

(i) In the first case, we assume that the government keeps constant the (*ex-ante*) total amount of fiscal receipts and just switches some fiscal resources (1 billion €) from somewhat "unproductive" public consumption to investments in medical R&D.

(ii) In the second case, we finance the one billion € increase in medical-R&D public investment, by rising the tax rate on labor and capital income.

6.2.1 Scenario 1

In this case, the policy shock we simulate, is a *permanent transfer of an amount of 1 billion € from public consumption to public medical R&D*. Such a transfer raises the share of public spending in medical R&D from $\tilde{\sigma}_m = 0.372\%$ to about $\tilde{\sigma}'_m = 0.498\%$, while the public consumption share decreases from $\tilde{\sigma}_b = 75.873\%$ to $\tilde{\sigma}'_b = 75.747\%$ ($\tilde{\sigma}_a$ and $\tilde{\sigma}_n$ remaining constants); this corresponds to an extra-investment in medical R&D of one billion euros under the balanced-budget constraint $\tilde{\sigma}_a + \tilde{\sigma}'_b + \tilde{\sigma}'_m + \tilde{\sigma}_n = 1$.

The following table sums up the main results :

Year	Growth rate (%)	Discounted sum of GDP increases (billions €)	Discounted sum of GDP increases (% GDP 2006)	Increase of fiscal revenues (billions €)
0	2.000%	0.000	0.00%	0.000
1	2.241%	4.055	0.23%	1.907
5	2.091%	29.486	1.65%	3.892
10	2.054%	61.323	3.42%	5.158
Long term	2.048%	567.392	31.66%	-

Table 4. Scenario 1: results

The first year the growth rate increases from 2% to about 2.24 % corresponding to a short-term discounted GDP gain equal to 4.05 billions € generating an increase of fiscal receipts close to 2 billions €. Over a decade, the GDP discounted total benefit associated to the policy shock stands above 60 billions € corresponding to 3.42% of the 2006 GDP, *i.e.* about 1.7 years of growth; after ten years the amount of annual fiscal receipts amounts to 5.157 billions €, higher that it would have been without the policy adjustment.

6.2.2 Scenario 2

In this case, the policy shock we simulate, is a *permanent €1 billion increase of public medical-R&D expenditures, totally funded by a rise of the tax rate on labor and capital incomes.*

Let us first compute the increase of the tax rate τ_q on labor and capital income in order to compensate the 1 billion € increase of public medical-R&D expenditures and keep a balanced budget.

We know that total fiscal receipts are given by $T = \tau Y = [\tau_p s_p + \tau_q (1 - s_p)] Y$. Then, the variation of T associated to a rise of the tax rate from τ_q to τ'_q is given by $\Delta T = (\tau'_q - \tau_q)(1 - s_p) Y$. Setting $\Delta T = 1$ (billion €), we can easily compute the rise of the tax rate τ_q on labor and capital income which ensures a balanced budget:

$$\tau'_q = \tau_q + \frac{1}{(1-s_p)Y} = 44.34\%$$

since Y, the 2006 GDP, is 1792 billions €.

Eventually, the new shares, into the total amount of fiscal revenues, of public consumption, public investment, health-related R&D and "other" health expenditures, are simply given by:

$$\begin{aligned}(\tilde{\sigma}'_a, \tilde{\sigma}'_b, \tilde{\sigma}'_m, \tilde{\sigma}'_n) &= \left(\frac{\tilde{\sigma}_a T}{T+1}, \frac{\tilde{\sigma}_b T}{T+1}, \frac{\tilde{\sigma}_m T + 1}{T+1}, \frac{\tilde{\sigma}_n T}{T+1}\right) \\ &= (7.57\%, 75.78\%, 0.5\%, 16.15\%)\end{aligned}$$

since T, the 2006 tax receipt, is 792.49 billions €.

Table 5 below sums up the main results due to this policy shock:

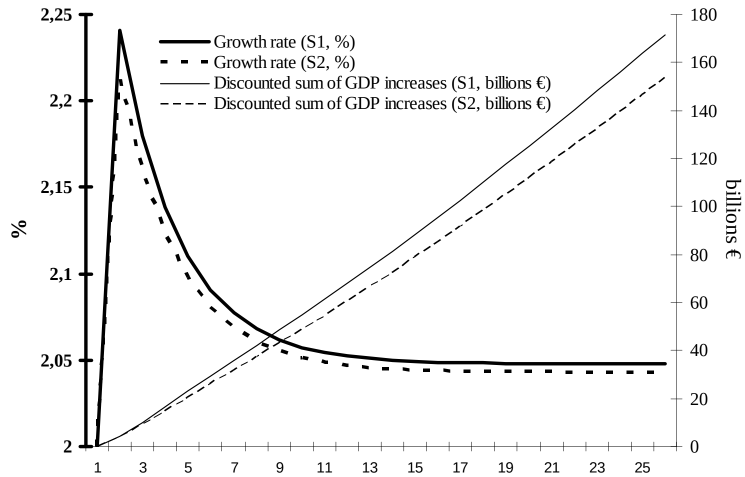
Year	Growth rate (%)	Discounted sum of GDP increases (billions €)	Discounted sum of GDP increases (% GDP 2006)	Increase of fiscal revenues (billions €)
0	2.000%	0.000	0.00%	0.000
1	2.213%	3.585	0.20%	1.686
5	2.081%	26.142	1.46%	3.459
10	2.049%	54.564	3.04%	4.612
Long term	2.043%	505.755	28.39%	-

Table 5. Scenario 2: results

The first year the growth rate increases from 2% to about 2.21% corresponding to a short-term discounted GDP gain equal to 3.59 billions €, which is less than what we got with the first scenario; the positive impact of the policy shock on GDP, generates an increase of total fiscal revenues standing around 1.7 billions € after one year. Over a decade the GDP discounted total benefit associated with the policy shock is close to 55 billions € corresponding to more or less 3% of 2006 GDP, *i.e.* 1.5 year of economic growth; after ten years the amount of annual fiscal receipts is 4.612 billions € higher than it would have been without any policy shock.

Results associated to scenarios 1 and 2 are however close to each other: the macroeconomic impact on GDP of a 1 billion € increase in public investment in medical-R&D is clearly *positive and strong, whatever the funding process.* Nevertheless the impact of rising publicly funded medical-R&D appears to be higher when the supplementary investment is financed by a transfer from public consumption, than when it is financed by a rise of the tax rate on labor and capital income. Unsurprisingly, in the latter case, the rise of the overall tax rate of the economy affects negatively labor and investment incentives, and then the

GDP and fiscal revenues; over a decade the difference between the two cases, measured by the gap between the two discounted sums of GDP increases, stands around 6.75 billions €. The graph below provides a quick comparison of both the scenarios.



Graph 1. Comparison of scenarios 1 and 2

7 Conclusion

The general equilibrium endogenous growth model presented in the paper emphasizes the key role played by public health R&D investments in determining the long-run rate of economic growth and welfare.

From a theoretical point of view we obtain four main results: *(i)* the equilibrium path is unique, *(ii)* market imperfections – externalities and taxes – make the equilibrium inefficient under arbitrary policies, *(iii)* an appropriate fiscal policy (tax rate and public spending shares) can restore the second best, *(iv)* health R&D matters more than other health expenditures in order to achieve the social welfare target in the long run. This last point is crucial as it stresses how the health-related research and development, as a productive externality, is a powerful engine for growth, compared to alternative policies, such as, public consumption for example. This depends on the fact that in an endogenous growth framework, knowledge accumulation has a dramatic and unbounded impact on factors productivity.

From an empirical point of view, the numerical simulations provided in the paper can be seen as a fruitful illustration of what could be the benefits associated to a moderate public spending reallocation in favor of medical R&D. More precisely, we find that a limited 1 billions euros permanent reallocation of public spending, from "unproductive" public consumption toward medical R&D investments, induces a GDP discounted benefit that stands around 60 billions € over a decade.

8 Appendix

8.1 Appendix 1: Proofs of propositions

Proof of Proposition 1. We want to prove that the steady state is a saddle point. Since the system is two-dimensional with one predetermined variable, saddle-path stability entails equilibrium uniqueness under rational expectations (with or without transition).

In the (T, D)-plane, the saddle points match with the two cones:

$$\begin{aligned} -T-1 &< D < T-1 \\ T-1 &< D < -T-1 \end{aligned}$$

As $\varepsilon_2 < 0$, (35) implies:

$$D = T - 1 - \frac{1}{\gamma}\frac{y}{x\eta}\left(\frac{\varphi'}{\gamma}\tau x - \frac{\rho}{\rho+\Delta_k}\varepsilon_2\varepsilon_u\right) < T-1 \tag{38}$$

To show that the stationary state is a saddle point, one needs only to prove that $D > -T-1$.

Substituting formulas (34) and (35) into $D > -T-1$, one gets the following condition:

$$D > \frac{1}{2}\frac{1}{\gamma}\frac{y}{x\eta}\left(\frac{\rho}{\rho+\Delta_k}\varepsilon_2\varepsilon_u - \frac{\varphi'}{\gamma}\tau x\right) - 1$$

or, equivalently,

$$\gamma + \Delta_k + \frac{\varphi'}{\eta}\left(1 - \tau - \tau x\eta - \frac{\tau}{2}\frac{y}{\gamma}\right) > \frac{1}{2}\frac{y}{x\eta}\frac{\rho}{\rho+\Delta_k}\varepsilon_2\varepsilon_u$$

Since $\varepsilon_2 < 0$, it is sufficient to prove that

$$\gamma + \Delta_k + \frac{\varphi'}{\eta}\left(1 - \tau - \tau x\eta - \frac{\tau}{2}\frac{y}{\gamma}\right) > 0 \tag{39}$$

From equation (29) and definition (32), using $\gamma = \Delta + \tau\varphi$ we observe that

$$y = \varphi(1-\tau) - \eta x(\gamma - \Delta_k) \tag{40}$$

Replacing (40) in (39), we get:

$$\begin{aligned} & \gamma + \Delta_k + \frac{\varphi'}{\eta}\left(1 - \tau - \tau x\eta - \frac{\tau}{2}\frac{y}{\gamma}\right) \\ = \quad & \Delta + (1-\tau)\frac{\varphi'}{\eta}\left(1 - \frac{1}{2}\frac{\tau\varphi}{\Delta+\tau\varphi}\right) + \left(\frac{1}{2}\frac{\Delta+\tau\varphi-\Delta_k}{\Delta+\tau\varphi} - 1\right)\frac{\varphi' x}{\varphi}\tau\varphi + \tau\varphi + \Delta_k \\ \geq \quad & \Delta + (1-\tau)\frac{\varphi'}{\eta}\left(1 - \frac{1}{2}\frac{\tau\varphi}{\Delta+\tau\varphi}\right) + \left(\frac{1}{2}\frac{\Delta+\tau\varphi-\Delta_k}{\Delta+\tau\varphi} - 1\right)\varepsilon_1\tau\varphi + \varepsilon_1(\tau\varphi + \Delta_k) \\ = \quad & \Delta + (1-\tau)\frac{\varphi'}{\eta}\left(1 - \frac{1}{2}\frac{\tau\varphi}{\Delta+\tau\varphi}\right) + \left(\frac{1}{2}\tau\varphi + \left(1 - \frac{1}{2}\frac{\tau\varphi}{\Delta+\tau\varphi}\right)\Delta_k\right)\varepsilon_1 > 0 \end{aligned}$$

where

$$\varepsilon_1 \equiv \frac{\varphi' x}{\varphi} \in (0,1)$$

■

Proof of Proposition 2. Before maximizing, we need to compute the welfare function (utility function) along the balanced growth path: $(c_t, b_t, m_t, n_t) = (c_0, b_0, m_0, n_0)\,\gamma^t$, where γ is the common (regular) growth factor:

$$e_t \equiv e(m_t, n_t) = e\left(m_0\gamma^t, n_0\gamma^t\right) = e(m_0, n_0)\,\gamma^t = e_0\gamma^t$$

(notice that the health good production function is supposed to be homogeneous of degree one). Denoting $e_0 \equiv e(m_0, n_0)$, one gets under restriction (36):

$$\begin{aligned} W &= c_u \sum_{t=0}^{\infty} \beta^t \ln\left(c_0\gamma^t\right) + c_v \sum_{t=0}^{\infty} \beta^t \ln\left(b_0\gamma^t\right) + c_w \sum_{t=0}^{\infty} \beta^t \ln\left(e_0\gamma^t\right) \\ &= \left(c_u \ln c_0 + c_v \ln b_0 + c_w \ln e_0\right) \sum_{t=0}^{\infty} \beta^t + \left(c_u + c_v + c_w\right) \ln\gamma \sum_{t=0}^{\infty} \beta^t t \\ &= \frac{1}{1-\beta}\left(c_u \ln c_0 + c_v \ln b_0 + c_w \ln e_0 + \frac{\beta}{1-\beta}\ln\gamma\right) \end{aligned}$$

Equilibrium uniqueness under rational expectations (Proposition 1) requires c_0, b_0, e_0 to be compatible with the regular growth factor γ. Definition (10) details the economic policy and implies at the beginning: $(a_0, b_0, m_0, n_0) = (\sigma_a, \sigma_b, \sigma_m, \sigma_n)\,g_0$ and $e_0 = e(m_0, n_0) = e(\sigma_m g_0, \sigma_n g_0) = e(\sigma_m, \sigma_n)\,g_0$. From definition (22) one gets $c_0 = y g_0$. The endogenous growth of steady state implies a regular growth path; under restriction (36) we obtain:

$$\begin{aligned} W &= \frac{1}{1-\beta}\left(c_u \ln(y g_0) + c_v \ln(\sigma_b g_0) + c_w \ln\left[e(\sigma_m, \sigma_n)\,g_0\right] + \frac{\beta}{1-\beta}\ln\gamma\right) \\ &= \frac{1}{1-\beta}\left[c_u \ln y + c_v \ln\sigma_b + c_w \ln e(\sigma_m, \sigma_n) + \ln g_0 + \frac{\beta}{1-\beta}\ln\gamma\right] \end{aligned}$$

where $g_0 \equiv a_0 + b_0 + m_0 + n_0$ is an initial condition.

As β and g_0 are not choice variables, the problem of maximizing W turns out to be equivalent to the following:

$$\max\left[c_u \ln y + c_v \ln\sigma_b + c_w \ln e(\sigma_m, \sigma_n) + \frac{\beta}{1-\beta}\ln\gamma\right] \tag{41}$$

Under Assumption 4, the policy of public spending (10) entails:

$$\begin{aligned} \tilde{f}(\kappa_t, \pi_t, a_t, m_t, \bar{p}_t) &= \theta\kappa_t^{s_k}\pi_t^{s_p}a_t^{s_a}m_t^{s_m}\bar{p}_t^{1-s_k-s_p-s_a-s_m} \\ f(\kappa_t, \pi_t, a_t, m_t) &= \theta\kappa_t^{s_k}\pi_t^{1-s_k-s_a-s_m}a_t^{s_a}m_t^{s_m} \\ \varphi(x) &= \frac{f(\kappa_t, \pi_t, a_t, m_t)}{g_t} = f\left(x, (\eta-1)\,x, \sigma_a, \sigma_m\right) \\ &= \theta\sigma_a^{s_a}\sigma_m^{s_m}(\eta-1)^{1-s_k-s_a-s_m}\,x^{1-s_a-s_m} \qquad (42) \\ \varphi'(x) &= (1-s_a-s_m)\,\theta\sigma_a^{s_a}\sigma_m^{s_m}(\eta-1)^{1-s_k-s_a-s_m}\,x^{-s_a-s_m} \end{aligned}$$

where now

$$\eta \equiv 1 + \frac{s_p}{s_k}\frac{1-\tau_p}{1-\tau_q}$$

Still under Assumption 4 we have: $\varepsilon_1 \equiv x\varphi'/\varphi = 1 - s_a - s_m$.

Since $\varepsilon_u = 1$, one gets from (28) an implicit equation defining the stationary state x:

$$\Delta + \tau\theta\sigma_a^{s_a}\sigma_m^{s_m}(\eta-1)^{1-s_k-s_a-s_m}x^{1-s_a-s_m} = \beta\left[\Delta_k + (1-\tau_q)s_k\theta\sigma_a^{s_a}\sigma_m^{s_m}(\eta-1)^{1-s_k-s_a-s_m}x^{-s_a-s_m}\right] \tag{43}$$

where now, according to (19):

$$\tau = s_p\tau_p + (1-s_p)\tau_q \tag{44}$$

Taking into account that $\tau\varphi = \gamma - \Delta$, equation (28) becomes

$$\gamma = \beta\left[\Delta_k + \frac{s_k}{s_k+s_\pi}\frac{1-\tau_q}{\tau}\frac{1}{x}\frac{x\varphi'}{\varphi}(\gamma-\Delta)\right] \tag{45}$$

Substituting ε_1 into equation (45), noticing that $s_\pi \equiv 1 - s_k - s_a - s_m$ and solving for γ, the growth factor is now explicitly computed:

$$\gamma = \beta\frac{\Delta s_k(1-\tau_q) - \Delta_k\tau x}{\beta s_k(1-\tau_q) - \tau x} \tag{46}$$

Under Assumption 4, the implicit equation (43) becomes,

$$\theta\sigma_a^{s_a}\sigma_m^{s_m}(\eta-1)^{1-s_k-s_a-s_m} = \frac{(\Delta - \beta\Delta_k)x^{s_a+s_m}}{\beta s_k(1-\tau_q) - \tau x} \tag{47}$$

Instead of maximizing welfare with respect to policy tools $(\sigma_a, \sigma_m, \sigma_n, \tau_p, \tau_q)$, one maximizes it indirectly with respect to an alternative vector $(\sigma_a, \sigma_m, \sigma_n, \eta, h)$, where h is given by (37), and finally compute $(\tau_p, \tau_q)^*$ using $(\sigma_a, \sigma_m, \sigma_n, \eta, h)^*$.

σ_b is given by (11) and program (41) becomes

$$\max\left[c_u \ln y + c_v \ln(1-\sigma_a-\sigma_m-\sigma_n) + c_w \ln e(\sigma_m,\sigma_n) + \frac{\beta}{1-\beta}\ln\gamma\right] \tag{48}$$

Let us express y in terms of $(\sigma_a, \sigma_m, \sigma_n, \eta, h)$.

Using (32) and (44), we find that

$$\begin{aligned} 1-\tau_p &= \frac{1-\tau}{1-s_p+(\eta-1)s_k}\frac{s_k}{s_p}(\eta-1) \\ 1-\tau_q &= \frac{1-\tau}{1-s_p+(\eta-1)s_k} \end{aligned} \tag{49}$$

From (42) and (47), we know that

$$\varphi(x) = \frac{(\Delta-\beta\Delta_k)x}{\beta s_k(1-\tau_q) - \tau x} \tag{50}$$

From (28), we know also that

$$\tau = \frac{\gamma - \Delta}{\varphi(x)} \tag{51}$$

or, equivalently,

$$\varphi(x) = \frac{\gamma - \Delta}{\tau} \tag{52}$$

Replacing (49) in (50) and (50) so modified in (51) and solving for τ, we get

$$\tau = \left[1 + x\frac{\gamma - \beta\Delta_k}{\gamma - \Delta}\frac{1 - s_p + (\eta - 1)\, s_k}{\beta s_k}\right]^{-1} \tag{53}$$

Substituting (52) and (53) in (33), we get

$$y = x\left[\frac{1 - s_p + (\eta - 1)\, s_k}{\beta s_k}(\gamma - \beta\Delta_k) - \eta(\gamma - \Delta_k)\right] \tag{54}$$

Replacing (54) in (48) and using (37), we obtain

$$\begin{aligned} & \tilde{W}(\sigma_a, \sigma_m, \sigma_n, \eta, h) \\ \equiv\ & c_u \ln x + c_u \ln\left[\frac{1 - s_p + (\eta - 1)\, s_k}{\beta s_k}(h + (1 - \beta)\Delta_k) - \eta h\right] \\ & + c_v \ln(1 - \sigma_a - \sigma_m - \sigma_n) + c_w \ln e(\sigma_m, \sigma_n) + \frac{\beta}{1 - \beta}\ln(h + \Delta_k) \end{aligned}$$

We observe that x is determined by (47), where we have substituted (37), (49) and (53):

$$\theta\sigma_a^{s_a}\sigma_m^{s_m}(\eta - 1)^{1 - s_k - s_a - s_m} x^{1 - s_a - s_m} = h + \Delta_k - \Delta + x(h + (1 - \beta)\Delta_k)\frac{1 - s_p + (\eta - 1)\, s_k}{\beta s_k} \tag{55}$$

where

$$\Delta = \Delta_b + \sigma_a(\Delta_a - \Delta_b) + \sigma_m(\Delta_m - \Delta_b) + \sigma_n(\Delta_n - \Delta_b) \tag{56}$$

From (55), according to the Implicit Function Theorem, we locally define

$$x = x(\sigma_a, \sigma_m, \sigma_n, \eta, h)$$

with partial derivatives:

$$(x_a, x_m, x_n, x_\eta, x_h) = \left(\frac{\partial x}{\partial \sigma_a}, \frac{\partial x}{\partial \sigma_m}, \frac{\partial x}{\partial \sigma_n}, \frac{\partial x}{\partial \eta}, \frac{\partial x}{\partial h}\right)$$

These partial derivatives can be computed by totally differentiating (55).

$$\frac{x_a}{x} = \frac{\Delta_a - \Delta_b + \varphi\frac{s_a}{\sigma_a}}{(s_a + s_m)\,\varphi - z} \tag{57}$$

$$\frac{x_m}{x} = \frac{\Delta_m - \Delta_b + \varphi\frac{s_m}{\sigma_m}}{\varphi\,(s_a + s_m) - z} \tag{58}$$

$$\frac{x_n}{x} = \frac{\Delta_n - \Delta_b}{\varphi\,(s_a + s_m) - z} \tag{59}$$

$$\frac{x_\eta}{x} = \frac{\varphi\frac{1-s_k-s_a-s_m}{\eta-1} - (z + \Delta - \beta\Delta_k)\,\frac{x}{\beta}}{\varphi\,(s_a + s_m) - z} \tag{60}$$

$$\frac{x_h}{x} = -\frac{1 + \frac{1-s_p+(\eta-1)s_k}{s_k}\frac{x}{\beta}}{\varphi\,(s_a + s_m) - z} \tag{61}$$

where $z \equiv \gamma - \Delta = h + \Delta_k - \Delta$.

The optimal policy implements a vector $(\sigma_a, \sigma_m, \sigma_n, \eta, z)^*$ satisfying the following system:

$$\left(\frac{\partial\tilde{W}}{\partial\sigma_a}, \frac{\partial\tilde{W}}{\partial\sigma_m}, \frac{\partial\tilde{W}}{\partial\sigma_n}, \frac{\partial\tilde{W}}{\partial\eta}, \frac{\partial\tilde{W}}{\partial h}\right) = 0$$

Noticing that, under Assumption 2,

$$\frac{\partial e}{\partial\sigma_m}\frac{1}{e} = \frac{\beta_m}{\sigma_m},\ \ \frac{\partial e}{\partial\sigma_n}\frac{1}{e} = \frac{\beta_n}{\sigma_n}$$

we obtain

$$\frac{\partial\tilde{W}}{\partial\sigma_a} = c_u\frac{x_a}{x} - c_v\frac{1}{1-\sigma_a-\sigma_m-\sigma_n} = 0$$

$$\frac{\partial\tilde{W}}{\partial\sigma_m} = c_u\frac{x_m}{x} - c_v\frac{1}{1-\sigma_a-\sigma_m-\sigma_n} + c_w\frac{\beta_m}{\sigma_m} = 0$$

$$\frac{\partial\tilde{W}}{\partial\sigma_n} = c_u\frac{x_n}{x} - c_v\frac{1}{1-\sigma_a-\sigma_m-\sigma_n} + c_w\frac{\beta_n}{\sigma_n} = 0$$

$$\frac{\partial\tilde{W}}{\partial\eta} = c_u\frac{x_\eta}{x} + c_u\frac{\frac{1}{\beta}\,(z + \Delta - \beta\Delta_k) - (z + \Delta - \Delta_k)}{\frac{1-s_p+(\eta-1)s_k}{\beta s_k}\,(z + \Delta - \beta\Delta_k) - \eta\,(z + \Delta - \Delta_k)} = 0$$

$$\frac{\partial\tilde{W}}{\partial h} = c_u\frac{x_h}{x} + c_u\frac{\frac{1-s_p+(\eta-1)s_k}{\beta s_k} - \eta}{\frac{1-s_p+(\eta-1)s_k}{\beta s_k}\,(z + \Delta - \beta\Delta_k) - \eta\,(z + \Delta - \Delta_k)} + \frac{\beta}{1-\beta}\frac{1}{z+\Delta} = 0$$

Substituting equations (57-61) and taking into account equation (55), we get the following system. The optimal policy is a vector $(\sigma_a, \sigma_m, \sigma_n, \eta, z)^*$ such that $(\sigma_a, \sigma_m, \sigma_n, \eta, z, \varphi)^*$ is solution of:

$$\begin{aligned}
\theta \sigma_a^{s_a} \sigma_m^{s_m} (\eta - 1)^{1-s_k-s_a-s_m} x^{1-s_a-s_m} &= \varphi && (62)\\
c_u \frac{\Delta_a - \Delta_b + \frac{s_a}{\sigma_a}\varphi}{(s_a + s_m)\varphi - z} - c_v \frac{1}{1 - \sigma_a - \sigma_m - \sigma_n} &= 0 && (63)\\
c_u \frac{\Delta_m - \Delta_b + \frac{s_m}{\sigma_m}\varphi}{(s_a + s_m)\varphi - z} - c_v \frac{1}{1 - \sigma_a - \sigma_m - \sigma_n} + c_w \frac{\beta_m}{\sigma_m} &= 0 && (64)\\
c_u \frac{\Delta_n - \Delta_b}{(s_a + s_m)\varphi - z} - c_v \frac{1}{1 - \sigma_a - \sigma_m - \sigma_n} + c_w \frac{\beta_n}{\sigma_n} &= 0 && (65)
\end{aligned}$$

$$\frac{\frac{1-s_k-s_a-s_m}{\eta-1}\varphi - x(z + \Delta - \beta\Delta_k)\frac{1}{\beta}}{(s_a + s_m)\varphi - z} + \frac{(1-\beta)(z+\Delta)}{\frac{1-s_p-s_k}{s_k}(z + \Delta - \beta\Delta_k) + \eta(1-\beta)(z+\Delta)} = 0 \quad (66)$$

$$-c_u \frac{1 + x\frac{1-s_p+(\eta-1)s_k}{\beta s_k}}{(s_a + s_m)\varphi - z} + c_u \frac{\frac{1-s_p+(\eta-1)s_k}{\beta s_k} - \eta}{\frac{1-s_p+(\eta-1)s_k}{\beta s_k}(z + \Delta - \beta\Delta_k) - \eta(z + \Delta - \Delta_k)} + \frac{\beta}{1-\beta}\frac{1}{z+\Delta} = 0 \quad (67)$$

where

$$\varphi = h + \Delta_k - \Delta + x(h + (1-\beta)\Delta_k)\frac{1 - s_p + (\eta - 1)s_k}{\beta s_k} \quad (68)$$

Replacing (62) in (63) and (64), we obtain

$$\begin{aligned}
\sigma_m &= \frac{\beta_m \frac{c_w}{c_u}((s_a + s_m)\varphi - z) + s_m\varphi}{\Delta_a - \Delta_m + \varphi\frac{s_a}{\sigma_a}} && (69)\\
\sigma_n &= \frac{\beta_n \frac{c_w}{c_u}((s_a + s_m)\varphi - z)}{\Delta_a - \Delta_n + \varphi\frac{s_a}{\sigma_a}} && (70)
\end{aligned}$$

Substituting (69) and (70) in (62) and solving for z, we find

$$z(\varphi, \sigma_a) \equiv \varphi(s_a + s_m) - \frac{c_u\left(\Delta_a - \Delta_b + \varphi\frac{s_a}{\sigma_a}\right)\left(1 - \sigma_a - \frac{\varphi s_m}{\Delta_a - \Delta_m + \varphi\frac{s_a}{\sigma_a}}\right)}{c_v + c_w\left(\Delta_a - \Delta_b + \varphi\frac{s_a}{\sigma_a}\right)\left(\frac{\beta_m}{\Delta_a - \Delta_m + \varphi\frac{s_a}{\sigma_a}} + \frac{\beta_n}{\Delta_a - \Delta_n + \varphi\frac{s_a}{\sigma_a}}\right)} \quad (71)$$

Replacing (71) in (69) and (70), we obtain

$$\sigma_n(\varphi, \sigma_a) = \frac{\beta_n}{\Delta_a - \Delta_n + \varphi\frac{s_a}{\sigma_a}} \frac{c_w\left(\Delta_a - \Delta_b + \varphi\frac{s_a}{\sigma_a}\right)\left(1 - \sigma_a - \frac{\varphi s_m}{\Delta_a - \Delta_m + \varphi\frac{s_a}{\sigma_a}}\right)}{c_v + c_w\left(\Delta_a - \Delta_b + \varphi\frac{s_a}{\sigma_a}\right)\left(\frac{\beta_m}{\Delta_a - \Delta_m + \varphi\frac{s_a}{\sigma_a}} + \frac{\beta_n}{\Delta_a - \Delta_n + \varphi\frac{s_a}{\sigma_a}}\right)} \quad (72)$$

$$\sigma_m(\varphi, \sigma_a) \equiv \frac{\varphi s_m}{\Delta_a - \Delta_m + \varphi\frac{s_a}{\sigma_a}} + \sigma_n(\varphi, \sigma_a)\frac{\beta_m}{\beta_n}\frac{\Delta_a - \Delta_n + \varphi\frac{s_a}{\sigma_a}}{\Delta_a - \Delta_m + \varphi\frac{s_a}{\sigma_a}} \quad (73)$$

From (68), we get

$$x=\frac{\varphi-z}{z+\Delta-\beta\Delta_k}\frac{\beta s_k}{1-s_p+(\eta-1)s_k} \tag{74}$$

Replacing in (67) and solving for η, we have

$$\eta(\varphi,\sigma_a)\equiv\frac{1}{1-\beta}\frac{1-s_p-s_k}{s_k}\frac{1-\frac{\varphi+\Delta(\varphi,\sigma_a)-\beta\Delta_k}{\varphi(s_a+s_m)-z(\varphi,\sigma_a)}+\frac{1}{c_u}\frac{\beta}{1-\beta}\frac{z(\varphi,\sigma_a)+\Delta(\varphi,\sigma_a)-\beta\Delta_k}{z(\varphi,\sigma_a)+\Delta(\varphi,\sigma_a)}}{\frac{\varphi+\Delta(\varphi,\sigma_a)-\beta\Delta_k}{\varphi(s_a+s_m)-z(\varphi,\sigma_a)}\frac{z(\varphi,\sigma_a)+\Delta(\varphi,\sigma_a)}{z(\varphi,\sigma_a)+\Delta(\varphi,\sigma_a)-\beta\Delta_k}-\frac{1}{c_u}\frac{\beta}{1-\beta}-1} \tag{75}$$

where

$$\Delta(\varphi,\sigma_a)\equiv\Delta_b+\sigma_a(\Delta_a-\Delta_b)+\sigma_m(\varphi,\sigma_a)(\Delta_m-\Delta_b)+\sigma_n(\varphi,\sigma_a)(\Delta_n-\Delta_b)$$

Substituting (74) in (65), we find

$$\begin{aligned}\varphi &= \theta\sigma_a^{s_a}\sigma_m(\varphi,\sigma_a)^{s_m}(\eta(\varphi,\sigma_a)-1)^{1-s_k-s_a-s_m}\\ &\quad *\left[\frac{\varphi-z(\varphi,\sigma_a)}{z(\varphi,\sigma_a)+\Delta(\varphi,\sigma_a)-\beta\Delta_k}\frac{\beta s_k}{1-s_p+(\eta(\varphi,\sigma_a)-1)s_k}\right]^{1-s_a-s_m}\end{aligned} \tag{76}$$

Eventually, replacing (74) in (66), we obtain

$$\begin{aligned}0 &= \frac{\varphi\frac{1-s_k-s_a-s_m}{\eta(\varphi,\sigma_a)-1}-(\varphi-z(\varphi,\sigma_a))\frac{s_k}{1-s_p+(\eta(\varphi,\sigma_a)-1)s_k}}{\varphi(s_a+s_m)-z(\varphi,\sigma_a)}\\ &\quad+\frac{(1-\beta)(z(\varphi,\sigma_a)+\Delta(\varphi,\sigma_a))}{\frac{1-s_p-s_k}{s_k}(z(\varphi,\sigma_a)+\Delta(\varphi,\sigma_a)-\beta\Delta_k)+\eta(\varphi,\sigma_a)(1-\beta)(z(\varphi,\sigma_a)+\Delta(\varphi,\sigma_a))}\end{aligned} \tag{77}$$

The optimal policy is $(\varphi,\sigma_a)^*$ is solution of the two-dimensional system (76-77), where $(z,\sigma_m,\sigma_n,\eta)^*$ are given by (71), (72), (73) and (75), respectively. ■

8.2 Appendix 2: Calibration procedures

Calibrating θ. Using (28) and (42), we get

$$\theta=\frac{\gamma-\Delta}{\tau\sigma_a^{s_a}\sigma_m^{s_m}(\eta-1)^{1-s_k-s_a-s_m}x^{1-s_a-s_m}} \tag{78}$$

Replacing in (43) and solving for x, we obtain

$$x=\beta s_k\frac{1-\tau_q}{\tau}\frac{\gamma-\Delta}{\gamma-\beta\Delta_k}$$

Replacing in (78), we find

$$\theta=\frac{\gamma-\Delta}{\tau\sigma_a^{s_a}\sigma_m^{s_m}(\eta-1)^{1-s_k-s_a-s_m}\left(\beta s_k\frac{1-\tau_q}{\tau}\frac{\gamma-\Delta}{\gamma-\beta\Delta_k}\right)^{1-s_a-s_m}} \tag{79}$$

that is the right way of calibrating the unobserved θ, given the observed γ.

From (14), the breakdown of the total amount of *public spending* into its four components (investment without health, consumption, health R&D and other health expenditures) takes the form:

$$(\tilde{\sigma}_a, \tilde{\sigma}_b, \tilde{\sigma}_m, \tilde{\sigma}_n) = \left(\frac{a_{t+1}/a_t - \Delta_a}{g_{t+1}/g_t - \Delta}\sigma_a, \frac{b_{t+1}/b_t - \Delta_b}{g_{t+1}/g_t - \Delta}\sigma_b, \frac{m_{t+1}/m_t - \Delta_m}{g_{t+1}/g_t - \Delta}\sigma_m, \frac{n_{t+1}/n_t - \Delta_n}{g_{t+1}/g_t - \Delta}\sigma_n \right)$$

At the steady state (regular growth), we obtain

$$(\tilde{\sigma}_a, \tilde{\sigma}_b, \tilde{\sigma}_m, \tilde{\sigma}_n) = \left(\frac{\gamma - \Delta_a}{\gamma - \Delta}\sigma_a, \frac{\gamma - \Delta_b}{\gamma - \Delta}\sigma_b, \frac{\gamma - \Delta_m}{\gamma - \Delta}\sigma_m, \frac{\gamma - \Delta_n}{\gamma - \Delta}\sigma_n \right) \quad (80)$$

Using (13) and solving for $(\sigma_a, \sigma_b, \sigma_m, \sigma_n)$, we obtain $(\sigma_a, \sigma_b, \sigma_m, \sigma_n)^T = M (\tilde{\sigma}_a, \tilde{\sigma}_b, \tilde{\sigma}_m, \tilde{\sigma}_n)^T$, where

$$M \equiv \gamma \begin{bmatrix} \gamma - (1 - \tilde{\sigma}_a) \Delta_a & \tilde{\sigma}_a \Delta_b & \tilde{\sigma}_a \Delta_m & \tilde{\sigma}_a \Delta_n \\ \tilde{\sigma}_b \Delta_a & \gamma - (1 - \tilde{\sigma}_b) \Delta_b & \tilde{\sigma}_b \Delta_m & \tilde{\sigma}_b \Delta_n \\ \tilde{\sigma}_m \Delta_a & \tilde{\sigma}_m \Delta_b & \gamma - (1 - \tilde{\sigma}_m) \Delta_m & \tilde{\sigma}_m \Delta_n \\ \tilde{\sigma}_n \Delta_a & \tilde{\sigma}_n \Delta_b & \tilde{\sigma}_n \Delta_m & \gamma - (1 - \tilde{\sigma}_n) \Delta_n \end{bmatrix}^{-1}$$

Numerically, we set $\gamma = 1.02$, $\tilde{\sigma}_a = 0.0758432283057199$, $\tilde{\sigma}_b = 0.758\,728\,816\,8$, $\tilde{\sigma}_m = 0.00371613521937185$, $\tilde{\sigma}_n = 0.161711819707504$, $\Delta_a = 1 - \delta_a$, $\Delta_b = 1 - \delta_b$, $\Delta_m = 1 - \delta_m$, $\Delta_n = 1 - \delta_n$, $\Delta_k = 1 - \delta_k$, $\delta_a = 0.05$, $\delta_b = 1$, $\delta_m = 0.08$, $\delta_n = 0.61$, $\delta_k = 0.08$, in order to find $\sigma_a = 0.510\,790\,285\,2$, $\sigma_b = 0.350\,679\,408\,5$, $\sigma_m = 0.017\,519\,244\,2$, $\sigma_n = 0.121\,011\,062\,1$. Notice that $\sigma_a + \sigma_b + \sigma_m + \sigma_n = 1$.

Using (13) and setting also $\tau = 0.442\,237\,723\,3$, $\tau_q = 0.442857763$, $\tau_p = 0.38085379464$, $s_k = 0.75$, $s_a = 0.23$, $s_m = 0.01$, $s_p = 0.01$, $\beta = 0.961\,538\,461\,5$ and using (32) with $\tau_k = \tau_q$, eventually, we get from (79): $\theta = 0.563096\,6639$.

8.3 Appendix 3: Public spending in France

Public spending and percentages in 2006.

YEAR 2006		Billions €	% of Overall Taxes	Type of Public Expenditures
National Budget [3]		**280,75**	**35,43%**	
After transfers to Local Public Administrations	Health expenditures [2]	3,70	0,47%	
	including Medical R&D expenditures [1]	*2,95*	*0,37%*	Medical R&D
	Other health expenditures	*0,76*	*0,10%*	Health expenditures (excl. R&D)
	Other expenditures	277,05	34,96%	
	including Investment [4]	*10,27*	*1,30%*	Public investment
	Consumption	*266,79*	*33,66%*	Public consumption
Local Public Administrations [3]		**101,32**	**12,79%**	
	Health expenditures [2]	1,50	0,19%	Health expenditures (excl. R&D)
	Other expenditures	99,82	12,60%	
	including Investment [4]	*43,51*	*5,49%*	Public investment
	Consumption	*56,31*	*7,11%*	Public consumption
Social Security Administrations [3]		**405,75**	**51,20%**	
	Health expenditures [2]	125,90	15,89%	Health expenditures (excl. R&D)
	Other expenditures	279,85	35,31%	
	including Investment [4]	*6,33*	*0,80%*	Public investment
	Consumption	*273,52*	*34,51%*	Public consumption
European Union (U.E.) [3]		**4,67**	**0,59%**	Public consumption
Overall Taxes [3]		**792,49**	**100,00%**	

Table 6. Breakdown of taxes paid by French citizens by type of expenditures (2006)

Sources:

(1) Ministry of National Education, Advanced Instruction, and Research, quoted in Fenina & Geffroy [2007], p. 43.

(2) National Institute for Statistics and Economic Studies, INSEE, National accounts (base 2000), in http://www.insee.fr/fr/themes/comptes-nationaux/.

(3) Report on 2006 National Accounts, quoted in *Ministère de l'intérieur, de l'outre-mer et des collectivités territoriales* [2008], p. 34.

(4) 2006 National Accounts, quoted in *Ministère de l'intérieur, de l'outre-mer et des collectivités territoriales* [2008], p. 40.

9 References

Agenor P.R. [2005], Fiscal Policy and Endogenous Growth with Public Infrastructure, Center for Growth and Business Cycle Research, University of Manchester, *Working Papers* 59.

Agenor P. R. [2005], Health and Infrastructure in Models of Endogenous Growth, Center for Growth and Business Cycle Research, University of Manchester, *Working Papers* 62.

Agenor P.R. and B. Moreno-Dodson [2006], Public Infrastructure and Growth: New Channels and Policy Implications, The World Bank, *Policy Research Working Papers Series* 4064.

Aghion P. and P. Howitt [1992], A Model of Growth Through Creative Destruction, *Econometrica* 60, 323-351.

Aghion P. and P. Howitt [1997], *Endogenous Growth Theory*, MIT Press.

Arrazola M. and J. de Hevia [2004], More on the Estimation of the Human Capital Depreciation Rate, *Applied Economics Letters* 11, 145-148.

Arrazola M., J. de Hevia, M. Risueño and J.F. Sanz Sanz [2005], A Proposal to Estimate Human Capital Depreciation: Some Evidence for Spain, *Hacienda Pública Española* 127, 9-22.

Barro R.J. [1990], Government Spending in a Simple Model of Endogenous Growth, *Journal of Political Economy* 98, 103-126.

Blanchard O.J. and D. Quah [1989], The Dynamic Effects of Aggregate Demand and Supply Disturbances, American Economic Review 79, 655-673.

Barro R.J. and X. Sala-i-Martín [1995], *Economic Growth*, MIT Press.

Benos N. [2005], Fiscal Policy and Economic Growth: Empirical Evidence from OECD Countries, Department of Economics of Cyprus University, *Discussion Papers* 2005-01.

Bloom, D.E., D. Canning and J. Sevilla [2004], The Effect of Health on Economic Growth: A Production Function Approach, *World Development* 32(1), 1-13.

Chari, V.V., P. Kehoe and E. McGrattan [1997], The Poverty of Nations: A Quantitative Investigation, Federal Reserve Bank of Minneapolis, *Working Papers*.

Corsetti G. and N. Roubini [1996], Optimal Government Spending and Taxation in Endogenous Growth Models, National Bureau of Economic Research, *Working Papers* 5851.

Erosa A., T. Koreshkova and D. Restuccia [2006], On the Aggregate and Distributional Implications of Productivity Differences Across Countries, University of Toronto, *Working Papers*.

Fenina A. and Y. Geffroy [2007], Comptes nationaux de la santé 2006, DREES, Ministère de l'emploi, du travail et de la cohésion sociale, *Document de Travail Série Statistiques* n°114, September.

Gyimah-Brempong K. and M. Wilson [2004], Health, Human Capital, and Economic Growth in Sub-Saharan African and OECD Countries, *Quarterly Review of Economics and Finance* 44, 296-320.

Grossman G. and E. Helpman [1991], *Innovation and Growth in the Global Economy*, Cambridge, MIT Press.

Hall B.H. [2006], R&D, Productivity, and Market Value, University of California at Berkeley, *mimeo.*

Hall B.H. [2007], Measuring the Returns to R&D: The Depreciation Problem, National Bureau of Economic Research, *Working Papers* 13473.

Hosoya K. [2003], Tax financed government health expenditure and growth with capital deepening externality, *Economics Bulletin* 5, 1-10.

Howitt P. [2000], Endogenous Growth and Cross-Country Income Differences, *American Economic Review* 90, 829-846.

Jamison D.T., L.J. Lau and J. Wang [2005], Health's Contribution to Economic Growth in an Environment of Partially Endogenous Technical Progress, in *Health and Economic Growth: Findings and Policy Implications*, López-Casasnovas G., Berta Rivera and Luis Currais eds., MIT Press.

Jones C.I. [2007], TheWeak Link Theory of Economic Development, University of Berkeley, *Working Papers.*

Klenow P.J., A. Rodriguez-Clare [2005], Externalities and Growth, in Aghion P. and S. Durlauf eds., *Handbook of Economic Growth*, Elsevier, Amsterdam.

Knowles S. and P.D. Owen [1997], Education and Health in an Effective-Labour Empirical Growth Model, The Economic Society of Australia, *The Economic Record* 73, 314-328.

Mankiw N.G., D. Romer and D.N. Weil [1992], A Contribution to the Empirics of Economic Growth, *Quarterly Journal of Economics* 107, 407-437.

Manuelli R. and A. Seshadri [2005], Human Capital and the Wealth of Nations, University of Wisconsin, *Working Papers.*

Mead C.I. [2007], R&D Depreciation Rates in the 2007 R&D Satellite Account, Bureau of Economic Analysis, NSF, *2007 R&D Satellite Account Background Paper.*

Ministère de l'éducation nationale, de l'enseignement supérieur et de la recherche [2004], *Les dépenses de R&D en santé en France en 2001*, Direction de l'évaluation et de la prospective, *Note de recherche.*

Ministère de l'intérieur, de l'outre-mer et des collectivités territoriales [2008], Les collectivités locales en chiffres 2008, Direction Générale des Collectivités locales/DESL, Mars

Mushkin S.J. [1962], Health as an Investment, *Journal of Political Economy* 70, 129-157.

Nadiri M.I. and I.R. Prucha [1996], Estimation of the Depreciation Rate of Physical and R&D Capital in the U.S. Total Manufacturing Sector, *Economic Inquiry* 34, 43-56.

Romer, P. [1990], Endogenous Technological Change, *Journal of Political Economy* 98, 71-102.

Sala-i-Martin X., Doppelhofer G. and R.I. Miller [2004], Determinants of

Long-Term Growth: A Bayesian Averaging of Classical Estimates (BACE) Approach, *American Economic Review* 94, 813-35.

Sécurité sociale [2006], ONDAM et dépense nationale de santé, *Projet de loi de financement de la sécurité sociale*, Annexe 7.

Turnovsky S.J. [2000], Fiscal Policy, Elastic Labor Supply, and Endogenous Growth, *Journal of Monetary Economics* 45, 185-210.

Vazeille O. and Sonnette C. [2006] Dépenses sociales, premières dépenses publiques, Division Synthèse générale des comptes, *Insee première* 1102.

Weil D.N. [2005], Accounting for the Effect of Health on Economic Growth, Department of Economics, Brown University, *Working Papers* 2005-07.

Zagler M. and G. Durnecker [2003], Fiscal Policy and Economic Growth, *Journal of Economic Surveys* 17, 397-418.

Zon A.H. and J. van Muysken [1997], Health, education and endogenous growth, MERIT, Maastricht, *Research Memoranda* 6.

Zon A.H. and J. van Muysken [2001], Health and Endogenous Growth, *Journal of Health Economics* 20, 169-85.