EPEE
UNIVERSITE D'EVRY
VAL D'ESSONNE

DOCUMENT DE RECHERCHE

EPEE

CENTRE D'ETUDE DES POLITIQUES ECONOMIQUES DE L'UNIVERSITE D'EVRY

Consumption externalities, Heterogeneous Agents and Cycles

Riham BARBAR & Jean-Paul BARINCI

09 - 02

www.univ-evry.fr/EPEE

UNIVERSITE D'EVRY – VAL D'ESSONNE, 4 BD. FRANÇOIS MITTERRAND, 91025 EVRY CEDEX

Consumption Externalities, Heterogeneous Agents and Cycles

Riham Barbar*and Jean-Paul Barinci†

February 12, 2009

Abstract

This paper studies the effect of consumption externalities in a Ramsey model with heterogeneous agents and borrowing constraints. We consider two types of agents who differ in their initial wealth and discount factors. Further, we leave open the possibility of increase in consumption of one agent producing a positive (Keeping-Up with the Joneses) or a negative (Running-Away from the Joneses) effet over the marginal utility from own consumption of the other agent. We show that, at a steady-state equilibrium, inequality arises across agents which in turn implies that, agents have different consumption levels. Moreover, we show that, whenever the preferences display Keeping-Up with the Joneses feature, low elasticity of production factors substitution is *no longer needed* for the emergence of flip cycles and instability. Instead, only the external effects in consumption that plays a crucial role and promotes instability and the appearence of these endogeneous cycles.

Key words: Consumption externalities; borrowing constraints; heterogeneous agents; saddle-path stability; endogeneous cycles.

JEL Classification: E22; E23; O40.

*EPEE, University of Evry Val d'Essonne, riham.barbar@univ-evry.fr

†EPEE, University of Evry Val d'Essonne, jean-paul.barinci@univ-evry.fr

1 Introduction

The study of consumption externalities is considered as one of the important issues of economic behaviour. In fact, agents care about their relative position in the society.[1] Several empirical studies confirme that happiness of agents depends not only on own consumption (or wealth), but also on how it compares with the consumption (or wealth) of others, for example, Frank (1985), Easterlin (1995), Cole, Mailath and Postlewaite (1992, 1995), and Clark and Oswald (1996), among others.

Consumption of others results in an increase or a reduction in the marginal utility from own consumption; then we say that preferences display *Keeping-Up with the Joneses* feature or *Running-Away from the Joneses* feature, respectively.

Consumption externalities have been widely studied in dynamic macroeconomic. For example, Abel (1990) and Gali (1994) examine how consumption externalities modify the price of financial assets. More recent works explore the impact of consumption externalities on optimal taxation (Ljungqvist and Uhlig, 2002), equilibrium efficiency (Liu and Turnovsky, 2005), and the emergence of endogeneous fluctuations and indeterminacy (Alonso-Carrera, Caballé and Raurich, 2007; Chen and Hsu, 2007).

Remarkably, a common feature of most previous works is the assumption that agents are indentical. Consumption externalities in representative-agent economies can be formally introduced in a way that, an individual agent's utility depends on his consumption and on the average-consumption level in the economy. At equilibrium, the individual-consumption and the average-consumption levels must be equal.

However, if all agents are identical and their consumption levels coincide then, for a rational agent who realize it, the external effect from other's consumption should disappear. Therefore, heterogeneity across agents is a fundamental component of consumption externalities.[2]

Only few works have studied the role of consumption externalities on dynamic behaviour of the economy under the assumption that agents are heterogeneous. Garcia-Penalosa and Turnovsky (2007) consider a neoclassical growth model with consumption externalities and assume that agents are heterogeneous with respect to initial wealth and reference consumption levels. They show that the aggregate behaviour of the economy is independent of wealth distribution and is equivalent to that in representative-agent models in two cases: whenever all agents have the same reference consumption or whenever the reference consumption level enters the utility as a separable homogeneous function. In addition, the steady state is a saddle point.

In this paper, we are interested in studying the effect of consumption externalities on the saddle-path stability of a Ramsey model whenever agents are heterogeneous. In a way, we are close to Garcia-Penalosa and Turnovsky (2007)

[1] We say that they are *status-seeker*. The origin of such a concept is referred to Smith (1759) and Veblen (1899).

[2] Garcia-Penalosa and Turnovsky (2007).

model except that we consider a simpler framework as described shortly. We assume that heterogeneity across agents stems from different initial wealth and different discount factors. For the sake of simplicity, we reduce agents' heterogeneity to two types; one is more patient than the other. Further, each agent is assumed to supply one unit of labor inelastically and get wage-income. In addition, as assumed in Becker (1980) and Becker and Foias (1987, 1994) and Sorger (1994), agents in our model are not allowed to borrow against their future income; we impose a borrowing constraint on agents.

Under the above assumptions, around a steady state, we can characterize an equilibrium at which only the patient agent holds positive amount of capital while the impatient agent chooses not to do so and he consumes only his wage-income. Therefore, contrary to Garcia-Penalosa and Turnovsky (2007), at a steady-state equilibrium *inequality* arises across agents which in turn implies that, agents have different consumption levels.

Further, as our model is based on Becker and Foias (1994) framework, it worth clarifying the added value of this paper with respect to theirs. Our main result states that the steady state is always determinate but looses its stability through two-period cycles. Moreover, contrary to Becker and Foias (1994),[3] we show that, whenever the preferences display Keeping-Up with the Joneses feature, instability *does not* require a negative response of the patient agent's income to a rise in the capital stock. In other words, low elasticity of production factors substitution is *no longer needed* for the emergence of flip cycles. Instead, only the external effects in consumption that plays a crucial role on the appearence of these endogeneous cycles.

This paper is organised as follows. In the next section, we present the model. Section 3 defines the intertemporal equilibrium and shows the uniqueness of the steady state. In section 4, we analyze the local dynamics. The last section concludes.

2 The model

We consider a Ramsey model in which agents are assumed to be heterogeneous with respect to their time-preference rates and initial endowments. For simplicity but without loss of generality, we reduce agents' heterogeneity to two types of infinitely-lived agents. Further, both types supply one unit of labor inelastically. Finally, denote the size of type i by n^i which is exogeneously given.

In addition, we assume that they are status seekers in the sense that one's utility at each period depends on his consumption as well as on the other agent's consumption. Formally, let the preferences of agent i be represented by the instantaneous utility function $u^i\left(c_t^i, c_t^j\right)$, for $i = 1, 2$, where c_t^i is agent i's consumption and c_t^j is the other agent's consumption.

[3] Becker and Foias (1994) show that the occurrence of two-period cycles requires sufficiently weak capital-labor substitution (or equivalently, the patient agent's income is decreasing in the capital stock) and a very small elasticity of intertemporal substitution.

Assumption 1 *The instantaneous utility function* $u^i\left(c_t^i, c_t^j\right)$*, for* $i = 1, 2$*, is twice continuously differentiable and satisfies the following conditions:*

$$u_1^i\left(c^i, c^j\right) > 0 > u_{11}^i\left(c^i, c^j\right) \tag{1}$$

Assumption 2 *We leave open the possibility of increase in consumption of agent* j *producing a positive or a negative effet over the marginal utility from own consumption of agent* i*. If the external effect is positive,* $u_{12}^i\left(c^i, c^j\right) > 0$ *(negative* $u_{12}^i\left(c^i, c^j\right) < 0$*), preferences display Keeping-Up with the Joneses feature (Running-Away from the Joneses feature).*

Assume that agent i is initially endowed by $k_0^i > 0$. Given a sequence of real interest rates on capital $\{r_t\}$ and wages rate $\{w_t\}$, agent i chooses a pattern of consumption and capital holdings at each time that maximizes his lifetime utility subject to a sequence of budget constraints and a sequence of borrowing constraints. Formally, i's maximization problem is written as

$$\max_{c_t^i, k_{t+1}^i} \sum_{t=0}^{+\infty} \beta^{i,t} u^i\left(c_t^i, c_t^j\right) \tag{2}$$

subject to

$$c_t^i + k_{t+1}^i - (1-\delta)\, k_t^i \leq r_t k_t^i + w_t \tag{3}$$

$$k_{t+1}^i \geq 0 \tag{4}$$

where the constraint (4) states that agents must have non-negative wealth at each time t; they are not allowed to finance present consumption by borrowing against future income.

One can easily show that the necessary first order conditions imply Euler equation

$$u_1^i\left(c_t^i, c_t^j\right) \geq \beta^i\left(r_{t+1} + 1 - \delta\right) u_1^i\left(c_{t+1}^i, c_{t+1}^j\right) \tag{5}$$

which holds with equality if $k_{t+1}^i > 0$. Morover, the monotonicity of the utility function gives rise to a binding budget constraint

$$c_t^i + k_{t+1}^i - (1-\delta)\, k_t^i = r_t k_t^i + w_t \tag{6}$$

In a framework where agents have different time-preference rates, only the most patient agent owns a positive capital stock in the long-run. This property has already been formally proved by Becker (1980) and Becker and Foias (1987).

In the following, it will be assumed that agent 2 is more impatient than agent 1, that is, he discounts the future more heavily:

Assumption 3

$$0 < \beta^2 < \beta^1 < 1 \tag{7}$$

Notice that, near the steady state, the real gross return on capital should be $1/\beta^1$ since a firm prefers to have the capital with the lowest cost. Thus equation (5) for agent 2 shows that, around the steady state, agent 2 will choose to hold no capital provided that he is sufficiently impatient.

Formally, around the steady state, an equilibrium of the optimization problem (2)-(4) is described by a sequence $\left\{c_t^i, k_t^i\right\}$ for $i = 1, 2$ such that only the patient agent 1 has a positive capital stock $k_t^1 > 0$ while the impatient agent 2 consumes only his wage $k_t^2 = 0$. At this equilibrium, agent 1's optimal behavior is fully characterized by

$$\frac{u_1^1\left(c_t^1, c_t^2\right)}{u_1^1\left(c_{t+1}^1, c_{t+1}^2\right)} = \beta^1\left(r_{t+1} + 1 - \delta\right) \tag{8}$$

$$c_t^1 + k_{t+1}^1 - (1-\delta)\, k_t^1 = r_t k_t^1 + w_t \tag{9}$$

The transversality condition must be satisfied for the patient agent

$$\lim_{t \to +\infty} \beta^t u_1^1\left(c_t^1, c_t^2\right) k_{t+1}^1 = 0 \tag{10}$$

However, as $k_t^2 = 0$, agent 2's Euler equation holds with inequality and he consumes only his wage. Therefore, agent 2's optimal behavior is described by

$$\frac{u_1^2\left(c_t^2, c_t^1\right)}{u_1^2\left(c_{t+1}^2, c_{t+1}^1\right)} > \beta^2\left(r_{t+1} + 1 - \delta\right) \tag{11}$$

$$c_t^2 = w_t \tag{12}$$

Trivially, the transversality condition is satisfied for the impatient agent.

Some elasticities are introduced and evaluated at the steady state. The elasticity of intertemporal substitution in consumption is $-1/\varepsilon_{11}^i$ where $\varepsilon_{11}^i \equiv u_{11}^i c^i / u_1^i < 0$, for $i = 1, 2$. In addition, the elasticities of externalities are given by $\varepsilon_{12}^i \equiv u_{12}^i c^j / u_1^i \lesseqgtr 0$, for $i = 1, 2$.

In contrast to the consumers' side, the production sector is homogeneous. Assume that a representative firm produces the final good using a constant return to scale technology $y_t = F\left(K_t, L_t\right)$, where K_t and L_t are the aggregate capital and labor. Let $k_t \equiv K_t / L_t$ be the capital per capita, using homogeneity of degree one, the production function could be written as $F\left(K_t, L_t\right) \equiv f\left(k_t\right) L_t$. Suppose that the representative firm maximizes the profit $\pi_t \equiv y_t - r_t K_t - w_t L_t$ taking factor prices (the real interest rate r_t and the real wage w_t) and the technology as given.

Assumption 4 *The technology $f(k)$ is a continuous function of the capital per capita $k \geq 0$, positive-valued and differentiable. Furthermore, $f''(k) <$*

$0 < f'(k)$, for $k > 0$, and $f(0) = 0$, $\lim_{K\to 0} f'(k) = +\infty$ and $\lim_{K\to +\infty} f'(k) = 0$.

Under assumption (4), profit maximization implies

$$\begin{aligned} r_t &= f'(k_t) \\ w_t &= f(k_t) - k_t f'(k_t) \end{aligned}$$

We define the following elasticities. The elasticity of capital-labor substitution is given by $\sigma \equiv [kf'(k)/f - 1] f'(k)/kf''(k)$. The capital share of the total income is given by $s \equiv f'(k)k/f(k) \in (0,1]$. Finally, the elasticities of the interest rate with respect to capital and labor are $F_{KK}K/F_K = -F_{KL}L/F_K = -(1-s)/\sigma$, and the elasticities of the real wage with respect to capital and labor are $F_{LL}L/F_L = -F_{LK}K/F_L = -s/\sigma$.

3 Intertemporal equilibrium

First, we define an equilibrium for the economy described above:

Definition 1 *An equilibrium of the economy $E = \left(F, \left(k_0^i, \beta^i, u^i, n^i\right)_{i=1}^2\right)$ is an intertemporal sequence $\left(r_t, w_t, K_t, L_t, \left(c_t^i, k_t^i\right)_{i=1}^2\right)_{t=0}^{+\infty}$ which satisfies the following conditions:*

1. *Given the sequence $(r_t, w_t)_{t=0}^{+\infty}$, $\left(c_t^i, k_t^i\right)_{t=0}^{+\infty}$ solves the ith consumer's program for $i = 1, 2$;*
2. *Given the sequence $(r_t, w_t)_{t=0}^{+\infty}$, K_t solves the firm's program for $t = 0, 1, \ldots, \infty$;*
3. *The capital market clears $K_t = n^1 k_t^1 + n^2 k_t^2$, for $t = 0, 1, \ldots, \infty$;*
4. *The labor market clears $L_t = n^1 + n^2$, for $t = 0, 1, \ldots, \infty$;*
5. *By Walras' Law, the product market also clears*

$$\sum_{i=1}^{2} \left(k_{t+1}^i - (1-\delta) k_t^i + c_t^i\right) n^i = F(K_t, L_t) \tag{13}$$

In the next proposition, we characterize the intertemporal equilibrium with perfect forseight.

Proposition 1 *Let an economy E satisfying assumptions $1-4$. Consider the conditions below for $t = 0, 1, \ldots, \infty$* and $i = 1, 2$;

(P1) $K_t > 0,\ L_t > 0,\ Y_t = F(K_t, L_t) > 0,\ k_t^i \geq 0,\ c_t^i \geq 0$;

(P2) $r_t = f'(k_t)$ *and* $w_t = f(k_t) - k_t f'(k_t)$;

(P3) $u_1^i \left(c_t^i, c_t^j\right) \geq \beta^i (r_{t+1} + 1 - \delta) u_1^i \left(c_{t+1}^i, c_{t+1}^j\right)$, *with equality when* $k_{t+1}^i > 0$;

(P4) $c_t^i + k_{t+1}^i - (1-\delta) k_t^i = r_t k_t^i + w_t$;

(P5) $K_t = n^1 k_t^1 + n^2 k_t^2$;

(P6) $L_t = n^1 l^1 + n^2 l^2$;

Then if the sequance $\left(r_t, w_t, K_t, L_t, \left(c_t^i, k_t^i\right)_{i=1}^2\right)_{t=0}^{+\infty}$ *is a competitive equilibrium, the conditions (P1)-(P6) hold. Furthermore, if the sequence* $\left(r_t, w_t, K_t, L_t, \left(c_t^i, k_t^i\right)_{i=1}^2\right)_{t=0}^{+\infty}$ *satisfies (P1)-(P6) and the transversality condition*

$$\lim_{t \to +\infty} \beta^t u_1^i \left(c_t^i, c_t^j\right) k_{t+1}^i = 0$$

for $i = 1, 2$, *it is an equilibrium for the economy* E.

Now we can characterize the steady state in which only the most patient agent owns positive capital.

Proposition 2 *Under assumptions* $1 - 4$ *and* $K_0 > 0$, *there exists a steady state, with constant prices* r *and* w, *defined by the following properties:*

(S1) $\beta^1 (1 - \delta + r) = 1 \geq \beta^2 (1 - \delta + r)$, that is, $k^1 > 0$ and $k^2 = 0$;

(S2) $c^1 = (r - \delta) k^1 + w$ and $c^2 = w$;

(S3) $K = n^1 k^1$;

(S4) $L = n^1 + n^2 = n$,

(S5) $Y = F\left(n^1 k^1, L\right)$;

Consider the following notation:

$$N^i \equiv \frac{n^i}{n^1 + n^2} \in [0, 1]$$

Then, around a steady state, the capital per capita is

$$k_t = \frac{K_t}{L_t} = \frac{n^1 k_t^1}{n^1 + n^2} = N^1 k_t^1$$

Now we can characterize the two-dimensional system around a steady state $(S1) - (S5)$.

Proposition 3 *Let assumptions* $1-4$ *hold. Then, an intertemporal equilibrium with perfect forsight is a sequence of* $\left\{c_t^1, k_t^1\right\}_{t=0}^{+\infty}$ *that solves the two-dimensional dynamic system that is given by*

$$\frac{u_1^1\left(c_t^1, f\left(N^1 k_t^1\right) - N^1 k_t^1 f'\left(N^1 k_t^1\right)\right)}{u_1^1\left(c_{t+1}^1, f\left(N^1 k_{t+1}^1\right) - N^1 k_{t+1}^1 f'\left(N^1 k_{t+1}^1\right)\right)} = \beta^1\left[1-\delta+f'\left(N^1 k_t^1\right)\right] \tag{14}$$

$$c_t^1 + k_{t+1}^1 - (1-\delta) k_t^1 = f\left(N^1 k_t^1\right) + \left(1-N^1\right) k_t^1 f'\left(N^1 k_t^1\right) \tag{15}$$

subject to the initial aggregate endowment $k_0^1 > 0$ *and the transversality condition (10).*

Equation (14) is the intertemporal Euler equation of agent 1. Here we observe that agent 1's marginal rate of substitution between consumption at different dates (LHS of equation (14)) is affected by agent 2's consumption. As this latter ignores his effect on agent 1's decision, this may lead agent 1 to substitute inefficiently his consumption across periods. Therefore, agent 1's consumption equilibrium path could be inefficient. Such a result has been proved in previous literature such as Fisher and Hof (2000a, 2000b), Liu and Turnovsky (2005) and Alonso-Carrera et *al.* (2004, 2005, 2006). However, even if consumption externalities make the equilibrium path inefficient, they play no role in determining the steady-state as we will see below.

The two-dimensional system at the steady state is given by

$$1 = \beta^1\left[1-\delta+f'\left(N^1 k^1\right)\right] \tag{16}$$

$$c^1 + \delta k^1 = f\left(N^1 k^1\right) + \left(1-N^1\right) k^1 f'\left(N^1 k^1\right) \tag{17}$$

In steady-state equilibrium, the capital k^1 is determined by the standard Golden Rule (equation (16)). That is, the capital stock in the long-run depends only on the parameters of the model (the rate of time preferences $\beta^1 \in (0,1)$ and depreciation rate $\delta \in (0,1)$). Once k^1 is known from equation (16), the equilibrium consumption c^1 could be easily computed from equation (17). Obviously, the steasy state $\left(c^1, k^1\right)$, satisfying (16) and (17), is unique.

4 Local dynamics

In order to examine the effect of consumption externalities on the saddle-path stability, we study the local dynamics of the economy around the steady state. Contrary to Becker and Foias (1994),[4] we do not impose any restriction on the elasticity of production factors substitution σ, so that we could capture the role of consumption externalities on the appearence of endogeneous cycles.

[4]The economic intuition of Becker and Foias (1994) is based on the assumption that the patient agent's income is decreasing in capital stock.

We start by linearizing the system (14) and (15) around the steady state (16) and(17), we get $\left(dc^1_{t+1}/c^1, dk^1_{t+1}/k^1\right)^T = J\left(dc^1_t/c^1, dk^1_t/k^1\right)^T$ where J is the Jacobian matrix of linearized system.

Let

$$A \equiv (1-s)\left[1-\beta^1(1-\delta)\right]$$

The trace and the determinant of the Jacobian matrix J are given as follows

$$D = \frac{1}{\beta^1}\left(1-\frac{A\left(1-N^1\right)}{\sigma}-\frac{\left[s\left(1-\beta^1\right)+AN^1\right]\varepsilon^1_{12}}{\sigma}\left(-\frac{1}{\varepsilon^1_{11}}\right)\right) \tag{18}$$

$$T = 1+D+\frac{A\left[s\left(1-\beta^1\right)+AN^1\right]}{s\sigma\beta^1}\left(-\frac{1}{\varepsilon^1_{11}}\right) \tag{19}$$

Here we show that once we start to vary the elasticity of intertemporal substitution in patient 1's consumption, denote it by $x \equiv -1/\varepsilon^1_{11}$, from 0 to $+\infty$, the trace and the determinant represents a half-line in the (T, D) −plane. Formally, this half-line can be defined by the locus $\Delta_0 \equiv \{(T(x), D(x)) : x \geq 0\}$, where x is refered to as bifurcation parameter. The following lemma shows that Δ_0 is linear in x and so allows us to apply the geometrical method of Grandmond, Pintus and de Vilder (1998).

Lemma 1 *Δ_0 is linear in x. Its origin lies on the line (AC) and its slope is given by*

$$S = -\frac{s\varepsilon^1_{12}}{A - s\varepsilon^1_{12}}$$

Moreover, Δ_0 rotates clockwise when ε^1_{12} goes up around the initial point.

Proof. Consider the determinant and the trace given by (18) and (19). Our objective is to locate Δ_0 in the (T, D) −plane.

The initial point at $x = 0$ is $(T_0, D_0) = (1 + D_0, D_0)$ with $D_0 = \left[1 - A\left(1-N^1\right)/\sigma\right]/\beta^1$. That is, Δ_0 starts at a point on the line (AC).

Moreover, Δ_0 is linear in x with slope

$$S \equiv \frac{D'(x)}{T'(x)} = \frac{s\varepsilon^1_{12}}{s\varepsilon^1_{12} - A}$$

with $\partial S/\partial\varepsilon^1_{12} < 0$ for all ε^1_{12}. Thus Δ_0 makes a clockwise rotation with ε^1_{12} around the initial point. ■

We observe from (19) that $D(x) < T(x) - 1$, $\forall x$, that is, the locus Δ_0 lies to the right of AC. In other words, the economy never lies in the sink-stability ABC. In order to know whether the economy is characterized by saddle-path stability or a source configuration, we need to locate (T, D) with respect to the line AB.

Proposition 2 *Let assumptions (1)–(4) hold. Then, for x close to its critical value*

$$x^F = \frac{s}{s\left(1-\beta^1\right)+AN^1}\frac{\left(1+\beta^1\right)\sigma - A\left(1-N^1\right)}{s\varepsilon^1_{12}-A/2} \tag{20}$$

there generically exist Flip cycles.

(1) *For $\varepsilon^1_{12} \in (-\infty, 0)$ or $\varepsilon^1_{12} \in (0, A/2s)$, a flip bifurcation occurs if and only if $\sigma < A\left(1-N^1\right)/\left(1+\beta^1\right)$.*

(2) *For $\varepsilon^1_{12} \in (A/2s, A/s)$, there generically exists a flip bifurcation at $x = x^F$ if and only if $\sigma > A\left(1-N^1\right)/\left(1+\beta^1\right)$.*

(3) *For $\varepsilon^1_{12} \in (A/s, +\infty)$, there is no room for flip cycles.*

Proof. Recall that the occurence of Flip cycles requires that one of the eigenvalues is equal to -1. Solving $D(x) = -T(x) - 1$ for x, one obtains the critical value x^F given by equation (20).

(1) If preferences of the patient agent display *Running-Away from the Joneses*, $\varepsilon^1_{12} < 0$, then $S \in (0, 1)$ and Δ_0 makes an upward movement with x. In this case, flip cycles appear if and only if Δ_0 starts below the point A, that is, $D_0 < -1$ and σ is sufficiently low. More precisely,

(i) If $\sigma < A\left(1-N^1\right)/\left(1+\beta^1\right)$, the steady state is a source for $x < x^F$ and a saddle point for $x > x^F$. So the system undergoes a flip bifurcation whenever Δ_0 crosses the line (AB) at $x = x^F$.

(ii) If $\sigma > A\left(1-N^1\right)/\left(1+\beta^1\right)$, the steady state is a saddle point.

(2) If preferences of the patient agent display *Keeping-up with the Joneses*, $\varepsilon^1_{12} > 0$, then two subcases arise:

(i) For $\varepsilon^1_{12} \in (0, A/s)$ then $S < 0$ which implies that either $S < -1$ or $S > -1$.

(ia) $\varepsilon^1_{12} \in (A/2s, A/s)$ then $S \in (-1, 0)$: The appearence of flip cycles require that Δ_0 starts above the point A. That is, if $\sigma > A\left(1-N^1\right)/\left(1+\beta^1\right)$, the steady state is a saddle for $x < x^F$, a source for $x > x^F$; the system undergoes a flip bifurcation at $x = x^F$; otherwise, the steady state is a source.

(ib) $\varepsilon^1_{12} \in (0, A/2s)$ or equivalently $S > -1$: Flip cycles require $D_0 < -1$. That is, if $\sigma < A\left(1-N^1\right)/\left(1+\beta^1\right)$, the steady state is a source $x < x^F$, a saddle for $x > x^F$; so a flip bifurcation occurs at $x = x^F$; otherwise, the steady state is a saddle point.

(ii) For $\varepsilon_{12}^{1} \in (A/s, +\infty)$ then $S > 0$. Flip cycles appear iff Δ_0 starts below the point A that is, $D_0 < -1$ and $\sigma < A\left(1 - N^1\right) / \left(1 + \beta^1\right)$. However, this case implies a negative value for x^F in (20). Therefore, there is no room for flip cycles, $\sigma < A\left(1 - N^1\right) / \left(1 + \beta^1\right)$, and the steady state is always a saddle point.

■

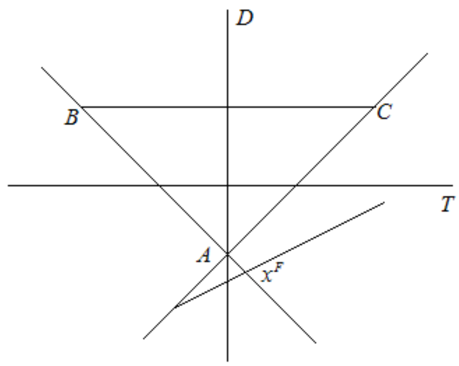

Figure 1: $S \in (0, 1)$

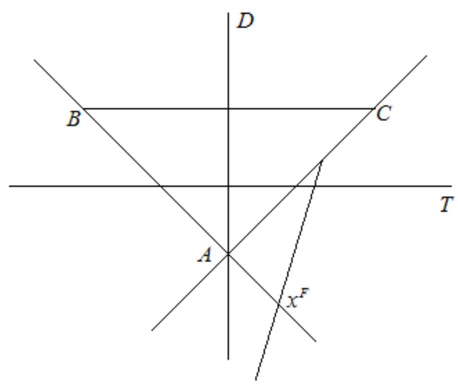

Figure 2: $S \in (-1, 0)$

4.1 Consumption externalities and cycles

In this subsection, we are interested in studying the effect of consumption externalities on the appearence of cycles. We start by differentiating (20) with respect to ε_{12}^{1}:

$$\frac{\partial x^F}{\partial \varepsilon_{12}^{1}} = -\frac{s^2}{s\left(1 - \beta^1\right) + AN^1} \frac{\left(1 + \beta^1\right)\sigma - A\left(1 - N^1\right)}{\left(s\varepsilon_{12}^{1} - A/2\right)^2} \tag{21}$$

We observe that the sign of $\partial x^F/\partial\varepsilon_{12}^1$ depends on σ. Assume first that the elasticity of capital-labor substitution is high, then $\partial x^F/\partial\varepsilon_{12}^1 < 0$. This means that the saddle-path stability range $\left(0, x^F\right)$ shrinks whenever the elasticity of consumption external effects ε_{12}^1 is higher. In other words, higher is the consumption external effects ε_{12}^1, lower is the elasticity of intertemporal substitution in consumption of patient agent needed in order for two-period cycles to appear.

Now assume that the elasticity of capital-labor substitution is low, then $\partial x^F/\partial\varepsilon_{12}^1 > 0$. That is, the saddle-path stability range $\left(x^F, +\infty\right)$ shrinks whenever the elasticity of consumption external effects ε_{12}^1 is higher. Thus, for higher external consumption effects, endogenous cycles are more ikely to appear.

In sum, consumption externalities promote instability and the appearence of two-period cycles.

4.2 Size of patient agent and cycles

Now we study the relation between the patient agent size and the stability of the economy. We differentiate (20) with respect to N^1:

$$\frac{\partial x^F}{\partial N^1} = \frac{sA}{\left[s\left(1-\beta^1\right)+AN^1\right]^2}\frac{s\left(1-\beta^1\right)-\left(1+\beta^1\right)\sigma+A}{s\varepsilon_{12}^1-A/2} \tag{22}$$

Whenever σ is sufficiently low, $\partial x^F/\partial N^1 < 0$. This means that, as the size of patient agent increases, the saddle-path stability range $\left(x^F, +\infty\right)$ widens. However, whenever σ is high, $\partial x^F/\partial N^1 > 0$ iff $\sigma < \left[s\left(1-\beta^1\right)+A\right]/\left(1+\beta^1\right)$. That is, the saddle-path stability range $\left(0, x^F\right)$ widens with the size of patient agent.

The interpretation is given as follows. Under keeping-up with the Joneses preferences and high elasticity of factors substitution σ, two-period cycles appear. However, as (22) shows, these endogenous cycles are less likely to appear whenevr the number of patient agents increases. In other words, more patient agents are, the economy becomes closer to the representative-agent one. In this later, the steady state is always a saddle point. Therefore, for larger size of patient agents type, more likely the economy is stable and so the saddle-path range is wider.

4.3 Interpretation

Case (2) of proposition (2) states that, whenever the preferences of the patient agent display Keeping-Up with the Joneses feature, the steady state looses its stability through two-period cycles. Moreover, the emergence of these cycles does not require a sufficiently low elasticity of production factors substitution σ. To understand the added value of this result, we compare it with Becker and Foias (1994).

In Becker and Foias (1994), the economic intuition is based on the assumption that the patient agent's capital income decreases in capital. Such an assumption requires a sufficiently low elasticity of capital-labor substitution. Further, a sufficiently weak elasticity of intertemporal substitution in consumption of the patient agent is necessary for the occurence of cycles and instability. This means that, the elasticity of production factors substitution plays an important role in the appearence of cycles. In addition, only the patient agent's preferences matter, as the dynamic system depends exclusively on his preferences, while the impatient agent's preferences play no role on the dynamics.

In Becker and Foias (1994), suppose that k_t^1 increases. Then, under a sufficiently low σ, the patient agent's income dercreases. This must imply to a reduction in his current consumption c_t^1. However, the patient agent 1 is willing to smooth his consumption c_t^1 since his elasticity of intertemporal substitution is sufficiently low. He thus reduces c_t^1 slightly. As a result, the reduction in income will be absorbed as a drop of k_{t+1}^1. Two-period cycles appear.

Notice that case (1) of the proposition recovers Becker and Foias (1994). Simply, set $\varepsilon_{12}^1 = 0$, then Δ_0 is horizontal while the initial point has the same expression that depends on σ. One can directly observe that, if σ is not sufficiently low, then the steady state is always a saddle point and never changes its stability. However, for very small values of σ, the half-line starts below the point A and therefore, two-period cycles emerge.

In our analysis, we depart from Becker and Foias (1994) as it does not allow for the possibility that the patient agent's capital income decreases in capital. In other words, we do not impose any restriction on the values of the elasticity of capital-labor substitution σ. Moreover, contrary to Becker and Foias (1994), our work captures the heterogeneity by introducing consumption externalities. Notice that, in the dynamic system (14)-(15), the marginal rate of substitution in patient agent's consumption is affected by the impatient's agent consumption. Therefore, our contribution consists in showing that consumption externalities play a crucial role in the appearence of endogeneous cycles (case (2) of the above proposition). To understand this point, the intuition is given as follows.

Suppose that k_t^1 goes up, the income of impatient agent $w_t = f\left(N^1 k_t^1\right) - N^1 k_t^1 f'\left(N^1 k_t^1\right)$ also moves up and so does his consumption $c_t^2 = w_t$. The patient agent's income also increases (since σ is not low). The positive external effect (KUJ, $\varepsilon_{12}^1 > 0$) induces agent 1 to raise his consumption c_t^1. The increase in c_t^1 can exceed the increase in income. Then according to the patient agent's budget constraint (15), next-period capital k_{t+1}^1 decreases. So two-period cycles appear.

5 Conclusion

This paper introduces consumption externalities in a Ramsey model with heterogeneous agents and borrowing constraints. It is shown that low elasticity of production factors substitution is no longer needed in order for endogenous cycles to emerge. Instead, only the consumption externalities that play a crucial role in the appearence of cycles. In particular, under Keeping-Up with the Joneses preferences, two-period cycles appear.

References

[1] Abel, A. (1990). "Assets Prices under Habit Formation and Catching up with the Joneses," *American Economic Review* 80, No. 2, pp. 38-42.

[2] Alonso-Carrera J., J. Caballé and X. Raurich (2007). "Can Consumption Spillovers be a Source of Equilibrium Indeterminacy?" CAMA Working Paper Series, No. 13, The Australian National University.

[3] Becker R. and C. Foias (1994). "The Local Bifurcation of Ramsey Equilibrium," *Economic Theory* 4, pp. 719-744.

[4] Becker R. and C. Foias (1987). "A Characterization of Ramsey Equilibrium," *Journal of Economic Theory* 41, pp. 173-184.

[5] Becker R. (1980). "On the Long-Run Steady State in a Simple Dynamic Model of Equilibrium with Heterogeneous Households," *The Quarterly Journal of Economics* 95, No. 2, pp. 375-382.

[6] Chen B. and M. Hsu (2007). "Admiration is a Source of Indeterminacy," *Economics Letters* 95, PP. 96-103.

[7] Clark A. and A. Oswald (1996). "Satisfaction and Comparison Income," *Journal of Public Economics* 61, pp. 359-381.

[8] Cole, H., G. Mailath and A. Postlewaite (1995). "Incorporating Concern for Relative Wealth into Economic Models," *Federal Reserve Bank of Minneapolis Quarterly Review* 19, No. 3, pp. 12-21.

[9] Cole H., G. Mailath and A. Postlewaite (1992). "Social Norms, Savings Behavior and Growth," *The Journal of Political Economy* 100, No. 6, pp. 1092-1125.

[10] Dupor B. and Liu, W.F (2003). "Jealousy and Equilibrium Overconsumption," *American Economic Review* 93, No. 1, pp. 423-428.

[11] Easterlin R. (1995). "Will Raising the Incomes of all increase the Happiness of all?" *Journal of Economic Behavior and Organisation* 27, pp. 35-47.

[12] Frank R. (1985). "Are Positional Externalities Different from other Externalities?"

[13] Gali J. (1994). "Keeping up with the Joneses: Consumption Externalities, Portfolio Choice, and Asset Prices." *Journal of Money, Credit and Banking* 26, No. 1, pp. 1-8.

[14] Garcia-Penalosa, C. and S. Turnovsky (2008). "Consumption Externalities: A Representative Consumer Model When Agents are Heterogeneous." *Economic Theory* 37, No. 3, pp. 439-467.

[15] Grandmont J.M., P. Pintus and R. de Vilder (1998). "Capital-Labor Substitution and Competitive Nonlinear Endogenous Business Cycles," *Journal of Economic Theory* 80, pp. 14-59.

[16] Liu W. and J. Turnovsky Stephen (2005). "Consumption Externalities, Production Externalities and long-run macroeconomic efficiency." *Journal of Public Economics* 89, pp 1097-1129.

[17] Ljungqvist L. and H. Uhlig (2000) "Tax Policy and Aggregate Demand Management under Catching up with the Joneses," *American Economic Review* 90, pp. 356-366.

[18] Oswald A. (1997). "Happiness and Economic Performance," *The Economic Journal* 107, No. 445, pp. 1815-1831.

[19] Smith A. (1759). *The Theory of Moral Sentiments*. London: A. Millar.

[20] Veblen T. (1899). *The Theory of the Leisure Class*. Boston: Houghton Mifflin.