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CENTRE D'ETUDE DES POLITIQUES ECONOMIQUES DE L'UNIVERSITE D'EVRY

**Search Frictions, Real Wage Rigidities and the Optimal Design
of Unemployment Insurance : A Study in a DSGE
Framework**

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09 – 03R

Search frictions, real wage rigidities and the optimal design of unemployment insurance: a study in a DSGE framework

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June 2008

Revised July 2009

Abstract

In this paper, we feature the optimal unemployment benefits financing scheme when the economy is subject to labor market failures characterized by search frictions and wages rigidity. We show how policy instruments should interact with labor market imperfections. The US unemployment insurance financing is such that firms are taxed in proportion of their layoffs. The question of the optimal tax schedule naturally arises. Using the DSGE methodology, we investigate the optimal design of the unemployment benefits financing scheme. The welfare gains and the stabilizing effects of this policy are evaluated. If unemployment benefits and layoff taxes are chosen in an appropriate way, welfare and labor market performances can be improved. Search externalities and wage rigidities cause sizeable welfare losses and influence the optimal design of the financing scheme. Without wage rigidities the efficient layoff tax corresponds to the expected fiscal cost of an unemployed worker. Conversely, when wages are rigid, the cost firms should support is much higher.

Keys-words : DSGE models, search and matching friction, layoff tax, wage rigidities.

JEL Classification : E61; E65, J63, J65.

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³We are grateful to François Langot for helpful comments on a previous version of this paper.

1 Introduction

What is the optimal level of unemployment benefits and how should they be financed? This question is often discussed without taking into consideration labor market rigidities nor the potential role of labor market institutions for short-run stabilization, leaving the job to macroeconomic policy, and especially monetary and budget policy. Indeed, labor market is characterized by search frictions and wage rigidities which distort agents jobs acceptance behavior and firms jobs posting. They may generate inefficiencies and affect labor market performances as well as the social welfare. Furthermore, they influence the response of macroeconomic variables to aggregate shocks and can magnify fluctuation costs. The question of using labor market policy to reduce fluctuation costs and to offset labor market failures naturally arise. Taking inspiration from the US unemployment insurance system, we wonder if firms should be taxed in proportion of their layoffs to finance the cost incurred by the unemployment benefit fund. This paper investigates the optimal design of the unemployment benefits financing scheme in a DSGE framework.

While it is often argued that labor market institutions can affect long-run labor market performances, they have received little attention in the field of short-run stabilization. The strong intensity of business cycles in the US and the high volatility of unemployment and vacancies may mean it is quite relevant to assert the usefulness of stabilization policies based on the search for an optimal design of the unemployment insurance. Moreover, the existence of labor market rigidities gives rise to a complementary motivation. These rigidities can be summed up into two categories. Those which limit quantities adjustment and those which limit price adjustment.

Matching frictions typically represent the first ones. They capture the time-consuming search process and generate congestion externalities. They influence the average duration of unemployment and therefore the fiscal cost associated to a dismissal. The second ones corresponds to real wage rigidities. They have been pointed out by many authors (Hall (2005), Shimer (2005), Christoffel and Linzert (2005)) to solve the unemployment volatility puzzle. They prevent wages from adjusting instantaneously to economic fluctuations. Consequently, shocks translate into quantities such as employment, job creations and job destructions. Furthermore, they capture rigidities coming from the existence of wage norms. They reduce the ability of firms and workers to use taxes and benefits as a threat in the wage bargaining. Following Abbritti and Weber (2008), the nature of adjustments in the US labor market may be linked to its institutions allowing strong quantities adjustments associated to significant real wage rigidity⁴. The idea that matching frictions and wage rigidities could interact arise. These labor market

⁴Abbritti and Weber (2008) estimate the degree of real wage rigidity on OECD countries data. Their estimates suggest that flexible labor market are associated to a high degree of real wage rigidity.

rigidities introduce some inefficiencies and leave a room for policy instruments to reduce inefficiency and stabilize labor market fluctuations.

There is a growing literature about fluctuations stabilization and labor market imperfections. However, most of this literature is centered on the design of the optimal monetary policy (see Christoffel and Linzert (2005), Blanchard and Gali (2005) and Abbritti and Weber (2008)). Zanetti (2007), Joseph, Pierrard, and Sneessens (2004) and Faia (2008) introduce some labor market institutions such as unemployment benefits, firing costs or a minimum wage in a DSGE model and study their implications for the business cycle dynamics. Despite they highlight their non-negligible impact on fluctuations, none of them characterize the optimal design of these institutions.

In the US, the rapid increase of unemployment after bad shocks can be related to the weakness of the employment protection legislation and the extend to which firms can layoff workers at no cost. Intuition suggests that if firms do not pay the entire cost of their dismissals, their incentive to fire is higher. The role of unemployment insurance in magnifying permanent and temporary layoffs have been illustrated by a large body of papers (Feldstein (1976), Topel (1983), Topel (1983) and Card and Levine (1994)). In this system based on the experience rating principle, individual employers' contribution rates are varied on the basis of the firm's history of generating unemployment. Basically, the more dismissal, the higher the firms' contribution to unemployment insurance. Blanchard and Tirole (2008) point out the question of the design of unemployment insurance and its link with employment protection. The respective levels of the layoff tax and of the unemployment benefits may play a key role to achieve an optimal allocation. In this line we explore how unemployment benefits and payroll tax should be designed to reduce fluctuation costs. We compare the properties of an optimal scheme with the current one and discuss what reforms should be engaged to improve social welfare.

To feature optimally the unemployment insurance, we build a DSGE model with search and matching frictions and where job creation and job destruction are both endogenous. It has been widely recognized that as firms bear a small share of the total cost of job destructions (due to imperfect experience rating), the unemployment insurance induces too many layoffs. Moreover the existence of search frictions and wage norms can strongly affect the firm's hiring and firing behavior. Then we wonder how much firms should be taxed in proportion of their layoffs to finance the fiscal cost induced by their redundancies. Using the DSGE methodology we compute the Ramsey allocation and determine the optimal tax schedule. Under this setup we study (i) how the optimal policy can offset labor market imperfections (ii) how it reduces the welfare cost of fluctuations and (iii) how it affects business cycles.

We show that an optimal combination of unemployment benefits and layoff taxes is welfare-enhancing and can improve labor market performances. Wage rigidities and search externalities have both a strong impact on the welfare cost

of fluctuations and on the optimal level of policy instruments. The cost firms should support is higher than the entire burden of the expected fiscal cost of an unemployed worker when wages are rigid. This effect is magnified by search externalities. The optimal policy strongly dampens macroeconomic fluctuations whatever the level of labor market rigidities. It also reduces the persistence of output and unemployment as well as the intensity and the length of the cycle.

The rest of the paper is organized as follows. Section 2 presents the model and the unemployment insurance system. The equilibrium and the optimal policies are defined in section 3. Section 4 is devoted to simulation exercises and section 5 concludes.

2 The economic environment and the model

We build a discrete time DSGE model including a Non-Walrasian labor market and endogenous job destructions in the spirit of Mortensen and Pissarides (1994) and Den Haan, Ramey, and Watson (2000). Following Shimer (2005), we focus on workers flows between employment and unemployment. Workers “out of the labor force” are thus not taken into account and all unemployed workers are unemployment insurance (UI thereafter) eligible. The economy is populated by *ex ante* homogeneous households and firms. There is a continuum of households on the interval $[0, 1]$. Each household consists of a unitary mass of identical infinitely lived workers. Household workers may be employed or unemployed. There are an infinite number of identical firms. There are several jobs per firm and endogenous separations occur because of jobs specific productivity shocks. There are search and matching frictions in the labor market. Wages are determined through a Nash bargaining process. Following Hall (2005), we introduce real wage rigidities through a wage norm constraining wage adjustments. There is no other market failures.

2.1 The labor market

Search process and recruiting activity are costly and time-consuming for both, firms and workers. A firm offers jobs which may either be filled and productive or unfilled and unproductive. To fill its vacant jobs, the firm posts its vacancies and incurs a cost κ per posted vacancy. Workers are *ex ante* identical, they may either be employed or unemployed. Unemployed workers are engaged in a search process. The number of matches M_t is given by the following Cobb-Douglas matching function :

$$M_t = \chi(1 - N_t)^\psi V_t^{1-\psi} \text{ with } \psi \in]0, 1[, \chi > 0 \quad (1)$$

with V_t the vacancies and $1 - N_t$ the unemployed workers. The labor force is normalized to 1, the number of unemployed workers satisfied $U_t = 1 - N_t$. The matching function (1), satisfying the usual assumptions, is increasing, concave and homogenous of degree one. A vacancy is filled with probability $q_t = M_t/V_t$. Let $\theta_t = V_t/(1 - N_t)$ be the labor market tightness, the probability an unemployed worker finds a job is $\theta_t q_t = M_t/(1 - N_t)$. It is useful to rewrite these probabilities as follows :

$$q_t = \chi \left(\frac{1 - N_t}{V_t} \right)^\psi \quad (2)$$

$$\theta_t q_t = \chi \left(\frac{V_t}{1 - N_t} \right)^{1-\psi} \quad (3)$$

At the beginning of each period, separations occur for two reasons. Firstly, they are exogenous separation at rate ρ^x . Secondly, jobs productivity is subject to idiosyncratic shocks *i.i.d.* drawn from a time-invariant distribution $G(\cdot)$ defined on $[0, \bar{\varepsilon}]$. If the job specific productivity component ε_t falls below an endogenous threshold $\underline{\varepsilon}_t$, the job is destroyed and the employment relationship ceases. Endogenous separations occur at rate :

$$\rho_t^n = P(\varepsilon_t < \underline{\varepsilon}_t) = G(\underline{\varepsilon}_t) \quad (4)$$

2.2 The production technology and the employment laws of motion

At each date, a job is characterized by its specific productivity level ε_t drawn from the distribution $G(\cdot)$. The job productivity is also subject to an aggregate productivity shock z_t . The production level is given by :

$$y_t = z_t \varepsilon_t \quad (5)$$

The productivity shock z_t has a mean equal to $z > 0$ and follows the random process :

$$z_t = (1 - \rho_z)z + \rho_z z_{t-1} + \varepsilon_t^z$$

ε_t^z is *iid* and normally distributed, that is $\varepsilon_t^z \sim N(0, \sigma_{\varepsilon^z}^2)$ and ρ_z is the persistence parameter and satisfies $|\rho_z| < 1$.

We now describe the sequence of events and the labor market timing, we mainly follow Zanetti (2007). Employment in period t has two components : new and old workers. New employment relationships are formed through the matching process. Matches formed at period $t - 1$ contribute to period t employment. New

jobs begin with the highest productivity level $\bar{\varepsilon}$, thus, all the new employment relationships are productive (at the first period). Let $N_t^N = M_{t-1}$ denote the new employment relationships. At the beginning of period t , N_{t-1} jobs are inherited from period $t-1$ and $\rho^x N_{t-1}$ jobs are exogenously destroyed. Thereafter, idiosyncratic shocks are drawn and firms observe jobs specific component ε_t . If the specific component is below the threshold $\underline{\varepsilon}_t$, the employment relationship is severed. Otherwise, the employment relationship goes on. A fraction ρ_t^n of the remaining jobs $(1 - \rho^x)N_{t-1}$ is destroyed. Let $n_t^C(\varepsilon)$, with $\varepsilon \in [\underline{\varepsilon}_t, \bar{\varepsilon}]$, denotes the number of continuing productivity ε employment relationships. It satisfies :

$$n_t^C(\varepsilon) = (1 - \rho^x)N_{t-1}G'(\varepsilon) \quad (6)$$

The total number of continuing employment relationships is given by $N_t^C = \int_{\underline{\varepsilon}_t}^{\bar{\varepsilon}} n_t^C(\varepsilon)d\varepsilon = (1 - \rho^x)(1 - \rho_t^n)N_{t-1}$ and the total separation rate is defined as follows :

$$\rho_t = \rho^x + (1 - \rho^x)\rho_t^n \quad (7)$$

Finally, the employment law of motion is described by the following equations :

$$N_{t+1}^N = M_t \quad (8)$$

$$N_t = \int_{\underline{\varepsilon}_t}^{\bar{\varepsilon}} n_t^C(\varepsilon)d\varepsilon + N_t^N \quad (9)$$

$$N_t = (1 - \rho^x)(1 - \rho_t^n)N_{t-1} + N_t^N \quad (10)$$

2.3 The large family program

Each household may be viewed as a large family. There is a perfect risk sharing, family members pool their incomes (labor incomes and unemployment benefits) that are equally redistributed. The expected intertemporal utility of a large family writes :

$$V_t^M = E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{(C_s + (1 - N_t)h)^{1-\sigma}}{1 - \sigma} \quad (11)$$

$\beta \in]0, 1[$ is the discount factor and $\sigma \in]0, 1[\cup]1, \infty[$ is the intertemporal elasticity of substitution. h denotes unemployed workers home production. Family consumption is the sum of the total home production $(1 - N_t)h$ and of the market consumption goods C_t . The dynamic optimization problem consists of choosing a sequence of consumption $\{C_s\}_t^\infty$ maximizing the expected intertemporal utility subject to the budget constraint and a set of equations describing the employment motion. The large family's choice problem takes the following recursive form :

$$\begin{aligned}
V^M(\Theta_t) &= \max_{C_t} \left\{ \frac{(C_t + (1 - N_t)h)^{1-\sigma}}{1 - \sigma} + \beta E_t V^M(\Theta_{t+1}) \right\} & (12) \\
\text{s.t.} \quad & \begin{cases} -C_t + \int_{\underline{\varepsilon}_t}^{\bar{\varepsilon}} n_t^C(\varepsilon) w_t(\varepsilon) d\varepsilon + N_t^N w_t^N(\bar{\varepsilon}) + (1 - N_t)b_t + \Pi_t = 0 & (\lambda_t) \\ -N_t + \int_{\underline{\varepsilon}_t}^{\bar{\varepsilon}} n_t^C(\varepsilon) d\varepsilon + N_t^n = 0 & (\mu_t^1) \\ -N_{t+1}^N + \theta_t q_t (1 - N_t) = 0 & (\mu_t^2) \\ (1 - \rho^x) N_{t-1} G'(\varepsilon) - n_t^C(\varepsilon) = 0 & (\mu_t(\varepsilon)), \forall \varepsilon \in [\underline{\varepsilon}_t, \bar{\varepsilon}] \end{cases}
\end{aligned}$$

with the state vector $\Theta_t = (N_{t-1}, N_t^N; z_t)$. b_t is the unemployment benefit perceived by an unemployed worker and $w_t(\varepsilon)$ denotes the wage associated to the productivity level ε . New jobs begin with the highest productivity level, the associated wage writes $w_t^N(\bar{\varepsilon})$. Finally, the large family receives instantaneous profits for an amount Π_t . The first constraint is the budget constraint and the three other constraints describe the employment motion. The third constraint expresses the fact that the large family takes the job finding probability as given. The consumption optimality condition writes :

$$(C_t + (1 - N_t)h)^{-\sigma} = \lambda_t \quad (13)$$

The envelop conditions can be expressed as follows :

$$\begin{aligned}
V_2^M(\Theta_t) &= \lambda_t w_t^N(\bar{\varepsilon}) - \lambda_t b_t - \lambda_t h - \theta_t q_t \beta E_t V_2^M(\Theta_{t+1}) \\
&+ (1 - \rho^x) \beta E_t \int_{\underline{\varepsilon}_{t+1}}^{\bar{\varepsilon}} \mu_{t+1}(\varepsilon) dG(\varepsilon) & (14)
\end{aligned}$$

$$\begin{aligned}
\mu_t(\varepsilon) &= \lambda_t w_t(\varepsilon) - \lambda_t b_t - \lambda_t h - \theta_t q_t \beta E_t V_2^M(\Theta_{t+1}) \\
&+ (1 - \rho^x) \beta E_t \int_{\underline{\varepsilon}_{t+1}}^{\bar{\varepsilon}} \mu_{t+1}(\varepsilon) dG(\varepsilon) & (15)
\end{aligned}$$

Equations (14) and (15) respectively provide the family's marginal values of a new job and of a continuing job of productivity ε .

2.4 The large firm program

A firm may be viewed as a large firm having many jobs and employing many workers. The expected discount sum of instantaneous profits of the large firm writes :

$$\begin{aligned}
V_t^F &= E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{\lambda_s}{\lambda_t} \left[\int_{\underline{\varepsilon}_s}^{\bar{\varepsilon}} z_s \varepsilon n_s^C(\varepsilon) d\varepsilon + N_s^N z_s \bar{\varepsilon} - \int_{\underline{\varepsilon}_s}^{\bar{\varepsilon}} w_s(\varepsilon) n_s^C(\varepsilon) d\varepsilon \right. \\
&\quad \left. - \kappa V_t - N_s^N w_s^N(\bar{\varepsilon}) - (1 - \rho^x) \rho_s^n N_{s-1} \tau_s^E \right] & (16)
\end{aligned}$$

New jobs are always productive, obviously no separation occurs. Conversely, old jobs do not continue (recall the decision to continue is taken after observing the specific productivity shock) if their specific productivity level is below a threshold $\underline{\varepsilon}_t$. If the large firm terminates an employment relationship, it has to pay a firing tax τ_t^E , this is expressed by the term $(1 - \rho^x)\rho_t^n N_{t-1}\tau_t^E$. The dynamic optimization problem consists of choosing sequences of vacancies, thresholds and the number of continuing employment relationships, that is $\mathbf{C}_t = (V_t, \underline{\varepsilon}_t, \{n_t^C\}_{\varepsilon \in [\underline{\varepsilon}_t, \bar{\varepsilon}]})$, maximizing the expected discount sum of instantaneous profits subject to the constraints describing the employment motion. The large firm problem takes the following recursive form :

$$\begin{aligned}
V^F(\Delta_t) = \max_{\mathbf{C}_t} & \left\{ \int_{\underline{\varepsilon}_t}^{\bar{\varepsilon}} z_t \varepsilon n_t^C(\varepsilon) d\varepsilon + N_t^N z_t \bar{\varepsilon} - \int_{\underline{\varepsilon}_t}^{\bar{\varepsilon}} w_t(\varepsilon) n_t^C(\varepsilon) d\varepsilon - N_t^N w_t^N(\bar{\varepsilon}) - \kappa V_t \right. \\
& \left. - (1 - \rho^x) N_{t-1} \tau_t^E + \tau_t^E \int_{\underline{\varepsilon}_t}^{\bar{\varepsilon}} n_t^C(\varepsilon) d\varepsilon + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} V^F(\Delta_{t+1}) \right\} \quad (17) \\
\text{s.t.} & \begin{cases} -N_t + \int_{\underline{\varepsilon}_t}^{\bar{\varepsilon}} n_t^C(\varepsilon) d\varepsilon + N_t^n = 0 & (\Lambda_t^1) \\ -N_{t+1}^N + q_t V_t = 0 & (\Lambda_t^2) \\ (1 - \rho^x) N_{t-1} G'(\varepsilon) - n_t^C(\varepsilon) = 0 & (\zeta_t(\varepsilon)), \forall \varepsilon \in [\underline{\varepsilon}_t, \bar{\varepsilon}] \end{cases}
\end{aligned}$$

with the state vector $\Delta = (N_{t-1}, N_t^N; z_t)$. The second constraint means that the large firm takes the probability to fill a job as given. The optimality conditions may be written as follows :

$$-\frac{\kappa}{q_t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} V_2^F(\Delta_{t+1}) = 0 \quad (18)$$

$$-V_2^F(\Delta_t) + z_t \bar{\varepsilon} - z_t \underline{\varepsilon}_t - (w_t(\bar{\varepsilon}) - w_t(\underline{\varepsilon}_t)) - \tau_t^E = 0 \quad (19)$$

$$\begin{aligned}
& w_t(\underline{\varepsilon}_t) - z_t \underline{\varepsilon}_t - \tau_t^E - (1 - \rho^x) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\int_{\underline{\varepsilon}_{t+1}}^{\bar{\varepsilon}} (z_{t+1} \varepsilon - z_{t+1} \underline{\varepsilon}_{t+1}) \right. \\
& \left. - (w_{t+1}(\varepsilon) - w_{t+1}(\underline{\varepsilon}_{t+1})) dG(\varepsilon) - \tau_{t+1}^E \right) = 0 \quad (20)
\end{aligned}$$

Equations (18) and (19) provide the employment creation condition whereas equation (20) is the destruction condition. The envelop condition writes :

$$\begin{aligned}
V_2^F(\Delta_t) &= z_t \bar{\varepsilon} - w_t^N(\bar{\varepsilon}) + (1 - \rho^x) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \int_{\underline{\varepsilon}_{t+1}}^{\bar{\varepsilon}} \zeta_{t+1}(\varepsilon) dG(\varepsilon) \\
&- (1 - \rho^x) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \tau_{t+1}^E \quad (21)
\end{aligned}$$

$$\begin{aligned}
\zeta_t(\varepsilon) &= z_t \varepsilon - w_t(\varepsilon) + \tau_t^E + (1 - \rho^x) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \int_{\underline{\varepsilon}_{t+1}}^{\bar{\varepsilon}} \zeta_{t+1}(\varepsilon) dG(\varepsilon) \\
&- (1 - \rho^x) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \tau_{t+1}^E \quad (22)
\end{aligned}$$

Equations (21) and (22) respectively give the large firm's marginal values of a new job and of a continuing job.

2.5 Wage setting mechanism

At equilibrium, filled jobs generate a net return which is the total surplus of the match. The total surplus of a match is equal to the sum of the firm's and family's job marginal values. Furthermore, we have to distinguish two surplus, the surplus of a new job and the surplus of a continuing job, that is :

$$S_t^N(\bar{\varepsilon}) = \frac{V_2^M(\Theta_t)}{\lambda_t} + V_2^F(\Delta_t) \quad (23)$$

$$S_t(\varepsilon_t) = \frac{\mu_t(\varepsilon)}{\lambda_t} + \zeta_t(\varepsilon) \quad (24)$$

The two wages are determined through an individual Nash bargaining process between a worker and a firm who share the total surplus. The outcome of the bargaining process is given by the solution of the following maximization problem :

$$w_t^N(\bar{\varepsilon}) = \arg \max_{w_t^N(\bar{\varepsilon})} \left(\frac{V_2^M(\Theta_t)}{\lambda_t} \right)^{1-\xi} (V_2^F(\Delta_t))^\xi \quad (25)$$

$$w_t(\varepsilon_t) = \arg \max_{w_t(\varepsilon_t)} \left(\frac{\mu_t(\varepsilon)}{\lambda_t} \right)^{1-\xi} (\zeta_t(\varepsilon))^\xi \quad (26)$$

where $\xi \in]0, 1[$ and $1 - \xi$ denote the bargaining power of firms and workers respectively. Using the free entry condition, the optimality conditions of the above problems may be written as follows :

$$\xi \frac{V_2^M(\Theta_t)}{\lambda_t} = (1 - \xi) V_2^F(\Delta_t) \quad (27)$$

$$\xi \frac{\mu_t(\varepsilon)}{\lambda_t} = (1 - \xi) \zeta_t(\varepsilon) \quad (28)$$

Using equations (14), (15), (18), (21) and (22) to substitute values in (27) and (28) by their expression, wages are given by :

$$w_t^N(\bar{\varepsilon}) = (1 - \xi) \left\{ z_t \bar{\varepsilon} + \kappa \theta_t - \beta(1 - \rho^x) E_t \frac{\lambda_{t+1}}{\lambda_t} \tau_{t+1}^E \right\} + \xi(b_t + h) \quad (29)$$

$$w_t(\varepsilon_t) = (1 - \xi) \left\{ z_t \varepsilon_t + \kappa \theta_t + \tau_t^E - \beta(1 - \rho^x) E_t \frac{\lambda_{t+1}}{\lambda_t} \tau_{t+1}^E \right\} + \xi(b_t + h) \quad (30)$$

The structure of the wage equations is the same as in the standard matching theory. It contains the weighted contribution of both parties. Both equations take into account the expected firing costs (τ_t^E). During the bargaining, firms internalize that hiring a worker may be costly if the job is destroyed. The burden of the expected firing costs is thus subtracted from the worker's contribution to firm's output. Equations (29) and (30) differ because of the firing costs. Concerning an old job, firing costs should be paid in case of separation. Each part may use the cost of layoffs as a threat.

Real wage rigidity Following Shimer (2005) and Hall (2005), real wage rigidities are introduced. There exists a wage norm \tilde{w}_t constraining wage adjustment. The real wage paid for a given productivity level job is a weighted average of the Nash bargaining process wage and the wage norm \tilde{w}_t . One has :

$$w_t^N(\bar{\varepsilon}) = \gamma \left[(1 - \xi) \left\{ z_t \bar{\varepsilon} + \kappa \theta_t - \beta(1 - \rho^x) E_t \frac{\lambda_{t+1}}{\lambda_t} \tau_{t+1}^E \right\} + \xi(b_t + h) \right] + (1 - \gamma) \tilde{w}_t \quad (31)$$

$$w_t(\varepsilon_t) = \gamma \left[(1 - \xi) \left\{ z_t \varepsilon_t + \kappa \theta_t + \tau_t^E - \beta(1 - \rho^x) E_t \frac{\lambda_{t+1}}{\lambda_t} \tau_{t+1}^E \right\} + \xi(b_t + h) \right] + (1 - \gamma) \tilde{w}_t \quad (32)$$

with $\gamma \in [0, 1]$. The higher $1 - \gamma$, the higher the real wages are rigid. The wage norm \tilde{w}_t can be defined in different ways. Usually, it is equal to the steady state value of the average wage, that is $\tilde{w}_t = \bar{w}$, or to the past average wage, that is $\tilde{w}_t = \bar{w}_{t-1}$. All the numerical simulations are made taking the wage norm equal to the steady state value of the average wage. It should be stressed that we studied the case with a wage norm equal to the past average wage. As a whole, results do not significantly differ.

2.6 Job creation and job destruction

Job creation and job destruction are determined by equations (18) — (19). Using the wage setting equations (31) and (32), the job creation and job destruction conditions respectively write :

$$-\frac{\kappa}{q_t} + (1 - \gamma(1 - \xi)) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (z_{t+1} \bar{\varepsilon} - z_{t+1} \underline{\varepsilon}_{t+1} - \tau_{t+1}^E) = 0 \quad (33)$$

$$(1 - \gamma(1 - \xi))(z_t \underline{\varepsilon}_t + \tau_t^E) - \gamma(1 - \xi) \kappa \theta_t - (1 - \gamma) \tilde{w}_t - \gamma \xi(b_t + h) + (1 - \gamma(1 - \xi))(1 - \rho^x) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\int_{\underline{\varepsilon}_{t+1}}^{\bar{\varepsilon}} (z_{t+1} \varepsilon - z_{t+1} \underline{\varepsilon}_{t+1}) dG(\varepsilon) - \tau_{t+1}^E \right) = 0 \quad (34)$$

Equation (33) says the expected gain from hiring a new worker is equal to the expected cost of search (which is κ times the average duration of a vacancy $1/q_t$). It defines the relationship between the labor market tightness and the threshold value of idiosyncratic productivity. Equation (34) is the job destruction condition, it teaches us that the critical value of a job productivity depends on the reservation wages and on firing costs. It states that higher firing costs lower the reservation productivity because separations are more costly.

2.7 The unemployment insurance financing

An unemployed worker receives a benefit b_t . Unemployment benefits are financed through a layoff tax and a lump-sum tax paid by the large family. The layoff tax is paid by employers when an endogenous separation occurs. We impose the unemployment benefits may not be financed by debt. The unemployment insurance fund budget constraint is thus balanced every period :

$$\underbrace{(1 - N_t) b_t}_{\text{Unemployment benefits}} = \underbrace{T_t}_{\text{Lump sum tax}} + \underbrace{(1 - \rho^x) \rho_t^n N_{t-1} \tau_t^E}_{\text{Layoff tax}} \quad (35)$$

The sequences followed by T_t , τ_t^E and b_t may be chosen following different ways, provided they satisfy the above budget constraint. Our aim is to evaluate an unemployment benefits financing scheme (calibrated on US data) close to the US labor market institutions and to study its optimality. We have to describe an institutional rule setting taxes and unemployment benefits levels being close to the US system. An unemployed worker receives a benefit b_t equal to a proportion of the average wage \bar{w}_t , that is :

$$b_t = \rho_R \bar{w}_t \quad (36)$$

$\rho_R < 1$ is the average replacement rate. The average wage of the economy \bar{w}_t is given by :

$$\bar{w}_t = \frac{N_t^N}{N_t} w_t^N(\bar{\varepsilon}) + \frac{N_{t-1}}{N_t} (1 - \rho_x) \int_{\bar{\varepsilon}_t}^{\bar{\varepsilon}} w_t(\tilde{\varepsilon}) dG(\tilde{\varepsilon}) \quad (37)$$

Using equations (29) and (30), the above equation may usefully be rewritten as follows :

$$\begin{aligned} N_t \bar{w}_t &= N_t \gamma \xi (b_t + h) + N_t \gamma (1 - \xi) \left\{ \kappa \theta_t - \beta (1 - \rho^x) E_t \frac{\lambda_{t+1}}{\lambda_t} \tau_{t+1}^E \right\} \\ &+ \gamma (1 - \xi) \left\{ N_t^N z_t \bar{\varepsilon} + N_{t-1} (1 - \rho^x) z_t \int_{\bar{\varepsilon}_t}^{\bar{\varepsilon}} \varepsilon dG(\varepsilon) \right\} \\ &+ N_{t-1} (1 - \rho^x) (1 - \rho_t^n) \gamma (1 - \xi) \tau_t^E + N_t (1 - \gamma) \tilde{w}_t \end{aligned} \quad (38)$$

Concerning the setting rule of the layoff tax τ_t^E , the US unemployment insurance is characterized by a system known as experience rating. We adopt a simplified representation based on Cahuc and Malherbet (2004). The layoff tax is proportional to the expected fiscal cost of an unemployed worker Q_{t+1} . Let $e > 0$ be the experience rating index (ERI), the firing tax τ_t^E satisfies :

$$\tau_t^E = eQ_t \quad (39)$$

where

$$Q_t = b_t + \beta E_t \{ \theta_t q_t \times 0 + (1 - \theta_t q(\theta_t)) Q_{t+1} \} \quad (40)$$

The above equation recursively determines the expected cost of an unemployed worker. The layoff tax corresponds to a share of the expected fiscal cost of an unemployed worker paid by the firm. The higher the index e , the higher the firm contribution to the unemployment insurance. If $e = 1$, firms fully take care the expected fiscal cost of an unemployed worker. Despite the complexity of the current US legislation, the above representation allows to approximate in a simple way the US unemployment insurance system. It captures the fact that firms contribute to the fiscal cost they induce and are made responsible of their layoff decisions.

3 The equilibrium and the unemployment benefit financing policies

This section is devoted to the definition of the equilibrium and to the description of the unemployment insurance financing policies. In our economy, the equilibrium is not a Pareto optimum. This come from the real wage rigidity and the fact that, in our benchmark calibration, the Hosios condition is not satisfied. Economic policies allowing to implement the Pareto allocation (first-best) or improving welfare (second-best) are defined. Before defining the equilibrium, it is necessary to explicit the aggregate resources constraint.

3.1 The aggregate resource constraint

The aggregate output Y_t is obtained through the sum of individual productions :

$$Y_t = (1 - \rho_x) N_{t-1} \int_{\underline{\varepsilon}_t}^{\bar{\varepsilon}} z_t \tilde{\varepsilon} dG(\tilde{\varepsilon}) + N_t^N z_t \bar{\varepsilon} \quad (41)$$

The aggregation of the individual profits provides the amount of profits Π_t received by the large family, that is :

$$\Pi_t = Y_t - \bar{w}_t N_t - \kappa V_t - \tau_t^E (1 - \rho^x) \rho_t^n N_{t-1}$$

The above equation together with the large family budget constraint (program (12)) and the government budget constraint (equation 35) gives the aggregate resource constraint :

$$Y_t = C_t + \kappa V_t \quad (42)$$

3.2 Definition of the equilibrium

Our aim is to compare the allocation implied by the institutional unemployment benefits financing scheme (described in section 2.7) to the ones implied by any other tax process. Two definitions of the equilibrium, corresponding to the different financing schemes, need to be given. Let now define the equilibrium for any tax and unemployment benefit process.

Definition 1 *For given lump sum tax rate T_t and firing tax τ_t^E processes, and for a given exogenous stochastic process z_t , the competitive equilibrium is a sequence of prices and quantities $N_t, N_t^N, C_t, V_t, \underline{\varepsilon}_t, \theta_t, \lambda_t, q_t, \bar{w}_t, w_t^N, Y_t, \rho_t^n, M_t$ and b_t satisfying equations (1)-(4), (8),(10), (13), (31), (33)-(35),(38), (41) and (42).*

Finally, if taxes and benefits are set according to equations (36), (39) and (40), the equilibrium definition writes as follows :

Definition 2 (Experience rating system) *For given parameters ρ_R and e and for a given exogenous stochastic process z_t , the competitive equilibrium is a sequence of prices and quantities $N_t, N_t^N, C_t, V_t, \underline{\varepsilon}_t, \theta_t, \lambda_t, q_t, \bar{w}_t, w_t^N, Y_t, \rho_t^n, M_t, b_t, \tau_t^E, T_t$ and Q_t satisfying equations (1)-(4), (8),(10), (13), (31), (33)-(35),(38), (41), (42), (36), (39) and (40).*

3.3 The Pareto allocation

The Pareto allocation should be derived from the central planner's problem. However, it can directly be determined. To do it, consider equations (33) and (34). Suppose the Hosios condition ($\xi = 1 - \psi$) be satisfied and there is no wage rigidities ($\gamma = 1$), and set unemployment benefits and taxes equal to 0. One gets the following creation and destruction conditions :

$$-\frac{\kappa}{1 - \psi} \frac{V_t}{M_t} \lambda_t + \beta E_t \{ \lambda_{t+1} (z_{t+1} \bar{\varepsilon} - z_{t+1} \underline{\varepsilon}_{t+1}) \} = 0 \quad (43)$$

$$\begin{aligned} & \lambda_t (z_t \underline{\varepsilon}_t - h) - \kappa \frac{\psi}{1 - \psi} \frac{V_t}{1 - N_t} \lambda_t \\ & + \beta (1 - \rho_x) E_t \left\{ \lambda_{t+1} \left(\int_{\varepsilon_{t+1}}^{\bar{\varepsilon}} (z_{t+1} \tilde{\varepsilon} - z_{t+1} \underline{\varepsilon}_{t+1}) dG(\tilde{\varepsilon}) \right) \right\} = 0 \end{aligned} \quad (44)$$

We can give the extensive definition of the Pareto allocation (or equivalently the first best), that is :

Definition 3 (The Pareto allocation) *For a given exogenous stochastic process, the Pareto allocation is a sequence of quantities $N_t, N_t^N, C_t, V_t, \underline{\varepsilon}_t, \lambda_t, Y_t, \rho_t^n, M_t$ satisfying equations (1),(4), (8), (10), (13) and (41)-(44).*

3.4 The unemployment insurance policies

In this subsection, we define the two unemployment insurance policies we evaluate. Firstly, we define the *Ramsey allocation*. The government chooses a taxes and benefits sequence maximizing the social welfare subject to a set of constraint defining the equilibrium and the unemployment insurance fund budget constraint. The effects of such a policy on the equilibrium are internalized. We show this policy allows to implement the Pareto allocation. Secondly, we define a *second best allocation*. in which the institutional environment is not modified. We determine the replacement rate ρ_R and the index e maximizing conditional welfare. Finally, using a welfare cost evaluation, we compare the different allocations and evaluate the ability of the second-best allocation to bring the equilibrium closer to the Pareto allocation.

3.4.1 The Ramsey allocation

As shown by equation (35), unemployment benefits may be financed through two ways : an experience rating tax (τ_t^E) and a lump-sum tax (T_t). The lump-sum tax adjusts to equilibrate, at each date, the unemployment benefit fund. If the Hosios condition is not satisfied or if the real wage is rigid, the decentralized equilibrium of the economy without unemployment benefits and taxes is not optimal. Our aim is to determine an optimal unemployment benefits financing scheme and to compare the equilibrium allocation obtained with the Pareto allocation. The Ramsey policy is the taxation policy under commitment maximizing the intertemporal welfare of the representative household.

Definition 4 (The Ramsey allocation) *The Ramsey equilibrium is a sequence of prices, quantities and taxes $N_t, N_t^N, C_t, V_t, \underline{\varepsilon}_t, \theta_t, \lambda_t, q_t, \bar{w}_t, w_t^N, Y_t, \rho_t^n, M_t, b_t, T_t, \tau_t^E$ maximizing the representative agent life-time utility :*

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{(C_{t+j} + (1-h)N_{t+j})^{1-\sigma}}{1-\sigma}$$

subject to the equilibrium conditions (1)-(4), (8),(10), (13), (31), (33)-(35),(38), (41) and (42) and given the exogenous stochastic processes z_t .

3.4.2 The Pareto allocation and the equivalence with the Ramsey allocation

Proposition 1 *The optimal policy (definition 4) allows to implement the Pareto allocation⁵ whatever the wage norm, that is : a simple norm $\tilde{w}_t = \bar{w}$ or the lagged average wage $\tilde{w}_t = \bar{w}_{t-1}$.*

Proof See appendix A.

Proposition 1 provides a simple way to determine the taxes and unemployment benefit processes implementing the Pareto allocation. The equilibrium values of $N_t, N_t^N, C_t, V_t, \underline{\varepsilon}_t, \theta_t, \lambda_t, q_t, Y_t, \rho_t^n$ and M_t are determined using equations (1),(4), (8), (10), (13) and (41)-(44). The exogenous stochastic process being given. The processes followed by the average wage, the taxes and unemployment benefits T_t, τ_t^E and b_t are then easily deduced from equations (38) and (33)-(35). Finally, w_t^N is provided by equations (29).

Corollary 1 *The optimal policy obtained by solving the Ramsey allocation problem is time-consistent.*

Proof See appendix A.

The Ramsey problem which credibly commit to long-run plans and choose the future policies today is equivalent to the policy interventions decided on a period-by-period optimization. In other word, the government has no interest in going back on his words and trying to deviate from the optimal policy rules chosen in the Ramsey problem as time goes by.

Proposition 2 *The Hosios condition ($\xi = 1 - \psi$) no longer achieves efficiency when wages are rigid i.e. $\gamma < 1$.*

Proof Straightforward when comparing (33) and (34) to (43) and (44) and setting $\xi = 1 - \psi$.

Corollary 2 *Tax and unemployment benefits are not optimally equal to zero when $\xi = 1 - \psi$.*

Proof Using equations (33), (34), (43) and (44) taken at the steady state, the optimal value of the layoff tax and the unemployment benefit (τ^{EO} and b^O respectively) are obtained:

$$\tau^{EO} = \frac{\kappa V}{\beta M} \frac{\psi - \gamma(1 - \xi)}{(1 - \gamma(1 - \xi))(1 - \psi)} \quad (45)$$

$$b^O = \frac{1}{\gamma\xi} \left[\kappa V \frac{\psi - \gamma(1 - \xi)}{1 - \psi} \left(\frac{1 - \beta(1 - \rho^x)}{\beta M} + \frac{1}{1 - N} \right) + (1 - \gamma)(h - \bar{w}) \right] \quad (46)$$

⁵Thereafter we use the term “First-best allocation” to name the Ramsey allocation.

It is easy to see that the usual Hosios condition doesn't imply $\tau^E = 0$ and $b = 0$ when $0 < \gamma < 1$. In our economy, there is no reason the equilibrium be a Pareto optimum. This occurs if there is real wage rigidity or, without a wage rigidity, if the Hosios condition is not satisfied, that is if $1 - \psi \neq \xi$.

3.4.3 The Second best allocation

The equilibrium allocation (definition 1) is defined conditionally to the unemployment benefits financing scheme (equations (36), (39) and (40)). This unemployment benefits financing scheme is a proxy of the American system. The key parameters: the replacement rate ρ_R and the index e , are set by the authorities. Thereafter, quantitative evaluations are made using a benchmark calibration based on US data, but there is no reason these two parameters be optimal. Here, we define a second best allocation where ρ_R and e are chosen to maximize the conditional welfare. Given initial conditions N_{-1} and N_0^N and given parameters ρ_R and e , let denote respectively by \tilde{C}_t and \tilde{N}_t the consumption and employment equilibrium allocation. The conditional welfare under the equilibrium allocation writes :

$$\tilde{\mathcal{W}}(\rho_R, e; N_{-1}, N_0^N) = E_0 \sum_{t=0}^{\infty} \beta^t \frac{(\tilde{C}_t + (1 - \tilde{N}_t)h)^{1-\sigma}}{1 - \sigma}$$

Optimal values for ρ_R^* and e^* are obtained solving the following problem :

$$\{\rho_R^*, e^*\} = \arg \max_{\rho_R, e} \tilde{\mathcal{W}}(\rho_R, e; N_{-1}, N_0^N)$$

The second best allocation is given by definition 2, knowing that $\rho_R = \rho_R^*$ and $e = e^*$.

3.4.4 The conditional welfare costs

In order to compare the different alternative allocations with the Ramsey allocation, we compute their welfare costs. We proceed as follows. The economy is initially at the steady state of an alternative allocation. We compute, at this point, the conditional welfare of the alternative allocation and the conditional welfare of the Ramsey allocation. To evaluate the welfare cost, we compute the fraction of consumption stream from an alternative policy to be added to achieve the Ramsey allocation welfare. Let \mathcal{W}_0^* be the conditional welfare under the Ramsey allocation and let C_t^a and N_t^a denote an alternative allocation and \mathcal{W}_0^a the associated welfare level. To evaluate these two conditional welfares, we take the steady state of the alternative allocation as initial conditions (N_{-1} and N_0^N). The welfare cost Ψ is obtained by solving the following equation :

$$\mathcal{W}_0^* = E_0 \sum_{t=0}^{\infty} \beta^t \frac{[(1 + \Psi)(C_t^a + (1 - N_t^a)h)]^{1-\sigma}}{1 - \sigma} \quad (47)$$

Ψ can be written as follows :

$$\Psi = \left(\frac{\mathcal{W}_0^*}{\mathcal{W}_0^a} \right)^{\frac{1}{1-\sigma}} - 1$$

with :

$$\mathcal{W}_0^a = E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t^a + (1 - N_t^a)h)^{1-\sigma}}{1 - \sigma}$$

Ψ is numerically computed using a second order approximation. Following Kim and Kim (2003), a first-order approximation may lead to inaccurate welfare approximations. First-order approximations do not take into account uncertainty effects and significant approximation errors may occur. To avoid the spurious welfare reversals phenomenon underlined by Kim and Kim (2003), we use a perturbation method (Schmitt-Grohé and Uribe (2004)) to compute second-order approximations of the policy rules. This approximation method allows to capture uncertainty effects and avoids spurious welfare reversals.

4 Quantitative evaluation of the model

4.1 Calibrating and solving the model

The benchmark economy is calibrated according to quarterly frequencies over the period 1951Q1-2004Q4. We follow Shimer (2005) to set the US labor market parameters. His approach concerns only transitions between employment and unemployment and starts from a simple measure of the job finding and separation probabilities. There is an unemployment insurance. Baseline parameters are reported in table 1.

We set the discount factor to 0.99, which gives an annual steady state interest rate close to 4%. The risk aversion coefficient σ is set to 2. The aggregate productivity shock follows a first-order autoregressive process : $\log z_{t+1} = \rho_z \log z_t + \varepsilon_{t+1}^z$. ρ_z corresponds to the autocorrelation coefficient; it is equal to 0.95 as in Den Haan, Ramey, and Watson (2000). ε_{t+1}^z is a random variable whose realizations are *i.i.d.* and drawn from a time-invariant Gaussian distribution with mean zero and standard deviation $\sigma_z = 0.007$. The distribution $G(\cdot)$ of idiosyncratic productivity shocks is *i.i.d.* and log-normal with mean zero. Its upper bound is located at 95 percentile of the distribution as in Zanetti (2007).

The probability of being unemployed is 3.51 percent on average in the US. We suppose as in Den Haan, Ramey, and Watson (2000), Zanetti (2007) and Algan (2004) that exogenous separations are two times higher than endogenous ones. Consequently, $\rho^x = 0.0236$ and $\underline{\varepsilon}$ is fixed in such a way that at the steady state $\rho^n \equiv 1/2\rho^x = 0.0118$. We keep the traditional value of 0.5 for the workers bargaining power. Following Shimer's estimations, the elasticity of the matching function with respect to unemployment is 0.7.

The equilibrium unemployment rates U is calibrated to 5.5%. At the steady state, the number of matches must be equal to the number of separations: $M = \rho N$. Following Andolfatto (1996), the rate at which a firm fills a vacancy is 0.9. Therefore, it takes 1 quarter and one week to fill a vacancy. We can deduce the number of vacancies $V = M/q_t$ and the job finding probability of about 0.61. Then, it takes a little bit more than one and a half quarter on average for an unemployed worker to find a job. χ is calculated in such a way that $M = \chi(1 - N)^\psi V^{1-\psi}$. Statistics from the Census Bureau of labor exhibit an average ERI across states and over the period 1988-2007 of about 0.65. According to the OECD, the US net replacement rate is 0.32. The remaining parameters κ and h are only given by solving the system of equations (33), (34) and (37) after eliminating \bar{w} . In this way, the expected cost of a vacant job κ/q represents 6% of the average annual wage, which is broadly consistent with empirical findings. Finally we set σ_ε and the rigidity wage parameter γ to catch up with the observed cyclical properties of labor market. We focus on the ratio of the standard deviation of the labor market tightness on the output standard deviation, that is (σ_θ/σ_Y) . The obtained γ is 0.35 and $\sigma_\varepsilon = 0.14$. Some US business cycle properties are reported on table ??.

Variables	Symbol	Value
Discount factor	β	0.99
Autocorrelation coefficient	ρ_z	0.95
Std. dev. of aggregate shock	σ_z	0.007
Std. dev. of idiosyncratic shock	σ_ε	0.14
Risk aversion coefficient	σ	2
95 percentile upper bound	$\bar{\varepsilon}$	1.2672
Matching elasticity	ψ	0.7
Total separation rate	ρ	0.0351
Exogenous separation rate	ρ^x	0.0236
Endogenous separation rate	ρ^n	0.0118
Worker bargaining power	ξ	0.5
Replacement rate	ρ^R	0.32
Experience rating index	e	0.65
Vacancy cost	κ	0.11
Wage rigidity parameter	γ	0.35

Table 1: **BASELINE PARAMETERS.**

We solve the model with a second order perturbation method (see Schmitt-Grohé and Uribe (2004)). State variables are N_t , N_t^N and z_t . Changing parameters lead up to a new steady state. It is calculated with a Newton algorithm. To evaluate integrals we use Gauss-Chebyshev quadratures with a 100 nodes grid.

4.2 The optimal labor market policy

Labor market frictions induces the non-optimality of the competitive equilibrium. In our model, non-optimality occurs because of search frictions and real wage rigidities. But how do layoff taxes and unemployment benefits work to restore Pareto optimality? What are the welfare gains of optimal policies? Do they significantly improve long-run labor market performances? We answer these questions by inspecting the optimal design of labor market policies and their interactions with labor market imperfections. Simulations are reported in table 2.

	BENCHMARK	1 st BEST ALLOCATION		SECOND BEST
	ECONOMY	Pareto	Ramsey	ALLOCATION
Experience rating index	0.6500	0	1.5790	1.5460
Replacement rate	0.3200	0	0.2429	0.2224
Output	100.00		102.06	102.10
Average wages	100.00		99.69	99.61
Employment	100.00		102.64	102.67
Welfare	100.00		100.46	100.44
Vacancies	100.00		100.32	108.24
Job finding rate	60.23 %		70.93 %	73.24 %
Separation rate	3.51 %		2.36 %	2.36 %
Welfare loss	0.4440		0	0.0184

Table 2: **OPTIMAL LABOR MARKET POLICY (WAGE RIGIDITY $\tilde{w}_t = \bar{w}$ AND $\gamma = 0.35$).** *Output, consumption, employment and welfare have been standardized. e and ρ^R have been recalculated when we compute the Ramsey. Percentage welfare losses are relative to the Ramsey allocation.*

The first-best optimal labor market policy exhibits two main features : (i) an experience rating index that is strongly increased and (ii) an average replacement rate that is roughly thirty percent lower. Output, employment are increased by 2.06% and 2.64% respectively while wages are reduced by 0.31%. The second-best allocation leads to the same effects on aggregate variables except for the number of vacancies which is 7.89% higher than in the Ramsey economy. Furthermore, the job finding rate is 3.25% higher than in the first-best. In the two allocations, the separation rate is strongly reduced. As mentioned by Algan (2004), L’Haridon and Malherbet (2008), Joseph, Pierrard, and Sneessens (2004) and Zanetti (2007) turnover costs introduce a labor hoarding phenomenon. As long as firing is costly, firms prefer continue the relation with a low productivity level than pay for the layoff tax. They cut back the reservation productivity to reduce endogenous separations. The reservation productivity falls up to a point where endogenous separations are close to zero. Then, ins and outs of employment are almost only governed by exogenous separations. The welfare is enhanced by 0.46% compare to the benchmark.

What are the welfare costs of wage rigidities and search externalities?

Let us now investigate how search externalities and real wage rigidities affect the welfare cost. Search externalities are captured by the difference between ψ and $1 - \xi$ ⁶ while wage rigidities are captured by $1 - \gamma$. When the Hosios condition

⁶Note that if $\gamma = 1$ search externalities are in favor of firms if the bargaining power of workers is weak: $1 - \xi < \psi$. Conversely, they are in favor of workers if $1 - \xi > \psi$. Most of the empirical studies find a value of ψ higher than $1 - \xi$.

is satisfied, the only labor market imperfection comes from real wage rigidities. Results are depicted in figure 1.

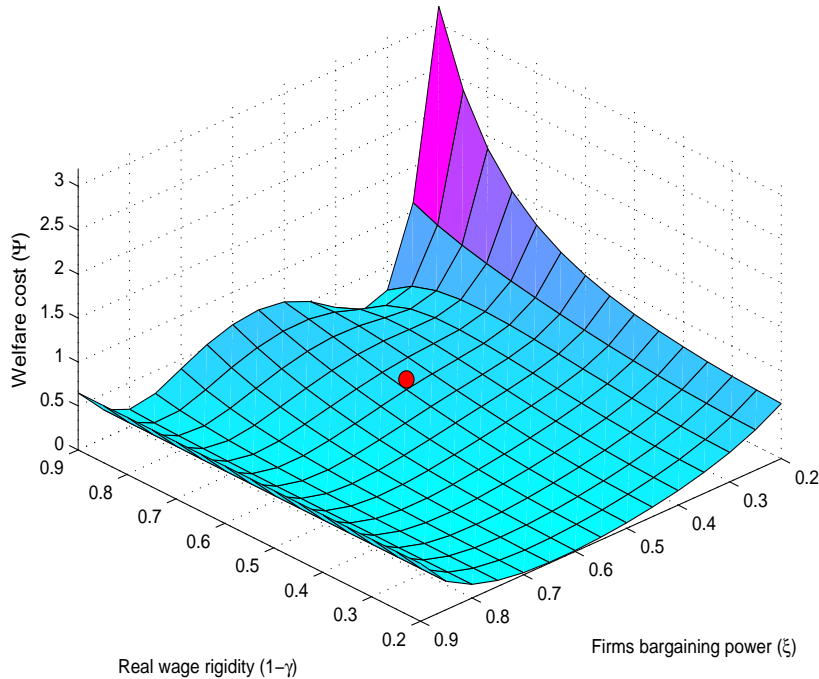


Figure 1: **Welfare cost** (in percentage). The red dot point correspond to the benchmark calibration: $\xi = 0.5$ and $\gamma = 0.35$.

It is shown that the optimal policy can reduce important welfare losses, from 0 to more than 3%. In our benchmark set up ($\xi = 0.5$ and $\gamma = 0.35$), the welfare loss (Ψ) is of about 0.44% relative to the optimal policy (see table 2 and the red dot in figure 1). The alternative policy (second-best) displays a very weak loss (0.0184%). Figure 1 highlights the non-monotone relation between ξ , γ and Ψ . While high wage rigidities lead to high welfare costs⁷ search frictions involve ambiguous effects. When the level of wage rigidities is low search frictions raise the welfare cost. Recall that it occurs when $|\xi - (1 - \psi)|$ is high. In other words, one of the two parties takes advantage of a too high share of the economic rent if $\xi > 1 - \psi$ or if $1 - \xi < \psi$. This is represented by the U-shaped form of figure 1. When the level of wage rigidities is high it becomes quite difficult to identify the consequences of search frictions on welfare costs. Indeed, the surface preserves its U-shaped form and is always increasing when $\xi \rightarrow 1$ or $\xi \rightarrow 0$, but the welfare

⁷Except when the level of wage rigidity is high, the algorithm has some difficulties to converge.

cost also increases when the firm bargaining power lies between 0.5 and 0.7. The reason can be found through the link between search externalities and real wage rigidities. The first ones allow only one agent to take advantage of the bargaining while the second reduce the ability of both, firms and workers, in using taxes and benefits in the wage bargaining. The two labor market rigidities interact and affect differently the wage bargaining. As a consequence, one externality may amplify the other or, on the contrary, dampens its effects on the welfare cost.

How policy instruments should interact with labor market imperfections? The optimal policy described in table 2 highlights the need to use policy instruments to offset labor market imperfections. But equations (45) and (46) stress that there is no evidences that one distortion amplifies or reduces the other. To scrutinize the impact of externalities on the optimal design of the policy we have to map the efficient layoff tax and unemployment benefits as a function of γ and ξ . Results are depicted in figure 2 and 3.

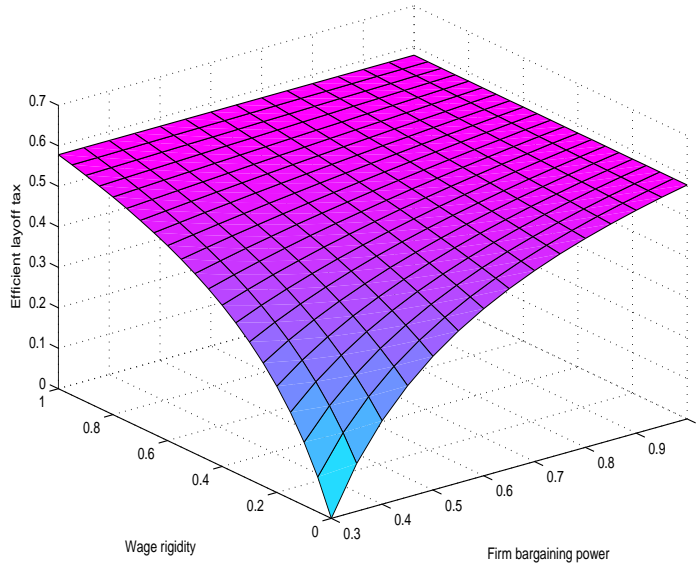


Figure 2: **Efficient layoff tax (τ^{EO})**

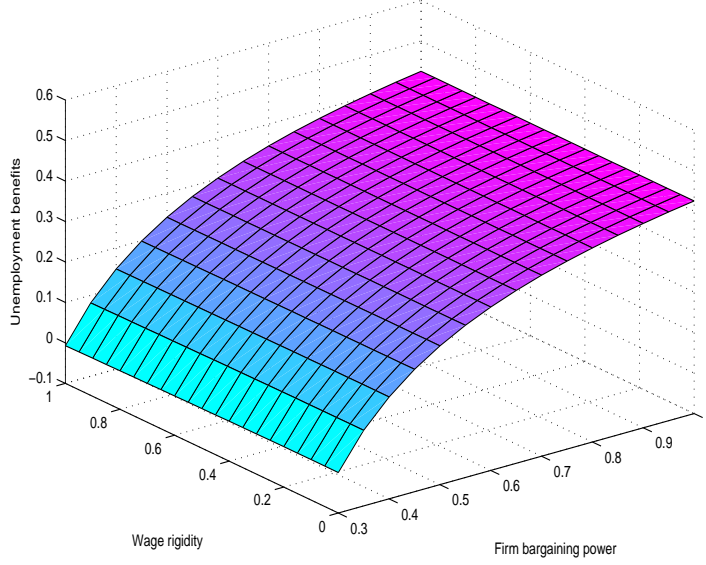


Figure 3: **Efficient unemployment benefits (b^O)**

When $\gamma = 1$ and $\xi = 1 - \psi$ we are in a Pareto world where no policy instruments have to be used. In other cases, the efficient layoff tax is increasing in the level of search externalities and in the level of real wage rigidities. The intuition is as follows. The existence of a wage norm tends to reduce wages dispersion. Other things being equal, the large firm marginal value of a new job is increased (equation (19)), it follows that more vacancies are posted. Furthermore, wages associated to low productivity jobs are higher than under flexible wages. The threshold $\underline{\varepsilon}$ and the job destruction rate are enhanced. To counterbalance this effect the efficient layoff tax should be higher. We conclude that wage rigidities magnify the initial labor market trade externalities. It is worth noting that when one of the two externalities is low the amplification effect is high. In other words when ξ is close to $(1 - \psi)$, an increase in the level of wage rigidities has a strong impact on τ^{EO} .

The efficient level of unemployment benefits is an increasing function of search externalities and a decreasing function of wage rigidities⁸. The interpretation is quite similar and arise from the wage dispersion effect. Other things being equal, the family's marginal value of a new job tends (14) to decline when the wage rigidity increases because new wages are reduced. The outside option becomes relatively higher. Then, to offset this effects the efficient unemployment benefits have to decrease.

⁸However, this effect is very weak and doesn't appears sizeable compare to the variation of with respect to ξ . But b^O is decreasing with γ , whatever the level of ξ .

Policy implementation Welfare may be enhanced through a labor market reform. Our numerical experiments suggest the experience rating index and the replacement rate of the US economy are away from their optimal levels. In terms of welfare cost, the second-best appears to be a good approximation of the first-best. The second-best provides a good way to implement a labor market reform through a change in parameters e and ρ_R allowing to enhance welfare.

4.3 Business cycle analysis

The stabilizing effects of macroeconomic policies is of a great concern among economists. While the vast majority of studies focuses on the optimal design of monetary policy, we show in this section that an optimal unemployment benefits financing scheme is able to smooth labor market fluctuations and to improve welfare. Intuition suggests the layoff tax increases the cost of separation and makes employers responsible for their dismissal decisions. Then, one can expect that it acts as an economic stabilizer, reducing employment fluctuations. We investigate how the optimal policy affects the propagation of shocks. To do it, we analyze the impulse response functions, second order moments, autocorrelations, correlations and the length and the intensity of the cycle. Results are reported in table 3.

	US Economy	Benchmark	1 st best	2 nd best
STANDARD DEVIATIONS				
Output	1.58	1.51	0.93	1.01
Employment	0.63	0.52	0.02	0.13
Average wage	0.43	0.41	0.97	0.66
Unemployment	7.83	9.00	0.62	3.99
Vacancy	8.83	10.09	1.94	12.20
Tightness	16.31	16.82	2.31	14.60
Job finding rate	6.79	5.05	0.69	4.38
Separation rate	3.58	6.05	0.01	0.01
AUTOCORRELATION (1)				
Output	0.845	0.830	0.720	0.767
Unemployment	0.866	0.895	0.820	0.815
Job finding rate	0.801	0.722	0.718	0.720
Separation rate	0.480	0.703	0.709	0.709
CORRELATION				
U_t, V_t	-0.916	-0.580	-0.513	-0.520
Length (quarters)	22.55	18.930	15.886	17.406
Intensity (%)	5.703	4.547	2.761	3.079

Table 3: *Cyclical properties* The model is simulated 500 times over 120 quarters horizon. Results are report in logs as deviations from an HP trend with smoothing parameter 1600 and ignoring the first 1000 observations. All standard deviation are relative to output (except output). Initial rate for $\xi = 0.5$ and $\gamma = .35$

We simulate a one percent negative productivity shock and investigate how labor market policies affect the way variables respond to the shock. As firms anticipate lower future aggregate productivity (recall the shock follows an autoregressive process), the recessionary shock leads to a reduction of the job posting activity and raises the separation rate. The number of hirings declines following the shock while the increase of unemployment (with a one-period lag) drives it above its steady state level. This "echo effect" is weaker when implementing the two optimal policies. The separation spike induced by the negative shock is close to zero while the fall in vacancies is only dampened in the Ramsey economy.

Indeed, in the second-best allocation, firms cut back vacancies during the recession rather than firing. The standard deviation of V_t is 21% higher than in the benchmark. The reason is that layoff taxes induce a labor hoarding phenomenon, reducing greatly separation fluctuations. Furthermore, the steady state level of vacancies is 9.90% higher than in the first-best. Then, the main difference between the two optimal policies lies in how the shock is propagated, *i.e.* through prices or through quantities. In the Ramsey allocation, productivity fluctuations are mainly absorbed by wages adjustments, while in the second-best allocation, the effects of aggregate shocks translate into the job posting activity. The cost of

separation magnifies the initial impact on vacancies while it dampens unemployment fluctuations. In the second-best, the volatility of the labor market tightness and the job finding rate are 10.48% and 35.48% weaker respectively than in the benchmark economy. In the first-best, the stabilization effect are quite stronger.

In the benchmark, output and consumption decline following the shock and follow a hump-shaped adjustment before returning to their equilibrium value. The Ramsey policy reduces the persistence of output by around 13.3%. The length⁹ of the cycle becomes shorter whereas the intensity of the cycle declines by 38.3%. These decreases are a little bit weaker in the second-best allocation.

The impact of the recessionary shock on the financing scheme depends on the sensitivity of two key variables: the average wage (governing the response of unemployment benefits) and unemployment. In the benchmark economy, the unemployment compensation is reduced according to its wage indexation. But the probability of finding a job falls, leaving the overall effect on the expected fiscal cost of an unemployed worker (equation (40)) undetermined. Simulations show that the increase in the average duration of unemployment has a higher impact on the fiscal cost Q_t (measured by the IRF of the layoff tax which is proportional to Q_t) than the decrease of benefits per unemployed worker. Consequently, the layoff tax jumps above its steady state level to cut back on the cost incurred by the unemployment benefit fund. The lump-sum tax decreases following the shock and overtakes its initial value as soon as unemployment increases. Since unemployment is persistent, the fiscal cost of an unemployed worker remains high for a long time.

In the Ramsey allocation, it is shown that taxes jump in the opposite direction. One can explain it by the dampened fluctuations of unemployment and the strong sensitivity of wages (and therefore of the unemployment compensation). As a consequence, unemployment insurance expenditures go down following the (negative) shock. Taxes have to decrease in order to balance the budget. In the second-best allocation and compared to the Ramsey allocation, unemployment benefits respond little to shifts in productivity while the rise of unemployment is stronger. Therefore, following a negative aggregate shock, taxes have to increase to balance the budget.

Except for the vacancies in the second-best, the stabilization effects of the two reforms are sizeable, especially concerning unemployment and the job separation rate. These results confirm the efficiency of the studied policy instruments in reducing the welfare cost of fluctuations and labor market fluctuations.

Robustness of results In this section we check if the two optimal policies display similar results concerning business cycle dynamics, whatever the initial level of labor market rigidities. Indeed, results clearly depends on the initial labor market imperfections because reforms aim at offsetting them. However, there are

⁹See appendix B for a definition of the length and intensity of the cycle.

no clear empirical evidences about the value of the degree of wage rigidity or the firms' bargaining power¹⁰. We consider that the key parameters lie between two values¹¹ and we simulate the model for these different calibrations. Concerning the firms' bargaining power in the US, we assume it lies between 0.4 and 0.75. The benchmark value is taken equal to 0.5, which corresponds to the most commonly admitted value. The wage rigidity parameter (γ) is most of time set to match the volatility of some key variables. New-Keneysian literature dealing with real wage rigidities (Blanchard and Gali (2005), Faia (2008), Krause and Lubik (2006), Christoffel and Linzert (2005)) reports values of the wage rigidity parameter lying between 0.4 and 0.9. The estimated value found by Abbritti and Weber (2008) is equal 0.826 over the period 1970-1999 in the US. Our benchmark value of the wage rigidity parameter $1 - \gamma = 0.65$ is broadly consistent with empirical findings. The lower bound and the upper bound of $1 - \gamma$ are equal to 0.3 and 0.9 respectively.

There are two calibration strategies to check the robustness of our results. We can simply compute the new steady state induced by a parameter change, simulate the model and study the cyclical properties. The second method consists, considering an alternative calibration, to impose some long-run levels be the same than in the benchmark calibration¹². We opt for the first one because the second would impose to change the value of other parameters like the value of leisure h or the cost of a vacancy κ (see section ?? about the model calibration). Furthermore we prefer to change the value of the firms' bargaining power (ξ) rather than the elasticity of the matching function with respect to unemployment (ψ). Indeed, ξ captures institutional features like union density, coordination and the degree of centralization of unions while ψ represents a structural parameter. Results are reported in table 4.3.

¹⁰or the elasticity of the matching function with respect to unemployment.

¹¹Estimations provide a range of admissible values.

¹²In that case, as explained in section ?? about the model calibration, we impose the steady state values of some endogenous variables. We thus reveal the value of parameters for which there is a lack of data

	$\xi = 0.4, \gamma = 0.6$		$\xi = 0.4, \gamma = 0.1$		$\xi = 0.75, \gamma = 0.6$		$\xi = 0.75, \gamma = 0.1$	
	1 st	2 nd	1 st	2 nd	1 st	2 nd	1 st	2 nd
STANDARD DEVIATIONS								
Y_t	0.94	0.96	0.93	1.17	0.93	1.03	0.92	1.11
N_t	0.02	0.07	0.02	0.26	0.02	0.13	0.02	0.22
\bar{w}_t	0.97	0.84	0.97	0.34	0.97	0.66	0.97	0.23
U_t	0.63	2.00	0.63	7.90	0.63	4.08	0.63	7.90
V_t	1.96	6.14	1.97	24.26	1.97	12.43	1.97	23.05
θ_t	2.36	7.34	2.36	29.14	2.36	14.91	2.36	27.93
$\theta_t q_t$	0.70	2.20	0.70	8.74	0.70	0.73	0.70	8.38
ρ_t	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.01
AUTOCORRELATION								
Y_t	0.72	0.74	0.72	0.82	0.72	0.77	0.72	0.80
U_t	0.82	0.82	0.82	0.82	0.81	0.82	0.81	0.78
$\theta_t q_t$	0.73	0.72	0.72	0.72	0.72	0.73	0.72	0.72
ρ_t	0.71	0.71	0.71	0.71	0.71	0.71	0.71	0.72
Length	16.04	16.46	16.11	18.26	15.97	16.82	15.82	17.65
Int.	2.78	2.87	2.78	3.50	2.76	3.01	2.74	3.33

Table 4: *Cyclical properties* The model is simulated 500 times over 120 quarters horizon. Results are report in logs as deviations from an HP trend with smoothing parameter 1600 and ignoring the first 1000 observations. All standard deviation are relative to output (except output). 1st stands for the first-best allocation (Ramsey) and 2st for the second-best allocation.

The main result is that stabilizing effects induced by the Ramsey policy are unchanged. Whatever the initial level of labor market imperfections, the first-best allocation strongly reduces the volatility of output and of labor market variables. The decrease in the persistence of output, unemployment and worker flows are exactly the same as in the initial calibration ($\xi = 0.5$ and $\gamma = 0.35$). Furthermore, no substantial changes can be observed concerning the length and the intensity of the cycles. The volatility of wages remains high, highlighting their important role in the propagation of shocks.

The second-best allocation displays weaker stabilization effects than the first-best, which is consistent with our previous findings. The standard deviation of vacancies and tightness in column 5 and 9 of table 3 are higher than those found in the benchmark economy ($\xi = 0.5$ and $\gamma = 0.35$). However, recall that when we choose an alternative calibration, the steady state of the benchmark, under which the optimal policy is calculated, is changed. The new benchmark, computed after setting $\xi = 0.4$ and $\gamma = 0.1$ (or $\xi = 0.75$ and $\gamma = 0.1$), implies a higher volatility of V and θ than in the second-best with this parametrization. Then, the second-best allocation always reduces labor market fluctuations. The decreases in the volatility of output, unemployment, vacancies and tightness are in the same order of magnitude as that found in the initial calibration. The persistence of output,

unemployment, the job finding rate and the separation rate are roughly similar. The impact of the second-best policy on the intensity and the average duration of the cycle is a little bit lower when the degree of wage rigidity is high ($\gamma \rightarrow 0$).

5 Conclusion

In this paper, the properties of an optimal unemployment benefits financing scheme are studied using the DSGE methodology. We wonder if firms should be taxed in proportion of their layoffs and if the layoff tax should correspond to a part or all of the cost incurred by the unemployment insurance. In particular, we investigate how the optimal policy can offset labor market failures generated by search frictions and wages rigidities. Furthermore, in a business cycle perspective, we evaluate the stabilizing properties of these policies and measure how they improve welfare.

We find that it is optimal that the employers should be fully responsible for their dismissal decisions. When an employer lays a worker off he should pay the entire expected fiscal cost of an unemployed worker. This result is magnified in the presence of wage rigidities. It is then optimal that the firms pay an amount greater than the fiscal cost of an unemployed worker. In each case, optimality imposes a replacement rate reduced by around one third. Our results highlight the sensitivity of the optimal design of the unemployment insurance to real wage rigidity. Significant real wage rigidities magnify the fluctuations of employment and unemployment over the cycle and the costs of fluctuations. An optimal design of the unemployment insurance allows to reduce significantly welfare cost fluctuations. As a whole, an optimal combination of unemployment benefits and layoff taxes is welfare-enhancing and improves labor market performances. Finally, we provide a way to implement a reform of the unemployment insurance. Numerical experiments suggest the layoff tax and the replacement rate of the US economy are away from their optimal values. An appropriate modification of their levels should bring the economy closer to optimum.

Our study may be extended in several directions. First, along the paper we assume that incomes pooled by family members are equally redistributed. The assumption of a perfect risk sharing limits the results. Indeed, considering individual risks would magnify welfare costs and give to the policy instruments a more important role. An extension in an heterogeneous agents framework would be worthwhile. Second, our study shows that the design of the unemployment insurance can contribute to fluctuation stabilization, which completes results obtained in the optimal monetary policy literature. In a more general model, other sources of distortions, such as imperfect competition and nominal rigidities, may be introduced. A greater number of distortions would require to use other policy instruments. In this case, the joint optimal design of unemployment insurance and monetary policy may be studied.

A Proof of result 1

Let's write the lagrangian of the Ramsey allocation problem taken at a date t :

$$\begin{aligned}
\mathcal{L}_t = & E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{(C_{t+j} + (1 - N_{t+j})h)^{1-\sigma}}{1-\sigma} \right. \\
& + \Omega_{t+j}^1 \kappa \frac{V_{t+j}}{M_{t+j}} - \Omega_{t+j-1}^1 (1 - \gamma(1 - \xi)) \lambda_{t+j} (z_{t+j} \bar{\varepsilon} - z_{t+j} \underline{\varepsilon}_{t+j} - \tau_{t+j}^e) \\
& + \Omega_{t+j}^2 \left(\gamma(1 - \xi) \kappa \lambda_{t+j} \frac{V_{t+j}}{1 - N_{t+j}} + \gamma \xi (b_{t+j} + h) \lambda_{t+j} - (1 - \gamma(1 - \xi)) (z_{t+j} \underline{\varepsilon}_{t+j} + \tau_{t+j}^e) \lambda_{t+j} \right. \\
& + (1 - \gamma) \tilde{w}_t \lambda_{t+j} - \Omega_{t+j-1}^2 (1 - \gamma(1 - \xi)) (1 - \rho_x) \xi \lambda_{t+j} \left(z_{t+j} \int_{\underline{\varepsilon}_{t+j}}^{\bar{\varepsilon}} (\varepsilon - \underline{\varepsilon}_{t+j}) dG(\varepsilon) - \tau_{t+j}^e \right) \\
& + \Omega_{t+j}^3 \left(-N_{t+j} \bar{w}_{t+j} \lambda_{t+j} + N_{t+j} \gamma \xi (b_{t+j} + h) \lambda_{t+j} \right. \\
& + N_{t+j} \gamma (1 - \xi) \kappa \frac{V_{t+j}}{1 - N_{t+j}} \lambda_{t+j} + \gamma (1 - \xi) \left\{ N_{t+j}^N z_{t+j} \bar{\varepsilon} + N_{t+1+j} (1 - \rho^x) z_{t+j} \int_{\underline{\varepsilon}_{t+j}}^{\bar{\varepsilon}} \varepsilon dG(\varepsilon) \right\} \lambda_{t+j} \\
& + N_{t-1+j} (1 - \rho^x) (1 - \rho_{t+j}^n) \gamma (1 - \xi) \tau_{t+j}^E \lambda_{t+j} \\
& + (1 - \gamma) ((1 - \rho^x) (1 - \rho_{t+j}^n) N_{t-1+j} + N_{t+j}^N) \tilde{w}_t \lambda_{t+j} \left. \right) \\
& - \Omega_{t+j-1}^3 N_{t-1+j} \gamma (1 - \xi) (1 - \rho^x) \lambda_{t+j} \tau_{t+j}^E \\
& + \Lambda_{t+j}^1 (Y_{t+j} - C_{t+j} - \kappa V_{t+j}) \\
& + \Lambda_{t+j}^2 (-N_{t+1+j}^N + M_{t+j}) + \Lambda_{t+j}^3 (-N_{t+j} + (1 - \rho_x) (1 - \rho_t^n) N_{t-1+j} + N_{t+j}^N) \\
& + \Lambda_{t+j}^4 (-M_{t+j} + \chi (1 - N_{t+j})^\varphi V_{t+j}^{1-\varphi}) + \Lambda_{t+j}^5 \left(-\rho_{t+j}^n + \int_0^{\underline{\varepsilon}_{t+j}} dG(\varepsilon) \right) \\
& + \Lambda_{t+j}^6 \left(-Y_{t+j} + (1 - \rho_x) N_{t-1+j} z_{t+j} \int_{\underline{\varepsilon}_{t+j}}^{\bar{\varepsilon}} \varepsilon dG(\varepsilon) + N_{t+j}^N z_{t+j} \bar{\varepsilon} \right) \\
& + \Lambda_{t+j}^7 ((C_{t+j} + (1 - N_{t+j})h)^{-\sigma} - \lambda_{t+j}) + \Lambda_{t+j}^8 (-(1 - N_{t+j})b_{t+j} + T_{t+j} + (1 - \rho_x) \rho_{t+j}^n N_{t-1+j} \tau_{t+j}^E) \left. \right]
\end{aligned}$$

As it is usual in this class of problem, the multipliers associated to the forward dynamic constraints have initial values equal to 0, that is $\Omega_{-1}^1 = \Omega_{-1}^2 = \Omega_{-1}^3 = 0$.

This optimization problem has potentially a time inconsistent solution. This occurs because of the forward dynamic constraints.

To begin, consider the initial period, that is $t = 0$. The optimality conditions with respect to T_0 , b_0 and τ_0^E write :

$$\frac{\partial \mathcal{L}_0}{\partial T_0} = \Lambda_0^8 = 0 \quad (48)$$

$$\frac{\partial \mathcal{L}_0}{\partial b_0} = \Omega_0^2 \gamma \xi \lambda_0 - \Lambda_0^8 (1 - N_0) + \Omega_0^3 N_0 \gamma \xi \lambda_0 = 0 \quad (49)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_0}{\partial \tau_0^E} &= -\Omega_0^2 (1 - \gamma(1 - \xi)) \lambda_0 + \Lambda_0^8 (1 - \rho^x) \rho_0^n N_{-1} \\ &+ \Omega_0^3 N_{-1} (1 - \rho^x) (1 - \rho_0^n) \gamma (1 - \xi) \lambda_0 = 0 \end{aligned} \quad (50)$$

Consider now the case with $t \geq 1$, the optimality conditions with respect to T_t , b_t and τ_t^E write :

$$\frac{\partial \mathcal{L}_t}{\partial T_t} = \Lambda_t^8 = 0 \quad (51)$$

$$\frac{\partial \mathcal{L}_t}{\partial b_t} = \Omega_t^2 \gamma \xi \lambda_t - \Lambda_t^8 (1 - N_t) + \Omega_t^3 N_t \gamma \xi \lambda_t = 0 \quad (52)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial \tau_t^E} &= -\Omega_{t-1}^1 (1 - \gamma(1 - \xi)) \lambda_t - \Omega_t^2 (1 - \gamma(1 - \xi)) \lambda_t \\ &- \Omega_{t-1}^2 (1 - \gamma(1 - \xi)) (1 - \rho^x) \xi \lambda_t \\ &+ \Omega_t^3 N_{t-1} (1 - \rho^x) (1 - \rho_t^n) \gamma (1 - \xi) \lambda_t \\ &- \Omega_{t-1}^3 N_{t-1} \gamma (1 - \xi) (1 - \rho^x) \lambda_t + \Lambda_t^8 (1 - \rho^x) \rho_t^n N_{t-1} \end{aligned} \quad (53)$$

Consider now the first case of wage rigidity with $\tilde{w}_t = \bar{w}$, \bar{w} being the steady state real wage. The optimality condition with respect to \bar{w}_t , for all $t \geq 0$ writes :

$$\frac{\partial \mathcal{L}_t}{\partial \bar{w}_t} = -\Omega_t^3 N_t \lambda_t = 0 \quad (54)$$

It follows from equations (48) — (54) that $\Lambda_t^8 = \Omega_t^2 = \Omega_t^3 = 0$ and $\Omega_t^1 = 0$, $\forall t \geq 0$.

Consider now the second case with $\tilde{w}_t = \bar{w}_{t-1}$. The optimality condition with respect to \bar{w}_t writes :

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial \bar{w}_t} &= \beta E_t \Omega_{t+1}^2 (1 - \gamma) \lambda_{t+1} - \Omega_t^3 N_t \lambda_t \\ &+ \beta E_t \Omega_{t+1}^3 (1 - \gamma) ((1 - \rho^x) (1 - \rho_t^n) N_t + N_{t+1}^N) \lambda_{t+1} \\ &= \beta E_t \Omega_{t+1}^2 (1 - \gamma) \lambda_{t+1} - \Omega_t^3 N_t \lambda_t + \beta E_t \Omega_{t+1}^3 (1 - \gamma) N_{t+1} \lambda_{t+1} = 0 \end{aligned} \quad (55)$$

Knowing that $\Lambda_t^8 = 0$ $t \geq 0$, conditions (52) and (55) imply that $\Omega_t^3 N_t \lambda_t = 0$. It follows that $\Omega_t^1 = \Omega_t^2 = 0$ $\forall t \geq 0$.

Consequently, the forward dynamic constraints vanish in all cases and the optimization problem is thus time consistent.

The others optimality conditions may then be written as follows :

$$\frac{\partial \mathcal{L}}{\partial C_t} = (C_t + (1 - N_t)h)^{-\sigma} - \Lambda_t^1 - \Lambda_t^7 \sigma (C_t + (1 - N_t)h)^{-\sigma-1} = 0 \quad (56)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = -\Lambda_t^7 = 0 \quad (57)$$

$$\frac{\partial \mathcal{L}}{\partial Y_t} = \Lambda_t^1 - \Lambda_t^6 = 0 \quad (58)$$

$$\frac{\partial \mathcal{L}}{\partial V_t} = -\Lambda_t^1 \kappa + \Lambda_t^4 \chi (1 - N_t)^\varphi (1 - \varphi) V_t^{-\varphi} = 0 \quad (59)$$

$$\frac{\partial \mathcal{L}}{\partial \rho_t^n} = -\Lambda_t^1 F (1 - \rho_x) N_{t-1} - \Lambda_t^3 (1 - \rho_x) N_{t-1} - \Lambda_t^5 = 0 \quad (60)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial N_t} &= -\beta E_t \Lambda_{t+1}^1 F (1 - \rho_x) \rho_{t+1}^n - \Lambda_t^3 + \beta E_t \Lambda_{t+1}^3 (1 - \rho_x) (1 - \rho_{t+1}^n) \\ &\quad - \Lambda_t^4 \chi \varphi (1 - N_t)^{\varphi-1} V_t^{1-\varphi} + \beta E_t \Lambda_{t+1}^6 (1 - \rho_x) z_{t+1} \int_{\underline{\varepsilon}_{t+1}}^{\bar{\varepsilon}} \varepsilon dG(\varepsilon) \\ &\quad + \Lambda_t^7 \sigma h (C_t + (1 - N_t)h)^{-\sigma-1} - h (C_t + (1 - N_t)h)^{-\sigma} = 0 \end{aligned} \quad (61)$$

$$\frac{\partial \mathcal{L}}{\partial N_{t+1}^N} = -\Lambda_t^2 + \beta E_t \Lambda_{t+1}^3 + \beta E_t \Lambda_{t+1}^6 z_{t+1} \bar{\varepsilon} = 0 \quad (62)$$

$$\frac{\partial \mathcal{L}}{\partial M_t} = \Lambda_t^2 - \Lambda_t^4 = 0 \quad (63)$$

$$\frac{\partial \mathcal{L}}{\partial \underline{\varepsilon}_t} = \Lambda_t^5 - \Lambda_t^6 (1 - \rho_x) N_{t-1} z_t \underline{\varepsilon}_t = 0 \quad (64)$$

The system formed by equations (56) — (64) seems untractable. However, it can easily be showed that it reduces to the equations system defining the Pareto allocation.

It immediately follows from equation (57), (56) and (58) that $\Lambda_t^7 = 0$, $\Lambda_t^1 = (C_t + (1 - N_t)h)^{-\sigma} = \lambda_t$ and $\Lambda_t^6 = \lambda_t$.

>From equations (59), (60), (63) and (64), is is easily deduced that :

$$\begin{aligned} \Lambda_t^4 &= \frac{\kappa}{1 - \varphi} \frac{V_t}{M_t} \lambda_t \\ \Lambda_t^5 &= (1 - \rho_x) N_{t-1} \lambda_t \underline{\varepsilon}_t \\ \Lambda_t^3 &= -\lambda_t F - \lambda_t z_t \underline{\varepsilon}_t \\ \Lambda_t^2 &= \Lambda_t^4 \end{aligned}$$

Substituting in equations (61) and (62) provides :

$$\begin{aligned}
& -\frac{\kappa}{1-\varphi} \frac{V_t}{M_t} \lambda_t - \beta E_t \{ \lambda_{t+1} (z_{t+1}(\bar{\varepsilon} - \underline{\varepsilon}_t) - F) \} = 0 \\
& \lambda_t (z_t \underline{\varepsilon}_t + F - h) - \kappa \frac{\varphi}{1-\varphi} \frac{V_t}{1-N_t} \lambda_t \\
& + \beta (1 - \rho_x) E_t \left\{ \lambda_{t+1} \left(z_{t+1} \int_{\underline{\varepsilon}_{t+1}}^{\bar{\varepsilon}} (\varepsilon - \underline{\varepsilon}_{t+1}) dG(\varepsilon) - F \right) \right\} = 0
\end{aligned}$$

The above equations are exactly equations (44) and (43). We thus have verified that the Ramsey allocation corresponds to the Pareto one.

B Length and intensity of the cycle

To compute the length and the intensity of the cycle we determine turning points using the Bry and Boschan (1971) algorithm also described in Harding and Pagan (2004). We apply the following conditions on the detrended output to extract peaks: $\Delta y_t > 0$, $\Delta^2 y_t > 0$, $\Delta y_{t+1} < 0$ and $\Delta^2 y_{t+2} < 0$ where y_t stands for the HP-filtered output, $\Delta y_t = y_t - y_{t-1}$ and $\Delta^2 y_{t+2} = y_{t+2} - y_t$. The following conditions apply for troughs: $\Delta y_t < 0$, $\Delta^2 y_t < 0$, $\Delta y_{t+1} > 0$ and $\Delta^2 y_{t+2} > 0$. We use a censoring rule ensuring that peaks and troughs alternate and turning points that are too close to the trend are eliminated. The last condition is obtained using: $y_t > \sigma_Y/2$. While this condition appears quite arbitrary, it allows to detect all turning points of the US economy over the period 1948-2007 according to the NBER definition. The length of the cycle corresponds to the average duration from peak to peak. The intensity of the cycle is equal the growth rate from a trough to a peak: $\log y_{peak} - \log y_{trough}$. The intensity and the length are a little bit lower in the model compared to the data. It is possible to modify the censoring rule ($y_t > \sigma_Y/1.5$ for example) to obtain the same length as in the data. However, for the sake of comparison we prefer to keep the initial rule ($\sigma/2$).

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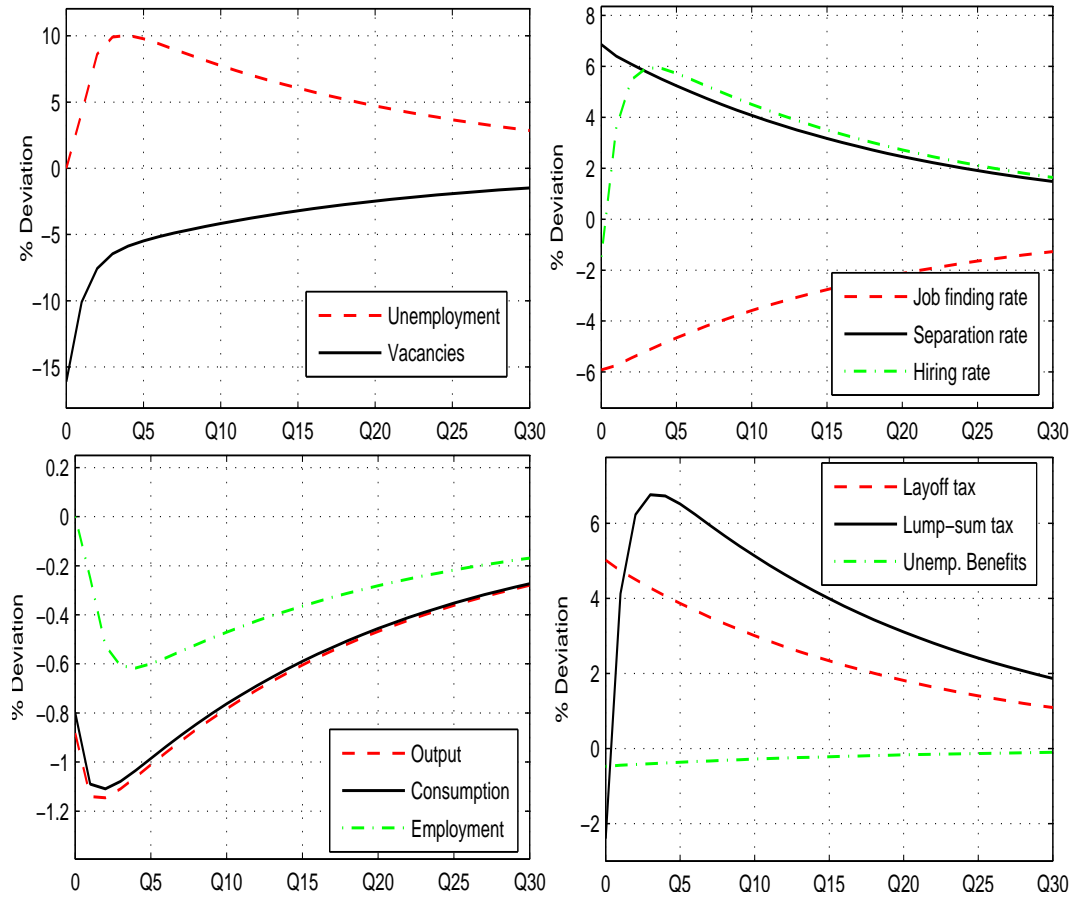
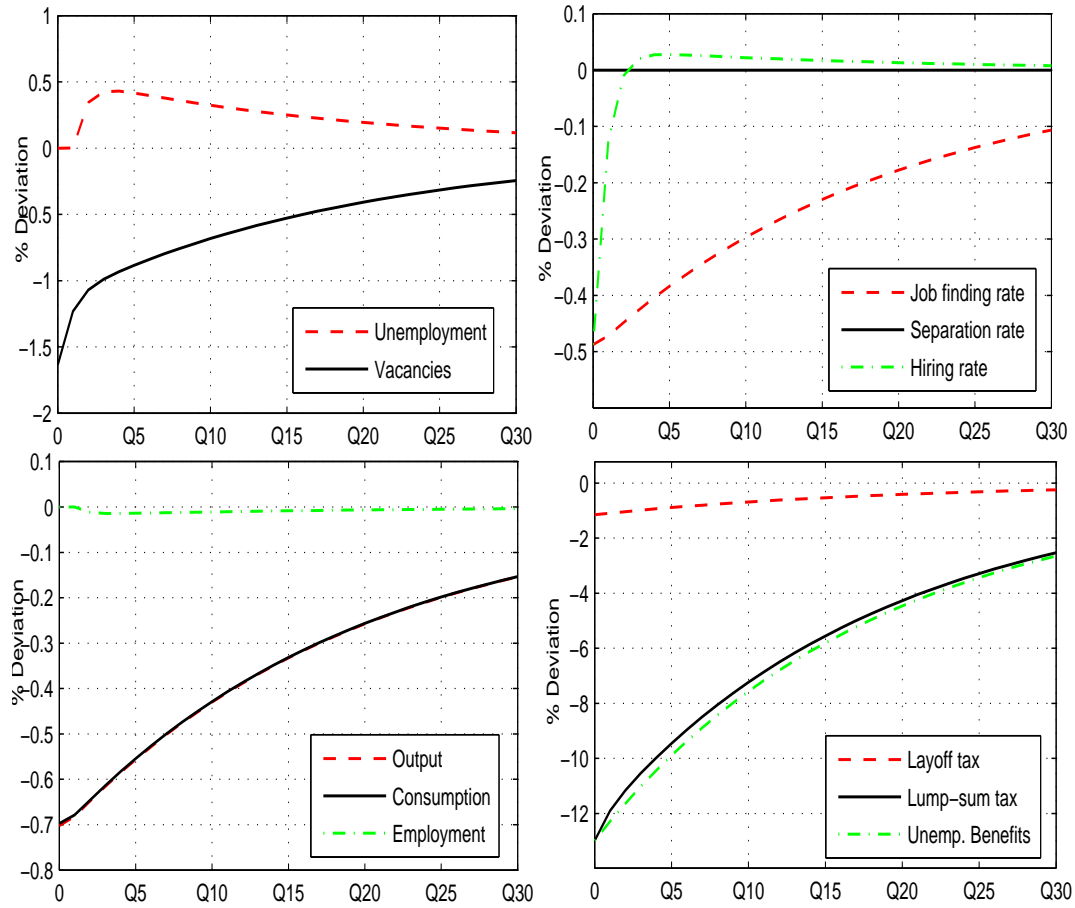


Figure 4: *Impulse response functions - Benchmark economy.* We simulate a one percent negative aggregate productivity shock.



Impulse response functions - Ramsey economy. We simulate a one percent negative aggregate productivity shock.

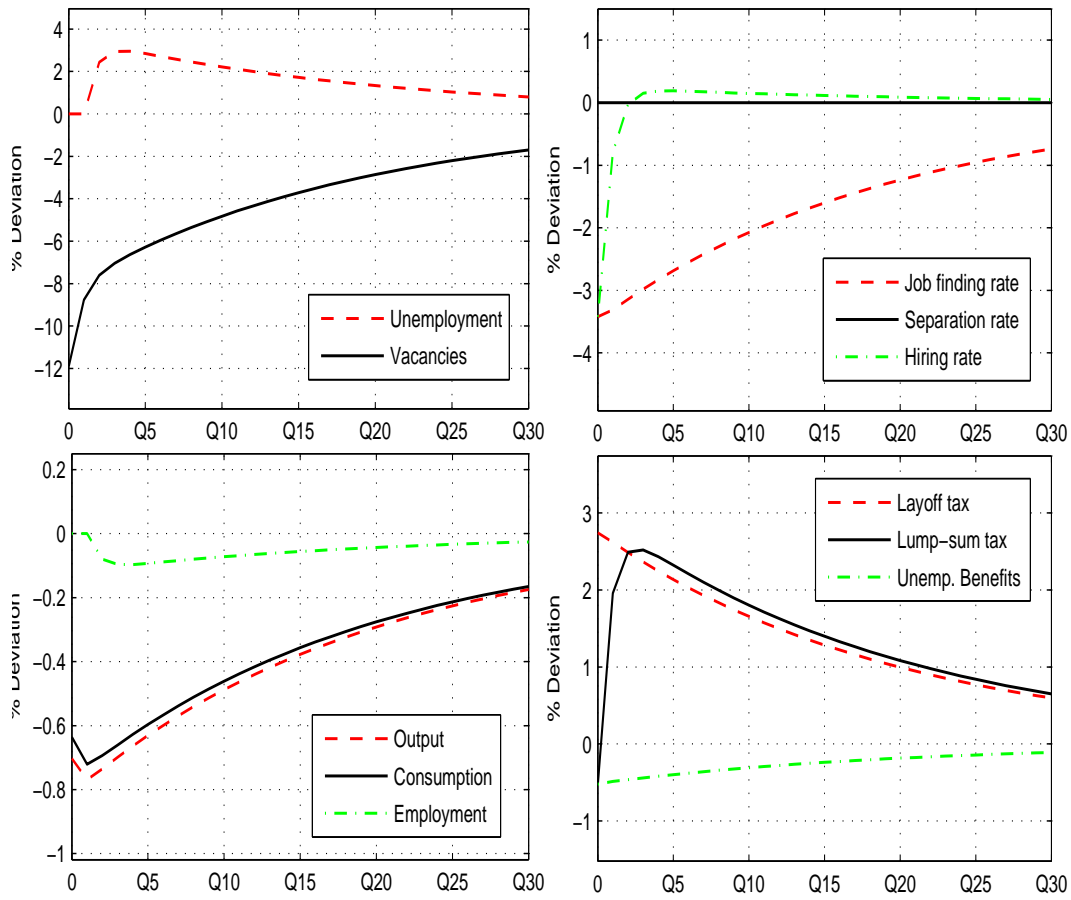


Figure 5: *Impulse response functions - Second best allocation.* We simulate a one percent negative aggregate productivity shock.