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Abstract

Ehrmann *et al.* (2003) proposed an IRF in the frame of Markov-Switching structurally VARs. Their IRF provides insights on the dynamics *within* the regime in which the shock occurs. We propose an IRF that captures the global response of the system and illustrate its use with examples.

Keywords: Structural VAR, Markov-Switching model, impulse-response function, state asymmetry, regime-dependent IRF.

JEL classification: C32, C52, C53.

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Introduction

Impulse-response functions (IRF hereafter) provide a global picture of what happens in a system hit by an exogenous shock within a given horizon. With nonlinear specifications, the analysis reaches the richer field of asymmetries (see for instance Hamilton, 1989, Tong, 1990, Terasvirtä, 1995 among many others). However, while widely explored in the frame of threshold representations with the seminal article by Koop *et al.* (1996) on Generalized IRFs, they are much less studied in Markov-Switching Vectoriel AutoRegressions (MS-VAR hereafter).

Up to now, two paths have been followed in the MS-VAR literature to question state asymmetry. The first one consists in calculating exact IRFs (EIRF hereafter) in very simple specifications (only the constant or the intercept is regime-dependent; *e.g.* Krolzig & Toro, 1998, Artis *et al.*, 2004, Ang & Bekaert, 2002). In this framework, the computation of IRFs is straightforward and with no cost. These IRFs are somehow different from the ones posed by the traditional VAR literature insofar as they study the impact of a change in regime or a variable and not of an identified structural shock. Furthermore, working with such model specification implies strong assumptions concerning state, sign and size asymmetries. The second approach was proposed by Ehrmann *et al.* (2003) in fully-parameterized MS-VARs and deals more specifically with the problem of state asymmetry. They proposed a *regime-dependent* IRF (RD-IRF hereafter) to study the response of the system conditionally to the regime in which the shock occurs. As they assume there is no more change in regime in the wake of the shock, their IRF provides insights on the dynamics at stake *within* the regime in which the shock occurs but cannot be considered as a general tool for MS-VARs evaluation.

In this paper, we propose a more general IRF that captures the global response of the system in the wake of an identified shock, whatever the states visited in the wake of the shock. It allows not only to deal with state asymmetry by investigating the possible different impact of a shock depending on the regime in which it occurs but also to calculate the unconditional response of the system to this shock.

The paper is organised as follows. First, we present the econometric context. Second, we present the EIRF and discuss their relations with the RD-IRF proposed by Ehrmann *et al.* (2003). Third, we illustrate the discussion with simple examples and conclude.

1 The model: the structural MS-VAR

The general standard MS-VAR is:

$$Y_t = \mu_{s_t} + \sum_{i=1}^{p} \Phi_{i,s_t} \cdot Y_{t-i} + u_t \tag{1}$$

with $\{Y_t\}_{t=1}^{t=T}$ a set of k variables, u_t *iid* gaussian with a variance-covariance matrix Ω_{u,s_t}. $s_t \in \{1, 2, \dots S\}$ is modeled as an unobserved one-order Markov chain of S regimes, with fixed transition probabilities p_{ij}:

$$P(s_t = j | s_{t-1} = i, s_{t-2} = k, \dots) = P(s_t = j | s_{t-1} = i) = p_{ij} \quad \text{with} \quad \forall i \in \{1, \dots S\} \quad \sum_{j=1}^{S} p_{ij} = 1$$

Every parameter is regime-dependent. Each regime is then characterized by its own propagation system. The set of unknown parameters, $\Theta = \{\mu_j, \Phi_{1,j}, \dots, \Phi_{p,j}, \Omega_{u,j}, p_{ij}\}$ $\forall j \in [1, S]$, $\forall i \in [1, S-1]$ is estimated by $\hat{\Theta}_T$.

For the identification of shocks, we can use the Choleski approach (see *e.g.* Weise, 1999 or Ehrmann *et al.*, 2003) or the semi-structural approach. If $u_t = D_{s_t} \cdot \varepsilon_t$ and ε_t gaussian with an identity variance-covariance matrix, we solve for each regime:

$$\Omega_{u,s_t} = D_{s_t} \cdot D_{s_t}' \tag{2}$$

where D_{s_t} is not necessarily lower triangular and Ω_{u,s_t} is provided by estimation.

2 Impulse-response functions

2.1 The common framework

We use the following equation from (1) and (2):

$$Y_h = \mu_{s_h} + \sum_{i=1}^{p} \Phi_{i,s_h} \cdot Y_{h-i} + D_{s_h} \cdot \varepsilon_h \tag{3}$$

At $h = 0$, we suppose a constant value $\overline{Y}$ for the Y_{h-i}. To assess state asymmetry, we suppose the regime (*e.g.* $s_1 = j$) and that a unit shock occurs at $h = 1$. Then, from equation (3), we have:

$$Y_1 = \hat{\mu}_j + \left[\sum_{i=1}^{p} \hat{\Phi}_{i,j}\right] \cdot \overline{Y} + \hat{D}_j \cdot \varepsilon_1$$

with probability 1. This response will be the same for both the IRFs of this paper since independent of their respective definition. From $h = 2$ to H, the forecast horizon, the regime is no longer observable. We then have to perform an inference on both s_h and Y_h conditionally to s_1.

2.2 Exact Impulse-Response Function (EIRF)

Knowing s_1, every possible path of the system (S^H) is computed, with its associated probability from the Markov chain:

$$\forall h \geq 2, \quad P(s_h = j, s_{h-1} = i, \ldots, s_2 = k \,|\, s_1, \hat{\Theta}_T) = \hat{p}_{ij} \cdot P(s_{h-1} = i, \ldots, s_2 = k \,|\, s_1; \hat{\Theta}_T)$$

Conditionally to each possible path, s_1 and ε_1, the optimal forecast of Y is calculated:

$$\forall h \geq 2, \quad E[Y_h \,|\, s_h, \ldots, s_2; s_1, \varepsilon_1, \hat{\Theta}_T] = \hat{\mu}_{s_h} + \sum_{i=1}^{p} \hat{\Phi}_{i,s_h} \cdot E[Y_{h-1} \,|\, s_{h-1}, \ldots, s_2; s_1, \varepsilon_1, \hat{\Theta}_T] \tag{4}$$

One then deduces the shocked trajectory as the mean of all possible responses weighted by their own probability:

$$\forall h \geq 2, \quad E[Y_h \,|\, s_1, \varepsilon_1, \hat{\Theta}_T] = \sum_{s_h=1}^{S} \cdots \sum_{s_2=1}^{S} E[Y_h \,|\, s_h, \ldots, s_2; s_1, \varepsilon_1, \hat{\Theta}_T] \cdot P(s_h, \ldots, s_2 \,|\, s_1, \hat{\Theta}_T) \tag{5}$$

As the framework is nonlinear, the same is done without shock at date 1 to calculate the baseline ($\varepsilon_1 = 0_{(k \times 1)}$). EIRF is the difference between the shocked trajectory and the baseline:

$$\forall h \geq 2, \quad EIRF_Y(h, s_1, \varepsilon_1, \hat{\Theta}_T) = E[Y_h \,|\, s_1, \varepsilon_1, \hat{\Theta}_T] - E[Y_h \,|\, s_1, 0, \hat{\Theta}_T] \tag{6}$$

Deriving the unconditional response of the system to a shock is straightforward. The main properties of this IRF are derived by Karamé & Olmedo (2007)[1]

2.3 RD impulse-response functions

Ehrmann *et al.* (2003) developed their RD-IRF by assuming there is no change in regime in the wake of the shock: $\forall h \geq 2$,

$$RD - IRF_Y(h, s_1, \varepsilon_1, \hat{\Theta}_T) = E\left[Y_h \middle| s_h = s_{h-1} = \ldots = s_1, \varepsilon_1, \hat{\Theta}_T\right] - E\left[Y_h \middle| s_h = s_{h-1} = \ldots = s_1, 0, \hat{\Theta}_T\right] \qquad (7)$$

It is straightforward that RD-IRF is a special case of EIRF, due to the extra assumption of no change in regime, which amounts to choosing one particular path among all possible.

3 Illustration

Suppose a bivariate MS-VAR with two regimes (table 1). The coefficients are quite different across regimes, implying two well differentiated regimes. The probabilities of remaining in regime 1 and 2 are respectively 0.9 and 0.6, which is usual in the literature and provides a clear sample split. Hence, regime 1 is prevailing and switching from the minor regime 2 is quite easy. The identification constraint assumes the responses of both variables to the second shock to display the same magnitude[2]. We deliberately choose short-run dynamics in both regimes. For simplicity, we suppose only one lag and no interaction between the variables of the system to fully understand the mechanisms at stake. The dynamics is stationary in regime 1 and we suppose two alternatives for regime 2. In case I, the variables display a stationary dynamics in regime 2 while in case II, the variables display respectively a unit-root and an explosive dynamics in regime 2 (Hall *et al.,* 1999). This allows covering all possible cases. The Karlsen (1990) criterion concludes that the representation is globally stationary in both cases, while in case II, the dynamics in regime 2 is not (table 2).

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[1] They demonstrate that under the assumption of fixed transition probabilities (FTP), EIRF does not depend either on the starting values $\overline{Y}$ before the shock or on the history of shocks. Besides, the model implies sign and size symmetry. Their properties are more general when the FTP assumption is relaxed.

[2] The whole parameterization (including the identification scheme) is neutral in the analysis since it is used to calculate both IRF.

Table 1: Parameters for MS-VAR(1)

	μ_{s_t}	Φ_{1,s_t}	Ω_{u,s_t}	D_{s_t}	P
Regime 1	$\begin{pmatrix} 0.006 \\ 1.601 \end{pmatrix}$	$\begin{pmatrix} 0.1 & 0 \\ 0 & 0.9 \end{pmatrix}$	$\begin{pmatrix} 0.174 & -0.03 \\ -0.03 & 0.193 \end{pmatrix}$	$\begin{pmatrix} -0.31 & 0.28 \\ 0.34 & 0.28 \end{pmatrix}$	$\begin{pmatrix} 0.9 & 0.4 \\ 0.1 & 0.6 \end{pmatrix}$
Regime 2	$\begin{pmatrix} 3.232 \\ 5.660 \end{pmatrix}$	Case I: $\begin{pmatrix} 0.9 & 0 \\ 0 & 0.1 \end{pmatrix}$ Case II: $\begin{pmatrix} 1 & 0 \\ 0 & 1.05 \end{pmatrix}$	$\begin{pmatrix} 0.199 & -0.452 \\ -0.452 & 1.481 \end{pmatrix}$	$\begin{pmatrix} -0.4 & 0.18 \\ 1.20 & 0.18 \end{pmatrix}$	

Table 2: Global stationarity and stationarity by regime

		Stationarity by regime	Global stationarity
		Largest eigenvalue in each regime	*Largest eigenvalue of the Karlsen companion matrix*
Case I	Regime 1	0.9	0.73
	Regime 2	0.9	
Case II	Regime 1	0.9	0.89
	Regime 2	1.05	

Figure 1 presents the IRFs when shocks occur in regime 1 for both cases since the parameterization is the same. We can see that both IRFs are quite close. This is because a possible change in regime has a low probability when the shock occurs in the prevailing regime. RD-IRF is just more (resp. less) persistent than EIRF, because persistence is less (resp. more) important in the other regime.

Figure 1: IRF (the shock occurs in regime 1, cases I and II)

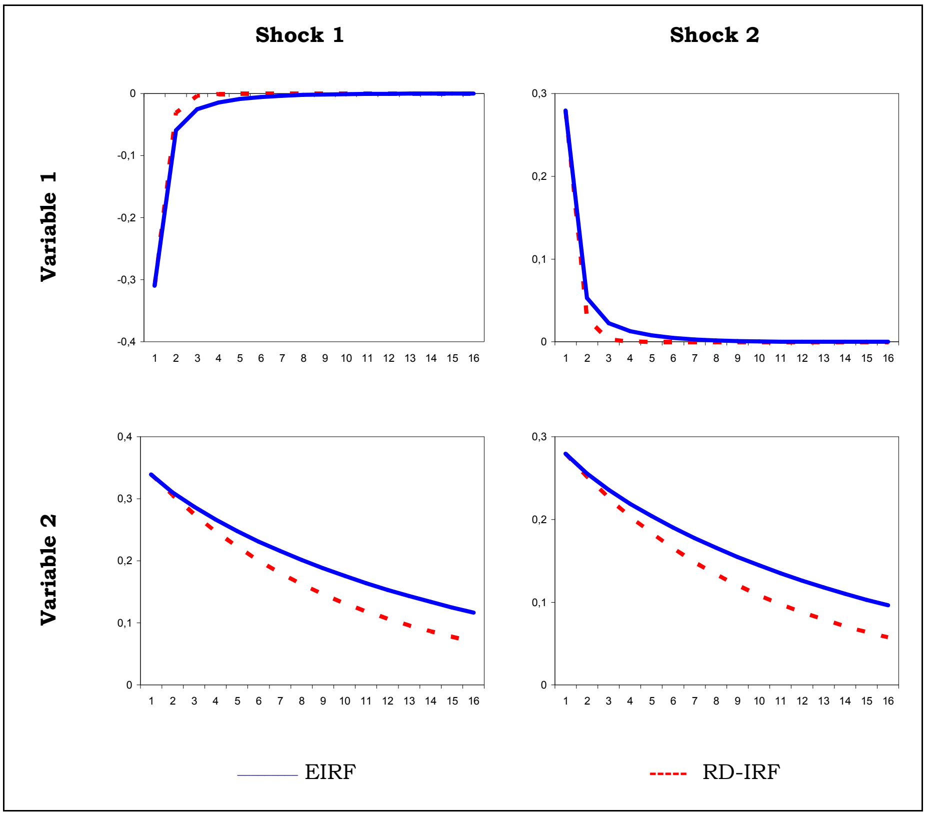

Figure 2: IRF (the shock occurs in regime 2, case I)

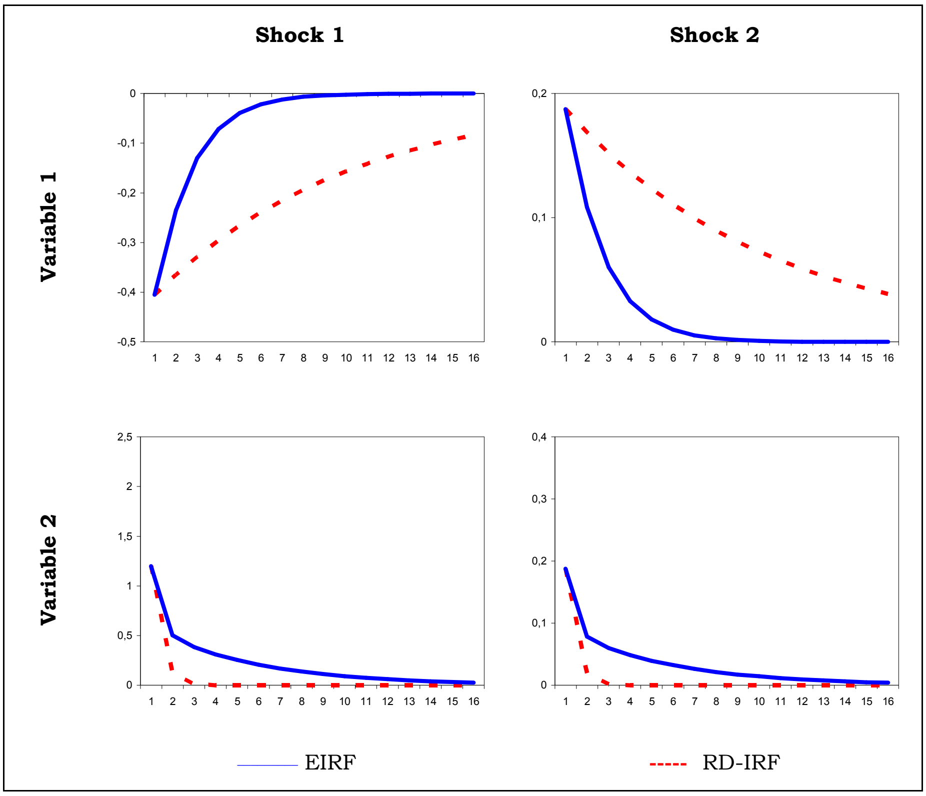

Figure 3: IRF ((the shock occurs in regime 2, case II, globally stationary example)

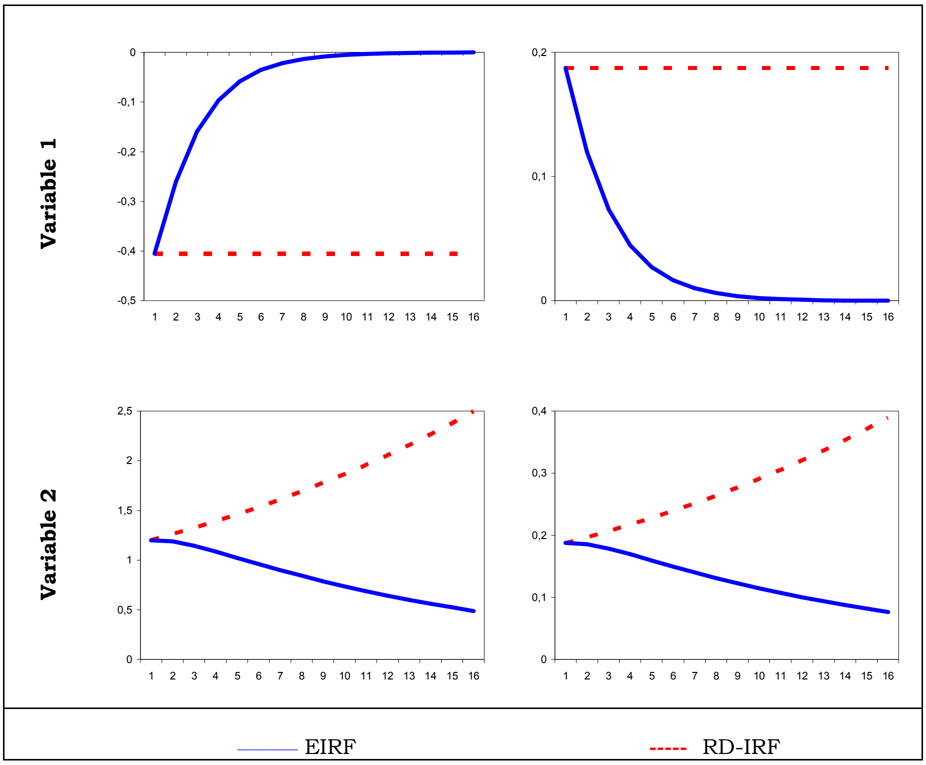

Figure 2 presents the calculated IRFs when shocks occur in regime 2 for case I. We can see that the differences between both IRFs are greater than in regime 1. This is due to the fact that change in regime has a higher probability of occurrence than in regime 1 and then highly influences the calculation of EIRFs in this case. In case II (figure 3), RD-IRFs capture the explosive dynamics within regime as expected while EIRFs display the global stationary pattern of the system due to the prevailing stationary regime.

Conclusion

This paper derived EIRF, an evaluation tool for identified shocks in structural MS-VAR framework. EIRF allows understanding of the dynamics of the *whole system* in presence of potential state asymmetry while RD-IRF allows understanding of the dynamics at stake *within* each regime in response to a shock. There is then no ambiguity between these tools that can be viewed as complementary. The extension of this tool to time-varying transition probabilities or its properties as regards the other kinds of asymmetries are left for further research.

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