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Asymmetric Properties of IRF in Structural Markov-Switching Vector AutoRegressions

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Abstract

We propose a methodology extending the structural VAR approach to nonlinear Markov-Switching framework. We present the exact IRFs and discuss their properties as regards the different types of asymmetries (sign, size, state) and assumptions on transition probabilities. We propose a statistical methodology for discriminating some asymmetric properties of the system.

Keywords: Structural VAR, Markov-switching model, asymmetries, impulse-response function.

JEL classification: C32, C52, C53.

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Introduction

In the Nineties, main macroeconomic time series (GDP, interest rates, unemployment, inflation, exchange rates, …) were found nonlinear, underlining the need to replace linear representations by nonlinear ones to model economic relations. Models like threshold autoregressions (*e.g.* Tong 1990, Terasvirta 1995) or Markov-Switching vector autoregressions (*MS-VAR hereafter*) (*e.g.* Hamilton 1989, Krolzig 1997) have then encountered a formidable success in modelling processes characterized by nonlinear dynamics. With these specifications, the analyst reaches the richer investigation field of asymmetries in the economic and financial mechanisms[1].

A simple and popular tool for dynamics investigation in applied macroeconomics and particularly the structural VAR literature is impulse-response functions (*IRF hereafter*). Indeed, it provides a reasonable picture of what happens to a system hit by an exogenous shock within a given horizon. With the seminal article by Koop *et al.* (1996), Generalized IRFs (*GIRF hereafter*) are widely employed in the frame of threshold representations. One is then able to investigate the difference in the variables response to a shock as regards *(i)* the *state* of the system at the date the shock occurs, *(ii)* the *size* of the shock and *(iii)* the *sign* of the shock.

Concerning this problematic in the MS-VAR framework, two issues are at stake. The first one consists in interpreting the studied shock. On the one hand, the literature considers the response to a change in regime or to a change in a variable and not to an identified structural shock (Krolzig 2006). On the other hand, papers use the Choleski approach to identify "structural" shocks (*e.g.* Weise 1999, Erhmann *et al.* 2003). We propose to extend the identification of structural shocks to the use of short-term restrictions in the nonlinear framework, that is more in line with the traditional structural VARs literature and less restrictive than Choleski. The second question is the specification of the nonlinear model: with fixed or time-varying transition probabilities? With regime-dependent dynamics? We show this point is crucial because it induces strong untested restrictions on the asymmetric behaviour of the variables.

In this paper, we propose a methodology extending the structural VAR approach to MS-VAR. In the first section we present the econometric framework. In the second section, we describe the identification of structural shocks with short-term

[1] See Hamilton (1989), Beaudry & Koop (2003), Engel & Hamilton (1990), Sensier *et al.* (2002) and Sichel (1994) among many others.

restrictions. In the third section, we present EIRF and study their asymmetric properties (sign, size, state) as regards the specification choices of the model. We then present a simple testing methodology for determining some asymmetric properties of the system.

1 The MS-VAR

Our standard MS-VAR is:

$$Y_t = \mu_{s_t} + \sum_{i=1}^{p} \Phi_{i,s_t} \cdot Y_{t-i} + u_t \tag{1}$$

with $\{Y_t\}_{t=1}^{t=T}$ a set of k variables, u_t *iid* gaussian with a variance-covariance matrix Ω_{u,s_t}. $s_t \in \{1, 2, \ldots S\}$ is modeled as an unobserved one-order Markov chain of S regimes, with fixed transition probabilities p_{ij}:

$$P(s_t = j | s_{t-1} = i, s_{t-2} = k, \ldots) = P(s_t = j | s_{t-1} = i) = p_{ij} \quad \text{with} \quad \forall i \in \{1, \ldots S\} \quad \sum_{j=1}^{S} p_{ij} = 1$$

Every parameter is regime-dependent. Each regime is then characterized by its own propagation system. $\Theta = \{\mu_j, \Phi_{1,j}, \ldots, \Phi_{p,j}, \Omega_{u,j}, p_{ij}\}$, $\forall j \in [1,S]$, $\forall i \in [1, S-1]$ is the set of unknown parameters estimated by $\hat{\Theta}_T$.

With time-varying transition probabilities (*TVTP hereafter*) (Filardo 1994), transition probabilities are assumed to depend on the observed lagged values of the system variables:

$$P(s_t = j | s_{t-1} = i, s_{t-2} = k, \ldots, \underline{z_{t-1}}) = P(s_t = j | s_{t-1} = i, \underline{z_{t-1}}) = p_{ij}(\underline{z_{t-1}}) \quad \text{still with}$$
$$\forall i \in [1,S] \quad \sum_{j=1}^{S} p_{ij}(\underline{z_{t-1}}) = 1$$

where $\underline{z_{t-1}}$ is the information set containing some delayed variables of Y affecting the transition probabilities at each date. This modification enables for instance to question the determinants of transition probabilities (*e.g.* Garcia & Schaller 2002 for the influence of monetary policy on recessions).

2 The identification of structural shocks

In the nonlinear empirical literature, several attempts have been made to identify economically interpretable shocks. For Krolzig (2006), the shock investigated corresponds to a change in regime or to a change in the system variable. Weise (1999) or Ehrmann *et al.* (2003) (among others) recursively identify shocks through a Choleski decomposition, respectively in a threshold VAR and in a MS-VAR. In these latter cases, it simply consists in Choleski decomposition regime by regime.

In traditional VAR's, the residual terms are generally correlated (their variance-covariance matrix Ω_u is not diagonal) and u_t can not be considered as economically interpretable innovations. Structural VARs introduce some general assumptions provided by economic theory in order to disentangle the structural shocks ε_t from the residuals estimated from the reduced-form. The relation between the residuals and the economic shocks is supposed of the following form:

$$u_t = D \cdot \varepsilon_t \tag{2}$$

where ε_t has an identity variance-covariance matrix[2]. From the variance of the processes, one deduces:

$$\Omega_u = D \cdot D'$$

This equality can be considered as a nonlinear system. D contains k^2 unknown parameters. Ω_u is an estimated symmetric matrix that provides $k(k+1)/2$ non-redundant equations. The system is then under-identified. One has then to assume at least $k(k-1)/2$ restrictions to make this system just-identified or over-identified. Choleski is a particular case where D is lower triangular: the shocks are ordered from the most important one (that affects all variables when it occurs) to the less important one (that only affects the last variable of the system when it occurs. That amounts to $k(k-1)/2$ "no instantaneous response" restrictions in the system. It seems natural to us to identify the shocks in the spirit of the traditional VAR methodology rather than in the Choleski approach that appears more restrictive. Let's suppose:

[2] It is a convention. This assumption can be relaxed by normalizing the k diagonal elements of D to unity, considering a diagonal variance matrix Ω_ε with k unknown parameters and making shocks with a magnitude of one standard-error. We adopt this presentation to draw a parallel with the Choleski approach that can be viewed as a particular case where D is lower triangular.

$$u_t = D_{s_t} \cdot \varepsilon_t \qquad (3)$$

We have to solve a nonlinear just-identified system in the S regimes:

$$\Omega_{u,s_t} = D_{s_t} \cdot D_{s_t}{'}$$

The resolution is conducted regime by regime through a numerical method consisting in minimizing:

$$\left[vech\left(\Omega_{u,s_t} - D_{s_t} \cdot D_{s_t}{'}\right)\right]' \cdot \left[vech\left(\Omega_{u,s_t} - D_{s_t} \cdot D_{s_t}{'}\right)\right].$$

with respect to the $k(k+1)/2$ unknown parameters and the $k(k-1)/2$ short-term restrictions in D_{s_t}, not necessarily zeros and not necessarily in a lower triangular form. Our approach clearly embeds the S Choleski decompositions. Furthermore, we suppose the same set of short-term restrictions across the regimes to identify the structural shocks but this can be relaxed.

3 Impulse-response functions

We use the following equation from (1) and (3):

$$Y_h = \mu_{s_h} + \sum_{i=1}^{p} \Phi_{i,s_h} \cdot Y_{h\text{-}i} + D_{s_h} \cdot \varepsilon_h \qquad (3)$$

At $h = 0$, we suppose a constant value $\overline{Y}$ for the Y_{h-i}. To assess state asymmetry, we suppose the regime (*e.g.* $s_1 = j$) and that a unit shock occurs at $h = 1$. Then, from equation (3), we have:

$$Y_1 = \hat{\mu}_j + \left[\sum_{i=1}^{p} \hat{\Phi}_{i,j}\right] \cdot \overline{Y} + \hat{D}_j \cdot \varepsilon_1$$

with probability 1.

From $h = 2$ to H, the forecast horizon, the regime is no more observable. An inference has to be performed on both s_h and Y_h conditionally to s_1. Knowing s_1, one can obtain from the Markov chain:

$$\forall h \geq 2, \quad P(s_h = j, s_{h-1} = i, \ldots, s_2 = k \left| s_1, \hat{\Theta}_T \right.) = \hat{p}_{ij} \cdot P(s_{h-1} = i, \ldots, s_2 = k \left| s_1; \hat{\Theta}_T \right.)$$

Conditionally to each possible path, s_1 and ε_1, the optimal forecast of Y is calculated:

$$\forall h \geq 2, \quad E[Y_h \left| s_h, \ldots, s_2; s_1, \varepsilon_1, \hat{\Theta}_T \right.] = \hat{\mu}_{s_h} + \sum_{i=1}^{p} \hat{\Phi}_{i,s_h} \cdot E[Y_{h-1} \left| s_{h-1}, \ldots, s_2; s_1, \varepsilon_1, \hat{\Theta}_T \right.] \tag{4}$$

One then deduces the shocked trajectory as the mean of all possible responses weighted by their own probability:

$$\forall h \geq 2, \quad E[Y_h \left| s_1, \varepsilon_1, \hat{\Theta}_T \right.] = \sum_{s_h=1}^{S} \cdots \sum_{s_2=1}^{S} E[Y_h \left| s_h, \ldots, s_2; s_1, \varepsilon_1, \hat{\Theta}_T \right.] \cdot P(s_h, \ldots, s_2 \left| s_1, \hat{\Theta}_T \right.) \tag{5}$$

As the framework is nonlinear, the same is done without shock at date 1 ($\varepsilon_1 = 0_{(k \times 1)}$) to calculate the baseline. The EIRF is the difference between the shocked trajectory and the baseline:

$$\forall h \geq 2, \quad EIRF_Y(h, s_1, \varepsilon_1, \hat{\Theta}_T) = E[Y_h \left| s_1, \varepsilon_1, \hat{\Theta}_T \right.] - E[Y_h \left| s_1, 0, \hat{\Theta}_T \right.] \tag{6}$$

Krolzig (2006) provides a simple way to calculate these EIRF, without limitations by horizon length.

As previously, EIRF can be interpreted as forecast-errors due to the occurrence of the unanticipated shock. Consequently, calculating forecast-error variance decomposition conditionally to the regime in which the shock occurs is straightforward.

4 Asymmetric properties of EIRF and a testing methodology

The asymmetric properties of the EIRF depend on the assumptions on the transition probabilities and on the specification of the model. They appear more clearly as equation (6) can be re-written *under the FTP assumption*: $\forall h \geq 2$,

$$\begin{aligned} &EIRF_Y(h, s_1, \varepsilon_1, \hat{\Theta}_T) = \\ &\quad \sum_{s_h=1}^{S} \cdots \sum_{s_2=1}^{S} \left\{ E[Y_h \left| s_h, \ldots, s_2; s_1, \varepsilon_1, \hat{\Theta}_T \right.] - E[Y_h \left| s_h, \ldots, s_2; s_1, 0, \hat{\Theta}_T \right.] \right\} \cdot P(s_h, \ldots, s_2 \left| s_1, \hat{\Theta}_T \right.) \end{aligned} \tag{7}$$

It is obvious from equation (7) that the EIRF does not depend on innovations history, *i.e.* the introduction of the same sequence of innovations in the shocked and baseline trajectories. Indeed, the regime's distribution is invariant to the history of innovations since there is no possible feedback from the system's variables to the transition probabilities and then to the forecast of the state of the system. As a first direct consequence, there is no need to simulate innovations' history to provide EIRF, as in Koop *et al.* (1996).

Suppose a FTP MS-VAR model where only the intercept μ_{s_t} is regime-dependent (or a MSI-VAR along Krolzig 1997). It is obvious from equation (7) that the EIRF does not depend on μ_{s_t} and consequently neither on s_t nor s_1:

$$\forall i \neq j,\ EIRF_Y(h, s_1 = i, \varepsilon_1, \hat{\Theta}_T) = EIRF_Y(h, s_1 = j, \varepsilon_1, \hat{\Theta}_T)$$

In this case, the system displays the same responses whatever the state in which the shock occurs.

On the other hand, the existence of other types of asymmetries (in the sign and in the size of shock) for all variables of the system *can not* be investigated in a general FTP representation such as equation (1) since it can be deduced from equation (7) that the response is linear in the value of the shock:

$$EIRF_Y(h, s_1, \lambda \cdot \varepsilon_1, \hat{\Theta}_T) = \lambda \cdot EIRF_Y(h, s_1, \varepsilon_1, \hat{\Theta}_T).$$

Consequently, the use of a general FTP model as in equation (1) allows the possibility of state asymmetry in the system variables and *imposes the presence of both sign and size symmetries.*

All the previous remarks do not stand any more for TVTP models, *i.e.* if transition probabilities can display a feedback effect from the variables of the system to the state of the system. In that case, equation (7) and its implications stand no more. The EIRF *does* depend on the innovations' history, such as GIRF in threshold models through the transition function. Random draws on the history of innovations should then be simulated in that case. The TVTP model allows investigating for the three kinds of asymmetries (in sign, size and state).

This opens the way for a very simple strategy for discriminating between the different kinds of asymmetries through models testing. Since there is no problem of nuisance parameters in this case (Hansen 1992), likelihood ratio tests can be considered as χ^2 distributed. Hence:

- the acceptation of a general FTP model (as in equation 1) against a general TVTP model amounts to *accept the null assumption of size* *<u>and</u>* *sign symmetry in the responses of* *<u>all</u>* *the system variables.*

- the rejection of a simple FTP model (with only state-dependent intercept) against a general FTP model (as in equation 1) amounts *to reject the null assumption of state symmetry in the responses of* *<u>all</u>* *the system variables.*

Such an approach can then lead to choose a specification more congruent to the data properties.

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