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An Algorithm for Generalized Impulse-Response Functions in Markov-Switching Structural VAR

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Abstract

We transpose the Generalized Impulse-Response Function (GIRF) developed by Koop *et al.* (1996) to Markov-Switching structural VARs. As the algorithm displays an exponentially increasing complexity as regards the prediction horizon, we use the collapsing technique to easily obtain simulated trajectories (shocked or not), even for the most general representations. Our approach encompasses the existing IRFs proposed in the literature and is illustrated with an applied example on gross job flows.

Keywords: structural VAR, Markov-switching regime, generalized impulse-response function.

JEL classification: C32, C52, C53.

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1 Introduction

In the Nineties, main macroeconomic time series (GDP, interest rates, unemployment…) were found nonlinear, underlining the need to replace linear representations by nonlinear ones to model economic relations. Models like threshold autoregressions (Tong 1990, Terasvirta 1995) or Markov-switching vector autoregressions (*MS-VAR* hereafter) (Hamilton 1989, Krolzig 1997) have encountered a huge success in modelling processes characterized by nonlinear dynamics. With these specifications and the introduction of economically-identified shocks, the analyst reaches the richer investigation field of state-, sign- or size asymmetries in the economic and financial mechanisms[1], which is not possible in the linear frame.

A simple and popular tool for dynamics investigation in applied macroeconomics, and particularly the structural VAR literature, is the impulse-response function (*IRF* hereafter). Indeed, it provides a global picture of what happens in a system hit by an exogenous shock within a given horizon. Potter (2000) and Koop *et al.* (1996) generalized the traditional linear IRF approach and derived the formal requirements for IRF in nonlinear representations: history- and shock- dependencies. With these seminal articles, generalized IRFs (*GIRF* hereafter) are widely employed in the frame of threshold representations, particularly to examine the three kinds of asymmetries (*e.g.* Weise 1999 among many others). However, this issue is less developed in the MS-VAR literature.

Up to now, Ehrmann *et al.* (2003) propose a regime-dependent IRF to study the response of the system conditionally to the regime in which the shock occurs and with no change in regime afterwards. Karamé (2010) generalizes their approach to whatever visited states in the wake of the shock. These approaches are very close to the traditional linear literature on IRF since they are based on several steps ahead predictions. However, they are unable to question sign or size asymmetries issues (Karamé 2011). Camacho & Perez-Quiros (2011) propose an IRF based on a one-step ahead prediction since it partially uses the updating step on regime probabilities of the Hamilton filter. It is able to examine the three kinds of asymmetries but fails to fulfil all the requirements defined by Koop *et al.* (1996), particularly shock-dependence.

The present paper fully transposes the approach developed by Koop *et al.* (1996) to structural MS-VAR. As GIRF calculation displays an exponentially increasing

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[1] See Hamilton (1989), Beaudry & Koop (2003), Engel & Hamilton (1990), Sensier *et al.* (2002) and Sichel (1994) among many others.

complexity as regards the prediction horizon, we use the collapsing technique as the practical solution to obtain simulated trajectories (shocked or not) even for the most general representations. We then show that our GIRF encompasses the existing IRFs in the literature.

The paper is organised as follows. First, we present the econometric context. Second, we present the algorithm to calculate the GIRF. At last, we illustrate this new tool with an applied example based on aggregate gross job flows.

2 The Markov-switching structural VAR

Our standard MS-VAR is:

$$y_t = \mu_{s_t} + \sum_{k=1}^{p} \Phi_{k,s_t} \cdot y_{t-k} + u_t \tag{1}$$

with $y_{\underline{1:T}} = \{y_1, y_2, \ldots y_T\}$ a sample of *N* variables, u_t *iid* gaussian $N(0, \Omega^u_{s_t})$, $s_t \in \{1, \ldots S\}$ an unobserved one-order Markov chain of *S* regimes, with fixed transition probabilities $P(s_t = j | s_{t-1} = i, s_{t-2} = k, \ldots) = P(s_t = j | s_{t-1} = i) = p_{ij}$ and $\forall i \in \{1, \ldots S\}$, $\sum_{j=1}^{S} p_{ij} = 1$. Every parameter is regime-dependent. Each regime is then characterized by its own propagation system. $\Theta = \{\mu_j, \Phi_{1,j}, \ldots, \Phi_{p,j}, \Omega^u_j, p_{ij}\}$, $\forall j \in \{1, \ldots S\}$, $\forall i \in \{1, \ldots S-1\}$ is the set of unknown parameters estimated by $\hat{\Theta}_T$.

In the nonlinear empirical literature, several attempts have been made to identify economically interpretable shocks (*e.g.* Krolzig & Toro 1998, Ehrmann *et al.* 2003 or Camacho & Perez-Quiros 2011 among others). Karamé (2010) extends this approach by supposing $u_t = D_{s_t} \cdot \varepsilon_t$ with ε_t *iid* gaussian $N(0, I)$ and solving:

$$\Omega^u_{s_t} = D_{s_t} \cdot D_{s_t}{}' \tag{2}$$

D_{s_t} is not necessarily triangular[2]. The same set of restrictions across the regimes is assumed to identify the shocks but this can be relaxed.

[2] See Davis & Haltiwanger (1999) for an example.

3 A generalized impulse-response function

3.1 Assumptions and initialization of the algorithm

As in Koop *et al.* (1996), four assumptions are necessary to run the exercise.

1. The dynamic nonlinear model and its parameters are known[3].

2. An identified shock $[\varepsilon_t]_i = 1$ (the i^{th} column of an identity matrix for shock i) occurs at date t. Otherwise, $\varepsilon_{t+h} = 0_{(N,1)}$ if $h > 0$.

3. We draw A subsamples from the data for the initial lagged values of the system variables $y^a_{\underline{t-p:t-1}}$ (a = 1,... A) at the beginning of the recursion. We also need the current regime probability $P(s^a_t = j \big| y^a_{\underline{1:t}};\Theta)$[4]. To assess state asymmetry, initial conditions can be drawn from '*homogeneous*' regimes, *i.e.* subsamples selected with a rule such as $P(s^a_t = j \big| y^a_{\underline{1:t}};\Theta) = \underset{i \in \{1...S\}}{Max} P(s^a_t = i \big| y^a_{\underline{1:t}};\Theta)$.

4. B sequences of future shocks e^b_{t+h} *iid* Gaussian $N(0, I_N)$ (h = 0,... H, b = 1,... B) hit the system at each date. Combined with the model prediction, it provides the '*realization*' of system variables[5].

3.2 The algorithm

We first predict the system variables using:

$$\hat{y}^{i,j}_{t+h} = E[y_{t+h} \big| s_{t+h} = j, s_{t+h-1} = i, y^a_{\underline{t-p:t+h-1}};\Theta] = \mu_j + \sum_{k=1}^{p} \Phi_{k,j}\, y^i_{t+h-k} + D_j \varepsilon_{t+h}$$

$\hat{y}^{i,j}_{t+h}$ depends on the current regime for parameters and on the previous regime for lagged variables y^i_{t+h-k} that will be defined soon. Simulated variables for this trajectory are obtained with:

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[3] Uncertainty on parameters estimation can also be taken into account by repeating the following algorithm for a large number of draws in the estimated parameters joint distribution.

[4] Practically, we randomly draw an observation number from $[p+1, T]$. Initial values $y^a_{\underline{t-p:t-1}}$ are the corresponding p previous observations. The probability of the current regime is initialized with the estimated filtered probability $P(s^a_t \big| y^a_{\underline{1:t}};\hat{\Theta}_T)$.

[5] As in the usual VAR or the threshold approaches, boostrap can be used in the non-gaussian case.

$$\forall h \geq 0, \qquad y_{t+h}^{i,j} = \hat{y}_{t+h}^{i,j} + D_j \, e_{t+h}^{b}$$

Every iteration multiplies the inference on system variables by *S*. The collapsing technique (Kim 1994, Karamé 2011) circumvents this problem by averaging $y_{t+h}^{i,j}$ as regards s_{t+h-1}:

$$y_{t+h}^{j} = \frac{\sum_{i=1}^{S} P(s_{t+h} = j, s_{t+h-1} = i \,|\, \underline{y_{t-p:t+h}^{a}}; \Theta) \, y_{t+h}^{i,j}}{P(s_{t+h} = j \,|\, \underline{y_{t-p:t+h}^{a}}; \Theta)}$$

It replaces a S^2-gaussian mixture representation by an *S*-Gaussian mixture. Thus, we only handle *S* inferences on variables that constitute the input for the next prediction step (figure 1).

Collapsing requires probabilities calculated with a filter *à la* Hamilton[6]. For $h \geq 1$, knowing $\hat{p}_{ij} = P(s_{t+h} = j \,|\, s_{t+h-1} = i)$ and $P(s_{t+h-1} = i \,|\, \underline{y_{t-p:t+h-1}^{a}}; \Theta)$, we calculate *prior* joint probabilities for s_{t+h} and s_{t+h-1}[7]:

$$\forall h \geq 1, \qquad P(s_{t+h} = j, s_{t+h-1} = i \,|\, \underline{y_{t-p:t+h-1}^{a}}; \Theta) = P(s_{t+h} = j \,|\, s_{t+h-1} = i) \times P(s_{t+h-1} = i \,|\, \underline{y_{t-p:t+h-1}^{a}}; \Theta)$$

The conditional density function of $y_{t+h}^{i,j}$ is provided by:

$$\begin{aligned} \forall h \geq 0, \qquad & f(y_{t+h}^{i,j} \,|\, s_{t+h} = j, s_{t+h-1} = i, \underline{y_{t-p:t+h-1}^{a}}; \Theta) = \\ & \qquad (2\pi)^{-N/2} \left|\Omega_j^u\right|^{-1/2} exp\left\{ -\frac{1}{2} (y_{t+h}^{i,j} - \hat{y}_{t+h}^{i,j})' \, [\Omega_j^u]^{-1} (y_{t+h}^{i,j} - \hat{y}_{t+h}^{i,j}) \right\} \end{aligned}$$

We form the joint and the marginal densities:

$$\begin{aligned} \forall h \geq 0, \;\; & f(y_{t+h}^{i,j}, s_{t+h} = j, s_{t+h-1} = i \,|\, \underline{y_{t-p:t+h-1}^{a}}; \Theta) = \\ & \qquad f(y_{t+h}^{i,j} \,|\, s_{t+h} = j, s_{t+h-1} = i, \underline{y_{t-p:t+h-1}^{a}}; \Theta) \times P(s_{t+h} = j, s_{t+h-1} = i \,|\, \underline{y_{t-p:t+h-1}^{a}}; \Theta) \\ & f(y_{t+h}^{i,j} \,|\, \underline{y_{t-p:t+h-1}^{a}}; \Theta) = \sum_{i=1}^{S} \sum_{j=1}^{S} f(y_{t+h}^{i,j}, s_{t+h} = j, s_{t+h-1} = i \,|\, \underline{y_{t-p:t+h-1}^{a}}; \Theta) \end{aligned}$$

The probabilities needed for the collapsing operation are updated, providing the filtered probabilities needed as input for the next iteration:

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[6] See also Hamilton (1994) or Krolzig (1997).

[7] We do not apply this step for $h = 0$ since the current probabilities $P(s_t^a \,|\, \underline{y_{1:t}^{a}}; \hat{\Theta}_T)$ is supposed at initialization. This assumption is made to encompass existing IRF.

$$\forall h \geq 0, \qquad P(s_{t+h}=j, s_{t+h-1}=i \,\middle|\, \underline{y^a_{t-p:t+h}};\Theta) = \frac{f(y^{i,j}_{t+h}, s_{t+h}=j, s_{t+h-1}=i \,\middle|\, \underline{y^a_{t-p:t+h-1}};\Theta)}{f(y^{i,j}_{t+h} \,\middle|\, \underline{y^a_{t-p:t+h-1}};\Theta)}$$

$$P(s_{t+h}=j \,\middle|\, \underline{y^a_{t-p:t+h}};\Theta) = \sum_{i=1}^{S} P(s_{t+h}=j, s_{t+h-1}=i \,\middle|\, \underline{y^a_{t-p:t+h}};\Theta)$$

The simulated trajectories (conditionally to initial values and future shocks) are obtained as:

$$\forall h \geq 0, \qquad y^{a,b}_{t+h} = \sum_{i=1}^{S}\sum_{j=1}^{S} P(s_{t+h}=j, s_{t+h-1}=i \,\middle|\, \underline{y^a_{t-p:t+h}};\Theta)\cdot y^{i,j}_{t+h}$$

These steps are repeated until horizon H for the shocked trajectory and the baseline, and for all possible combinations of a and b.

3.3 The output

For a sufficiently large number of repetitions, the GIRF is obtained from the shocked trajectory $\left\{ y^{a,b}_{t+h} \,\middle|\, \varepsilon_t, \underline{y^a_{t-p:t-1}}, \underline{e^b_{t:t+h}};\Theta \right\}_{h=0}^{H}$ and the baseline $\left\{ y^{a,b}_{t+h} \,\middle|\, \varepsilon_t = 0_{(N,1)}, \underline{y^a_{t-p:t-1}}, \underline{e^b_{t:t+h}};\Theta \right\}_{h=0}^{H}$ as: $\forall h = 0,\ldots H,$

$$GI_y(h,\varepsilon_t,\underline{y^a_{t-1p:t-1}};\Theta) \approx \frac{1}{B}\sum_{b=1}^{B}\left[\left(y^{a,b}_{t+h} \,\middle|\, \varepsilon_t, \underline{y^a_{t-p:t-1}}, \underline{e^b_{t:t+h}};\Theta \right) - \left(y^{a,b}_{t+h} \,\middle|\, \varepsilon_t = 0_{(N,1)}, \underline{y^a_{t-p:t-1}}, \underline{e^b_{t:t+h}};\Theta \right)\right]$$

State asymmetry can be illustrated using: $\forall h = 0,\ldots H,$

$$GI_y(h,\varepsilon_t,s^a_t = j;\Theta) \approx \frac{\displaystyle\sum_{a=1}^{A} GI_y(h,\varepsilon_t,\underline{y^a_{t-p:t-1}};\Theta)\cdot 1\left\{ P(s^a_t = j \,\middle|\, \underline{y^a_{1:t}};\Theta) = \underset{i\in\{1\ldots S\}}{Max}\, P(s^a_t = i \,\middle|\, \underline{y^a_{1:t}};\Theta) \right\}}{\displaystyle\sum_{a=1}^{A} 1\left\{ P(s^a_t = j \,\middle|\, \underline{y^a_{1:t}};\Theta) = \underset{i\in\{1\ldots S\}}{Max}\, P(s^a_t = i \,\middle|\, \underline{y^a_{1:t}};\Theta) \right\}}$$

with $1(x)$ an indicator function taking the value 1 if x is verified and 0 otherwise. One has thus to compare $GI_y(h,\varepsilon_t,s^a_t = j;\Theta)$ with $GI_y(h,\varepsilon_t,s^a_t = i;\Theta)$ for $j \neq i$.

'*Unconditional*' response is obtained by averaging on initial conditions:

$$\forall h = 0,\ldots H, \quad GI_y(h,\varepsilon_t;\Theta) \approx \frac{1}{A}\sum_{a=1}^{A} GI_y(h,\varepsilon_t,\underline{y^a_{t-p:t-1}};\Theta)$$

One has thus to compare $GI_y(h,\varepsilon_t;\Theta)$ with $GI_y(h,\alpha\varepsilon_t;\Theta)/\alpha$ to assess sign and/or size asymmetries of structural shocks.

3.4 Relation with existing IRFs

Previous IRFs can be expressed as special cases of ours. In Erhmann *et al.* (2003), there is no iteration on probabilities (there is no more change in regime). In Karamé (2010), the probabilities are initialized to 1 or 0, evolve but are not updated. Karamé (2011) show there is no need for drawing future shocks and initial values for these two IRFs. Furthermore, the use of collapsing renders the calculation of the Karamé's IRF instantaneous. Camacho & Perez-Quiros (2011) randomly draw initial conditions but set all future shocks to 0. Updating probabilities is then only possible at the date of the shock. Afterwards, their approach amounts to a naïve approach.

4 Illustration

We want to show that choosing an IRF is not neutral as regards the conclusions. We reproduce the Davis & Haltiwanger (1999) article on US manufacturing gross job flows (from Davis *et al.* 2006, figure 2) by replacing the linear structural VAR representation with a general bivariate MS structural VAR with two lags and two regimes (table 1). The dynamic coefficients are quite different, implying two well differentiated regimes. The probabilities of remaining in regime 1 and 2 are 0.95, which is usual in the literature. Regime 1 is prevailing and switching from the minor regime 2 is quite easy (figure 3). Following the Davis & Haltiwanger preliminary approach, we assume the responses of both creation and destruction to an allocation shock have the same magnitude[8]. We then identify the second shock as an aggregate shock. We compare our GIRF to the ones provided by Erhmann *et al.* (2003), Karamé (2010) and Camacho & Perez-Quiros (2011). Simulation parameters for initial values and future shocks are set respectively to A = 500 and B = 500, which allows to reach stable results.

IRFs deliver qualitatively the same message (figure 4). However, the linear IRF and the Camacho & Perez-Quiros IRF significantly over-estimate the magnitude and the persistence in the responses of job flows in our example.

5 Conclusion

We propose a generalized IRF for MS structural VARs verifying the requirements derived by Koop *et al.* (1996). As the complexity of the algorithm exponentially increases with the forecast horizon, we simulate the model using the collapsing technique. We show that our GIRF encompasses the existing IRFs and that

[8] The whole parameterization (including the identification scheme) is neutral in the analysis since it is used to calculate all IRFs.

conclusions can be influenced by the retained IRFs. Future research may use this tool to provide a formal statistical test for asymmetries.

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Table 1: Parameters for MS(2)-VAR(2) on $y_t = (c_t, d_t)'$

	Regime 1	Regime 2
μ_{s_t}	$\begin{pmatrix} 0.065 \\ 0.406 \end{pmatrix}$	$\begin{pmatrix} -0.843 \\ 2.710 \end{pmatrix}$
Φ_{1,s_t}	$\begin{pmatrix} 0.739 & 0.017 \\ -0.304 & 0.842 \end{pmatrix}$	$\begin{pmatrix} 0.541 & 0.0003 \\ -0.040 & 0.856 \end{pmatrix}$
Φ_{2,s_t}	$\begin{pmatrix} 0.188 & 0.035 \\ 0.377 & 0.0006 \end{pmatrix}$	$\begin{pmatrix} 0.316 & 0.276 \\ -0.025 & -0.22 \end{pmatrix}$
$\Omega^u_{s_t}$	$\begin{pmatrix} 0.044 & -0.023 \\ -0.023 & 0.06 \end{pmatrix}$	$\begin{pmatrix} 0.244 & -0.248 \\ -0.248 & 0.673 \end{pmatrix}$
D_{s_t}	$\begin{pmatrix} 0.171 & 0.121 \\ -0.220 & 0.121 \end{pmatrix}$	$\begin{pmatrix} 0.414 & 0.270 \\ -0.774 & 0.270 \end{pmatrix}$
P	$\begin{pmatrix} 0.958 & 0.041 \\ 0.042 & 0.959 \end{pmatrix}$	

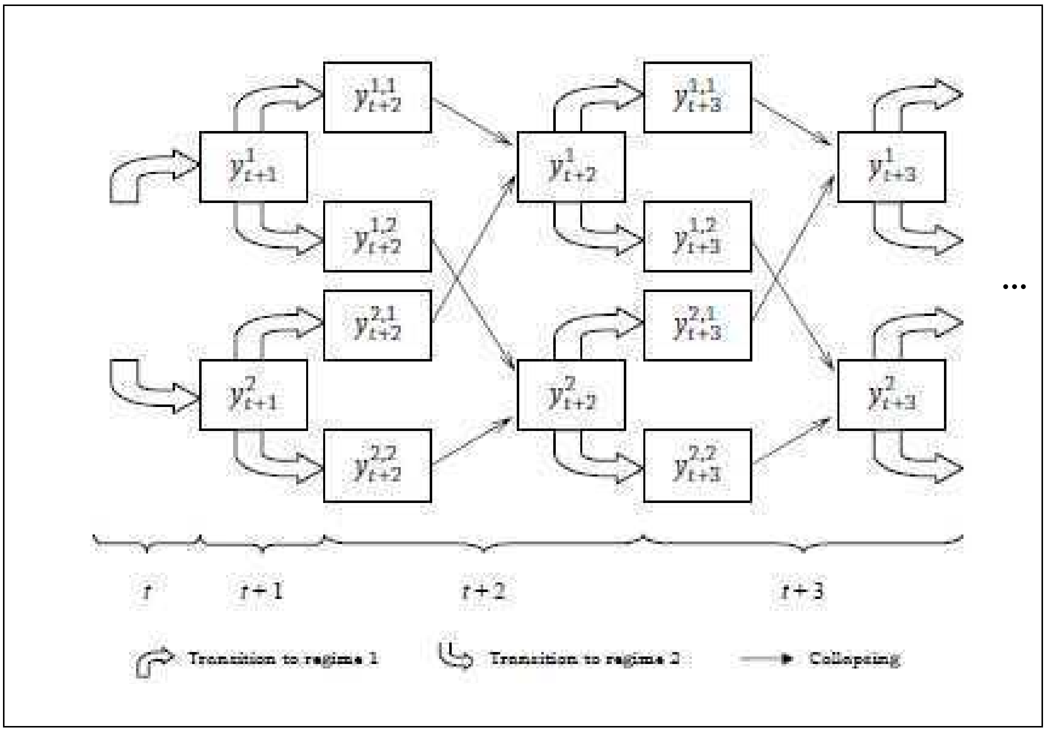

Figure 1: The collapsing technique

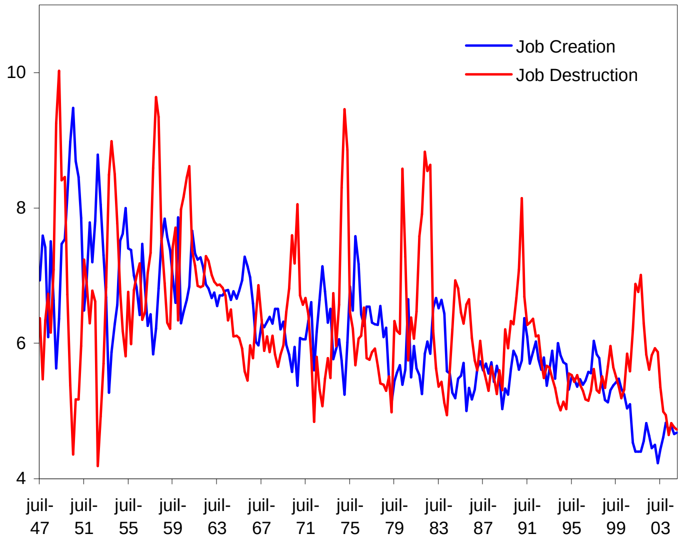

Figure 2: US aggregate gross job flows (1947Q1-2005Q1)

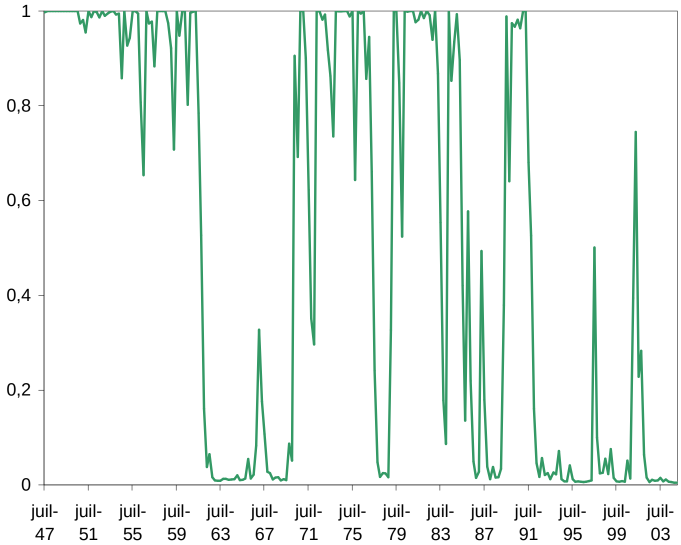

Figure 3: Smoothed probabilities (regime 2)

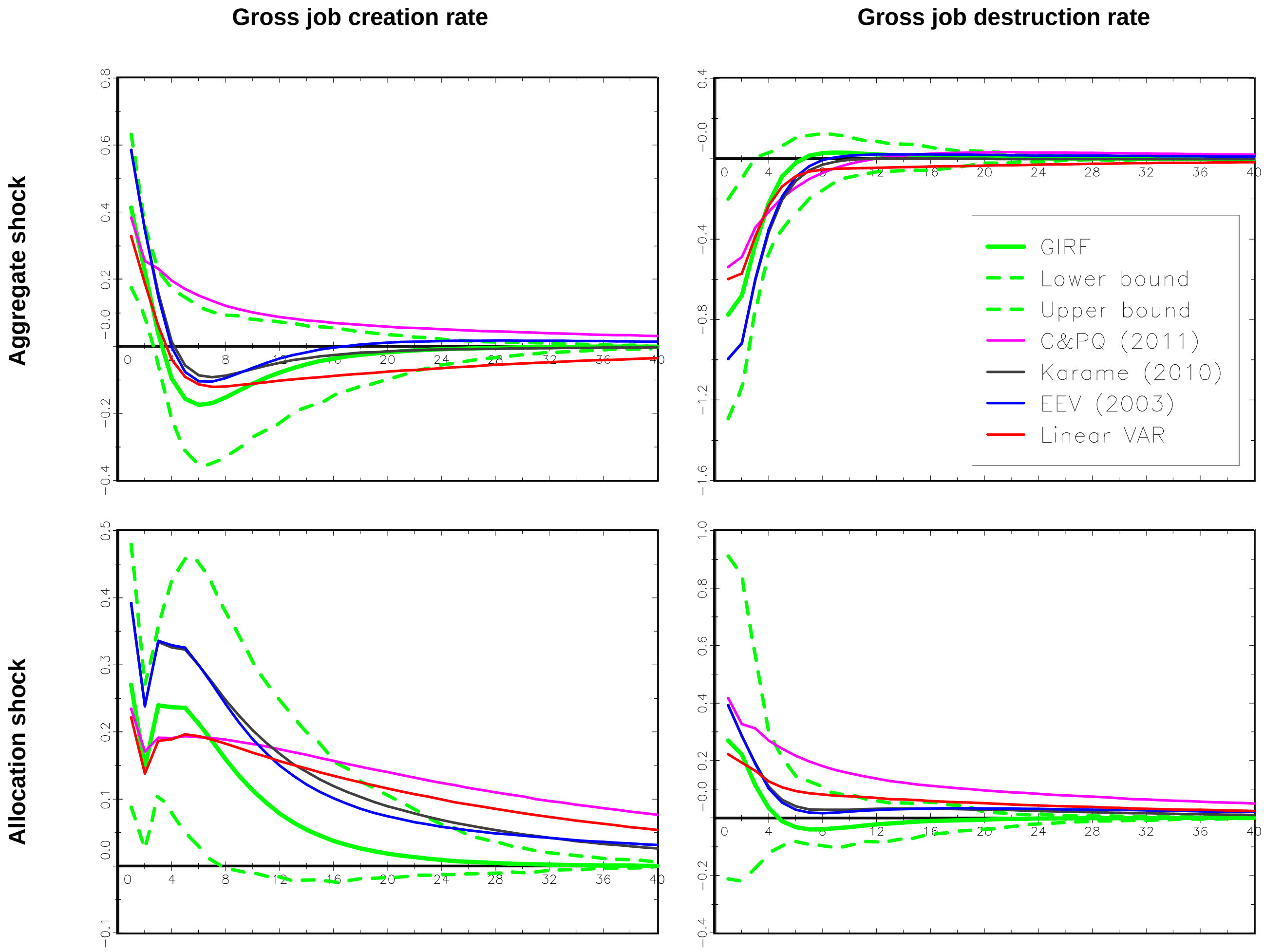

Figure 4: IRF[9] (with the GIRF 95% empirical confidence bands)

[9] C & PQ (2011) and EEV (2003) stand for the IRF proposed respectively by Camacho & Perez-Quiros (2011) and Erhmann *et al.* (2003).