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Monitoring and efficiency wage versus profit sharing in a revolutionary context *

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Abstract

We propose to study the trade off between two incentive strategies (monitoring and efficiency wage versus profit sharing), operated by one firm to induce more efforts among employees. We deal first with a normal context where shirkers bear the risk to be fired. We consider second a particular revolutionary context where employees, even when they go on infinite strikes, would not be dismissed as inspired by the tunisian revolution and more precisely by social movements and general strikes occurring among tunisian workers after revolution. We prove, in the first context, that the profit share to be distributed at equilibrium is positive and depending on the monitoring strategy. In this first context, the two strategies are shown to be strategic complements for low values of risk aversion and strategic substitutes for high ones. We show in the second framework, the emergence of a particular case where the capital holder increases the profit share distributed to employees relative to the one in the first context. This equilibrium profit share is proven to be independent of the monitoring strategy.

Keywords: monitoring, efficiency wage, profit sharing, strikes, risk aversion.
JEL Classification: J41, J52.

1 Introduction

The Arab Spring arisen in 2011 further to the Tunisian revolution, had several consequences especially on the economy of the countries where revolutions occur and more generally all over the economic world. One of its consequences is the multiplication of strikes and social movements among employees who become more affluent and much less controllable by companies hiring them.

The question which arises is related to the most efficient incentive measure that could be implemented by one firm to induce its employees to exert the optimal efforts in this particular context.

In our paper, we propose to study for one firm, the trade off between two incentive strategies: monitoring and efficiency wage versus profit sharing. Indeed, the worker

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may be induced to exert more effort through two different ways. According to the first one, the shareholder may design an incentive contract or a profit sharing scheme such that the worker's remuneration is positively linked to the firm's result; this is the standard incentive theory. According to the second strategy, the shareholder may monitor the worker, so that when shirking he/she is caught and fired with some positive probability, and may then let worker's equilibrium utility exceed his/her reservation utility by some strictly positive amount, in order for the worker to incur a penalty when fired; this is known as the efficiency wage approach.

We deal first with a normal context where the shirker bears the risk to be fired. We consider second a particular revolutionary context where the employee, even when he/she goes on infinite strike, would not be dismissed as inspired by the tunisian revolution and more precisely by social movements and general strikes occurring among tunisian workers after revolution. We prove, in the first ordinary context, that for low levels of employee's risk aversion, the shareholder chooses to induce the maximal effort through only the profit sharing scheme. For high risk aversion level, the shareholder uses both incentive systems. We show in the second revolutionary framework, the emergence of a particular case where the capital holder increases the profit share distributed to the employee relative to the one in the first context. This equilibrium profit share is proven to be independent of the monitoring parameter.

The related literature

Our paper is linked to the efficiency wage literature pioneered by Shapiro and Stiglitz (1984) who argue that paying employees more than the market-clearing wage is a potential way to increase their productivity by reducing shirking among them. There are other measures operated by firms to combat shirking among agents as monitoring (Baker and Jensen, 1988; Varian, 1990; etc.) and incentive contracts, studied by the managerial incentive theory (Freshtman and Judd, 1988), the profit sharing literature (Fitzroy and Kraft, 1987; Cahuc and Dormont, 1997), the agency theory (Jensen and Meckling, 1976), the ownership rights theory (Alchian and Demsetz, 1973), etc. The main purpose of these streams of the literature is to provide the appropriate motivation or/and threat to agents to make sure that they act in the way principals wish. This is in line with one of our aims as we propose among others, to study the most efficient way to prevent shareholder from a possible empowerment of their employee who can go on infinite strikes without bearing the risk to be fired. The difference is that in our model, we combine these different tools (efficiency wage system and monitoring versus profit sharing) and determine under which circumstances the incentive strategy entails additional benefit in terms of employees incentives for effort relative to the other strategies.

An important side of our paper is related to the employee's risk aversion and its impact on the monitoring and incentive systems used by the shareholder. Harris and Raviv (1979) approach this aspect and find a result intimately linked to ours. Indeed, they consider a principal-agent model in an asymmetric information context and propose to characterize the Pareto optimal contract and to evaluate how the alternatives for both parties to acquire information affect the structure of the incentive contract and the monitoring process. The authors prove that there are no monitoring advantages for the principal if the agent is risk neutral and/or when there is no uncertainty about the relationship between the principal's payoff and the agent's action. In all the other

cases, it has been shown that monitoring provide potential gains to the principal. Even if the main purpose of this paper is quite different from ours, the first result shown for a risk neutral agent has been generalized in our paper where we show that for low/neutral risk aversion level, no monitoring through the efficiency wage system is used by the shareholder.
The closest literature to our paper is the one analyzing the trade off between different incentive mechanisms employed by a principal to induce optimal effort from the agent. Holmstrom and Tirole (1993) point off the comparison operated by a publicly traded firm, between using the takeover threat resulting from going public, as a manager incentive device and employing a managerial incentive contract based on the information content of the stock prices available in the stock market. Indeed, according to these authors, inducing managers to exert more efforts in a publicly traded firm can be done through two mechanisms. The first one is the takeover threat implying for managers the risk to be fired and inducing thus less managerial misbehavior. The second one is related to the increasing effect of market liquidity on the stock price informativeness which enables the firm to align the managerial contract to this performance improvement. The authors conclude among others, that both incentive systems are not substitutes and they affect the managers incentives to efforts in different ways. Our paper is different from Holmstrom and Tirole's work as the agents we consider are employees, the firm modeled is not a publicly traded one and the incentive systems used in our analysis are thus different. In the same stream of the literature, Lazear and Rosen (1981) compare between two employees compensation schemes: a piece rate system and a rank-order payment scheme according to which the employees, losers and winners of labor market contests called tournaments, are paid prizes on the basis of their rank order. These prizes are determined in advance and independent of the firm's output contrary to the piece rate system implying that the employee's remuneration is contingent to the firm's revenue. Lazear and Rosen (1981), contrary to our paper, do not completely give the conditions under which rank order tournaments dominate piece rates and vice versa. Among this literature, Demougin and Fluet (2001) approach the more our work as it analyzes the trade off between monetary incentives and monitoring in a principal agent model. In their model, the authors suppose that the agent is risk neutral and faces a limited liability constraint. They prove that the principal, to induce more effort from the agent, employs stronger incentives or monitoring or both. In particular, the principal is proved to proceed to more incentives and less monitoring if monitoring costs are high and/or the agent's limited liability constraint is relaxed. Our approach differs from this paper in numerous ways. First, the agent in our model is supposed to be risk averse which is a more realistic assumption in an asymmetric information context. Second, the objective function of the principal in Demougin and Fluet (2001) is to minimize the total costs of inducing a level of employee's effort which is assumed to be exogenous. These hypothesis are quite different from ours as we consider that the employee's effort is endogenously chosen and affects the principal's revenue representing his/her objective function.

Our paper is organized as follows. In section 2, we present the benchmark model and give the market outcome. Section 3 provides the additional hypothesis and the main results of our model in the revolutionary framework. In section 4, we conclude and give some possible extensions.

All proofs are relegated to an appendix.

2 Benchmark model

2.1 Basic setting

We consider a dynamic framework and a two-period model where a risk-neutral shareholder (the principal) proposes to induce his/her risk-averse employee (the agent) to exert the optimal effort.
The employee's utility function at period (t= $1, 2$) is CARA one:

$$U_t = -\exp[-r\ R_t]$$

where r is the agent's coefficient of absolute risk aversion, R_t his/her income net of monetary cost of effort at period t and U_t his/her utility. The firm's output level qe_t is linearly increasing in the employee's effort level which takes only two possible values, i.e. $e_t \in \{\mathbf{0}, \mathbf{1}\}$. The employee may shirk ($e_t = 0$) or not ($e_t = 1$). The output level is imperfectly observed ($qe_t + \varepsilon$) where ε is an error term assumed to be an iid variable following a normal law $(0, \sigma^2)$.
The employee may be induced by the shareholder to exert a positive effort through two strategies. The first one is a profit sharing scheme i.e. an incentive contract. Indeed, the employee's net income R is the sum of a base wage (μ) and the revenues from the profit share λ of the firm, minus the effort's cost ce_t:

$$R_t = (1 - \lambda)\mu_t + \lambda(qe_t + \varepsilon) - ce_t,$$

with $\lambda \in [0, 1]$ and $c > 0$.

The second possible strategy for the shareholder, is to monitor the employee who may be caught shirking, i.e exerting the minimal effort ($e_t = 0$)) and fired with a strictly positive probability, ($g \in (0, 1]$), and thus loose the surplus of equilibrium utility over the reservation utility, $-\exp[-r\underline{\mu}]$, where $\underline{\mu}$ is the net sure income which can be earned outside the firm (reservation wage for short). Assume that $\underline{\mu} + c < q$ as $\underline{\mu}$ and c represent respectively the direct and indirect costs borne by the shareholder.

We analyze a two-period game occurring in four stages:

- In period 1, in the first stage, the shareholder determines the incentive wage contract parameters (λ and μ_1), given that the employee's share of profits λ is determined for the two periods [1].
- In period 1, in the second stage, the employee chooses his/her effort (e_1).
- In period 2, in the third stage, the shareholder determines the value of the fixed wage (μ_2).

[1] We suppose that the choice of employee's profit share is made for both periods as it is a middle-term decision that will not be changed in each period. This implies the interdependency between both periods.

- In period 2, in the fourth stage, the employee chooses his/her effort e_2.

In that benchmark case, the probability (g) to catch an employee shirking and to fire him/her, is identical over both periods, by opposition to the revolutionary case where this probability is strictly positive in the first period and supposed to become null in the second one. Nevertheless, in both cases, if the employee is caught shirking in period 1, the game ends up for all the parties.

We suppose [2] H_0:

$$\begin{cases} q > \sup[4(c + g\underline{\mu}), \sqrt{4cgr\sigma^2}, \frac{(2-g)gr\sigma^2(4g-1)}{2(1-g)}, \frac{\sqrt{q^2+12cgr\sigma^2}-\sqrt{q^2-4cgr\sigma^2}}{4}], \\ 0 < \underline{\mu} < \frac{q(q-2c+\sqrt{q^2-4cgr\sigma^2})^2+c^2gr\sigma^2}{2q^2+q\sqrt{q^2-4cgr\sigma^2}-2cgr\sigma^2}. \end{cases}$$

2.2 Solution

Proposition 1 In an ordinary context (without revolution), the shareholder chooses to induce the employee to exert the maximal effort ($e_2^* = e_1^* = 1$) and distributes at equilibrium:

- $\lambda^* = \frac{2c}{q+\sqrt{q^2-4cgr\sigma^2}}$ if $r\sigma^2 < \frac{(1-g)(3-g)q^2}{4gc(2-g)^2}$. In that case, the participation constraint is binding and the shareholder uses only the profit sharing scheme to induce the employee to exert a positive effort.

- $\lambda^* = \frac{(1-g)q}{2(2-g)gr\sigma^2}$ if $r\sigma^2 \geq \frac{(1-g)(3-g)q^2}{4gc(2-g)^2}$. In that case, the participation constraint is not binding and the shareholder uses both mechanisms (profit sharing and efficiency wage).

For low risk aversion ($r\sigma^2 < \frac{(1-g)(3-g)q^2}{4gc(2-g)^2}$), the shareholder distributes at equilibrium a positive profit share ($\lambda^* = \lambda' = \frac{2c}{q+\sqrt{q^2-4cgr\sigma^2}}$) as mentioned in Proposition 1 and represented in Fig. 1. Note that the shareholder chooses the profit share λ^* to be distributed at equilibrium to his/her employee so as to maximize his/her gain function G under the employee's participation and incentive constraints.
In that case, both constraints are binding meaning that the shareholder, uses only the profit sharing scheme to induce the employee to exert the maximal effort.

For high risk aversion among employees ($r\sigma^2 \geq \frac{(1-g)(3-g)q^2}{4gc(2-g)^2}$), the shareholder gives at equilibrium ($\lambda^* = \lambda^+ = \frac{(1-g)q}{2(2-g)gr\sigma^2}$) profit shares to his/her employee as represented in Fig. 2.

In that case, the participation constraint is not binding, i.e the worker's equilibrium utility or his/her efficiency utility is higher than the reservation income which represents another way for the shareholder to incite his/her employee to exert the maximal effort.

The intuition behind this result may be related to the costs for the shareholder, of both systems and how these costs vary with respect to the employee's risk aversion, the

[2] H_0 is a sufficient technical hypothesis supposed to ensure that the second period fixed wage at equilibrium is lower than the shareholder's revenue (Proof of Proposition 1).

profits' uncertainty and/or the probability for a shirker to be caught. Indeed, under an incentive contract, the worker's earnings fluctuate with the firms' results, assumed to be affected by a random shock. A risk averse employee requires to receive a risk premium [3] ($\frac{1}{2}r\lambda^2\sigma^2$) over his/her expected earning outside the firm at each period. This risk premium paid by the shareholder, is all the more important as the worker is more risk averse and the firm's result are more variable. An efficiency utility (equilibrium utility), on the contrary, may be constant and independent of the firm's fluctuating result but in order to induce the worker not to shirk, it has to be larger than the worker's reservation wage. The gap between the two, borne by the shareholder, has obviously to be the larger the lower the probability for the worker to be detected when shirking.
When the worker is risk neutral (and/or profits are certain), the incentive contract (profit sharing scheme) is costless for the shareholder. The fraction of marginal profits received by a worker has just to be enough to induce him not to shirk and the fixed component of his/her remuneration may be fixed so as to equalize the worker's equilibrium income and the reservation utility. In that case, monitoring plays no role. This continues to be true under small risk aversion and/or a small variance of profits ($r\sigma^2 < \frac{(1-g)(3-g)q^2}{4gc(2-g)^2}$). However, the risk premium increases with worker's risk aversion. Beyond a given critical value ($r\sigma^2 \geq \frac{(1-g)(3-g)q^2}{4gc(2-g)^2}$), it becomes profitable for the shareholder to limit this risk premium by giving more to the worker than his/her reservation utility through the fixed part of his/her remuneration.

3 Revolutionary context

3.1 Basic setting

We consider the same hypothesis as the benchmark model. We suppose in addition that at period 1, a revolution occurs and the shareholder thus anticipates that it will have two major consequences in period 2. The first one is an increase in the employee's negotiation power, supposed to be null at period 1, and which becomes strictly positive at period 2. The second consequence is the elimination for the shareholder of the possibility to dismiss the unproductive employee (the one infinitely on strike) which we model through the assumption that g becomes null [4] at period 2.
In period 2, due to the increase of the employee's bargaining power, we introduce a Nash bargaining process occurring between the employer and the employee over the fixed part of the employee's utility. The principal's bargaining power is $(1-\alpha)$ versus a bargaining power of ($\alpha \in]0,1]$) for the agent.

Decisions would thus take place through the following game:

[3] As outlined in the proof of Proposition 1.

[4] We had observed these consequences for instance in Cablitec, Yazaki, Ezzahra Leoni's plant, etc which were established in Tunisia and officially closed due to the uncontrolled strikes of workers after tunisian revolution.

- In period 1, in the first stage, the shareholder fixes the incentive wage contract modalities (λ and μ_1)
- In period 1, in the second stage, the employee chooses his/her effort (e_1).
- In period 2, in the third stage, the shareholder and the employee bargain to determine the values of the fixed wage (μ_2).
- In period 2, in the fourth stage, the employee chooses his/her effort e_2.

3.2 Solution

Numerical Result: In the revolutionary framework, a particular case arises in which the shareholder increases the profit share to be distributed to his/her employee, relative to the context without revolution.
At equilibrium, this part is given by:

$$\lambda^* = \frac{c}{q}(> \lambda^+ = \frac{(1-g)q}{2(2-g)gr\sigma^2})$$

under the necessary condition

$$r\sigma^2 \geq \frac{(1-g)q^2}{2cg(2-g)}(> \frac{(1-g)(3-g)q^2}{4gc(2-g)^2})$$

This result is proved numerically $\forall g$ and for the particular values of parameters ($c = 2$, $q = 5$, $r = 0.6$, $\sigma = 5.2$).

This particular equilibrium case where the shareholder distributes $\lambda^* = \frac{c}{q}$, emerges for high risk aversion levels ($gr\sigma^2 \geq \frac{(1-g)q^2}{2c(2-g)}$).
As we can see in Fig. 3, this equilibrium profit share $\lambda^* = \frac{c}{q}$, is higher than the equilibrium part distributed in the benchmark case for high risk aversion levels which is $\lambda^* = \lambda^+ = \frac{(1-g)q}{2(2-g)gr\sigma^2}$. The equilibrium profit share ($\lambda^* = \frac{c}{q}$) is independent of the monitoring parameter g. This can be explained by the fact that ($\lambda^* = \frac{c}{q}$) represents the profit share bounding the incentive constraint of period 2 where the shareholder cannot dismiss his/her employee even if he/she shirks (g is supposed to be null at period 2). In that particular case, monitoring does not play any role and it is completely inefficient whereas the profit sharing scheme entails additional benefit in terms of employee's incentive for effort. We can conclude from that particular case, that profit sharing may be a potential incentive tool to prevent a firm from employee's shirking. Indeed, it may induce the employee to exert maximal effort in a revolutionary context where monitoring is inefficient as the employee may go on strike without bearing any risk to be dismissed.

4 Conclusion

In our paper, we studied the trade off between two incentive strategies (monitoring and efficiency wage versus profit sharing), operated by one firm to induce more efforts among employees. We considered that the worker may be induced not to shirk through two different ways. According to the first one, the shareholder may design a profit sharing scheme such that the worker's remuneration is positively linked to the firm's result; this is the standard incentive theory. According to the second strategy, the shareholder may monitor the worker, so that when shirking he/she is caught and fired with some positive probability, and let worker's equilibrium utility exceed his/her reservation utility by some strictly positive amount, in order for the worker to incur a penalty when fired; this is known as the efficiency wage approach.

We deal first with a normal context where the shirker bears the risk to be fired. We consider second a particular revolutionary context where the employee, even when he/she goes on infinite strikes, would not be dismissed as inspired by the tunisian revolution and more precisely by social movements and general strikes occurring among tunisian workers after revolution.

We prove in the first context that for low levels of employee's risk aversion, the shareholder uses only the profit sharing scheme to induce the employee to exert the maximal effort. Indeed, for low risk aversion, the risk premium to be paid to the employee is low and the incentive scheme is thus not very costly. For high level of risk aversion, we prove that the worker's equilibrium utility is higher than the reservation income which represents the efficiency wage system operated by the shareholder, besides the profit sharing one to incite his/her employee to exert the maximal effort. Indeed, as the risk premium increases with the worker's risk aversion it becomes profitable for the shareholder to limit this risk premium by giving more to the worker than his/her reservation utility through the fixed part of his/her remuneration.

In the revolutionary context, we prove that for high risk aversion level, the equilibrium profit share is higher than the equilibrium part distributed in the benchmark case. In that case, monitoring does not play any role and it is completely inefficient whereas the profit sharing scheme is shown to entail additional benefits in terms of employee's incentive for effort. Profit sharing may be thus a potential incentive tool to prevent a firm from employee's shirking as it may induce the employee to exert maximal effort in a revolutionary context where monitoring is inefficient as the employee may go on strike without bearing any risk to be dismissed.

Several extensions may be considered to enrich this work. We may first make monitoring endogenous and see what would be the consequences on the market outcome. We can second consider, instead of a profit sharing scheme, an employee ownership system where the employee, even fired, continues to receive dividends. It would third be interesting to extend our model to more than two periods and see what will be the effects on the equilibrium outcome.

5 Appendix

Proof of Proposition 1.

The problem being solved by Backward Induction, the proof will be presented in four parts.

- Second period effort's choice

  In the second period, the employee chooses his/her effort by comparing the certainty equivalent of his/her second period utility when he/she exerts the minimal effort $e_2 = 0$ (which we denoted by the case S as shirking) to the one when he/she chooses to exert the maximal effort $e_2 = 1$ (which we denoted by the case N). If he/she chooses to shirk he/she bears the risk to be fired with a probability g.
  • $EC[U_2^S] = g\underline{\mu} + (1-g)(1-\lambda)\mu_2 - \frac{1}{2}r(1-g)^2\lambda^2\sigma^2$
  • $EC[U_2^N] = (1-\lambda)\mu_2 + \lambda q - c - \frac{1}{2}r\lambda^2\sigma^2$
  The condition of doing effort $e_2 = 1$ is thus given by:

  $$EC[U_2^N] > EC[U_2^S]$$

  yielding to

  $$g(1-\lambda)\mu_2 + \lambda q - c - g\underline{\mu} - \frac{1}{2}rg(2-g)\lambda^2\sigma^2 \geq 0$$

- Second period fixed wage

  The second period fixed wage (μ_2) is determined by the shareholder so as to induce the employee to exert the effort $e_2 = 1$ which yields:

  $$\mu_2^+ = \frac{g\underline{\mu} + c - \lambda q + \frac{1}{2}rg(2-g)\lambda^2\sigma^2}{g(1-\lambda)}.$$

  μ_2^+ has to be lower than the second period shareholder's revenue when the second period effort is maximal (q).

- First period efforts' choice

  In the first period, the employee chooses the effort e_1 satisfying his/her incentive constraint:

  $$[(1-\lambda)\mu_1 + \lambda q - c - \frac{1}{2}r\lambda^2\sigma^2] + [(1-\lambda)\mu_2^+ + \lambda q - c - \frac{1}{2}r\lambda^2\sigma^2]$$
  $$\geq$$
  $$2g\underline{\mu} + [(1-g)(1-\lambda)\mu_1 - \frac{1}{2}r\lambda^2(1-g)^2\sigma^2] +$$

$$[(1-g)(1-\lambda)\mu_2^+ + (1-g)\lambda q - c(1-g) - \frac{1}{2}r\lambda^2(1-g)^2\sigma^2]$$

$$\text{CI}: \ g(1-\lambda)[\mu_1 + \mu_2^+] + (1+g)[\lambda q - c] - 2g\underline{\mu} - rg(2-g)\lambda^2\sigma^2 \geq 0$$

- First period remuneration system

At this step, the shareholder maximizes the total certainty equivalent of both periods EC_{total} with respect to both λ and μ_1 under both incentive and participation constraints.

- The objective function:

$$\sum_{i=1}^{2} EC[G_i] = (1-\lambda)(2q - \mu_1 - \mu_2^+)$$

- The participation constraint (CP):

$$[(1-\lambda)\mu_1 + \lambda q - c - \frac{1}{2}r\lambda^2\sigma^2] + [(1-\lambda)\mu_2^+ + \lambda q - c - \frac{1}{2}r\lambda^2\sigma^2] \geq 2\underline{\mu}$$

$$\Leftrightarrow$$

$$2\lambda q - 2c + (1-\lambda)[\mu_1 + \mu_2^+] - r\lambda^2\sigma^2 - 2\underline{\mu} \geq 0$$

- The incentive constraint (CI):

$$g(1-\lambda)[\mu_1 + \mu_2^+] + (1+g)[\lambda q - c] - 2g\underline{\mu} - rg(2-g)\lambda^2\sigma^2 \geq 0$$

Where $\mu_2^+ = \frac{g\underline{\mu} + \frac{1}{2} - \lambda q + \frac{1}{2}rg(2-g)\lambda^2\sigma^2}{g(1-\lambda)}$.
To study which constraint is active, we compute the difference between the first period wage resulting from the incentive constraint $\mu_1^{CI}(\lambda)$ and the first period wage resulting from the participation constraint $\mu_1^{CP}(\lambda)$:

$$\mu_1^{CI}(\lambda) - \mu_1^{CP}(\lambda) = \frac{(1-g)(c - q\lambda + cgr\lambda^2\sigma^2)}{2g(1-\lambda)}$$

having the same sign as the polynomial $P(\lambda) = c - q\lambda + cgr\lambda^2\sigma^2$.

  - For $gr\sigma^2 > \frac{q^2}{4c}$, this polynomial does not have real roots. In that case, the polynomial is positive and the incentive constraint is active $\forall\lambda \in [0,1]$.
    The gain function is thus given by:

    $$G = (1-\lambda)(2q - \mu_2^+ - \mu_1^{CI})$$

    which is equal to:
    $G = (1-\lambda)(2q - \frac{g\underline{\mu} + c - \lambda q + \frac{1}{2}rg(2-g)\lambda^2\sigma^2}{g(1-\lambda)} - \frac{-12 + c(1+g) - 2\alpha + 2g\underline{\mu} - 2gq\lambda + (2-g)gr\lambda^2\sigma^2}{2g(1-\lambda)})$.
    This gain function is concave $\forall\lambda \in [0,1]$ and reaches its maximum on $\lambda^+ = \frac{(1-g)q}{2(2-g)gr\sigma^2}$. Note that $\lambda^+ < 1 \,\forall\, gr\sigma^2 > \frac{q^2}{4c}$.

- For $gr\sigma^2 \leq \frac{q^2}{4c}$, the real roots of the polynomial $P(\lambda)$ are given by $\lambda' = \frac{2c}{q+\sqrt{q^2-4cgr\sigma^2}}$ and $\lambda'' = \frac{2c}{q-\sqrt{q^2-4cgr\sigma^2}}$.

For $gr\sigma^2 \leq \frac{q^2}{4c}$, two cases have to be distinguished:

1. $\lambda' \leq 1 < \lambda''$ if $gr\sigma^2 \leq q - c (< \frac{q^2}{4c})$.
2. $\lambda' < \lambda'' \leq 1$ if $q - c < gr\sigma^2 \leq \frac{q^2}{4c}$ and $q > 2c$ (which is verified under H_0).

Note that the case $1 < \lambda' < \lambda''$ occurring for $q - c < gr\sigma^2 \leq \frac{q^2}{4c}$ and $q \leq 2c$ does not hold under H_0.
We study the first case: $\lambda' \leq 1 < \lambda''$ for $gr\sigma^2 \leq q - c \; (< \frac{q^2}{4c})$
• $\mu_1^{CI}(\lambda) - \mu_1^{CP}(\lambda) \geq 0 \;\; \forall \lambda \in [0, \lambda']$. In that subcase, the incentive constraint is active.
• $\mu_1^{CI}(\lambda) - \mu_1^{CP}(\lambda) \leq 0 \;\; \forall \lambda \in [\lambda', 1]$. In that subcase, the participation constraint is active.

We represent this result in Fig. 4.

As we can see from Fig. 4, the incentive constraint (CI) is active for $\lambda \in [0, \lambda']$ and the participation constraint (CP) is active for $\lambda \in [\lambda', 1]$. For $\lambda = \lambda'$, both constraints are binding.

The determination of the first period remuneration system is thus equivalent for the shareholder to maximize this function:

$$G = \begin{cases} (1-\lambda)[2q - \frac{g\underline{\mu}+c-\lambda q+\frac{1}{2}rg(2-g)\lambda^2\sigma^2}{g(1-\lambda)} - \frac{-12+c(1+g)-2\alpha+2g\underline{\mu}-2gq\lambda+(2-g)gr\lambda^2\sigma^2}{2g(1-\lambda)}] & \text{if } 0 \leq \lambda \leq \lambda', \\ (1-\lambda)[2q - \frac{g\underline{\mu}+c-\lambda q+\frac{1}{2}rg(2-g)\lambda^2\sigma^2}{g(1-\lambda)} - \frac{2q\lambda-1-2\alpha+g(4c+2\underline{\mu}+\lambda(-4q+gr\lambda\sigma^2))}{2g(1-\lambda)}] & \text{if } \lambda' \leq \lambda \leq 1. \end{cases} \quad (1)$$

where $\lambda' = \frac{2c}{q+\sqrt{q^2-4cgr\sigma^2}}$.
We study this function as follows:

- If $\lambda \leq \lambda'$, then:
  • $\frac{\partial G}{\partial \lambda} = \frac{q(1-g)-2(2-g)gr\lambda\sigma^2}{g}$.
  • $\frac{\partial^2 G}{\partial \lambda^2} = -2(2-g)r\sigma^2 < 0$. The gain function is concave at this interval and reaches its maximum on $\lambda = \lambda^+$.

- If $\lambda \geq \lambda'$, then $\frac{\partial G}{\partial \lambda} = -2r\lambda\sigma^2 < 0$. The gain function is decreasing at this interval.

Two cases have to be distinguished:

1. $\lambda^+ > \lambda'$ implying that $r\sigma^2 < \frac{(3-g)(1-g)q^2}{4cg(2-g)^2}$,

2. $\lambda^+ \leq \lambda'$ implying that $r\sigma^2 \geq \frac{(3-g)(1-g)q^2}{4cg(2-g)^2}$,

where $\lambda^+ = \frac{(1-g)q}{2(2-g)gr\sigma^2} \in [0,1] \; \forall \, r\sigma^2 \leq \frac{q(1-g)}{2g(2-g)}$.

- For $\lambda^+ > \lambda^{'}$, the equilibrium is given by $\lambda^* = \lambda^{'} = \frac{2c}{q+\sqrt{q^2-4cgr\sigma^2}}$ inducing a maximal effort at period 2. The effort in period 1 is maximal too and the fixed wages at equilibrium are given by:

$$\begin{cases} \mu_1^* = \frac{(q^2(-1+2c-2\alpha+2g\underline{\mu})+2cgr(1-3cg+2\alpha-2g\underline{\mu})\sigma^2+q(-1+2c-2\alpha+2g\underline{\mu})\sqrt{q^2-4cgr\sigma^2})}{(g(q+\sqrt{q^2-4cgr\sigma^2})(-2c+q+\sqrt{q^2-4cgr\sigma^2}))}, \\ \mu_2^* = \frac{2(q^2\underline{\mu}-cgr(c+2\underline{\mu})\sigma^2+q\underline{\mu}\sqrt{q^2-4cgr\sigma^2})}{(q+\sqrt{q^2-4cgr\sigma^2})(-2c+q+\sqrt{q^2-4cgr\sigma^2})} < q \text{ under } \mathrm{H}_0. \end{cases}$$

The equilibrium where $\lambda^* = \lambda^{'} = \frac{2c}{q+\sqrt{q^2-4cgr\sigma^2}}$ is represented in Fig. 1.

- For $\lambda^+ \leq \lambda^{'}$, the equilibrium is characterized by $\lambda^* = \lambda^+ = \frac{(1-g)q}{2(2-g)gr\sigma^2}$ inducing a maximal effort at period 2. The effort in period 1 is maximal too and the fixed wages at equilibrium are given by:

$$\begin{cases} \mu_1^* = \frac{(q^2(-1+2c-2\alpha+2g\underline{\mu})+2cgr(1-3cg+2\alpha-2g\underline{\mu})\sigma^2+q(-1+2c-2\alpha+2g\underline{\mu})\sqrt{q^2-4cgr\sigma^2})}{(g(q+\sqrt{q^2-4cgr\sigma^2})(-2c+q+\sqrt{q^2-4cgr\sigma^2}))}, \\ \mu_2^* = \frac{8(2-g)gr(c+g\underline{\mu})\sigma^2-(1-g)(3+g)q^2}{8(2-g)g^2r\sigma^2-4(1-g)gq} < q \text{ under } \mathrm{H}_0. \end{cases}$$

The equilibrium where $\lambda^* = \lambda^+ = \frac{(1-g)q}{2(2-g)gr\sigma^2}$ is represented in Fig. 2.

We proceed the same way for the cases $\lambda' < \lambda'' \leq 1$ and we obtain the same results as the previous case.
To summarize, we prove that for $\lambda^* = \lambda^{'}$, both constraints are binding meaning that the shareholder, uses only the profit sharing scheme to induce the employee to exert the maximal effort. When $\lambda^* = \lambda^+$, the participation constraint is not binding.

Proof of the numerical result.

The problem being solved by Backward Induction, the proof will be presented in four parts.

- Second period effort's choice

  In the second period, the employee chooses his/her effort by comparing the certainty equivalent of his/her second period utility when he exerts the minimal effort $e_2 = 0$ (case S) to the one when he chooses to exert the maximal effort $e_2 = 1$ (case N).
  • $EC[U_2^S] = (1-\lambda)\mu_2 - \frac{1}{2}r\lambda^2\sigma^2$
  • $EC[U_2^N] = (1-\lambda)\mu_2 + \lambda q - c - \frac{1}{2}r\lambda^2\sigma^2$
  The condition of doing effort $e_2 = 1$ is thus given by:

$$EC[U_2^N] \geq EC[U_2^S]$$

  yielding to

$$\lambda \geq \frac{c}{q} \, \forall q > c$$

The two cases to be distinguished are thus given by:

$$X = \begin{cases} N & \text{if } \lambda \geq \frac{c}{q}, \\ S & \text{if } \lambda < \frac{c}{q}, \end{cases} \quad (2)$$

- Second period fixed wage

The second period fixed wage (μ_2) is determined through a bargaining process occurring between the shareholder and the employee. The function to be maximized is given by:

$$[EC[U_2] - \underline{\mu}]^{\alpha} * EC[G_2]^{(1-\alpha)}$$

- $EC[U_2] - \underline{\mu} = \mu_2 + \lambda(qe_2 - \mu_2) - ce_2 - \frac{1}{2} r\lambda^2\sigma^2 - \underline{\mu}$
- $EC[G_2] = (1-\lambda)(qe_2 - \mu_2)$

The bargaining process yields to:

$$\mu_2^+ = \frac{2ce_2(1-\alpha) - 2qe_2(\lambda - \alpha) + r(1-\alpha)\lambda^2\sigma^2 + 2\underline{\mu}(1-\alpha)}{2(1-\lambda)}$$

When replacing e_2 by its possible values in both cases N and S, we obtain:
$\mu_2^+[e_2 = 1] = \frac{2c(1-\alpha) - 2q(\lambda-\alpha) + r(1-\alpha)\lambda^2\sigma^2 + 2\underline{\mu}(1-\alpha)}{2(1-\lambda)}$
$\mu_2^+[e_2 = 0] = \frac{r(1-\alpha)\lambda^2\sigma^2 + 2\underline{\mu}(1-\alpha)}{2(1-\lambda)}$.

- First period efforts' choice

In the first period, the employee chooses the effort e_1 satisfying his/her incentive constraint and taking into account his/her effort at the second period.
At the second period, remind that the employee exerts ($e_2 = 1$) which corresponds to the case ($X = N$) if $\lambda \geq \frac{c}{q}$. He/she shirks ($e_2 = 0$) otherwise which corresponds to the case ($X = S$):

$$X = \begin{cases} N & \text{if } \lambda \geq \frac{c}{q}, \\ S & \text{if } \lambda < \frac{c}{q}, \end{cases} \quad (3)$$

– For the case X=N ($e_2 = 1$), the incentive constraint (CI) writes:

$$[(1-\lambda)\mu_1 + \lambda q - c - \frac{1}{2} r\lambda^2\sigma^2] + [(1-\lambda)\mu_2^+[e_2 = 1] + \lambda q - c - \frac{1}{2} r\lambda^2\sigma^2]$$

$$\geq$$

$$2g\underline{\mu} + [(1-g)(1-\lambda)\mu_1 - \frac{1}{2} r\lambda^2(1-g)^2\sigma^2] +$$

$$[(1-g)(1-\lambda)\mu_2^+[e_2 = 1] + (1-g)\lambda q - c(1-g) - \frac{1}{2} r\lambda^2(1-g)^2\sigma^2]$$

$$\text{CI}: \; g(1-\lambda)[\mu_1 + \mu_2^+[e_2 = 1]] + (1+g)[\lambda q - c] - 2g\underline{\mu} - rg(2-g)\lambda^2\sigma^2 \geq 0$$

– For the case X=S ($e_2 = 0$), the incentive constraint after simplifications, writes:

$$g(1-\lambda)[\mu_1 + \mu_2^+[e_2 = 0]] + \lambda q - c - r\lambda^2\sigma^2 g(2-g) - 2g\underline{\mu} \geq 0$$

- First period remuneration system

At this step, the shareholder maximizes the total certainty equivalent of both periods EC_{total} with respect to both λ and μ_1 under both incentive and participation constraints.

1. For the case X=N where ($\lambda \geq \frac{c}{q}$).

   - The objective function:

$$EC_{total} = \sum_{i=1}^{2} EC[G_i] = (1-\lambda)(2q - \mu_1 - \mu_2^+[e_2 = 1])$$

   - The participation constraint (CP):

$$[(1-\lambda)\mu_1 + \lambda q - c - \frac{1}{2}r\lambda^2\sigma^2] + [(1-\lambda)\mu_2^+[e_2 = 1] + \lambda q - c - \frac{1}{2}r\lambda^2\sigma^2] \geq 2\underline{\mu}$$

$$\Leftrightarrow$$

$$2\lambda q - 2c + (1-\lambda)[\mu_1 + \mu_2^+[e_2 = 1]] - r\lambda^2\sigma^2 - 2\underline{\mu} \geq 0$$

   - The incentive constraint (CI):

$$g(1-\lambda)[\mu_1 + \mu_2^+[e_2 = 1]] + (1+g)[\lambda q - c] - 2g\underline{\mu} - rg(2-g)\lambda^2\sigma^2 \geq 0$$

   Where $\mu_2^+[e_2 = 1] = \frac{2c(1-\alpha) - 2q(\lambda-\alpha) + r(1-\alpha)\lambda^2\sigma^2 - 2\underline{\mu}(1-\alpha)}{2(1-\lambda)}$.
   To see which constraint is active, we compute the difference:

$$\mu_1^{CI[X=N]}(\lambda) - \mu_1^{CP[X=N]}(\lambda) = \frac{(1-g)(c - q\lambda + cgr\lambda^2\sigma^2)}{2g(1-\lambda)}$$

   having the same sign as the polynomial $P(\lambda) = c - q\lambda + cgr\lambda^2\sigma^2$.

2. For the case X=S where ($\lambda \leq \frac{c}{q}$):

- The objective function:

$$\sum_{i=1}^{2} EC[G_i] = (1-\lambda)(q - \mu_1 - \mu_2^+[e_2 = 0])$$

- The participation constraint (CP):

$$\lambda q - c + (1-\lambda)[\mu_1 + \mu_2^+[e_2 = 0]] - r\lambda^2\sigma^2 - 2\underline{\mu} \geq 0$$

- The incentive constraint (CI):

$$g(1-\lambda)[\mu_1 + \mu_2^+[e_2 = 0]] + \lambda q - c - r\lambda^2\sigma^2 g(2-g) - 2g\underline{\mu} \geq 0$$

Where $\mu_2^+[e_2 = 0] = \frac{r(1-\alpha)\lambda^2\sigma^2 - 2\underline{\mu}(1-\alpha)}{2(1-\lambda)}$.
We compute the difference:

$$\mu_1^{CI[X=S]}(\lambda) - \mu_1^{CP[X=S]}(\lambda) = \frac{(1-g)(c - q\lambda + cgr\lambda^2\sigma^2)}{2g(1-\lambda)}.$$

As for the case ($X = S$), this difference has the same sign as the polynomial $P(\lambda) = c - q\lambda + cgr\lambda^2\sigma^2$.

We study the sign of the polynomial $P(\lambda)$ as follows:

- For $gr\sigma^2 > \frac{q^2}{4c}$, $P(\lambda)$ does not have real roots. In that case and as in the benchmark model, the polynomial is positive and the incentive constraint is active $\forall\lambda \in [0, 1]$.
The gain function is thus concave $\forall\lambda \in [0, 1]$ and reaches its maximum on $\lambda^+ = \frac{(1-g)q}{2(2-g)gr\sigma^2}$. Note that $\lambda^+ < 1 \,\forall\, gr\sigma^2 > \frac{q^2}{4c}$.

- For $gr\sigma^2 \leq \frac{q^2}{4c}$, the real roots of $P(\lambda)$ are given by $\lambda' = \frac{2c}{q+\sqrt{q^2-4cgr\sigma^2}}$ and $\lambda'' = \frac{2c}{q-\sqrt{q^2-4cgr\sigma^2}}$.

For $gr\sigma^2 \leq \frac{q^2}{4c}$ and as in the benchmark model, two cases have to be distinguished:

1. $\lambda' \leq 1 < \lambda''$ if $gr\sigma^2 \leq q - c (< \frac{q^2}{4c})$.
2. $\lambda' < \lambda'' \leq 1$ if $q - c < gr\sigma^2 \leq \frac{q^2}{4c}$.

We study the first case where $\lambda' \leq 1 < \lambda''$ for $gr\sigma^2 \leq q - c$ $(< \frac{q^2}{4c})$.
• $P(\lambda) = (c - q\lambda + cgr\lambda^2\sigma^2) \geq 0 \;\; \forall\lambda \in [0, \frac{c}{q}[$. In that subcase, the incentive constraint is active.
• $P(\lambda) = (c - q\lambda + cgr\lambda^2\sigma^2) \geq 0 \;\; \forall\lambda \in [\frac{c}{q}, \lambda']$[5]. In that subcase, the incentive constraint is active.
• $P(\lambda) = (c - q\lambda + cgr\lambda^2\sigma^2) < 0 \;\; \forall\lambda \in [\lambda', 1]$. In that subcase, the participation constraint is active.

[5] As the incentive constraint of the second period is binding for $\lambda = \frac{c}{q}$, we suppose that this profit share is sufficient to induce the employee to exert the effort $e_2 = 1$

For $\lambda' \leq 1 < \lambda''$, the determination of the first period remuneration system is equivalent for the shareholder to maximize this function:

$$G = \begin{cases} (1-\lambda)[q - \mu_2[e_2=0] - \mu_1^{CI[X=S]}] & \text{if } \lambda < \frac{c}{q} (1) \\ (1-\lambda)[2q - \mu_2[e_2=1] - \mu_1^{CI[X=N]}] & \text{if } \frac{c}{q} \leq \lambda \leq \lambda' (2), \\ (1-\lambda)[2q - \mu_2[e_2=1] - \mu_1^{CP[X=N]}] & \text{if } \lambda \geq \lambda' (3). \end{cases} \quad (4)$$

Which implies:

$$G = \begin{cases} (1-\lambda)[q - \frac{r(1-\alpha)\lambda^2\sigma^2 + 2\underline{\mu}(1-\alpha)}{2(1-\lambda)} - \frac{2c+2g(1+\alpha)\underline{\mu} - 2q\lambda - gr(-3+2g-\alpha)\lambda^2\sigma^2}{2g(1-\lambda)}] & \text{if } (1) \\ (1-\lambda)[2q - \frac{2c(1-\alpha)-2q(\lambda-\alpha)+r(1-\alpha)\lambda^2\sigma^2+2\underline{\mu}(1-\alpha)}{2(1-\lambda)} - \frac{-2(c+cg\alpha+g(-q\alpha+\underline{\mu}+\alpha\underline{\mu})-q\lambda)+gr(-3+2g-\alpha)\lambda^2\sigma^2}{2g(1-\lambda)}] & \text{if} (2) \\ (1-\lambda)[2q - \frac{2c(1-\alpha)-2q(\lambda-\alpha)+r(1-\alpha)\lambda^2\sigma^2+2\underline{\mu}(1-\alpha)}{2(1-\lambda)} - \frac{2c(1+\alpha)-2q(\alpha+\lambda)+(1+\alpha)(2\underline{\mu}+r\lambda^2\sigma^2)}{2(1-\lambda)}] & \text{if } (3). \end{cases} \quad (5)$$

With $\lambda' = \frac{2c}{q+\sqrt{q^2-4cgr\sigma^2}}$.

This gain function is discontinuous for $\lambda = \frac{c}{q}$ as

$$G^{CI}_{X=N}[\lambda = \frac{c}{q}] - G^{CI}_{X=S}[\lambda = \frac{c}{q}] = q - c > 0 \forall q > c.$$

We study the concavity of the gain function as follows:

- If $\lambda < \frac{c}{q}$, then:

$$\begin{cases} \frac{\partial G^{CI}_{X=S}}{\partial \lambda} = \frac{q(1-g)-2(2-g)gr\lambda\sigma^2}{g}, \\ \frac{\partial^2 G^{CI}_{X=S}}{\partial \lambda^2} = -2(2-g)r\sigma^2 < 0 \end{cases} \quad (6)$$

  The gain function is concave at this interval and reaches its maximum on $\lambda = \lambda^+$.

- If $\frac{c}{q} \leq \lambda \leq \lambda'$, then:

$$\begin{cases} \frac{\partial G^{CI}_{X=N}}{\partial \lambda} = \frac{q(1-g)-2(2-g)gr\lambda\sigma^2}{g}, \\ \frac{\partial^2 G^{CI}_{X=N}}{\partial \lambda^2} = -2(2-g)r\sigma^2 < 0. \end{cases} \quad (7)$$

  The gain function is concave at this interval and reaches its maximum on $\lambda = \lambda^+$.

- If $\lambda > \lambda'$ then $\frac{\partial G^{CP}_{X=N}}{\partial \lambda} = -2r\lambda\sigma^2 < 0$. The gain function is decreasing at this interval.

Three cases have to be distinguished:

1. $\lambda^+ > \lambda'$ implying that $rg\sigma^2 < \frac{(3-g)(1-g)q^2}{4c(2-g)^2}$
2. $\lambda^+ \in [\frac{c}{q}, \lambda']$ implying that $\frac{(3-g)(1-g)q^2}{4c(2-g)^2} \leq rg\sigma^2 \leq \frac{(1-g)q^2}{2c(2-g)}$
3. $\lambda^+ < \frac{c}{q}$ implying that $rg\sigma^2 > \frac{(1-g)q^2}{2c(2-g)}$

Where $\lambda^+ = \frac{(1-g)q}{2(2-g)gr\sigma^2} \in [0,1] \, \forall \, rg\sigma^2 \leq \frac{q(1-g)}{2(2-g)}$

1. For $\lambda^+ > \lambda'$, the equilibrium is given by $\lambda^* = \lambda' = \frac{2c}{q+\sqrt{q^2-4cgr\sigma^2}}$ inducing a maximal effort at period 2. The second and first period efforts are in their maximal level and the equilibrium fixed wages are given by:

$$\begin{cases} \mu_1^* = \dfrac{\frac{gr(-3+2g-\alpha)\sigma^2}{(q+\sqrt{q^2-2gr\sigma^2})^2} - 2(c+cg\alpha+g(-q\alpha+\underline{\mu}+\alpha\underline{\mu}) - \frac{q}{q+\sqrt{q^2-2gr\sigma^2}})}{2g(1-\frac{1}{q+\sqrt{q^2-2gr\sigma^2}})}, \\ \mu_2^* = \dfrac{2c(1-\alpha)+(1-\alpha)(2\underline{\mu}+\frac{r \ \sigma^2}{(q+\sqrt{q^2-2gr\sigma^2})^2}) - 2q(\alpha-1/(q+\sqrt{q^2-2gr\sigma^2}))}{2(1-\frac{1}{(q+\sqrt{q^2-2gr\sigma^2})^2})} < q \text{ under } \mathrm{H}_0. \end{cases}$$

We represent this equilibrium where $\lambda^* = \lambda' = \frac{2c}{q+\sqrt{q^2-4cgr\sigma^2}}$ in Fig. 5.

2. For $\lambda^+ \in [\frac{c}{q}, \lambda']$, the equilibrium is characterized by $\lambda^* = \lambda^+ = \frac{(1-g)q}{2(2-g)gr\sigma^2}$. The second and first period efforts are in their maximal level and the equilibrium fixed wages are given by:

$$\begin{cases} \mu_1^* = \dfrac{(1-g)q^2(-5+2g^2+\alpha-g(1+\alpha))-8(2-g)^2gr(c+cg\alpha+g(-q\alpha+\underline{\mu}+\alpha\underline{\mu}))\sigma^2}{4(2-g)g(2(2-g)gr\sigma^2)-q(1-g)}, \\ \mu_2^* = \dfrac{(2-g)gr[2c(1-\alpha)+2\underline{\mu}+\alpha(2q-2\underline{\mu})-\frac{(-1+g)(q^2)[1-\alpha+g(-9+4g+\alpha)]}{4(-2+g)^2g^2r\sigma^2}]\sigma^2}{2(2-g)gr\sigma^2-q(1-g)} < q \text{ under } \mathrm{H}_0. \end{cases}$$

We represent the equilibrium where $\lambda^* = \lambda^+ = \frac{(1-g)q}{2(2-g)gr\sigma^2}$ in Fig. 6.

3. For $\lambda^+ < \frac{c}{q}$, as represented in Fig. 7, the equilibrium is either given by $\lambda^* = \lambda^+ = \frac{(1-g)q}{2(2-g)gr\sigma^2}$ or by $\lambda^* = \frac{c}{q}$ depending on:

$$G^{CI}_{X=N}[\lambda = \frac{c}{q}] - G^{CI}_{X=S}[\lambda = \lambda^+]$$

The equilibrium is given by $\lambda = \frac{c}{q}$ if

$$G^{CI}_{X=N}[\lambda = \frac{c}{q}] - G^{CI}_{X=S}[\lambda = \lambda^+] \geq 0$$

$$\Leftrightarrow$$

$$F = c(2 - \frac{1}{g}) + \frac{1}{4}q(-4 + \frac{(1-g)^2 q}{(2-g)g^2 r\sigma^2}) - \frac{-2c^2 r\sigma^2 + c^2 gr\sigma^2}{q^2} \leq 0$$

We prove that for the particular values of parameters ($c = 2$, $q = 5$, $r = 0.6$ and $\sigma = 5.2$), the polynomial F is negative $\forall g \in [0,1]$ as represented in Fig. 8.

For these particular values of parameters, the polynomial F is negative which implies that the equilibrium value of profit share to be distributed to the employee is equal to $\lambda^* = \frac{c}{q}$.
This equilibrium value is higher than the equilibrium value of the profit share found under the same condition $gr\sigma^2 \geq \frac{(1-g)q^2}{2c(2-g)}$ in the ordinary case without revolution.

Calculations are analogous for the case $\lambda' < \lambda'' \leq 1$ occurring if $q - c < gr\sigma^2 \leq \frac{q^2}{4c}$. We obtain the same results.

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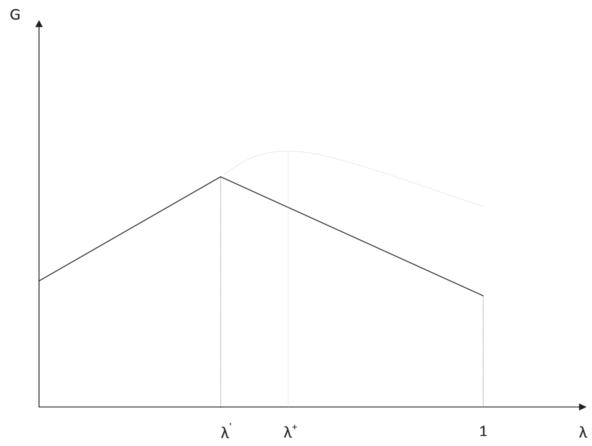

Fig. 1: Equilibrium for $r\sigma^2 < \frac{(1-g)(3-g)q^2}{4gc(2-g)^2}$ in the ordinary context

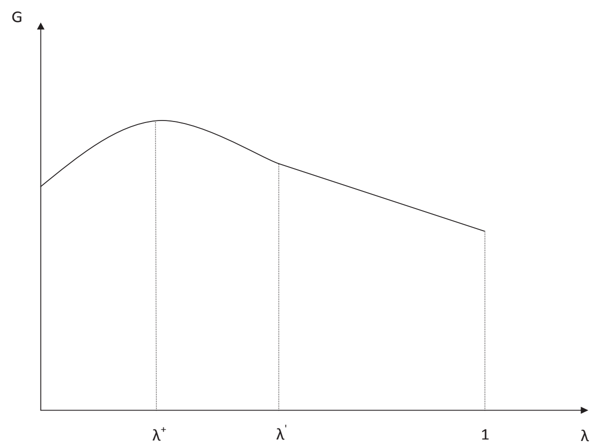

Fig. 2: Equilibrium for $r\sigma^2 \geq \frac{(1-g)(3-g)q^2}{4gc(2-g)^2}$ in the ordinary context

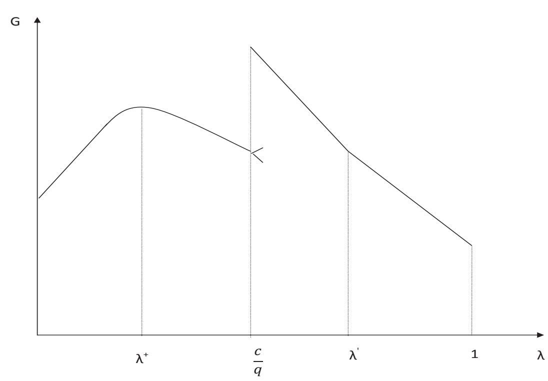

Fig. 3: Equilibrium for $r\sigma^2 \geq \frac{(1-g)q^2}{2cg(2-g)}$ in the revolutionary context

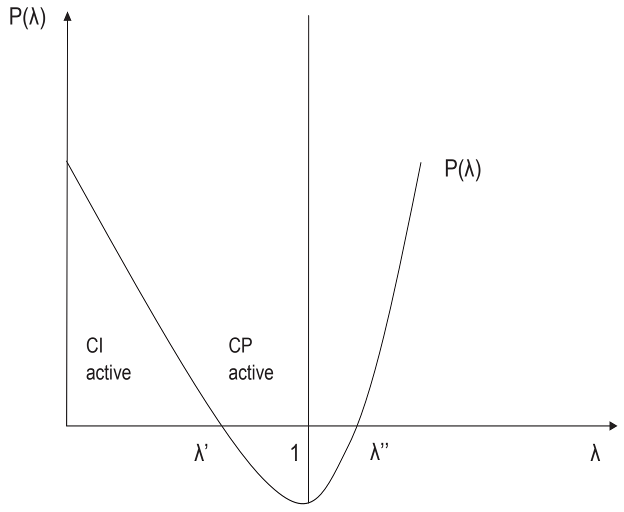

Fig. 4: Incentive and particpation constraints for $\lambda' \leq 1 < \lambda''$

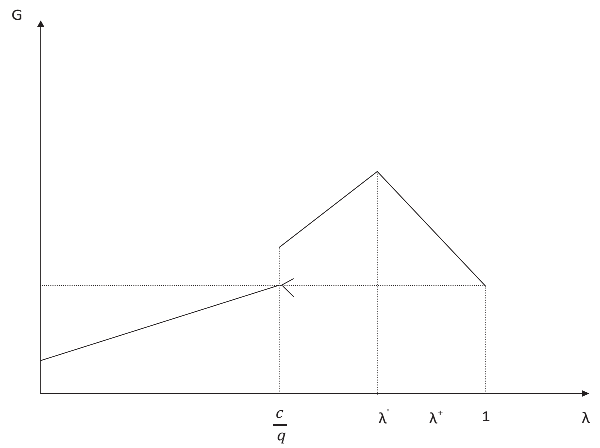

Fig. 5: Gain function for $\lambda^{+} > \lambda^{'}$ in the revolutionary context

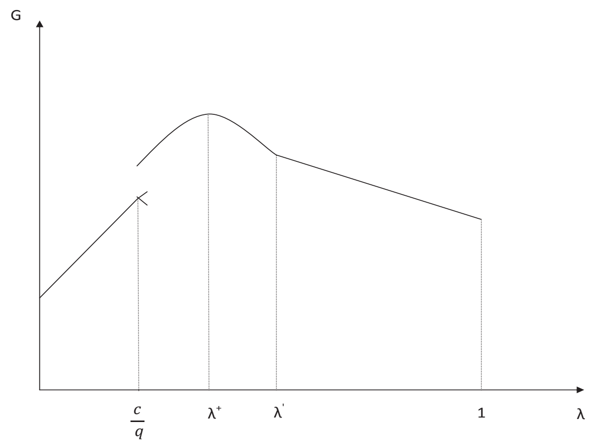

Fig. 6: Gain function for $\lambda^{+} \in [\frac{c}{q}, \lambda^{'}]$ in the revolutionary context

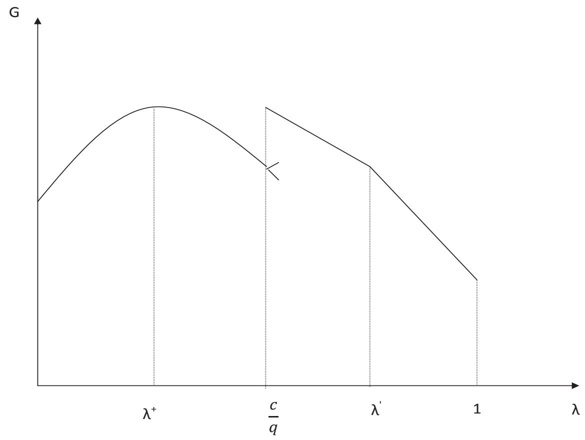

Fig. 7: Gain function for $\lambda^{+} < \frac{c}{q}$ in the revolutionary context

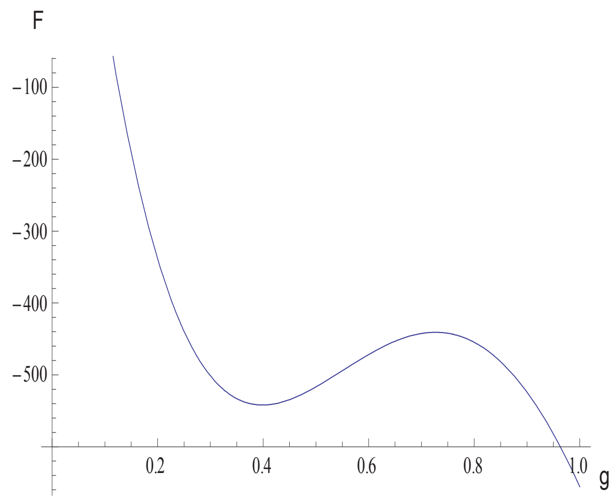

Fig. 8: Polynomial F for the particular values of parameters ($c = 2$, $q = 5$, $r = 0.6$ and $\sigma = 5.2$)