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Risk Factors, Copula Dependence and Risk Sensitivity of a Large Portfolio

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Abstract

In this paper we propose a flexible tool to estimate the risk sensitivity of a high-dimensional portfolio composed of different classes of assets, especially in extreme risk circumstances. We build a so-called Cvine Risk Factors Model (CRFM), which is a non-linear version of a risk factor model in a copula framework. Our tool allows us to decompose the risk of any asset and any portfolio into specific risk directions depending on the context. As an application, we compare the sensitivity of different types of portfolios to extreme risks. We also give an example of a view-type analysis as usually performed by portfolio managers who examine what their portfolio becomes under specific circumstances: here we examine the case of a low inflation context. These analyses allow us to detect changes in the diversification opportunities over time.

Keywords: Regular vine copula, Factorial model, Extreme Risks, Risk Management, Portfolio Management, Diversification. *JEL classification*: G11; G17; G32

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1 Introduction

Diversification opportunities across asset classes can be limited, depending on the market configuration (e.g., Clarke et al., 2005, Bender et al., 2010). As a consequence, the portfolios become more and more complex, with an increasing number of asset classes. The portfolio manager has therefore to deal with multiple risk sources: without being exhaustive, equity, interest rates, inflation, business cycle, emerging market, credit, or liquidity related risks. In such circumstances, it is worth guiding portfolio managers in evaluating their risk exposure especially in the case of extreme risks.

Thus we are facing three challenges:

- it is impossible to summarize all risk sources with a single factor representative of "bad times"[1] (e.g., Ilmanen, 2011);

- correlations, variances and beta coefficients are not relevant risk measures when the assets belong to different classes and particularly in situations of extreme risks;

- finally, the risk sources affect the assets in different ways depending on the period under study. For instance, inflation risk can prompt a positive correlation between stock and nominal bond returns during high unexpected inflation periods (via positive risk premium). However, during low unexpected inflation periods, nominal bonds are used to hedge equity risk (e.g., Campbell et al., 2013).

To deal with the first challenge we refer to multiple risk factor models, but in non-linear structures.

For the second one, we refer to the copula's theory to model the links between the returns of different types of assets.

Finally, concerning the problem of time varying risk exposures, we focus on a limited period (2001-2013 i.e. the two last financial cycles) to avoid the estimation of complex and unstable dynamic models possibly including regime shifts[2]. In particular, we do not consider time-varying risk discount rates (e.g., Cochrane, 2011) which are for example driven by time-varying risk aversion (e.g., Cambpell and Cochrane, 1999) or by the busi-

[1]Bad times refer for example to negative growth, high inflation, deflation, high volatility or correlations between assets, illiquidity spiral, debt crises, etc...

[2]Risk factors are indeed reputed to be regime dependent (e.g., Page and Taborsky, 2011).

ness cycle (e.g., Fama and French, 1989). More generally speaking we do not aim at modelizing the dynamics of the risk premia associated with different risk sources. However our tool allows us to account indirectly for time varying premia as we are able to develop views corresponding to changes in some of them. In the following of the paper, an example of inflation related views is illustrated. Thus, analyzing the effects of such changes in the risk premia is very useful for diversification issues. Indeed, changes in the risk premia involve changes in the diversification opportunities and it is important for a portfolio manager to asses how the performance of his portfolio may be affected by changes in the risk premia and to decide how it should be re-balanced.

More generally speaking, beyond diversification issues, being able to assess multiple interacting financial risk exposures is nowadays crucial since a poor risk management in a financial institution can lead to an individual or potentially systemic default as witnessed during the last crisis. Improving the risk measure of a portfolio is henceforth at the core of regulation issues.

In this regard, the question we mainly address in this paper is the following: what is the sensitivity of a large and complex multi-asset portfolio to extreme shocks related to multiple interacting risk sources? We propose a flexible tool to help answer this question whilst questioning the standard stress testing methods which do not allow to take into account the increased co-movements between markets during critical periods (e.g., Alexander and Sheedy, 2008).

Usual risk measures such as correlations and variance are obviously not relevant for that purpose. Indeed the Gaussian framework is not adapted to characterize the risk of a complex portfolio, in particular in extreme situations. Non-normality and fat tails can be captured by ARCH-type models Engle (1987) for single series and by DCC (e.g., Engle, 2002) for multivariate cases. Here we choose to use the copula's theory which offers, to our opinion, a more flexible framework. Indeed, Heinen and Valdesogo (2009) outline some limitations in using the DCC approach. Moreover we do not aim at proposing a better description of returns than existing downside risk measures (e.g., Ang et al., 2006). Comparing our results with the ones of these alternative methods, like Weiss (2013) for example, is beyond the scope of our paper.

More precisely, the tool we propose allows to specify and estimate what we call a general C-Vine-Risk Factors (CVRF) dependence structure which is an extension of the Canonical Vine Market Sector (CVMS) specification introduced by Heinen and Valdesogo (2009) who are interested in decomposing returns into global (market related) and sector specific components. The C-Vine structure we use is thus organized and constrained according to a factorial structure which we specify a priori.

Several papers address the statistical issue of finding the factorial structure that offers the best fit to the data. For example, Tumminello et al. (2007) apply a hierarchical clustering procedure and estimate a hierarchically nested factor model. Brechmann and Czado (2013), who model market returns with a R-vine copula structure, use a maximum spanning tree and adopt a pure statistical approach to discover the link between the assets. However the resulting relationships between financial series are often not easy to interpret. Contrary to these authors, we do not aim at finding the best (statistical) factorial structure, but we rather aim at proposing a tractable factorial dependence structure which combines a C-Vine factorization with asset's return decompositions that are meaningful from a financial point of view. We do not formally test the statistical fit of the factorial structure to the data, but we check that the simulated returns obtained with our factorial structure are close to the ones obtained with a standard C-Vine structure. Moreover we validate our factorial model because it provides us with results which are consistent with the economic interpretation.

More precisely we work with 35 indexes which cover the main risk sources. Thus, we identify eight risk factors from eight of the 35 indexes which can be viewed as common components for our 35 assets and are in the same time mainly driven by the different risk factors we want to identify: we retain three global indexes that are mostly related to three risk factors, denoted in the following as real interest rates, inflation, and market risk factors and five additional indexes which are specifically affected by (European) sovereign crisis, credit, emerging, commodities and USD related risks. The five latter indexes are used to emphasis the possibility of dealing with "custom risks" which are more specific to an investor's portfolio.

We thus successively focus on extreme shocks to the indexes retained as the main common components of the returns of our assets. Each time, we decompose the response of any asset (more specifically, the change in its expected return induced by the shock) into the contributions of the risk factors we want to capture. Accordingly we are able to quantify the sensitivity of any asset to any extreme shock and to jointly decompose this sensitivity into the marginal contributions of the risk factors.

This decomposition requires simulations of the returns of all assets after drawing extreme values of unconditional and conditional distribution functions in the CVRF model framework. For the latter case we develop an original algorithm.

Our CVRF's core application is thus to propose risk sensitivity analyses for different benchmark portfolios.

The rest of the paper is organized as follows. In section 2, we present the principles of C-vine copulas and our C-vine risk factor model. Section 3 is devoted to the practical implementation with a presentation of the data, a description of the factorial structure and an explanation of how the different types of simulations are implemented. In Section 4 we develop risk sensitivity analyses for the returns of different benchmark portfolios and compare their reactions to different types of extreme shocks. Section 5 concludes.

2 The CRVF structure

After recalling some definitions in Copulas' theory which are useful for what follows, we make a point on the characterization of conditional independence inside a n-dimensional copula and finally illustrate how to build a CVRF model.

2.1 Canonical Vine

A n-dimensional Copula $C(u_1, ...u_n)$ is a cumulative distribution function (cdf) with uniformly distributed marginals $U(0,1)$ on [0,1].

First, a copula is useful to characterize the dependence structure of several random variables whatever their marginal distribution. Indeed, according to the Sklar's theorem (Sklar, 1959) a multivariate cdf F of n random variables $\mathbf{X} = (X_1, ..., X_n)$ with marginals $F_1(x_1)$,...,$F_n(x_n)$ can be written as:

$$F(x_1, ..., x_n) = C(F_1(x_1), ..., F_n(x_n)), \tag{1}$$

where $C(F_1(x_1), ..., F_n(x_n)) = F(F_1^{-1}(u_1), ..., F_n^{-1}(u_n))$ is some appropriate n-dimensional copula and the F_i^{-1}s denote the quantile functions of the marginals. Accordingly, modelling of margins and dependence can be separated. Moreover, for an absolutely continuous F with strictly increasing, continuous marginal cdf F_i, we get the joint density function f by differentiating (1),

$$f(x_1, ..., x_n) = c_{1:n}(F_1(x_1), ..., F_n(x_n)) \cdot f_1(x_1) \cdots f_n(x_n), \tag{2}$$

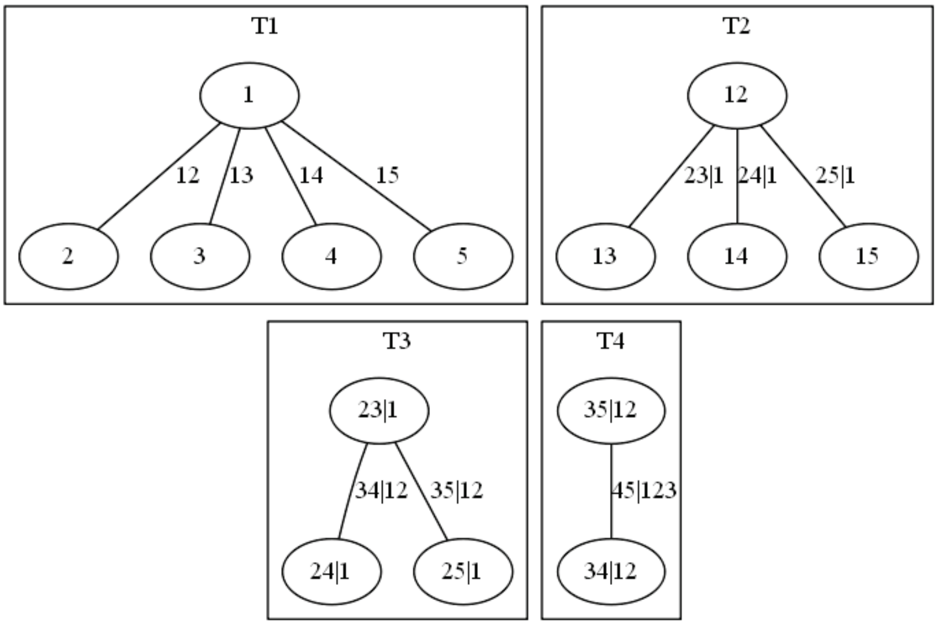
which is the product of the n-dimensional copula density $c_{1:n}(\cdot)$ and the marginal densities $f_i(\cdot)$.

Second, the $n-$dimensional density $c_{1:n}$ can be decomposed as a product of bivariate copulas. The decomposition is not unique (See a possible decomposition in the trivariate case in Appendix). To help organize the possible factorization of the joint density, Bedford and Cooke (2001, 2002) have introduced a graphical model denoted the regular vine. Regular vines (R-vines) are a convenient graphical model to hierarchically structure pair copula constructions. A special case of regular vines is the canonical vine where certain variables play a leading role. Figure 1 shows a canonical vine with five variables. From the figure, we observe that the variable 1 at the root node is a key variable that plays a leading role in governing interactions in the data set.

In the first tree, all nodes are associated with the $X_1, ..., X_5$ variables. For example, the edge 12 corresponds to the copula $c(F_1(x_1), F_2(x_2)$. In the second tree, the edge $23|1$ denotes the copula $c(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1))$. The following trees are built according to the same rules.

Next, in order to organize the dependence structure, it is useful to recall how to

Figure 1: A five dimensional canonical vine tree



characterize the independence of two variables in terms of copula.

2.2 Conditional independence in canonical vine

For a complete n-dimensional canonical vine, there are $n(n-1)/2$ bivariate copulas. This means that the numbers of parameters to estimate is very high for a large size portfolio. In order to simplify the structure, some conditional independence assumptions may be useful.

If one refers to the three dimensional case (see B), assuming that X_1 plays a leading role leads to the following factorization:

$$c_{23|1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)) = 1$$

which means that x_2 and x_3 are independent, conditionally on x_1. Hence, the structure simplifies to:

$$c(F_1(x_1), F_2(x_2), F_3(x_3)) = c_{12}(F_1(x_1), F_2(x_2)) \cdot c_{13}(F_1(x_1), F_3(x_3)).$$

Generally speaking, for a set of conditioning variables, υ and two variables X, Y, assuming that X and Y are conditionally independent given υ, leads:

$$c_{xy|\upsilon}(F_{x|\upsilon}(x|\upsilon), F_{y|\upsilon}(y|\upsilon)) = 1. \tag{3}$$

Heinen and Valdesogo (2009) use this property to develop a simplified version of canonical vine, the *Canonical Vine Market Sector (CVMS) model*. This two-factor model assumes that each asset depends on the market and on its own sector. To include this model into a canonical vine structure with the market and the sectors as root nodes, some conditional independence assumptions need to be introduced: conditionally on the market, sectoral returns are assumed to be independent and asset returns are independent once they belong to different sectors. The remaining conditional dependence between asset returns given the market and the respective sectors is modeled with a multivariate Gaussian copula. Our CVRF model is an extension of the CVMS model.

2.3 CVine-Risk-Factors model(CVRF)

Referring to Heinen and Valdesogo (2009), we introduce a C-Vine copula based factor model. Thus we assume that asset returns depend on several risk factors which mainly explain their dependence structure. Further, we loosen the usual conditional independence assumptions and assume that the risk factors can depend on each other while asset returns can depend on one or several risk factors at the same time. The specification of the factorial dependence structure is therefore more flexible than in the CVMS setting and can be used in accordance with any particular view of a portfolio manager.

As shown for example in table 1, the unconditional and conditional dependence structure can be specified in a symmetric matrix with dummy variables. Among the $n = 8$ assets in the table, we distinguish between CC-type assets which denotes indexes that are common components - or "factors" in the usual sense - for all assets and a-type ones which refer to the other asset of the database. The random variables are the corresponding returns, $r_i, i = 1, ..., 8$. If the dummy variable in the ith row and jth column d_{ij}

Table 1: Factorial dependence matrix M^s

	CC_1	CC_2	CC_3	CC_4	a_1	a_2	a_3	a_4
f_1	1							
f_2	1	1						
f_3	1	0	1					
f_4	1	0	1	1				
a_1	1	1	0	0	1			
a_2	1	0	1	0	0	1		
a_3	1	0	1	1	0	0	1	
a_4	1	0	1	1	0	1	0	1

is equal to 1, the return of asset a_j (or of common component CC_j) is related to the return of asset a_i (or common component CC_i), conditionally on the returns of any asset (or common component) preceding a_j (e.g., r_{j-1},r_{j-2},...,r_1). If $d_{ij} = 0$, the pair is conditionally independent, and the density of the associated copula is equal to one.

Constraining the previous matrix M^s allows us to impose any dependence structure specified "a priori". All diagonal entries are equal to 1 since each asset is obviously linked with itself, but imposing that all elements of the first column are equal to 1, $d_{i,1} = 1$ means that the returns of all assets (including the ones of the common components CC_2, CC_3 and CC_4) depend on the first common component CC_1. We can impose conditional independence or dependence between the common components; here, $d_{3,2} = 0$ and $d_{4,3} = 1$, respectively mean that CC_2 and CC_3 are independent, conditionally on CC_1, and CC_4 and CC_3 are dependent, conditionally on CC_1.

Moreover, each asset can share just one common component as well as several ones: for example, a_1 is only related to CC_2 conditionally on CC_1 while a_3 is related to CC_3 and CC_4, conditionally on CC_1. In the same way, assets can be dependent or independent on each other given the common components: for example, $d_{8,6} = 1$ means that a_2 is related to a_4 given the 4 common components while $d_{8,7} = 0$ indicates that a_3 and a_4 are conditionally independent. Moreover, for each pair of related assets ($d_{i,j} = 1$), the dependence is further characterized by one copula chosen in a set of various bivariate copulas.

In what follows, we retain the simplified structure which is summarized by Table 3 below and Table 5 in D. We describe it in details in the following section.

3 Practical implementation

We work with a database composed of 35 indexes (stocks, bonds, currencies and commodities) from Bloomberg, the observation frequency is weekly over the period January 5, 2001 to September 27, 2013. A description of the data set is given in the Table 4 in the Appendix. We define a particular (factorial) structure capturing the following risk directions: real (interest) rates, inflation, global equity, credit, emerging equity, commodities, USD. Within the C-vine structure, each conditioning is associated with an underlying factor and independence assumptions imply that assets earn only up to 4 risk premiums according to the ”ladder” structure presented in Table 3 given hereafter.

In order to estimate our CVRF model, we use a step-wise procedure. We specify the marginal distributions for each index in the first step. Next, we look for the best bivariate copulas which (conditionally or not) characterize the joint distribution of the returns of all indexes.

3.1 Marginal Distribution

Concerning the marginal distributions, there are different approaches. We have retained a usual GARCH specification to characterize the dynamics of the demeaned returns. [3]

As mentioned before, any other characterization of the marginal distributions could be retained. The details are given in E (See in particular Table 6). For all indexes, we find that the GARCH(1,1) with the GED for the residuals give the best specification. We also observe that the parameters (ν) of the GED distributions are smaller than two in most cases. This means that most of the distributions have thicker tails than the normal distribution.

3.2 The choice of copulas and the tail dependencies

Once the marginal parameters are estimated, we transform the standardized residuals into uniform residuals by using the approach proposed by Meucci (2007). In the second

[3]Some returns display a weak auto-regressive dynamics but the simulation results are weakly affected.

step, we fit a C-Vine copula structure to the set of the standardized residuals while taking into account the constraints imposed by the factorial structure we retain a priori. The bivariate copulas are chosen from a set of families: Gaussian, Student t, Clayton, Frank. Let us recall that the Gaussian and Frank copulas do not allow for any tail dependencies contrary to the Student and Clayton ones which allow for symmetric and lower tail dependencies respectively. Note that we do not have retained dynamic copulas. Dealing with a large portfolio of assets can become very difficult owning to the complexity of joint multivariate modelling. Some approaches have been proposed (Giot and Laurent, 2003, for a review), but most of them are rather complicated to implement and can give similar results to simpler methods (Jin and Lehnert, 2011). Based on the dependence structure, we assume that most tail dependencies are captured by the first three global indexes. In the second column of Table 2, we report the results about the choice of the bivariate copulas between each of the three global indexes and the other indexes. About 50% of the bivariate copulas are found with a tail dependence; indeed, among them, we find about 10% that are Clayton copulas with lower tail dependence and 40% that are Student t copula with tail dependencies in both sides. Accordingly, we find evidences of tail dependence between the indexes in our database. In the third column of Table 2, we have the copula choice between each of the next 5 indexes with others indexes. We can observe that, while conditionally on the previous indexes, tail dependencies become less frequent.

Table 2: Families of bivariate copulas

	Copula with first 3 indexes	Copula with next 5 indexes
Gaussian	33%	36%
Student t	40%	30%
Clayton	7%	3%
Frank	19%	30%
Total	100%	100%

3.3 Factorial structure and dependence

In this section, we present the "ex ante" factorial structure as an "input" needed to specify our model. This structure is undoubtedly not unique and depends on the viewpoints of the portfolio manager. However, economic theory can be useful to specify the factorial structure.

Asset returns can be decomposed as the sum of a risk free rate and risk premium which depend on the exposures to several risks. Specific realized bond premium (e.g., Credit) and realized equity premium are often calculated as the spread between the corresponding asset returns and the nominal bond return of a benchmark (US, AAA-rated, etc.). Regarding commodities and currencies, the link with risk free rate is not clear or nil and must be confirmed empirically. We use a long term Treasury bond instead of cash to get the "risk-free" rate as it better matches the investment horizon of a strategic allocation.

In what follows, we suppose that real rate and inflation risks affect all others indexes; that is why we consider the real rate risk as the first pivotal factor and then the inflation risk as a second one. Once we remove the nominal bond rate factor (split into real rate and inflation risk factors), we get a set of risk factors related to equity, credit, etc. We choose the equity risk as the next pivotal factor since stock markets best reveal the general risk aversion of the investors. This global market risk factor thus captures the last link with all remaining risk factors. Stressing the three first indexes allows us to spread a negative shock to all asset returns included in our database.

Furthermore, we define "residual" factors as those obtained once we have controlled asset returns from the first three risks (labeled global factors). Assets in our portfolio permit us to capture such residual risks like credit Euro debt, emerging market, commodities or USD related risks.

All in all, we retain eight risk factors with three global factors and five more specific ones (called "residual" hereafter). They can be represented by Table 3.

The first global risk factor is captured from the return of the World Inflation Linked

Table 3: Risk exposure and asset classes

Global Risk Factors			Residual Risk Factors	Related Assets
Real Rates				World Inflation Linked Index
Real Rates	Inflation			WGBI All Maturities
Real Rates	Inflation	Equity Market Risk Aversion		MSCI World Index
Real Rates	Inflation	Equity Market Risk Aversion	Euro Debt	Eurozone Sovereign
Real Rates	Inflation	Equity Market Risk Aversion	Credit	Average Corporate Bonds
Real Rates	Inflation	Equity Market Risk Aversion	Emerging	MSCI Emerging Market
Real Rates	Inflation	Equity Market Risk Aversion	Commidity	DJUBS Commodity
Real Rates	Inflation	Equity Market Risk Aversion	Long USD	Dollar Index

Bonds[4]. The second global factor is the inflation risk factor which is measured through the World Government Bond Index once the real rate risk is removed. The MSCI World Index is used to identify the third risk factor (equity risk factor). The five other risk factors are defined respectively from the indexes: IBOXX Euro Sovereigns, Average Corporate Bonds, MSCI Emerging, DJUBS Commodity and Dollar Index. See Table 3.

The structure can be specified in a symmetric matrix as proposed in section 2.2. It is described in Table 5. The first three indexes are linked with all the indexes. However, each index from the fourth to the eighth one is related only to some selected assets. In general, the latter indexes are connected to assets which are from the same asset class with common exposures to a given risk. Nevertheless, some particular assets can be linked to different specific indexes as they share multiple risk exposures. For example the FX Emerging index is not only linked with the emerging index but also with the currency

[4]Major exposures are US, UK and France. Inflation protected bonds index provides a good proxy for an imperfectly estimated real interest rate risk factor, especially because theses bonds can be temporarily prone to liquidity concerns as in 2008. TIPS (US case) are the assets that are considered as risk-free by long-term investors (US) who care about real return (e.g., Ilmanen, 2011).

index.

Table 5 displays Kendall's taus which are consist of average rank correlations between assets and the considered factors (labeled by column). For each column, we have the average rank correlation between the corresponding factor and the assets. Note that the Kendall's tau here in this case measures the conditional dependence between the index in each row and the index in each column with both of them conditioning on the indexes in previous columns (if they are not independent). By using these dependence measures, we can give an overview of the relationships between the risk factors and the assets.

The results we obtain for the Kendall's taus are globally consistent with what is expected from an economic point of view.

Let us focus first on the dependencies between the real rate factor (captured by the returns of inflation linked bonds) and the other assets. It is negatively related to equity returns and positively to bonds while, in absolute terms, the rank correlation is quite obviously much higher for bonds than for equities. Theoretically, we would expect a positive correlation between equities and the real rate factor as higher real rates make the discount rates rise, lowering the price of equities (expressed as the sum of discounted future cash flows). But given the low inflation and the occurrence of flight-to-quality periods observed through our sample, government bonds have been considered as safe haven assets, leading to this negative correlation between real rate factor and equity returns. On the contrary, high yield bond, commodity and currency returns do not exhibit significant correlation with real rate returns, specific risk premia contributing the most as developed thereafter. Only gold and European currencies display a significant dependence. Opportunity cost can explain the negative correlation between Gold and real rate since Gold is a non-interest bearing asset.

The second global factor accounts for the inflation effect. As expected, US and UK inflation linked bonds indexes (most represented in the world index) have no inflation sensitivity while Euro inflation linked bonds index shows some residual inflation sensitivity. However, this negative relation with inflation (positive rank correlation) is much lower than for traditional government bonds (nominal rate). This phenomenon may be due to higher liquidity concerns in the Euro inflation linked bonds index than in the world inflation linked bonds index. Government bond indexes are almost equally sensitive to inflation and to real rate. Nevertheless, we find higher Euro and German inflation sensibilities which are intuitive outcomes (central banks fear inflation).

Regarding risky assets, equities are similarly positively related to inflation. The link between equities and inflation depends mainly on the source of inflation: a demand driven inflation causes a positive relation with stock returns; a supply driven inflation causes a negative correlation (Lee, 2009). In our sample, inflation is mainly driven by demand. Interestingly enough, oil has the highest positive rank correlation with inflation. This result is consistent with the fact that oil has the best inflation hedging ability. Precious metals index (gold) has a weak link with inflation confirming the fact that gold is not really an inflation hedge. Gold is regarded as a safe haven against financial turmoil and US dollar weakness (Ilmanen, 2011).

The third global factor is useful to capture risk aversion through stock market (equity risk factor). All risky assets are positively related to that factor. We notice that government bonds have a weak but negative rank correlation with this risk. We expected such a relation because the studied period encompassed flight-to-quality episodes. On the contrary, Euro government bonds are positively related to risk aversion reflecting the Euro-area debt concerns which occurred at the end of the sample. As expected, corporate and emerging bonds (premia) have positive rank correlations with equity risk. The high-yield sensitivity is naturally higher than the one of the investment grade.

We now turn to residual risk factors[5]. We find a positive exposition of emerging bonds to the credit factor, which is consistent with the fact that emerging bond spread is generally viewed as a measure of an emerging economy's creditworthiness. Besides, the rank correlation is slightly higher than the one with the emerging factor. All currencies or any baskets of currencies have a negative correlation with the USD factor because all studied currencies are short USD whereas our factor is long USD. It is worth noting that gold could be seen as a currency and seems to be negatively related to the USD

[5]Except for emerging equity risk, the word "residual" is suitable because we have taken our risk aversion factor into account in each specific risk.

factor confirming its dollar hedge ability in case of USD weakness. Asian stocks show the most important rank correlation with emerging equity risk reflecting the weight of Asian countries within the emerging index. As stated previously, oil has the highest rank correlation with the commodity factor because of its weight in the global commodity index.

3.4 Return simulations

In this section, we implement two types of simulations with "unconditional" and "conditional" shocks.

3.4.1 General Simulation

To run the simulations, we proceed as follows. First, by using the estimated parameters for the different copulas and the algorithm 2 described in Aas et al. (2009), we simulate N samples from an I dimensional canonical vine for the next period, i.e. $\hat{u}_{1:I,T+1}$. Then, the inverse error distribution functions (G^{-1}) produce a sample of standardized residuals, i.e. $\hat{z}_{1:I,T+1} = G^{-1}(\hat{u}_{1:I,T+1})$. Finally, according to the GARCH equation (19) in E, the estimated GARCH parameters are used to compute the demeaned return forecasts for $i = 1 : I$,

$$\hat{r}_{i,T+1} = \hat{\mu}_i + \hat{\sigma}_{i,T+1}\hat{z}_{i,T+1}$$

with the variance forecast,

$$\hat{\sigma}^2_{i,T+1} = \hat{\omega}_i + \hat{\alpha}_i\hat{\sigma}^2_{i,T}\hat{z}^2_{i,T} + \hat{\beta}_i\hat{\sigma}^2_{i,T}$$

3.4.2 Simulation with extreme unconditional shocks

In the following, we focus on the simulations of uniforms from the vine structure. The process transformation from uniforms to returns remains the same as before. First, we introduce the simulations with unconditional shocks.

With the tool we can implement simulations in accordance to an extreme behavior of one index. Indeed, instead of drawing all the $\hat{u}_{1:I}$ between 0 and 1, we draw samples from an extreme zone (for example from 0 to 0.05) for the stressed variable $\hat{u}_i$, $i \in \{1, ..., I\}$. Since the dependence structure is supposed to be unaffected by the shock, a stress situation for one factor impacts not only the variables which are directly related to this factor but also the other variables in an indirect way, by affecting the key factor at the root node of the C-Vine (which is related to all variables). This means that a sharp decrease of one factor can cause the distress of the whole portfolio if other assets depend positively on this factor. The algorithm is given in Brechmann et al. (2013).

3.4.3 Simulation with extreme conditional shocks

Moreover we can apply shocks from conditional distributions which are interpreted as shocks to specific risk sources. First of all, some definitions of risk sources need to be clarified. The unconditional distribution of an index-factor f summarizes a set of different risk sources, whereas the conditional distribution of factor f_i given another factor f_j can be interpreted as a combination of the remaining risk sources when the risk associated with f_j has been removed. By considering the gap between the returns associated with the unconditional and conditional distributions, we can isolate the effect of a specific risk.

More generally speaking, if we want to apply a shock to the $i-th$ specific risk, it has to involve the conditional cumulative distribution functions $F(x_i|x_1, x_2, ..., x_c)$. In this case, we adapt the simulations for C-Vine copulas involving conditional distributions. For $j \leq c$ or $j > i$, the sampling procedure from $F(x_j|x_1, x_2, ..., x_{j-1})$ is the same as the one described before. However, sampling from $F(x_j), c < j \leq i$ given $F(x_i|x_1, x_2, ..., x_c)$ has to be modified. We develop a new algorithm to specifically deal with simulations involving conditional shocks (See F).

4 Applications to portfolio management in critical contexts

In this section we illustrate how to use the CVRF model for portfolio management. First, we show how to measure the sensitivity of any asset to extreme shocks to any other asset and we decompose the corresponding response into the marginal contributions of the different risk factors. In what follows we just focus on extreme shocks to the 7 first indexes[6] of our data base, since they mainly drive the co-movements of the assets and may consequently dramatically increase the risk of a portfolio in case of extreme events. Indeed, limiting the stress tests to this type of extreme shocks is natural when we extend the same type of analysis to portfolios as presented in a second stage.

4.1 Sensitivity of assets to extreme shocks

We focus on two types of sensitivity analyses. In the first one, we apply an extreme shock to each of the first eight indexes, we measure the global sensitivity of any asset to these shocks and we decompose the sensitivity into the marginal contributions of the different risk factors.

The second one consists in measuring the sensitivity of any asset to a specific risk. Thus we are rather interested in the response of any asset to an extreme shock applied to a relevant index i conditionally on the preceding indexes.

4.1.1 Marginal contributions of the risk factors

Our analyses refer to the risk factors represented by the ladder structure displayed in Table 3. For the first type of sensitivity analysis, we proceed as follows.

We successively consider extreme shocks to each of the 8 first indexes, $i = 1, ..., 8$. For each of these shocks, we decompose the responses of any index j of our data base into the marginal contributions of the different risk factors. Note that this decomposition

[6] We do not consider the Dollar index. A negative shock on the dollar index induces a positive effect on the other indexes. Besides, in general, currencies are indirectly invested in a portfolio through foreign assets.

depends on the origin of the shock.

Suppose that we want to measure the total sensitivity of index j to a shock to one of the first eight indexes, for example index i.

First we compute the "total sensitivity" ($\gamma(j/i)$) of index j to an extreme shock to index i :

$$\gamma(j/i) = E(R_j|F_i(R_i) < 5\%) - E(R_j) \tag{4}$$

where $E(R_j|F_i(R_i) < 5\%))$ denotes the expected return of asset j when the (unconditional) distribution $F_i(R_i)$ is stressed in its extreme negative part and $E(R_j)$ is the return obtained from a general simulation without shock.

The expected return $E(R_j)$ gives us a benchmark value corresponding to a situation without shock. Moreover, according to section 3.4.2, an extreme shock to i simply corresponds to a draw in the extreme (negative) part of $F_i(R_i)$ and one can obtain by simulation the expectation of the return of any index j conditionally on this shock. The sensitivity of index j to the extreme shock to index i is simply calculated as the difference between the conditional and the non-conditional expected returns.

At a second stage, we decompose the previous sensitivity into the marginal contributions of the different risk factors, which have an effective impact on the asset. [7] The decomposition of the total sensitivity can be obtained as follows, by using the simulation process described in section 3.4.3.

First, for any index j, we calculate the marginal contribution of the real rate risk (RR) to the sensitivity as:

$$\gamma_{RR}(j/i) = E(R_j|F_i(R_i) < 5\%) - E(R_j|F_{i|1}(R_i) < 5\%) \tag{5}$$

where $E(R_j|F_{i|1}(R_i) < 5\%)$ is the return of index j when the conditional distribution $F_{i|1}(R_i|R_1)$ is stressed. By conditioning on the return of the first index which is a proxy

[7] For example, if the shock comes from the first index, we can only identify the contribution of the real rate risk factor. In that case, we can not measure the contributions of the other risk factors because they do not impact the first index according to Table 3.

for the real rate risk, the corresponding risk is indeed removed from index i. For example, in the case where $i = 5$, there are only three risk factors underlying the conditional distribution $F_{i|1}(R_i|R_1)$): inflation, equity market and credit. Consequently, the sensitivity of index j to the conditional shock to index i given R_1 comes only from the exposition to the three remaining risk factors. The difference between the responses obtained for the two cases gives us the marginal contribution of the first real rate risk to the total sensitivity.

The simulations are performed according to the algorithm given in F. Similarly, we define the marginal contribution of inflation (INF) and market (M) risk as :

$$\gamma_{INF}(j/i) = E(R_j|F_{i|1}(R_i) < 5\%) - E(R_j|F_{i|1,2}(R_i) < 5\%) \tag{6}$$

$$\gamma_{M}(j/i) = E(R_j|F_{i|1,2}(R_i) < 5\%) - E(R_j|F_{i|1,2,3}(R_i) < 5\%) \tag{7}$$

Finally, the contributions of the residual risks (RES) to the total sensitivity are computed as:

$$\gamma_{RES}(j/i) = E(R_j|F_{i|1,2,3}(R_i) < 5\%) - E(R_j) \tag{8}$$

where $E(R_j|F_{i|1,2}(R_i) < 5\%)$ and $E(R_j|F_{i|1,2,3}(R_i) < 5\%)$ denote the expected return of index j conditionally on extreme draws from the conditional distributions $F_{i|1,2}$ and $F_{i|1,2,3}$.

In the following we denote the set of the previous equations (4) to (8) as (F_1).

The total sensitivity is obviously obtained as the sum of the previous marginal contributions. Note that the "Residual" contribution to the sensitivity includes the exposition of asset j to both specific risk factors (Credit Emerging, etc.) and idiosyncratic risks.

Tables 7 and 8 summarize the results for the sensitivity decompositions we obtain. All results reported in these tables concern deviations from the mean. It is worth outlining that our methodology captures complex linkages between financial assets as it enables to identify risk transmission channels of a specific stress. The following paragraphs gather the most interesting results together.

We notice that the contribution of the real rate risk to the sensitivities is negligible for the risky assets when the world equity market index is stressed (see the column "Real rate" of Table 8). However, in the same case, fixed income assets increase through the first two factors to the same extent (see the positive numbers in columns "Real Rate" and "Inflation" of Table 8). More precisely, we find from our simulations that US equity index should loose around -19.9% a month while US government bond (7-10 years) is expected to rise by +2.9% a month. US government bonds appear as safe haven assets in our sample. This performance of +2.9% is decomposed into +1.3% for real rate marginal effect, +1% for inflation marginal effect and 50bp for market risk marginal effect. The economic reason of this result lies in an increase of the recession risk, meaning lower Central Banks interest rates (lower inflation and less activity).

Such results are useful for risk diversification issues. In this case, higher diversification is obtained for the government bond indexes because the contributions of the two first risk factors to their total sensitivities are positive in the case of a negative shock to the MSCI World Index.

The way the shocks spread across risk factors are not the same for all indexes. For example, the expected return of the Euro Area Equity index (MSCI EMU) roughly decreases in the same magnitude (-1.5%) than the Germany Government Bond index (-1.8%) when we stress our corporate bond index (cf. column "Total" of Table 7). This observation is just a result of our conditional analysis of returns, but our methodology enables us to go a step further and to gauge the contribution of each risk factor. Indeed, we are thus able to highlight that the spreading is completely different. Indeed, the main channel for Germany Government bonds is the real rate risk while the market risk has no marginal effect. On the contrary, for the Euro Area Equity index, the negative corporate bond shock negatively spreads only through the equity market risk, a result that can be explained by an increase in the risk aversion or by growth concerns associated with a negative credit shock. This result is consistent with the economic intuition.

Some Government bonds shouldn't be considered as perfect hedges against equity risk. The dependence between the global risk aversion factor (market risk) and credit bond indexes proves that a negative shock to stocks (growing risk aversion) negatively spreads out to the assets exposed to credit risk. Indeed, the returns of these bonds decrease (as the negative numbers displayed in column "Equity Market/risk aversion" of Table 8, e.g. High Yield Bonds) while the (expected) sovereign bond returns increase (as the positive number in the same column, e.g. government bonds). However, the return of the Euro Sovereigns Index decreases because it can also be considered as exposed to the credit risk. This positive dependence with market risk is consistent with the strong positive link between the European equity markets and the "peripheral" sovereign spreads during the Euro debt crisis reflecting the fears this crisis placed on the future of the Euro zone.

Finally, when we stress the Dollar Index, we noticed that the Emerging Europe and Latin America equity indexes have the highest sensitivity among equity indexes to a falling USD. When we stress the commodity index, we have a negative impact for risky assets through the risk aversion factor. This is specific to our observation period along with commodity prices being driven by demand (rapid increase of consumptions in emerging countries).

To conclude this section, the results show us that the diversification opportunities provided by certain assets could be limited or even disappear. This could be due to the fact that the "hedging" risk factor effects reduce (those which positively react to a negative shock to the stressed assets). Another reason could be that the contribution of the risk factor becomes negative due to either temporary or structural critical economic situations (e.g. Euro Area "peripheral" sovereign bonds were likely to be unrelated to Market risk before the Debt Crisis and solvency issues but this is not the case over our sample period).

4.1.2 Benchmark risk sensitivities

In the previous sensitivity analysis, we measure the exposure of any asset to each risk factor for any stressed index. For example, we get the real rate risk exposures for a shock applied to each of the eight main indexes. This gives us several exposures of a given asset to the same risk. In what follows, we rather focus on the exposure of any asset to a given risk and we retain the asset's response to a shock to the one index which is the most representative for this risk.

Accordingly, we measure the sensitivities of any asset j to the different risk factors according to the following equations ((9) to (16)) denoted hereafter formulas (F_2).

The benchmark Real Rate (RR) risk sensitivity of asset j is given by:

$$\gamma_{RR}(j) = \gamma(j/1) = E(R_j|F_1(R_1) < 5\%) - E(R_j) \tag{9}$$

Similarly, the benchmark Inflation (INF) risk, is defined as:

$$\begin{aligned} \gamma_{INF}(j) &= \gamma(j/2) - \gamma_{RR}(j/2) \\ &= E(R_j|F_{2|1}(R_2) < 5\%) - E(R_j) \end{aligned} \tag{10}$$

while the benchmark Market(M) risk sensitivities is computed as follows:

$$\begin{aligned} \gamma_{M}(j) &= \gamma(j/3) - \gamma_{RR}(j/3) - \gamma_{INF}(j/3) \\ &= E(R_j|F_{3|1,2}(R_3) < 5\%) - E(R_j) \end{aligned} \tag{11}$$

Concerning the specific risk sensitivities, the definition depends on the type of risk. For the benchmark Euro Debt risk (ED) and the benchmark Credit Risk (CR), the sensitivities are specified as:

$$\begin{aligned} \gamma_{ED}(j) &= \gamma(j/4) - \gamma_{RR}(j/4) - \gamma_{INF}(j/4) \\ &= E(R_j|F_{4|1,2}(R_4) < 5\%) - E(R_j) \end{aligned} \tag{12}$$

$$\begin{aligned} \gamma_{CR}(j) &= \gamma(j/5) - \gamma_{RR}(j/5) - \gamma_{INF}(j/5) \\ &= E(R_j|F_{5|1,2}(R_5) < 5\%) - E(R_j) \end{aligned} \tag{13}$$

As a matter of fact, it is reasonable to remove the "nominal rate" component from bond index returns in order to evaluate these specific risk factors.

The benchmark Emerging Market risk (EM) sensitivity of asset i is given by:

$$\begin{aligned}\gamma_{EM}(j) &= \gamma(j/6) - \gamma_{RR}(j/6) - \gamma_{INF}(j/6) - \gamma_M(j/6) \\ &= E(R_j | F_{6|1,2,3}(R_6) < 5\%) - E(R_j) \qquad (14)\end{aligned}$$

In that case, an index or a portfolio are sensitive to this specific risk only if they include emerging assets. We make such a choice because if we didn't remove market risk factor, we wouldn't be able to disentangle global "beta" from emerging "beta".

Finally the benchmark Commodity Risk (CO) sensitivity is defined as the total sensitivity:

$$\gamma_{CO}(j) = \gamma(j/7) = E(R_j | F_7(R_7) < 5\%) - E(R_j) \qquad (15)$$

We make this choice because commodities do not exhibit a clear relationship with nominal interest rate, neither empirically nor theoretically. We can find a graphical representation of formulas (F_2) from Table 3. For example the Credit risk is obtained as the sum of the Equity Market Risk Aversion and the Credit Risk Factors on the fifth row.

In the next section we turn to sensitivity analyses for portfolios instead of single assets.

4.2 Sensitivity analysis for portfolios

In this section, we compare the sensitivity of different portfolios to different types of extreme shocks.

We perform the two previous types of sensitivity analyses by just replacing the return of an asset j by the return of a portfolio P in the formulas (F_1) and (F_2). Then the different types of sensitivity of portfolio P are obtained as linear combinations of the sensitivities of the individual assets:

$$\gamma_{Risk}(P/i) = \sum_{j=1}^{J} w_j \gamma_{Risk}(j/i) \tag{16}$$

where J is the total number of assets in the portfolio, the $w_j, j = 1, ..., J$ define the composition of the portfolio and the $\gamma_{Risk}(j/i)$ are the risk sensitivities of asset j to a shock to asset i for the different risks.

Similarly, we can define the different benchmark risk sensitivities as:

$$\gamma_{Risk}(P) = \sum_{j=1}^{J} w_j \gamma_{Risk}(j) \tag{17}$$

4.3 Sensitivities comparison for a panel of portfolios

In this section, we focus on different portfolios. We begin with describing how the portfolios are composed. Then we implement two types of sensitivity analyses.

First, we compare the reactions of the different portfolios of our panel to extreme shocks to the risk factors by using formulas (F_2) given in section 4.2. Second, we develop a view type analysis where the point is to examine the changes in the global and decomposed sensitivities of the portfolios when the inflation risk is supposed to be zero. Thus we refer to the decompositions of the global sensitivities into the marginal contributions of the different risk factors according to formulas (F_1) and the shock is applied to the Euro Sovereigns index: this case refers to the last sovereign crisis in the Euro area which happened in a low inflation context.

Let us precise which portfolios we retain for the sensitivity analysis.

4.3.1 Portfolio allocations

For the sake of simplicity, before constructing the portfolios, we gather assets into four baskets according to asset classes: Nominal Government Bonds, Other bonds (including Inflation linked, Corporate, High Yield Bonds), Equity, Commodity. For each group, we compute the equally weighted average return of the indexes in the basket. Portfolios are then composed of these four baskets.

More precisely, we examine traditional as well as risk-based portfolios:

- Minimum Variance Portfolio;
- Maximum Diversification Portfolio (e.g., Choueifaty et al., 2013);
- Three portfolios composed according to risk budgeting rules[8];
  - Low Risk Budgeting portfolio with weights 30%, 50%, 15%, 5%, respectively for Nominal Government Bonds, Other bonds, Equity and Commodity;
  - Equal Risk Contribution (ERC) Portfolio (e.g., Maillard et al. 2010);
  - High Risk Budgeting portfolio with weights 10%, 10%, 70%, 10%;
- Balanced allocation with weights 20%, 30%, 45%, 5%;
- High risk allocation with weights 10%, 20%, 60%, 10%.

All allocations in capital are reported in Figure 2. Note that the portfolios are ranked in the order of an increasing risk from left to right.

4.3.2 Comparison of the Benchmark Risk Sensitivities of the portfolios

In the following, we examine at first the sensitivities of the four baskets we have defined to extreme shocks to the different risk factors. Second, we examine the ones of the portfolios in our panel.

The sensitivities of the four baskets to the risk factors are given in Table 9.

We remark that the government bonds are immunized against all risks but negatively exposed to real rate and inflation risks, while the "other bonds" are less exposed to inflation risk but sensible to market and credit risks. In accordance with the economic intuition, credit assets ("other bonds") are jointly exposed to real rate and equity risk.

Equities are mainly affected by market risk but also positively exposed to real rate and inflation risks (with a higher sensitivity to inflation). As expected, equities are also

[8] In risk budgeting strategies the portfolio manager is interested in risk allocations rather than in capital allocations. Thus the weights associated with the different assets in the portfolio are chosen according to their contributions to the risk. Here we assess the risk of a position in terms of volatility.

exposed to emerging risk since our equity basket includes developed and emerging equity indexes. Finally, commodities are mainly related to the commodity risk factor but also positively react to inflation risks.

Concerning the portfolios, we can distinguish between defensive and aggressive portfolios. The allocation in risky assets (equities and commodities) ranges from 10% to 30% for defensive portfolios and from 50% to 70% for the aggressive one (Figure 2).

The risk-based allocations (Minimum Variance, Maximum Diversification, Low Risk Budgeting, ERC and High Risk Budgeting) are rather defensive as they correspond to higher investments in bonds which are less volatile than stocks and commodities, whereas our traditional portfolios (Balanced Allocation and High Risk Allocation) are aggressive portfolios because of their higher volatility.

Table 10 summarizes the benchmark risk sensitivities obtained for the different portfolios.

Concerning the benchmark REAL RATE risk, the defensive portfolios are significantly more sensitive, compared to the more traditional allocations which are little or not impacted by a shock to the real rate risk factor. It illustrates the stress observed in June 2013, which penalized risk-based funds (defensive allocation). Indeed, after Ben Bernanke announced the future reduction of the Federal Reserve Asset Purchase Program, long term interest rates spiked, although inflation remained limited. This rise in real rates did not affect equity markets durably but ERC-based portfolios suffered.

About the benchmark INFLATION risk, we notice that the ERC portfolio is nearly insensitive to this risk. The High (respectively Low) Risk Budgeting portfolio is positively (respectively negatively) exposed to inflation risk. These results indicate that investors who are likely to suffer from a rise in inflation should take into consideration the High Risk Budgeting portfolio methodology for their portfolio construction in order to obtain a balance mix of market risk and inflation risk (hedge).

For the benchmark MARKET risk, as expected, the more traditional allocations are significantly more exposed, compared to the Risk Based Allocations, because they are more invested in risky assets.

Concerning the benchmark CREDIT risk, it is worth emphasizing that the expected return of the aggressive portfolio decreases in a larger magnitude than the one of the defensive portfolios when we stress our corporate bond index (cf. column CREDIT). The reason is that the credit risk is related to the market risk.

Finally, for the EMERGING and COMMODITY risks, we observe that they have a higher impact on riskier portfolios because of the larger weights on these assets. However, when it comes to commodity risk, it is worth noting that we can have a portfolio with a relative small weight on a specific asset, which is however exposed to spillover effects through equity market risk. For example, if one focuses on the reaction of the Balanced portfolio to a commodity shock, one observes a 3.9% decrease in the return, but with a weight of only 5% for the commodities.

In the next section, we show how to develop a view type analysis where the point is to examine how the sensitivities of a portfolio change for a particular scenario, and, more specifically, for a scenario without inflation.

4.3.3 Example of view: scenario with low inflation risk

Here we investigate the behavior of the different portfolios under a zero inflation scenario. We refer to the computations of the global sensitivities and their decompositions into the marginal contributions of risk factors (according to formulas (F_1)). The results in Table 11 give us the total sensitivities and their decompositions when we apply an extreme shock to the Euro zone sovereign bonds (IBOXX Euro Sovereigns index). In the lower part of the table, we report the total sensitivities of the different portfolios, when there are no inflation risk sensitivity.

To give some intuitions on the view exercises, let us look at the situation of the Euro zone in 2014. The inflation declined to very low levels but remained positive (almost zero). In such an economic context, we could assume that inflation risk almost disappears (at least temporarily); thus, the contribution of the inflation risk factor should be zero (a significant increase in inflation normally implies a negative shock to the inflation factor). Ceteris paribus, a stress to the Euro Sovereigns index would cause a loss to the holder of the High Risk allocation portfolio (the portfolio with high capitalization in the risky assets). We indeed observe a 2.5% decrease in the monthly return, whereas the total gain is equal to 1.5% when inflation is not supposed to be zero. That discrepancy is mainly due to the negative contribution of the market risk (-2.7%) and the absence of positive effect of the inflation factor. This means that a critical context in the euro zone combined with a low inflation could seriously harm the risky portfolio by eliminating the benefits resulting from inflation.

It is worth noting that imposing views on the contributions of factors does not need any additional simulations; moreover, calculations are very simple since all effects are additive. Note that the view exercise, although based on a quantitative value, can initially be conceived qualitatively (e.g., Meucci, 2010). For instance, a view on a return can be discretized into five states (highly positive, positive, zero, negative, highly negative). Similarly, if we assume symmetric distributions, we could have views on the factors' contributions according to the following pattern: $\alpha \times$ marginal contribution with $\alpha = -2, -1, 0, 1, 2$.

5 Conclusion

The aim of this paper was to show the practical usefulness of vine copula based models for portfolio management in the case of a large number of assets. We have proposed a CVFR model combining a Canonical Vine and a factorial-type dependence structure specified a priori. Accordingly a portfolio manager can easily use this model to impose any dependence structure reflecting his own risk perception and to decompose the returns into risk factors which are crucial to his opinion (bond, equity, inflation, credit for example), while taking into account complex relationships between the different assets that can not be summarized by simple correlations.

As an application, we have examined the case of a set of 35 indexes of different types - stock, bonds, commodities, currencies. We used a GARCH approach for the marginal distributions. As to copula results, evidences of tail dependence are found between a significant number of indexes.

The factorial-type structure we have specified a priori includes eight indexes as common components from which we have identified eight different risk factors corresponding to real rate, inflation, market, credit, (European) sovereign debt, Emerging, Commodity and USD risks.

The core applications of our model are sensitivity analyses for each asset of our database to extreme shocks to any other asset and particularly to the eight first indexes which mainly account for the co-movements of the assets. Of particular importance is the decomposition we propose for each (total) sensitivity into the marginal contributions of the risk factors. Moreover our computations take into account the complex dependence structure among the 35 indexes we retain, including the tail dependencies which are particularly crucial in case of extreme shocks. All our results are obtained from simulations. In this regard, our approach is semi-parametric.

We have also applied two types of sensitivity analyses to portfolios. For that purpose we have chosen seven types of portfolios, the ones composed according to traditional rules, the others with risk-based allocations. First, we have compared the sensitivities of these portfolios to extreme shocks to the risk factors. Generally speaking, we can claim that the risk-based portfolios are more sensible to the real rate risk while the traditional ones are more exposed to the market risk and the specific risks. Moreover, the latter ones benefit from inflation shocks, which is not the case for the former ones. Besides, we show that the inflation risk could be diversified away with an appropriate balanced portfolio (ERC) while real rate risk and market (equity) risk remain thus the most important concerns which can only be diversified away within a more trivial and concentrated portfolio.

Second, referring to the recent sovereign crisis in Europe, in the general context of low inflation, we have developed a view-type analysis and examined the changes in the responses of the portfolios of our panel to extreme shocks applied to the Euro Sovereigns index when the inflation risk is supposed to be nil. In that case, our results tend to prove that risky portfolios loose their advantage compared to the risk-based portfolios in case of a Sovereign crisis in a low inflation context.

All these sensitivity analyses show that our model is well adapted to provide a portfolio manager with a general measure of the exposition of a wide range of assets and portfolios to various risk sources especially in critical (extreme risk) circumstances. Moreover, the decompositions of the sensitivities we propose into the contributions of the risk factors should help a portfolio manager to choose a mix of asset classes that best diversifies his risks while also reflecting his views on the global economy and financial markets, as summarized by the factorial-type structure he retains a priori.

These sensitivity analyses allow to detect diversification opportunities. Indeed, according to the results we obtain, certain positive risk contributions to the sensitivity under a negative shock may decrease or even disappear for some assets, depending on the economic context. This means that in this case these assets are less attractive in building a diversified portfolio.

Natural directions for future research are diverse. First of all, we can construct a portfolio under the constraint of being hedged against a given risk (factor). Indeed, using directly hedged allocations to a factor as a method of portfolio construction or as analysis tool provides target allocations which protect to some extent the portfolio from some financial markets shocks on certain residual factors. Moreover, if we refer to the drying-up of liquidity at the end of 2008 or the abundance carried by the Quantitative Easing (FED&ECB) it would be interesting to introduce an additional liquidity factor. In addition, for the sake of transparency and simplicity, we have decided to extract the factors from return series of representative and well diversified indexes. We could also refer to unobservable components issuing from statistical models (regressions, orthogonal components) instead of observable market indexes. We can even imagine to implement our analysis by using benchmark factor indexes that regulators could design in order to provide a common framework to measure the risks of financial institutions.

A CVine copulas

According to Kurowicka and Cooke (2007) a regular vine (R-vine) on n variables consists first of a sequence of linked trees $T_1, ..., T_{n-1}$ with nodes N_i and edges E_i for i = 1,..., n, where T_1 has nodes $N_1 = 1, ..., n$ and edges E_1, and for i = 2,..., n-1, T_i has nodes $N_i = E_{i-1}$. Moreover, two edges in tree T_i are joined in tree T_{i+1} only if they share a common node in tree T_i (See Brechman and Czado (2013) for a detailed presentation). A special case of R-vines which is often considered are canonical vines (C-vines). A C-vine is a R-vine if each tree T_i has a unique node with degree $d - i$, the root node.

The general n-dimensional canonical vine (CVine) copula density can be written as following:

$$c_{1:n}(F_1(x_1), ..., F_n(x_n)) = \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{j,j+i|1,...,j-1} \\ (F(x_j|x_1, ..., x_{j-1}), F(x_{j+i}|x_1, ..., x_{j-1})) \tag{18}$$

where $c_{j,j+i|1,...,j-1}$ denotes the bivariate copula between the distributions of x_j and x_{j+i} taken conditionally on $x_1, ..., x_{j-1}$.

B Factorization of a trivariate density

Here we present in details a possible factorization of a joint three-dimensional density function as a product of bivariate copulas and marginal densities.

$$f(x_1, x_2) = c_{12}(F_1(x_1), F_2(x_2)) \cdot f_1(x_1) \cdot f_2(x_2),$$

and the first conditional density is:

$$f(x_2|x_1) = c_{12}(F_1(x_1), F_2(x_2)) \cdot f_2(x_2).$$

In the same way, one possible decomposition of the second conditional density $f(x_3|x_1, x_2)$ is:

$$f(x_3|x_1, x_2) = c_{23|1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)) \cdot f(x_3|x_1),$$

where $c_{23|1}$ is the bivariate copula, applied to the transformed variables $F_{2|1}(x_2|x_1)$ and $F_{3|1}(x_3|x_1))$. Decomposing $f(x_3|x_1)$ further, leads to:

$$f(x_3|x_1, x_2) = c_{23|1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)) \cdot c_{13}(F_1(x_1), F_3(x_3)) \cdot f_3(x_3).$$

Combining the equations above, one obtains the joint density of the three variables as a product of marginal densities and bivariate conditional copulas:

$$\begin{aligned} f(x_1, x_2, x_3) = c_{23|1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)) \cdot c_{12}(F_1(x_1), F_2(x_2)) \\ \cdot c_{13}(F_1(x_1), F_3(x_3)) \cdot f_1(x_1) \cdot f_2(x_2) \cdot f_3(x_3). \end{aligned}$$

Note that Joe (1996) showed that conditional cdf's of the form $F(x|\upsilon)$ where υ is a vector, can be derived recursively from marginal cdf's by

$$h(x, \upsilon, \Theta) = F(x|\upsilon) = \frac{\partial C_{x,\upsilon_j|\upsilon_{-j}}(F(x|\upsilon_{-j}), F(\upsilon_j|\upsilon_{-j}))}{\partial F(\upsilon_j|\upsilon_{-j})},$$

where υ_{-j} denotes the set υ without the jth index. $h(\cdot)$ is the conditional distribution function and Θ denotes the set of parameters for the copula of the joint distribution function of x and υ. Let $h^{-1}(u, \upsilon, \Theta)$ be the inverse of the conditional distribution function that will be used in the simulation.

C Description of data set

Table 4: Description of data set

Factorial Model Data		
CLASS	**NAME**	**SOURCE/TICKER**
Fixed Income	Citigroup WGBI All Maturities	SBWGL index
Fixed Income	IBOXX Euro Sovereigns	QW1A Index
Fixed Income	Citigroup US GBI 7 to 10 Years	SBUS70L Index
Fixed Income	Citigroup UK GBI 7 to 10 Years	SBUK70l Index
Fixed Income	Citigroup Germany GBI All Matu	SBDML Index
Fixed Income	Barclays World Inflation Linked Bonds	BCIW1T Index
Fixed Income	Euro MTS Inflation Linked Bonds	EMTXIGC Index
Fixed Income	Citigroup US Inflation Linked Bonds	SBUSILSI index
Fixed Income	Barclays UK Inflation Linked Bonds	BCIU1T index
Fixed Income	Average Corporate Bonds	Author's calculation
Fixed Income	IBOXX Euro Investment Grade Bonds	IB8A Index
Fixed Income	IBOXX US Investment Grade Bonds	IBOXIG Index
Fixed Income	CS Western Euro High Yield Bonds	DLJWVLUE Index
Fixed Income	IBOXX US High Yield Bonds	IBOXHY Index
Fixed Income	JPM EMBI Global Diversified	JPGCCOMP Index
Equity	MSCI World	NDDLWI Index
Equity	MSCI North America	NDDLNA Index
Equity	MSCI EMU	NDDLEMU index
Equity	MSCI Europe Ex EMU	NDDLEXEU Index
Equity	MSCI Pacific Ex Japan	NDDLPXJ Index
Equity	MSCI Emerging	NDLEEGF Index
Equity	MSCI Emerging Latin America	MXLA index
Equity	MSCI Emerging Asia	MXMS index
Equity	MSCI Emerging Europe	MXMU index
Commodity	DJUBS Commodity	DJUBSTR Index
Commodity	DJUBS Prcioud Metal	DJUBSPR index
Commodity	DJUBS Indudtrial Metal	DJUBSIN index
Commodity	DJUBS Agriculture	DJUBSAG index
Commodity	DJUBS Petrolum	DJUBSPE index
FX	DOLLAR INDEX	DXY Index
FX	EUR-USD	EURUSDCR Curncy
FX	CHF-USD	CHFUSDCR Curncy
FX	GBP-USD	GBPUSDCR Curncy
FX	FX COMMO	Author's calculation
FX	FX EMERGENT	Author's calculation

Author's indexes		
CLASS	**NAME**	**SOURCE/TICKER**
Fixed	**Average Corporate Bonds**	**Average of 4 indexes***
Income	IBOXX Euro Investment Grade Bonds	IB8A Index
	IBOXX US Investment Grade Bonds	IBOXIG Index
	CS Western Euro High Yield Bonds	DLJWVLUE Index
	IBOXX US High Yield Bonds	IBOXHY Index
FX	**FX COMMO**	**Average of 5 indexes****
	AUD-USD	AUDUSDCR Curncy
	NZD-USD	NZDUSDCR Curncy
	NOK-USD	NOKUSDCR Curncy
	CAD-USD	CADUSDCR Curncy
	ZAR-USD	ZARUSDCR Curncy
FX	**FX EMERGENT**	**Average of 8 indexes****
	BRL-USD	BRLUSDCR Curncy
	MXN-USD	MXNUSDCR Curncy
	KRW-USD	KRWUSDCR Curncy
	THB-USD	THBUSDCR Curncy
	IDR-USD	IDRUSDCR Curncy
	PHP-USD	PHPUSDCR Curncy
	TRY-USD	TRYUSDCR Curncy
	PLN-USD	PLNUSDCR Curncy

* volatility-inverse-weighted
** equally-weighted

D Dependence structure

Table 5: Kendall's tau

	Real rate	Inflation	Equity Market Risk Aversion	Euro Debt	Credit	Emerging	Commodity	Long USD
Citigroup WGBI All Maturities	0.60							
MSCI World	-0.21	-0.20						
IBOXX Euro Sovereigns	0.51	0.54	0.17					
Average Corporate Bonds	0.38	0.17	0.29	-[1]				
MSCI Emerging	-0.12	-0.19	0.50	-	-			
DJUBS Commodity	ns[2]	-0.20	0.15	-	-	0.17		
DOLLAR INDEX	-0.14	0.05	-0.11	-	-	-	-	
Euro MTS Inflation Linked Bonds	0.45	0.14	0.12	0.40	-	-	-	-
Citigroup US Inflation Linked Bonds	0.68	ns	ns	-	-	-	-	-
Barclays UK Inflation Linked Bonds	0.62	ns	-0.08	-	-	-	-	-
IBOXX Euro Investment Grade Bonds	0.46	0.34	0.16	0.33	0.42	-	-	-
IBOXX US Investment Grade Bonds	0.47	0.30	0.15	-	0.42	-	-	-
CS Western Euro High Yield Bonds	ns	-0.07	0.24	ns	0.43	-	-	-
IBOXX US High Yield Bonds	ns	-0.07	0.29	-	0.48	-	-	-
Citigroup US GBI 7 to 10 Years	0.55	0.47	-0.09	-	-	-	-	-
Citigroup UK GBI 7 to 10 Years	0.56	0.42	-0.07	-	-	-	-	-
Citigroup Germany GBI All Matu	0.54	0.54	-0.06	0.36	-	-	-	-
EUR-USD	0.12	ns	0.09	-	-	-	-	-0.81
CHF-USD	0.18	ns	ns	-	-	-	-	-0.67
GBP-USD	0.08	ns	0.08	-	-	-	-	-0.52
MSCI North America	-0.19	-0.17	0.82	-	-	-	-	-
MSCI EMU	-0.23	-0.18	0.71	-	-	-	-	-
MSCI Europe Ex EMU	-0.21	-0.19	0.71	-	-	-	-	-
MSCI Pacific Ex Japan	-0.09	-0.17	0.49	-	-	-	-	-
JPM EMBI Global Diversified	0.15	ns	0.32	-	0.28	0.17	-	-
MSCI Emerging Latin America	-0.10	-0.17	0.50	-	-	0.38	-	-
MSCI Emerging Asia	-0.08	-0.17	0.40	-	-	0.73	-	-
MSCI Emerging Europe	-0.05	-0.18	0.38	-	-	0.38	-	-
FX COMMO	ns	-0.12	0.25	-	-	0.22	0.23	-0.41
FX EMERGENT	ns	-0.11	0.33	-	-	0.27	-	-0.36
DJUBS Precious Metals	0.11	-0.09	0.06	-	-	0.17	0.28	-0.20
DJUBS Industrial Metals	ns	-0.17	0.24	-	-	0.19	0.35	-0.06
DJUBS Agriculture	ns	-0.11	0.12	-	-	-	0.43	-0.07
DJUBS Petrolum	0.06	-0.21	0.08	-	-	0.14	0.53	0.06

[1]"-" indicates an independence assumption.
[2]non significant, according to a standard non parametric significant test for the Kendall's tau.

E Characterization of the marginal distributions

Here we present the different results concerning the best marginal distributions for each of the 35 indexes.

Let $r_{i,t}$ denote the return of the ith asset at time t. We estimate the marginal distribution with demeaned return ($\tilde{r}_{i,t}$). For each index, a GARCH(1,1) with generalized error distribution (GED) residuals model can be described as follows:

$$\begin{aligned} r_{i,t} &= \mu_i + \epsilon_{i,t} \\ \epsilon_{i,t} &= \sigma_{i,t} z_{i,t}, z_{i,t} \sim GED_i(\nu) \\ \sigma^2_{i,t} &= \omega_i + \alpha_i \epsilon^2_{i,t-1} + \beta_i \sigma^2_{i,t-1} \end{aligned} \tag{19}$$

For each marginal model, we have a list of parameters for different equations: (μ) in the mean equation, (ω, α, β) in the variance equation and (ν) for the innovation distribution. Standardized residuals from the model are given by

$$\hat{z}_{i,t} = \frac{(r_{i,t} - \hat{\mu}_i)}{\hat{\sigma}_{i,t}}$$

By using the Bayesian information criterion (BIC), we select the best model from a list of possible models. The volatility specification we chose the most commonly used GARCH(1,1). Besides, the innovation distribution can be selected among Gaussian, Student t and generalized error distribution. Estimation results of marginal distributions can be found in 6.

Table 6: Marginal distribution parameters. The 35 indexes have the same specification: GARCH(1,1)-GED. From distribution parameters, we observe that nearly all the distributions have thicker tails than that of the normal distribution.

	1	2	3	4	5	6	7
μ	0.0011	0.0008	0.0008	0.0009	0.0011	0.0025	0.0008
ω	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
α	0.0650	0.0529	0.1801	0.0912	0.1427	0.1282	0.0609
β	0.9005	0.9082	0.7693	0.8498	0.8336	0.7949	0.9173
ν	1.4342	1.9641	1.5545	1.5719	1.4693	1.4459	1.5076
Distribution	GED	GED	GED	GED	GED	GED	GED

	8	9	10	11	12	13	14
μ	-0.0004	0.0010	0.0012	0.0013	0.0010	0.0012	0.0013
ω	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
α	0.0635	0.1341	0.1042	0.0722	0.0787	0.0757	0.1279
β	0.9005	0.8156	0.8517	0.9171	0.8872	0.9035	0.8758
ν	1.9646	1.5131	1.4429	1.6323	1.7285	1.3339	1.1749
Distribution	GED	GED	GED	GED	GED	GED	GED

	15	16	17	18	19	20	21
μ	0.0011	0.0012	0.0011	0.0010	0.0007	0.0008	0.0004
ω	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
α	0.1285	0.0444	0.0489	0.0629	0.0579	0.0845	0.1038
β	0.8628	0.9424	0.9418	0.9221	0.9150	0.8594	0.8469
ν	1.0000	1.6900	1.8114	1.9413	1.9377	1.8676	2.0295
Distribution	GED	GED	GED	GED	GED	GED	GED

	22	23	24	25	26	27	28
μ	0.0010	0.0005	0.0010	0.0016	0.0019	0.0029	0.0022
ω	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
α	0.1596	0.1557	0.2337	0.1247	0.2879	0.1094	0.1605
β	0.7938	0.8243	0.7277	0.8465	0.6732	0.8304	0.7787
ν	1.4232	1.5776	1.4394	1.5346	1.1229	1.5044	1.5654
Distribution	GED	GED	GED	GED	GED	GED	GED

	29	30	31	32	33	34	35
μ	0.0026	0.0012	0.0012	0.0024	0.0015	0.0005	0.0020
ω	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
α	0.1070	0.0948	0.1507	0.0570	0.0990	0.1121	0.0835
β	0.8355	0.8643	0.7909	0.9144	0.8789	0.8503	0.8826
ν	1.3569	1.4903	1.3839	1.3983	1.7459	1.6148	1.6667
Distribution	GED	GED	GED	GED	GED	GED	GED

F Algorithm for conditional simulations

Let us consider the $i-th$ specific risk characterized by the conditional cumulative distribution function $F(x_i|x_1, x_2, ..., x_c)$ with $1 < c < i$. First, sample ω_j, $j = 1, ..., i-1, i+1, ..., n$ independent uniform on [0,1], and ω_i uniform on a defined interval $[Min, Max] \in [0, 1]$.

Then, we set

$$\begin{aligned}
\omega_1 &= F(x_1) \\
\omega_2 &= F(x_2|x_1) \\
.. &= ... \\
\omega_c &= F(x_c|x_1, x_2, ..., x_{c-1}) \\
\omega_{c+1} &= F(x_{c+1}|x_1, x_2, ..., x_c, x_i) \\
.. &= ... \\
\omega_{i-1} &= F(x_{i-1}|x_1, x_2, ..., x_{i-2}, x_i) \\
\omega_i &= F(x_i|x_1, x_2, ..., x_c) \\
\omega_{i+1} &= F(x_{i+1}|x_1, x_2, ..., x_i) \\
.. &= ... \\
\omega_n &= F(x_n|x_1, x_2, ..., x_{n-1})
\end{aligned}$$

$V = (\nu_{j,k}), j \in 1, ..., n; k \in 1, ..., j$ is an lower triangular matrix to store the conditional distribution function and $\Theta = (\theta_{j,k}), j \in 1, ..., n; k \in 1, ..., j$ is the matrix of parameters. We can easily get $x_i = w_i$. Like in the general algorithm, the first for loop runs over the variables from 2 to c. In this for loop, we have two other sub-for loops. The first one samples the variable x_j, $j \in 2, ..., c$ with the h^{-1} function and the second one gives the conditional distribution function needed for sampling the x_{j+1} by using the h-function. The variable x_i is sampled in the following procedure $x_i = F^{-1}(w_i|x_1, x_2, ..., x_c)$. The second for loop runs over the variables from $c+1$ to n. For sampling the variable $x_j, c < j < i$, we need the corresponding conditional distribution which is computed in the If loop, $F(x_j|x_1, x_2, ..., x_{j-1}) = F^{-1}(w_j, F(x_i|x_1, x_2, ..., x_j))$ where the argument $F(x_i|x_1, x_2, ..., x_j)$ is computed at the end of the last for loop. Then, a for loop samples the variable $x_j, c < j < n$ with the h^{-1} function. The remaining part of the algorithm provides the conditional distribution functions as arguments required for sampling the next variable.

Algorithm 1 Simulate sample from a C-vine model given the conditional distribution. Generates one sample x_1,x_2...x_n.

Sample ω_1,ω_2,...,ω_{i-1},ω_{i+1},...,ω_n independent uniform on [0,1].
Sample ω_i uniform on a defined interval $[Min, Max] \in [0, 1]$.
$x_1 = \nu_{1,1} = w_i$
for $j \leftarrow 2, ..., c$
  $\nu_{j,1} = w_j$
  for $k \leftarrow j - 1, ..., 1$
    $\nu_{j,1} = h^{-1}(\nu_{j,1}, \nu_{k,k}, \theta_{j,k})$
  end for
  $x_j = \nu_{j,1}$
  for $l \leftarrow 1, ..., j - 1$
    $\nu_{j,l+1} = h(\nu_{j,l}, \nu_{l,l}, \theta_{j,l})$
  end for
  if j=c **then**
    $\nu_{i,1} = w_i$
    for $k \leftarrow c, ..., 1$
      $\nu_{i,1} = h^{-1}(\nu_{i,1}, \nu_{k,k}, \theta_{i,k})$
    end for
    $x_i = \nu_{i,1}$
    for $p \leftarrow 1, ..., c$
      $\nu_{i,p+1} = h(\nu_{i,p}, \nu_{p,p}, \theta_{i,p})$
    end for
  end if
end for
for $j \leftarrow c + 1, ..., i - 1, i + 1, ..., n$
  $\nu_{j,1} = w_j$
  if $j < i$ **then**
    $\nu_{j,1} = h^{-1}(\nu_{j,1}, \nu_{i,j}, \theta_{i,j})$
  end if
  for $k \leftarrow j - 1, ..., 1$
    $\nu_{j,1} = h^{-1}(\nu_{j,1}, \nu_{k,k}, \theta_{j,k})$
  end for
  $x_j = \nu_{j,1}$
  if $j < n$ **then**
    for $l \leftarrow 1, ..., j - 1$
      $\nu_{j,l+1} = h(\nu_{j,l}, \nu_{l,l}, \theta_{j,l})$
    end for
  end if
  if $j < i$ **then**
    $\nu_{i,j+1} = h(\nu_{i,j}, \nu_{j,j}, \theta_{i,j})$
  end if
end for

With this algorithm, we can sample from a C-vine model given that the conditional distribution $F(x_i|x_1, x_2, ..., x_c)$ belongs to a given interval. This means that we can capture the outcome for all variables (returns) in the extreme case where the value of the conditional distribution is drawn between 0% and 5% for instance.

G Sensitivity analysis for the assets

Table 7: Decomposition of sensitivity for different indexes following shocks to fixed income indexes

RELATED FIXED INCOME RISK FACTORS	WORLD INFLATION LINKED					Citigroup WGBI All Maturities					IBOXX € EZSOV					Average Corporate Bonds				
		GLOBAL			SPECIFIC		GLOBAL			SPECIFIC		GLOBAL			SPECIFIC		GLOBAL			SPECIFIC
	TOTAL	Real rate	Inflation	Market equity Risk aversion	-	TOTAL	Real rate	Inflation	Market equity Risk aversion	-	TOTAL	Real rate	Inflation	Market equity Risk aversion	Euro debt	TOTAL	Real rate	Inflation	Market equity Risk aversion	Credit
RISK FACTORS ASSETS																				
WORLD INFLATION LINKED	-6.3%	-6.3%				-5.0%	-5.0%	-			-4.5%	-4.5%	-	-	-	-2.4%	-2.4%	-	-	-
Citigroup WGBI All Maturities	-2.8%	-2.8%				-3.4%	-1.3%	-2.1%			-3.0%	-1.5%	-1.6%	-	-	-1.5%	-1.1%	-0.4%	-	-
MSCI World Index	4.4%	4.4%				6.9%	1.2%	5.7%			4.4%	3.1%	5.6%	-4.3%	-	-1.7%	2.6%	1.0%	-5.4%	-
IBOXX € EZSOV	-3.4%	-3.4%				-4.1%	-1.8%	-2.4%			-4.6%	-1.4%	-1.1%	-0.1%	-2.0%	-1.9%	-1.3%	-0.4%	-0.2%	-
Average Corporate Bonds	-1.8%	-1.8%				-2.0%	-1.3%	-0.7%			-2.0%	-1.2%	-0.5%	-0.4%	-	-4.7%	-0.6%	-0.1%	-0.3%	-3.7%
MSCI Emerging Market	2.8%	2.8%				5.8%	-0.5%	6.3%			3.7%	1.4%	5.8%	-3.5%	-	-1.5%	1.7%	1.2%	-4.3%	-
DJUBS Commodity	-1.6%	-1.6%				2.5%	-3.6%	6.1%			1.6%	-2.3%	5.0%	-1.1%	-	-0.8%	-0.6%	1.2%	-1.4%	-
DOLLAR INDEX SPOT	2.1%	2.1%				1.3%	2.1%	-0.8%			1.3%	1.7%	-0.7%	0.4%	-	1.2%	0.9%	-0.2%	0.5%	-
SPECIFIC ASSETS																				
Euro MTS Inflation Linked	-4.3%	-4.3%				-4.0%	-3.1%	-1.0%			-4.9%	-2.4%	0.2%	-0.1%	-2.6%	-2.1%	-1.6%	-0.2%	-0.3%	-
Citigroup US Inflation Linked	-6.6%	-6.6%				-5.2%	-5.3%	0.1%			-4.7%	-4.7%	0.0%	0.0%	-	-2.5%	-2.5%	0.0%	0.0%	-
Barclays UK Inflation Linked	-7.9%	-7.9%				-5.9%	-6.3%	0.4%			-5.2%	-5.5%	0.2%	0.1%	-	-2.8%	-3.0%	0.1%	0.2%	-
IBOXX € LQD CRP	-2.7%	-2.7%				-2.9%	-1.8%	-1.1%			-3.2%	-1.5%	-0.6%	-0.2%	-0.9%	-2.7%	-0.9%	-0.2%	-0.1%	-1.4%
iBoxx $ Liquid Investment Grade	-5.0%	-5.0%				-5.6%	-3.0%	-2.5%			-5.1%	-3.0%	-1.8%	-0.2%	-	-5.1%	-1.7%	-0.4%	-0.1%	-2.9%
CS Western Euro High Yield	0.2%	0.2%				0.6%	-0.3%	1.0%			0.1%	0.1%	1.1%	-0.8%	-0.3%	-4.8%	0.9%	0.4%	-0.8%	-5.2%
IBOXX USD Liquid High Yield	-1.0%	-1.0%				-0.2%	-1.1%	0.9%			-0.6%	-0.7%	1.0%	-1.0%	-	-6.1%	0.3%	0.4%	-1.1%	-5.7%
Citigroup US GBI 7 to 10 Year	-6.1%	-6.1%				-6.9%	-3.4%	-3.5%			-6.1%	-3.6%	-2.6%	0.1%	-	-2.8%	-2.4%	-0.6%	0.2%	-
Citigroup UK GBI 7 to 10 Year	-5.2%	-5.2%				-5.6%	-3.1%	-2.6%			-5.0%	-3.1%	-1.9%	0.1%	-	-2.4%	-2.0%	-0.5%	0.1%	-
Citigroup Germany GBI	-3.8%	-3.8%				-4.4%	-1.9%	-2.5%			-4.4%	-1.8%	-1.4%	0.1%	-1.2%	-1.8%	-1.5%	-0.4%	0.1%	-
EUR-USD Carry Return	-2.1%	-2.1%				-1.5%	-2.0%	0.5%			-1.5%	-1.6%	0.6%	-0.5%	-	-1.4%	-0.9%	0.1%	-0.6%	-
CHF-USD Carry Return	-3.5%	-3.5%				-2.6%	-3.1%	0.5%			-2.3%	-2.7%	0.4%	-0.0%	-	-1.4%	-1.5%	0.1%	-0.0%	-
GBP-USD Carry Return	-1.4%	-1.4%				-0.7%	-1.4%	0.7%			-0.8%	-1.1%	0.7%	-0.4%	-	-0.9%	-0.6%	0.1%	-0.5%	-
MSCI Daily TR Net North America	4.1%	4.1%				6.5%	1.1%	5.3%			3.9%	3.0%	5.3%	-4.4%	-	-2.0%	2.5%	1.0%	-5.5%	-
MSCI Daily TR Net EMU Local	6.1%	6.1%				8.8%	2.4%	6.5%			5.8%	4.5%	6.3%	-5.0%	-	-1.5%	3.5%	1.2%	-6.2%	-
MSCI Daily TR Net Europe Ex EM	4.6%	4.6%				7.2%	1.5%	5.7%			4.7%	3.3%	5.5%	-4.0%	-	-1.3%	2.7%	1.0%	-5.0%	-
MSCI Daily TR Net Pacific Ex Japan	1.9%	1.9%				4.3%	-0.4%	4.7%			2.5%	1.1%	4.4%	-2.9%	-	-1.6%	1.2%	0.9%	-3.6%	-
JPM EMBI Global Diversified	-3.5%	-3.5%				-2.9%	-2.7%	-0.2%			-3.0%	-2.2%	0.2%	-1.0%	-	-4.9%	-0.9%	0.0%	-1.1%	-3.0%
MSCI EM LATIN AMERICA	3.7%	3.7%				8.2%	-0.8%	9.0%			4.9%	2.1%	8.4%	-5.5%	-	-2.9%	2.3%	1.6%	-6.9%	-
MSCI EM ASIA	3.4%	3.4%				6.4%	-0.4%	6.9%			4.2%	1.6%	6.3%	-3.7%	-	-1.8%	1.6%	1.3%	-4.7%	-
MSCI EM EUROPE	2.0%	2.0%				7.1%	-2.3%	9.4%			4.3%	0.2%	8.8%	-4.8%	-	-2.7%	1.3%	1.9%	-5.9%	-
FX COMMO	-1.6%	-1.6%				0.2%	-2.1%	2.3%			-0.3%	-1.3%	2.2%	-1.1%	-	-1.4%	-0.4%	0.5%	-1.4%	-
FX EMERGENT	-0.2%	-0.2%				0.7%	-0.8%	1.5%			0.1%	-0.3%	1.5%	-1.1%	-	-1.0%	-0.0%	0.3%	-1.3%	-
DJUBS PrcMtl	-3.3%	-3.3%				-0.8%	-4.3%	3.5%			-1.1%	-3.3%	3.0%	-0.7%	-	-1.9%	-1.7%	0.7%	-0.9%	-
DJUBS IndMtl	1.1%	1.1%				5.1%	-2.1%	7.2%			3.5%	-0.5%	6.4%	-2.5%	-	-1.2%	0.5%	1.5%	-3.1%	-
DJUBS Agri	0.6%	0.6%				2.8%	-1.1%	3.9%			2.1%	-0.3%	3.4%	-1.0%	-	-0.3%	0.3%	0.8%	-1.3%	-
DJUBS Pet	-3.4%	-3.4%				3.9%	-7.6%	11.6%			3.0%	-5.1%	9.3%	-1.2%	-	-0.6%	-1.3%	2.3%	-1.5%	-

Table 8: Decomposition of sensitivity for different indexes following shocks to risky indexes

RELATED EQUITY RISK FACTORS	MSCI World Index					MSCI Emerging Market					DJUBS Commodity					DOLLAR INDEX SPOT				
		GLOBAL			SPECIFIC		GLOBAL			SPECIFIC		GLOBAL			SPECIFIC		GLOBAL			SPECIFIC
	TOTAL	Real rate	Inflation	Market equity Risk aversion	-	TOTAL	Real rate	Inflation	Market equity Risk aversion	Emerging	TOTAL	Real rate	Inflation	Market equity Risk aversion	Commodities	TOTAL	Real rate	Inflation	Market equity Risk aversion	USD
RISK FACTORS ASSETS																				
WORLD INFLATION LINKED	1.4%	1.4%	-	-		0.8%	0.8%	-	-	-	-0.4%	-0.4%	-	-	-	1.3%	1.3%	-	-	-
Citigroup WGBI All Maturities	1.3%	0.6%	0.6%	-		1.0%	0.4%	0.6%	-	-	0.4%	-0.2%	0.6%	-	-	0.4%	0.6%	-0.2%	-	-
MSCI World Index	-19.4%	-0.6%	-1.2%	-17.6%		-14.3%	-0.6%	-1.3%	-12.4%	-	-5.0%	0.4%	-1.5%	-3.9%	-	2.2%	-1.3%	0.4%	3.1%	-
IBOXX € EZSOV	1.0%	0.7%	0.8%	-0.5%		0.8%	0.4%	0.7%	-0.4%	-	0.4%	-0.2%	0.7%	-0.1%	-	0.6%	0.7%	-0.2%	0.1%	-
Average Corporate Bonds	-0.3%	0.6%	0.3%	-1.2%		-0.3%	0.3%	0.3%	-0.9%	-	-0.2%	-0.2%	0.3%	-0.3%	-	0.7%	0.5%	-0.1%	0.2%	-
MSCI Emerging Market	-16.0%	-0.3%	-1.4%	-14.2%		-21.6%	-0.2%	-1.3%	-6.2%	-13.9%	-7.1%	0.3%	-1.7%	-3.1%	-2.5%	2.1%	-0.8%	0.5%	2.5%	-
DJUBS Commodity	-5.5%	0.5%	-1.7%	-4.3%		-7.1%	0.3%	-1.6%	-2.3%	-3.4%	-20.8%	-0.5%	-1.2%	-0.6%	-18.6%	1.6%	0.3%	0.5%	0.8%	-
DOLLAR INDEX SPOT	1.2%	-0.6%	0.1%	1.7%		1.0%	-0.3%	0.2%	1.2%	-	0.8%	0.2%	0.2%	0.4%	-	-9.5%	-0.2%	-0.0%	-0.2%	-9.0%
SPECIFIC ASSETS																				
Euro MTS Inflation Linked	0.5%	0.9%	0.4%	-0.8%		0.3%	0.5%	0.3%	-0.6%	-	-0.1%	-0.3%	0.3%	-0.2%	-	0.9%	0.8%	-0.1%	0.1%	-
Citigroup US Inflation Linked	1.5%	1.4%	-0.0%	0.1%		0.9%	0.8%	-0.0%	0.0%	-	-0.5%	-0.5%	-0.0%	0.0%	-	1.3%	1.3%	0.0%	-0.0%	-
Barclays UK Inflation Linked	1.9%	1.6%	-0.1%	0.4%		1.2%	1.0%	-0.1%	0.3%	-	-0.4%	-0.4%	-0.1%	0.1%	-	1.5%	1.5%	0.0%	-0.1%	-
IBOXX € LQD CRP	0.4%	0.6%	0.4%	-0.7%		0.3%	0.4%	0.4%	-0.5%	-	0.1%	-0.2%	0.4%	-0.2%	-	0.6%	0.6%	-0.1%	0.1%	-
iBoxx $ Liquid Investment Grade	1.1%	1.1%	0.8%	-0.8%		0.8%	0.6%	0.8%	-0.6%	-	0.2%	-0.4%	0.8%	-0.2%	-	1.0%	1.0%	-0.2%	0.2%	-
CS Western Euro High Yield	-3.2%	-0.0%	-0.1%	-3.1%		-2.5%	-0.0%	-0.2%	-2.3%	-	-0.9%	0.0%	-0.3%	-0.7%	-	0.6%	-0.1%	0.1%	0.6%	-
IBOXX USD Liquid High Yield	-3.6%	0.3%	-0.1%	-3.8%		-2.7%	0.1%	-0.1%	-2.7%	-	-1.2%	-0.1%	-0.2%	-0.9%	-	0.8%	0.1%	0.1%	0.7%	-
Citigroup US GBI 7 to 10 Year	2.9%	1.3%	1.0%	0.5%		2.2%	0.8%	1.0%	0.4%	-	0.7%	-0.5%	1.0%	0.1%	-	0.9%	1.3%	-0.3%	-0.1%	-
Citigroup UK GBI 7 to 10 Year	2.1%	1.1%	0.8%	0.3%		1.6%	0.6%	0.8%	0.2%	-	0.5%	-0.3%	0.8%	0.1%	-	0.8%	1.1%	-0.2%	-0.1%	-
Citigroup Germany GBI	1.7%	0.8%	0.7%	0.2%		1.3%	0.5%	0.7%	0.1%	-	0.5%	-0.2%	0.7%	0.0%	-	0.5%	0.8%	-0.2%	-0.0%	-
EUR-USD Carry Return	-2.1%	0.6%	0.0%	-2.8%		-1.5%	0.3%	-0.1%	-1.8%	-	-0.7%	-0.2%	-0.1%	-0.4%	-	10.6%	0.2%	0.0%	-0.2%	10.5%
CHF-USD Carry Return	0.6%	0.8%	-0.2%	-0.1%		0.3%	0.5%	-0.2%	-0.1%	-	-0.5%	-0.3%	-0.2%	-0.0%	-	10.9%	0.5%	-	-0.0%	10.5%
GBP-USD Carry Return	-1.8%	0.4%	-0.0%	-2.2%		-1.3%	0.2%	-0.1%	-1.4%	-	-0.7%	-0.1%	-0.2%	-0.3%	-	7.6%	0.1%	0.0%	-0.1%	7.5%
MSCI Daily TR Net North America	-19.9%	-0.6%	-1.1%	-18.2%		-14.6%	-0.6%	-1.2%	-12.8%	-	-5.0%	0.4%	-1.4%	-4.0%	-	2.4%	-1.2%	0.4%	3.2%	-
MSCI Daily TR Net EMU Local	-22.7%	-1.1%	-1.3%	-20.3%		-16.7%	-0.9%	-1.5%	-14.4%	-	-5.6%	0.6%	-1.7%	-4.5%	-	2.3%	-1.7%	0.5%	3.6%	-
MSCI Daily TR Net Europe Ex EM	-18.6%	-0.8%	-1.2%	-16.6%		-13.7%	-0.7%	-1.3%	-11.7%	-	-4.7%	0.5%	-1.5%	-3.7%	-	2.0%	-1.3%	0.4%	2.9%	-
MSCI Daily TR Net Pacific Ex Japan	-12.9%	-	-1.0%	-11.9%		-9.7%	-0.2%	-1.1%	-8.4%	-	-3.7%	0.2%	-1.3%	-2.6%	-	1.9%	-0.6%	0.4%	2.1%	-
JPM EMBI Global Diversified	-2.8%	0.6%	0.3%	-3.8%		-3.2%	0.4%	0.3%	-2.2%	-1.6%	-1.4%	-0.4%	0.1%	-0.9%	-0.3%	1.1%	0.4%	-0.0%	0.7%	-
MSCI EM LATIN AMERICA	-24.6%	-0.2%	-2.0%	-22.5%		-27.0%	-0.3%	-1.9%	-12.1%	-12.8%	-9.3%	0.4%	-2.4%	-5.0%	-2.3%	3.5%	-1.1%	0.7%	3.9%	-
MSCI EM ASIA	-16.6%	-0.4%	-1.6%	-14.7%		-25.4%	-0.3%	-1.2%	-5.2%	-18.7%	-8.4%	0.4%	-1.9%	-3.4%	-3.5%	2.3%	-0.8%	0.5%	2.6%	-
MSCI EM EUROPE	-21.4%	0.3%	-1.9%	-19.8%		-26.8%	0.0%	-1.7%	-9.6%	-15.5%	-9.6%	0.2%	-2.7%	-4.3%	-2.9%	3.5%	-0.7%	0.8%	3.4%	-
FX COMMO	-4.8%	0.5%	-0.4%	-4.8%		-5.7%	0.2%	-0.4%	-2.9%	-2.6%	-5.8%	-0.1%	-0.5%	-0.9%	-4.2%	6.8%	-0.1%	0.2%	0.8%	6.0%
FX EMERGENT	-4.5%	0.2%	-0.3%	-4.4%		-5.5%	0.1%	-0.3%	-2.3%	-3.0%	-1.9%	0.0%	-0.4%	-1.0%	-0.6%	3.5%	-0.0%	0.1%	0.8%	2.6%
DJUBS PrcMtl	-3.7%	1.2%	-0.8%	-4.2%		-7.2%	0.7%	-0.7%	-0.9%	-6.4%	-17.1%	-0.2%	-0.1%	-0.3%	-16.6%	5.8%	0.8%	0.3%	0.2%	4.6%
DJUBS IndMtl	-11.2%	0.2%	-1.7%	-9.7%		-12.7%	0.0%	-1.7%	-5.8%	-5.2%	-16.8%	0.0%	-1.6%	-1.9%	-13.3%	3.9%	-0.4%	0.6%	1.7%	2.0%
DJUBS Agri	-4.3%	-0.0%	-1.1%	-3.1%		-3.7%	-0.0%	-1.1%	-2.5%	-	-14.5%	0.0%	-0.6%	-0.5%	-13.4%	2.2%	-0.2%	0.3%	0.6%	1.4%
DJUBS Pet	-9.0%	1.1%	-3.1%	-7.1%		-11.3%	0.5%	-3.1%	-3.6%	-5.2%	-28.8%	-0.6%	-2.5%	-0.8%	-24.9%	0.6%	0.7%	0.9%	0.4%	-1.5%

H Sensitivity analysis for portfolios

Figure 2: Allocations

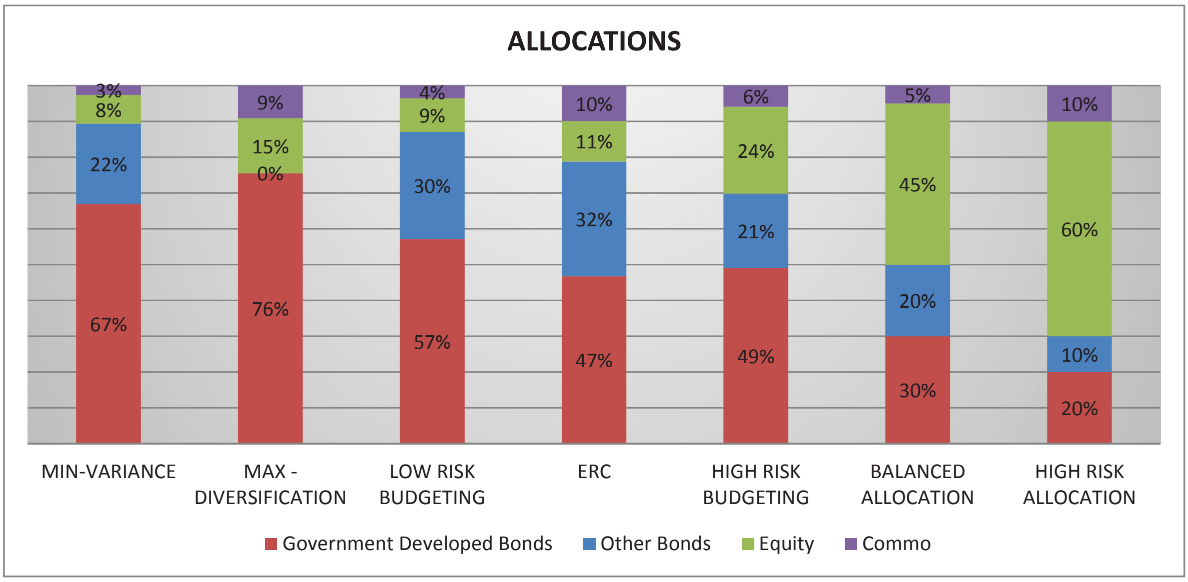


Table 9: Benchmark Factor Sensitivity of components

		A- FACTOR SENSITIVITY						
INDIVIDUAL COMPONENTS		REAL RATE	INFLATION	MARKET RISK	CREDIT	EMERGING	COMMODITIES	DEBT €
BASKETS	GOVERNMENT BONDS	-4.3%	-2.6%	0.1%	0.0%	0.0%	0.5%	-0.6%
	OTHER BONDS	-3.9%	-0.3%	-1.4%	-2.5%	-0.2%	-0.5%	-0.7%
	EQUITY	3.7%	6.6%	-17.3%	-5.3%	-6.8%	-6.5%	-4.2%
	COMMODITIES	-1.3%	6.5%	-5.7%	-1.7%	-4.1%	-19.6%	-1.3%

Table 10: Benchmark Factor Sensitivity of portfolios

		B- FACTOR SENSITIVITY						
PORTFOLIOS		REAL RATE	INFLATION	MARKET RISK	CREDIT	EMERGING	COMMODITIES	DEBT €
RISK-BASED ALLOCATION	MINIMUM VARIANCE	-3.5%	-1.1%	-1.8%	-1.0%	-0.7%	-0.8%	-0.9%
	MAXIMUM DIVERSIFICATION	-2.8%	-0.4%	-3.1%	-0.9%	-1.4%	-2.4%	-1.2%
	LOW RISK BUDGETING	-3.3%	-0.7%	-2.2%	-1.3%	-0.8%	-1.2%	-1.0%
	ERC	-2.9%	0.1%	-2.9%	-1.6%	-1.2%	-2.6%	-1.1%
	HIGH RISK BUDGETING	-2.1%	0.6%	-4.8%	-1.9%	-1.9%	-2.6%	-1.5%
TRADITIONAL ALLOCATION	BALANCED ALLOCATION	-0.5%	2.4%	-8.3%	-3.0%	-3.3%	-3.9%	-2.3%
	HIGH RISK ALLOCATION	0.8%	4.0%	-11.1%	-3.6%	-4.5%	-5.8%	-2.9%

Table 11: Sensitivity Decomposition under low inflation

		EURO GOV BONDS				
PORTFOLIOS		Total	REAL RATE	INFLATION	MARKET RISK	DEBT €
RISK-BASED ALLOCATION	MINIMUM VARIANCE	-3.4%	-2.0%	-0.5%	-0.4%	-0.5%
	MAXIMUM DIVERSIFICATION	-2.7%	-1.6%	0.1%	-0.7%	-0.5%
	LOW RISK BUDGETING	-3.2%	-1.9%	-0.2%	-0.5%	-0.5%
	ERC	-2.6%	-1.9%	0.4%	-0.7%	-0.4%
	HIGH RISK BUDGETING	-1.8%	-1.2%	1.0%	-1.2%	-0.4%
TRADITIONAL ALLOCATION	BALANCED ALLOCATION	0.0%	-0.3%	2.6%	-2.0%	-0.3%
	HIGH RISK ALLOCATION	1.5%	0.4%	4.0%	-2.7%	-0.2%

		EURO GOV BONDS				
PORTFOLIOS		Total	REAL RATE	INFLATION	MARKET RISK	DEBT €
RISK-BASED ALLOCATION	MINIMUM VARIANCE	-2.9%	-2.0%	0.0%	-0.4%	-0.5%
	MAXIMUM DIVERSIFICATION	-2.8%	-1.6%	0.0%	-0.7%	-0.5%
	LOW RISK BUDGETING	-2.9%	-1.9%	0.0%	-0.5%	-0.5%
	ERC	-3.0%	-1.9%	0.0%	-0.7%	-0.4%
	HIGH RISK BUDGETING	-2.8%	-1.2%	0.0%	-1.2%	-0.4%
TRADITIONAL ALLOCATION	BALANCED ALLOCATION	-2.6%	-0.3%	0.0%	-2.0%	-0.3%
	HIGH RISK ALLOCATION	-2.5%	0.4%	0.0%	-2.7%	-0.2%

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