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Abstract

We study the effectiveness of public investment in stimulating an economy stuck in a liquidity trap. We do so in the context of a tractable new-Keynesian economy in which a fraction of government spending increases the stock of public capital subject to a time-to-build constraint. Public investment projects typically entail significant time-to-build delays, which often span several years from approval to completion. We show that this feature implies that the spending multiplier associated with public investment can be substantially large — nearly twice as large as the multiplier associated with public consumption — in a liquidity trap. Intuitively, when the time to build is sufficiently long, and to the extent that public capital raises the marginal productivity of private inputs, the resulting disinflationary effect will occur after the economy has escaped from the liquidity trap. At the same time, the increase in households' expected wealth amplifies aggregate demand while the economy is still in the liquidity trap. Using a medium-scale model extended to allow for the accumulation of public capital, we quantify the multiplier associated with the spending component of the 2009's ARRA, which allocated roughly 40% of the authorized funds to public investment. We find a peak multiplier of 2.31. Our results also indicate that failing to account for the composition of the stimulus by overlooking its investment component would lead one to underestimate the spending multiplier by about 50%.

JEL classification: E4, E52, E62, H54
Key words: Public spending, Public investment, Time to build, Multiplier, Zero lower bound.

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"We can create some room to invest in things that make America stronger, like rebuilding America's infrastructure."

Timothy F. Geithner, The New York Times, December 4, 2012

"The U.S. must address its infrastructure, education, and training needs. Moreover, it must support aggregate demand to repair the damage caused by the Great Recession."

Barry Eichengreen, Secular Stagnation: Facts, Causes and Cures, 2014

1 Introduction

One of the most widely debated questions since the onset of the latest global recession has been the effectiveness of public spending as a tool to stimulate the economy. This effectiveness is commonly judged by the size the spending multiplier, that is, the dollar change in aggregate output that results from a dollar increase in public expenditures. From an empirical standpoint, estimates of the spending multiplier range from roughly 0 to well above 1, depending on the sample period and the identifying assumptions.[1] Most theoretical models, on the other hand, yield a spending multiplier in the neighborhood of 1.[2] The main assumption underlying the latter prediction, one that is also implicit in the empirical literature, is that the economy is in "normal times".

Recent theoretical research by Christiano et al. (2011), Eggertsson (2011), and Woodford (2011), however, shows that during sharp recessions that drive nominal interest rates down to their lower bound of zero — rendering conventional monetary policy useless — an increase in public spending can be very effective in stimulating economic activity. In this situation, often referred to as a liquidity trap, the spending multiplier is 2 to 3 times larger than in normal times, under plausible parameter values. Intuitively, by raising aggregate demand, government spending does what monetary policy cannot do: generate inflation. Since the nominal interest rate is stuck at zero, the real interest rate falls, thus further boosting aggregate demand. While very few economies had been plagued by liquidity traps before the Great Recession, there is an increasing fear among academics and policy-makers that such episodes will become recurrent events in the near future.[3] A series of recent papers collected in Baldwin & Teulings (2014) indeed discuss the specter of secular stagnation, i.e., a prolonged period of lackluster growth, that would keep interest rates close to their lower bound.

Perhaps surprisingly, existing studies of the effects of public spending when the zero lower bound (ZLB) on nominal interest rates is binding often assume that government expenditures are entirely

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[1]See, for example, Ramey & Shapiro (1998), Ramey (2011), Fatás & Mihov (2001), Blanchard & Perotti (2002), Perotti (2004a), Galí et al. (2007), Mountford & Uhlig (2009), Barro & Redlick (2011), Auerbach & Gorodnichenko (2012) and Bouakez et al. (2014) among others.

[2]See Perotti (2008) and Hall (2009) for comprehensive surveys of the theoretical literature.

[3]For instance, in his 2013 speech at the IMF Economic Forum, Lawrence Summers stated that *"We may well need, in the years ahead, to think about how we manage an economy in which the zero nominal interest rate is a chronic and systemic inhibitor of economic activity, holding our economies back below their potential"*.

wasteful and have no direct effect on the marginal productivity of private inputs. More specifically, those studies abstract from public investment despite the fact that it represents a non-negligible fraction of total public spending, averaging roughly 23 percent in the U.S., for example. More importantly, Bachmann & Sims (2012) show that, conditional on a positive government spending shock, the ratio of public investment to public consumption tends to rise more during recessions than during expansions. The largest fiscal stimulus plan in U.S. history — the American Recovery and Reinvestment Act (ARRA) of 2009 — allocated roughly 40% of non-transfer spending to public investment.

The purpose of this paper is to evaluate the effectiveness of public investment as a fiscal stimulus both in normal times and when the ZLB binds. We start by presenting an analytically tractable new-Keynesian model in which a fraction of government spending is in the form of public investment. The latter increases the stock of public capital, which is an external input in the production technology. In order to account for the implementation delays associated with the completion of investment projects, we assume that multiple periods are required to build new productive (public) capital.[4] Monetary policy sets the nominal interest rate according to a Taylor-type rule subject to a non-negativity constraint. As in Christiano et al. (2011), Eggertsson (2011), and Woodford (2011), we assume that states of the world in which the ZLB binds are the result of a large negative shock to the discount rate, which increases agents' desire to save.[5]

In addition to its direct effect on current aggregate demand, an increase in public investment has similar effects to those of an anticipated technology shock. On the one hand, it exerts downward pressure on real marginal cost and inflation once public capital becomes productive; we call this the supply-side effect. On the other hand, by increasing households' expected wealth, it raises aggregate demand, real marginal cost, and inflation even before public capital becomes productive. The effectiveness of public investment in stimulating output therefore depends on the relative importance of these two forces at different time horizons, which in turn crucially depends on the time-to-build delays.

We show that when these delays are short, the spending multiplier increases with the fraction of public investment in a stimulus plan in normal times, whereas the opposite result holds when the ZLB binds. Intuitively, when the time to build is relatively short, the disinflationary effect associated with the increase in productivity comes about rapidly, thus attenuating the increase in inflation resulting from higher aggregate demand and from the increase in households' expected wealth. In normal times, this implies a smaller initial increase in the long-term real interest rate, and therefore a larger spending multiplier, as the fraction of investment becomes larger. When the ZLB binds, the disinflationary effect persists for a prolonged period of time while the economy is

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[4]The assumption of time to build was first introduced by Kydland & Prescott (1982) in the context of a real business cycle model.

[5]Alternatively, Mertens & Ravn (2010) assume that the economy is plunged in a liquidity trap by a sudden change in agents' beliefs that is not motivated by fundamentals.

in the liquidity trap, which mitigates the decline in the long-term real interest rate and diminishes the multiplier to an extent that depends positively on the fraction of public investment.

When the time to build is relatively long, these results are reversed: the multiplier decreases with the fraction of public investment in normal times but increases with it when the ZLB binds. Intuitively, with a long time-to-build delay, the supply-side effect of public investment comes into play far in the future so that the inflationary pressure lasts for a long period of time. In normal times, this amplifies the increase in the long-term real interest rate and reduces the multiplier to an extent that depends positively on the fraction of public investment. When the ZLB binds, the further in time capital becomes productive, the more likely the resulting disinflationary pressure will occur after the economy has escaped from the liquidity trap. At the same time, the positive wealth effect associated with the expected increase in productivity adds to inflation while the economy is still in the liquidity trap. This tends to amplify the decline in the long-term real interest rate and, thus the multiplier, to an extent that increases with the fraction of public investment.

Importantly, we find that the spending multiplier can be substantially large at the ZLB when a significant fraction of public spending is invested in public capital and when the time-to-build delay is relatively long. With a time to build of 16 quarters, our simple model implies that the multiplier associated with public investment is roughly five times larger than in normal times and nearly twice as large as the multiplier associated with public consumption. Given that public investment projects entail significant time-to-build delays — often spanning several years — our results suggest that public investment can be a highly effective tool to stimulate an economy stuck in a liquidity trap.

In the last part of the paper, we extend the medium-scale model developed by Christiano et al. (2013) to allow for the accumulation of public capital and for time-to-build delays, and we use the extended model to gauge the output effects of the spending component of the ARRA. To this end, we calibrate the model based on the actual composition of the stimulus, the nature of public investment projects that it comprised, and their financing scheme. We find that the spending component of the ARRA had a peak multiplier of 2.31. Our results also indicate that counterfactually assuming that the stimulus was exclusively composed of public consumption spending would lead one to underestimate the multiplier by roughly 50%. These numbers change very little when we consider alternative empirically plausible assumptions about the way in which public capital affects the productivity of private inputs.

In addition to the literature cited above, this paper is closely related to those by Baxter & King (1993), Linnemann & Schabert (2006), Leeper et al. (2010), Coenen et al. (2012), Coenen et al. (2013), Leduc & Wilson (2013), Drautzburg & Uhlig (2013), and Albertini et al. (2014), who study the business-cycle implications of public investment in the context of general-equilibrium models. Baxter & King (1993) and Leeper et al. (2010) consider a neoclassical framework, whereas the remaining studies consider models with nominal rigidities. Coenen et al. (2012), Coenen et al.

(2013), Drautzburg & Uhlig (2013) and Albertini et al. (2014) study the effects of public investment shocks when monetary policy is accommodative (due, for instance, to a binding ZLB), but none of these papers takes into account the time-to-build delay associated with public investment projects, a feature which, as we show in this paper, has crucial implications for the sign and magnitude of those effects, particularly at the ZLB.[6]

The rest of this paper is organized as follows. Section 2 presents a simple new-Keynesian economy with public capital and time to build. Section 3 studies the relationship between the spending multiplier, the fraction of public investment in a stimulus, and the time to build. Section 4 discusses the empirical plausibility of the main assumption underlying our analysis, namely that public capital is productive, and provides some independent evidence supporting this hypothesis. Section 5 quantifies the effects of the spending component of the ARRA in a medium-scale new-Keynesian model and performs some sensitivity analysis. Section 6 concludes.

2 A Simple Model with Public Investment and Time to Build

We consider simple new-Keynesian economy without private capital but where a fraction of government spending is invested in public capital subject to a time-to-build requirement. The stock of public capital enters as an external input in the production of intermediate goods, which are used to produce an homogenous final good.

2.1 The model

2.1.1 Households

The economy is populated by a large number of identical households who have the following lifetime utility function:[7]

$$E_t \sum_{s=0}^{\infty} d_{t+s} U\left(C_{t+s}, N_{t+s}\right), \tag{1}$$

where C_t is consumption, N_t denotes hours worked, and d_t is a time-varying discount factor defined by

$$d_{t+s} = \begin{cases} \beta_{t+1}\beta_{t+2}...\beta_{t+s} & s \geq 1 \\ 1 & s = 0, \end{cases}$$

where $0 < \beta_{t+s} < 1$ is known in $t+s-1$ with certainty.

The representative household enters period t with $\mathcal{B}_{t-1}$ units of one-period riskless nominal bonds. During the period, it receives a wage payment, $\mathcal{W}_t N_t$, and dividends, $\mathcal{D}_t$, from the monopolistically competitive firms. This income is used to pay a lump-sum tax, $\mathcal{T}_t$, to the government, to

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[6]Although the model considered by Coenen et al. (2013) allows for multiple periods of time to build, these authors restrict their attention to the case with a single period of time to build.

[7]The equilibrium dynamics of this economy are identical to those of an economy in which public spending affects preferences in an additively separable manner (as in Christiano et al. (2011)). Since the focus of this paper is not on the normative implications of fiscal policy, we simply assume that public spending does not enter the utility function.

consumption, and to the purchase of new bonds. The household's budget constraint is therefore

$$P_t C_t + \mathcal{T}_t + \frac{\mathcal{B}_t}{1 + i_t} \leq \mathcal{W}_t N_t + \mathcal{D}_t + \mathcal{B}_{t-1}, \tag{2}$$

where P_t is the price of the final good, $\mathcal{W}_t$ is nominal wage rate, and $\frac{1}{1+i_t}$ is the price of a nominal bond purchased at time t, i_t being the nominal interest rate.

2.1.2 Firms

The final good is produced by perfectly competitive firms using the following constant-elasticity-of-substitution (CES) technology:

$$Y_t = \left[\int_0^1 X_t(z)^{1-1/\theta} dz \right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

where $X_t(z)$ is the quantity of intermediate good z and $\theta \geq 1$ is the elasticity of substitution between intermediate goods. Denoting by $P_t(z)$ the price of intermediate good z, demand for z is given by

$$X_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\theta} Y_t. \tag{4}$$

Firms in the intermediate-good sector are monopolistically competitive, each producing a differentiated good using labor as a direct input and public capital as an external input

$$X_t(z) = F(N_t(z), K_{G,t}). \tag{5}$$

Intermediate-good producers set their prices à la Calvo. That is, in each period, a given firm resets its price with probability $1 - \phi$. Denoting by P_t^o the optimal price chosen in period t, the firm's problem is

$$\max_{P_t^o} E_t \sum_{s=0}^{\infty} \phi^s Q_{t,t+s} \{ P_t^o X_{t,t+s} - (1-\tau) \mathcal{W}_{t+s} N_{t,t+s} \},$$

subject to

$$\begin{aligned} X_{t,t+s} &= F(N_{t,t+s}, K_{G,t+s}), \\ X_{t,t+s} &= \left(\frac{P_t^o}{P_{t+s}} \right)^{-\theta} Y_{t+s}, \end{aligned}$$

where $Q_{t,t+s} = d_{t+s} U_C(C_{t+s}, N_{t+s}) / d_t U_C(C_t, N_t)$ is the stochastic discount factor; $X_{t,t+s}$ and $N_{t,t+s}$ are, respectively, the quantity of intermediate good produced and labor demand in period $t+s$ if the price set at time t is still in effect; there is a subsidy $\tau = 1/\theta$ that corrects the steady-state distortion stemming from monopolistic competition in the goods market.

2.1.3 Fiscal and monetary authorities

The government levies lump-sum taxes to finance its expenditures and the subsidy given to firms in the intermediate-good sector. Its budget constraint is given by

$$P_t G_t + \tau \mathcal{W}_t N_t = \mathcal{T}_t, \tag{6}$$

where G_t is government spending, which is composed of two parts, public consumption, G_t^c, and public investment, G_t^i:

$$G_t = G_t^c + G_t^i. \tag{7}$$

Public investment increases the stock of public capital according to the following accumulation equation:

$$K_{G,t} = (1 - \delta) K_{G,t-1} + G_{t-T}^i, \tag{8}$$

where $T \geq 0$. This specification allows for the possibility that several periods may be required to build new productive capital, i.e., time to build (see Kydland & Prescott (1982)). This feature reflects the implementation delays typically associated with the different stages of public investment projects (planning, bidding, contracting, construction, etc.).

In normal times, public spending is determined by the following process:

$$G_t = (1 - \rho)G + \rho G_{t-1} + \epsilon_t, \tag{9}$$

where $0 \leq \rho < 1$, G is the steady-state level of public spending, and ϵ_t is a zero-mean serially uncorrelated disturbance.[8] Moreover, we assume that public investment is determined by the following policy rule:

$$G_t^i = G^i + \alpha \left(G_t - G \right), \tag{10}$$

where $0 \leq \alpha \leq 1$ and G^i is the steady-state level of public investment. The policy parameter α measures the fraction of public investment in a spending-based stimulus plan. Note that this fraction need not be equal to the steady-state share of public investment in total public expenditures, $G^i/G \equiv \bar{\alpha}$.

The monetary policy is described by a simple Taylor-type rule in which the inflation target is 0 and which is subject to a non-negativity constraint on the nominal interest rate:

$$i_t = \max \left(0; r_\beta + \phi_\pi \pi_t \right), \tag{11}$$

where $r_\beta = \beta^{-1} - 1$ and $\pi_t = P_t/P_{t-1} - 1$ is the inflation rate between $t - 1$ and t. To simplify the algebra, we assume that the nominal interest rate does not react to the output gap or to its past value. We will consider such extensions in the medium-scale model that we present in section 5.

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[8] The dynamics of public spending when the ZLB binds will be described in Section 3.2.

2.2 Functional forms

Following Christiano et al. (2011), we assume that the utility function is given by

$$
\begin{aligned}
U\left(C_t, N_t\right) &= \frac{\left(C_t^\gamma (1-N_t)^{1-\gamma}\right)^{1-\sigma} - 1}{1-\sigma} && \text{if } \sigma \neq 1 \\
&= \gamma \ln C_t + (1-\gamma)\ln(1-N_t) && \text{if } \sigma = 1,
\end{aligned}
\tag{12}
$$

where $\sigma > 0$ and $0 < \gamma \leq 1$.

The production function is given by

$$
F\left(N_t, K_{G,t}\right) = N_t^a K_{G,t}^b,
\tag{13}
$$

where $0 \leq a, b \leq 1$. This specification nests the linear technology assumed by Christiano et al. (2011) as a special case in which $a = 1$ and $b = 0$. The parameter b measures the elasticity of output with respect to public capital. When this elasticity is strictly positive, equation (13) implies that public capital improves total factor productivity, an assumption that we will discuss in detail in Section 4.

2.3 Linearized model

The model is solved by linearizing the equilibrium conditions around a deterministic zero-inflation steady state. For ease of exposition, we focus throughout the subsequent section on the case $U_{CN}(.) = 0$ (or, equivalently, $\sigma = 1$); the derivations for the general case $U_{CN}(.) \neq 0$ are provided in Appendix A. Let variables without a time subscript denote steady-state values and variables in lowercase denote percentage deviations from steady state ($z_t = (Z_t - Z)/Z$), except for $g_t = (G_t - G)/Y$. Defining $r_{\beta,t} = \beta_{t+1}^{-1} - 1$ and $\bar{g} = G/Y$, the linearized model is given by (see Appendix A for details)

$$
\begin{aligned}
c_t &= E_t c_{t+1} - \left(i_t - E_t \pi_{t+1} - r_{\beta,t}\right), \tag{14} \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa \left(\Theta_c c_t + \Theta_g g_t - \Theta_k k_{G,t}\right), \tag{15} \\
k_{G,t} &= (1-\delta) k_{G,t-1} + \alpha \tilde{\delta} g_{t-T}, \tag{16} \\
i_t &= \max\left(0; r_\beta + \phi_\pi \pi_t\right), \tag{17}
\end{aligned}
$$

where κ, $\tilde{\delta}$, and Θs are positive functions of the model parameters whose precise expressions are reported in Appendix A.

Model (14)–(17) nests the framework considered by Christiano et al. (2011), Eggertsson (2011) and Woodford (2011), in which public spending plays no productive role. This case can be recovered either by assuming that $F_K = 0$ (which implies that $\Theta_k = 0$) or by setting $\alpha = 0$. In the former case, public capital does not affect the marginal productivity of private inputs. In the latter, the fraction of additional public spending devoted to investment is nil, implying that the stock of public capital remains constant at its steady-state level.

3 Public Investment, Time to Build, and the Spending Multiplier

We use the model just described to quantify the spending multiplier and its dependence on (i) the share of public investment and (ii) the time-to-build delay. We start by studying the case in which the ZLB does not bind (i.e., normal times) before turning to the case in which the nominal interest rate hits the ZLB.

3.1 The spending multiplier in normal times

In normal times, the model is given by equation (14)–(17) with $i_t = r_\beta + \phi_\pi \pi_t$. Under the assumption $\phi_\pi > 1$, and given the process (9), the unique linear rational-expectation solution of the model is given by

$$c_t = \vartheta_k k_{G,t-1} + \sum_{\tau=0}^{T} \vartheta_g^\tau g_{t-\tau}, \tag{18}$$

$$\pi_t = \zeta_k k_{G,t-1} + \sum_{\tau=0}^{T} \zeta_g^\tau g_{t-\tau}, \tag{19}$$

where the coefficients ϑ_k, ζ_k, ϑ_g^τ, and ζ_g^τ ($\tau = 0, ..., T$) are given in Appendix A.6.1 . The impact multiplier, m^0, is then given by

$$m^0 = 1 + (1 - \bar{g}) \vartheta_g^0.$$

In order to study the way in which m^0 varies with the share of public investment, α, and the time-to-build delay, T, we need to assign values to the model parameters. Table 1 summarizes the chosen parameter values. We closely follow Christiano et al. (2011)'s calibration. In particular, we use their values for β, γ, a, κ, ϕ_π, and ρ, as well as for the steady-state ratio of government spending to output, $\bar{g}$. However, we set $\sigma = 1$ to ensure that $U_{CN}(.) = 0$.[9] We also need to assign values to three additional parameters that are absent from Christiano et al. (2011)'s model: the steady-state share of public investment in total public spending, $\bar{\alpha}$, the depreciation rate of public capital, δ, and the elasticity of output with respect to public capital, b. The parameter $\bar{\alpha}$ can be approximated by the historical average ratio of public investment to total public spending, which is roughly 0.23 in the U.S. (based on the sample period 1960–2014). We set the depreciation rate of public capital, δ, to 0.02 as in Leeper et al. (2010). The parameter b is less straightforward to parameterize, as available empirical estimates of the elasticity of output with respect to public capital vary considerably depending on the methodology and sample period considered. We use the meta-analysis estimate of 0.08 reported by Bom & Ligthart (2014), which lies between the two values considered by Leeper et al. (2010) (0.05 and 0.1).[10] In Section 5.4, we study the sensitivity of

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[9] Christiano et al. (2011) set $\sigma = 2$ in their benchmark calibration. See Bilbiie (2011) and Monacelli & Perotti (2008) for an analysis of the effects of public spending under the assumption of non-separability between consumption and leisure.

[10] See Section 4 for further discussion of existing empirical work and some independent evidence regarding the productivity of public capital.

our results with respect to the parameter b within a fully fledged medium-scale dynamic stochastic general-equilibrium (DSGE) model.

Table 1: Parameter Values

Discount factor	$\beta = 0.99$
Preference parameter	$\sigma = 1$
Preference parameter	$\gamma = 0.29$
Elasticity of output w.r.t labor	$a = 1$
Elasticity of output w.r.t public capital	$b = 0.08$
Depreciation rate of public capital	$\delta = 0.02$
Elasticity of inflation w.r.t real marginal cost	$\kappa = 0.03$
Inflation feedback parameter	$\phi_\pi = 1.5$
Autocorrelation coefficient of public spending	$\rho = 0.8$
Steady-state ratio of public spending to output	$\bar{g} = 0.2$
Steady-state ratio of public investment to total public spending	$\bar{\alpha} = 0.23$

Using the chosen parameter values, we compute m^0 as a function of α for $T = 1$, 4, 16. The results are depicted in Figure 1. The case $T = 1$ corresponds to the one-quarter delay assumed in Baxter & King (1993), Coenen et al. (2013), and Drautzburg & Uhlig (2013). The case $T = 4$ corresponds to quick-to-implement investment projects like minor maintenance of public infrastructure. Finally, the case $T = 16$ corresponds to investment projects that require a long time to complete, like major repairs or construction of new buildings and highways.
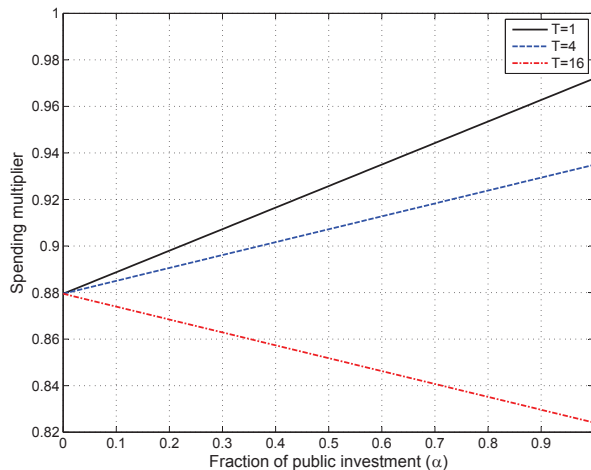

Figure 1: Spending multiplier as a function of the fraction of public investment and the time-to-build delay in normal times.

The first message that Figure 1 conveys is that the spending multiplier is monotonically increasing in α when the time to build is short or moderate, and monotonically decreasing in α when the time to build is long. To understand the intuition behind this result, it is useful to recall that the size of the multiplier depends on the response of consumption, which in our sticky-price

economy depends on the deviation of the long-term *ex ante* real interest rate from its steady-state value, $r_t^{LT} \equiv E_t \sum_{\tau=0}^{\infty}(i_{t+\tau} - \pi_{t+1+\tau} - r_\beta)$.[11] The latter is in turn determined by the entire path of (expected) inflation and ultimately by the dynamic response of real marginal cost (or, alternatively, the output gap).

In addition to its direct effect on current and future aggregate demand (since the shock is serially correlated), an increase in public investment has similar effects to those of an anticipated technology shock. On the one hand, it exerts downward pressure on real marginal cost and inflation once public capital becomes productive (supply-side effect). On the other hand, by increasing households' expected wealth, it raises aggregate demand, real marginal cost, and inflation even before public capital becomes productive.

Figure 2 depicts the dynamic effects of a positive government spending shock for $\alpha = 0$, $\{\alpha = 1,\ T = 4\}$, and $\{\alpha = 1,\ T = 16\}$. Owing to the wealth effect just described, inflation rises more in the short run in response to an increase in public investment ($\alpha = 1$) than in response to an increase in public consumption ($\alpha = 0$). With a time-to-build delay of 4 quarters, the disinflationary effect associated with the increase in labor productivity occurs rapidly so that inflation is short-lived. As a result, the initial response of the long-term real interest rate is smaller — implying a larger consumption response and thus a larger spending multiplier — than when $\alpha = 0$. In turn, this explains why the multiplier is increasing in α when the time to build is short. When the time to build is 16 quarters, the supply-side effect of public investment comes into play far in the future so that the inflationary pressure arising both from the direct increase in aggregate demand and from higher expected wealth lasts for a long period of time. In response to the public spending shock, the long-term real interest rate thus rises more and consumption falls more — implying a smaller multiplier — than when $\alpha = 0$. This also explains why the multiplier decreases with α in this case.

The second observation that emerges from Figure 1 is that the spending multiplier is always smaller than 1. Thus, under plausible parameter values, the spending multiplier in normal times remains numerically close to that predicted by a standard model in which all public spending is unproductive.

3.2   The spending multiplier in a liquidity trap

We now study the case in which the ZLB on the nominal interest rate binds, in which case, the model is given by equations (14)–(17) with $i_t = 0$. The ZLB becomes binding as a result of a shock

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[11]Iterating equation (14) forward yields

$$c_t = \lim_{j\to\infty} E_t c_{t+j} - E_t \sum_{\tau=0}^{\infty}(i_{t+\tau} - \pi_{t+1+\tau} - r_\beta) = \lim_{j\to\infty} E_t c_{t+j} - r_t^{LT}.$$

In response to a transitory shock, $\lim_{j\to\infty} E_t c_{t+j} = 0$ so that current consumption moves in an opposite direction to r_t^{LT}.
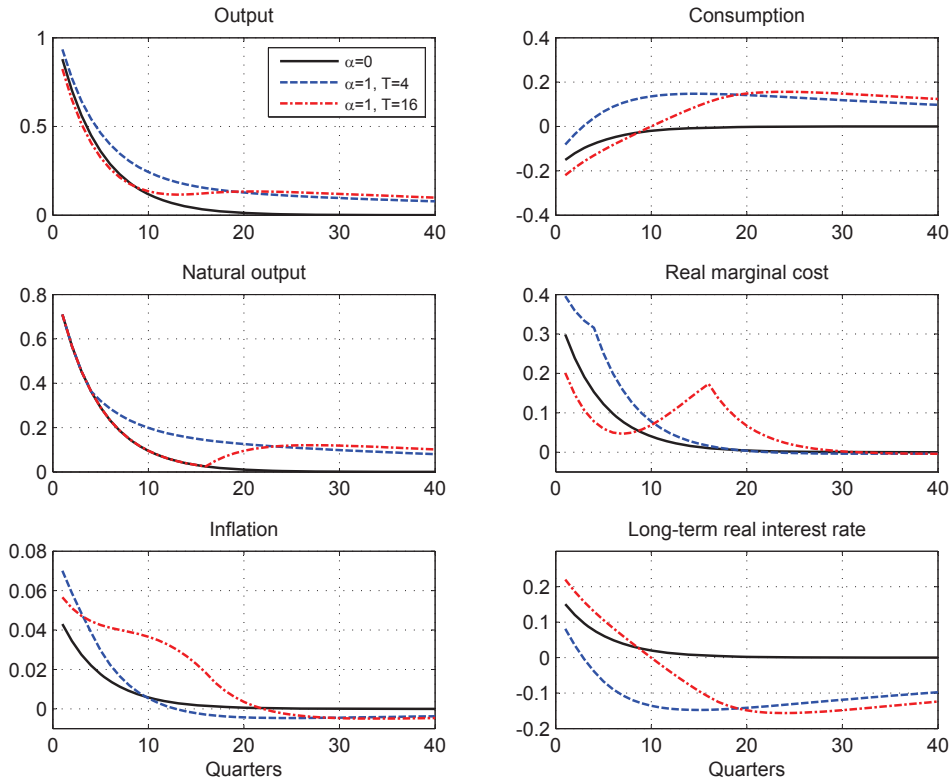
Figure 2: Dynamic effects of an increase in public spending equal to 1 percent of steady-state output in normal times.

Notes: All responses are expressed as percentage variations from steady state, except the responses of inflation and the long-term real interest, which are expressed as percentage points deviations from steady state. The long term real interest rate (in deviation from steady state) is given by $r_t^{LT} \equiv E_t \sum_{\tau=0}^{\infty} (i_{t+\tau} - \pi_{t+1+\tau} - r_\beta)$.

that raises the discount factor. As in Christiano et al. (2011), Eggertsson (2011) and Woodford (2011), we assume that the discount factor can take only two possible values, β and $\beta^l > \beta$, and evolves according to the following process:

$$
\begin{aligned}
Pr\left[\beta_{t+1} = \beta^l | \beta_t = \beta^l\right] &= p, \\
Pr\left[\beta_{t+1} = \beta^l | \beta_t = \beta\right] &= 0,
\end{aligned}
\tag{20}
$$

where p is the probability that the discount factor remains high. For simplicity, we also assume that $g_{t+1} = g_t > 0$ if the ZLB is still binding in $t+1$, and $g_{t+1} = 0$ otherwise. Finally, we assume that the ZLB will bind as long as the discount factor is high. Note that the economy does not immediately return to steady state when the ZLB ceases to bind, as the stock of public capital continues to adjust and converges to its steady state only gradually. Conditional on the economy

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being at the ZLB, the expected value of a given variable in $t+1$ is an average of its possible values in $t+1$ inside and outside the liquidity trap, weighted by the respective probabilities of being in these two states. The linear rational-expectation solution of the model when the ZLB binds is given by

$$c_t = \vartheta_i^l r_\beta^l + \vartheta_k^l k_{G,t-1} + \sum_{\tau=0}^{T} \vartheta_g^{l,\tau} g_{t-\tau}, \tag{21}$$

$$\pi_t = \zeta_i^l r_\beta^l + \zeta_k^l k_{G,t-1} + \sum_{\tau=0}^{T} \zeta_g^{l,\tau} g_{t-\tau}, \tag{22}$$

where the coefficients ϑ_k^l, ζ_k^l, $\vartheta_g^{l,\tau}$, and $\zeta_g^{l,\tau}$ ($\tau = 0, ..., T$) are given in Appendix A.6.2. The impact spending multiplier when the ZLB binds, $m^{l,0}$, is

$$m^{l,0} = 1 + (1 - \bar{g}) \vartheta_g^{l,0}.$$

We compute $m^{l,0}$ as a function of α for $T = 1, 4, 16$ using the parameter values in Table 1 and a value of 0.8 for p. The results, depicted in Figure 3, indicate that the spending multiplier is decreasing in α when the time to build is very short (1 quarter), but increases with α when the time to build is moderate or long.
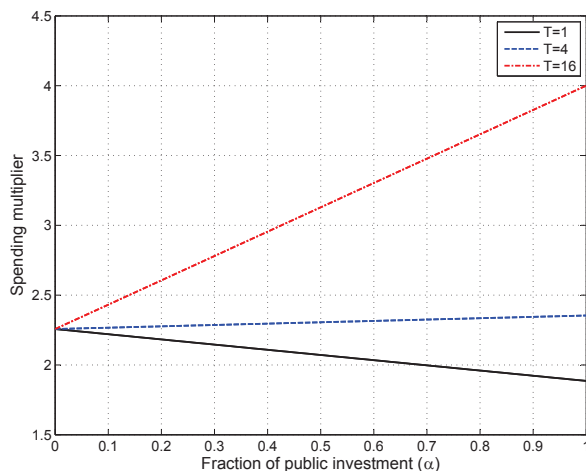

Figure 3: Spending multiplier as a function of the fraction of public investment and the time-to-build delay in a liquidity trap.

Intuitively, since the nominal interest rate is stuck at zero, the inflationary effect of an increase in public spending translates into a fall in the real interest rate, which further increases aggregate demand, thus creating a virtuous circle that results in a large increase in output. When the time to build is sufficiently short, the disinflationary (supply-side) effect associated with the increase in public capital comes about quickly after the initial shock and hence remains in effect for a prolonged period of time while the ZLB is still binding. This effect greatly attenuates the inflationary pressure

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stemming from the increase in aggregate demand and from the increase in households' expected wealth. As a result, the long-term real interest rate falls less compared with the case $\alpha = 0$, which in turn implies that the multiplier is a decreasing function of α.[12]

As the time to build increases, the disinflationary effect brought about by the increase in public capital is further delayed, becoming increasingly more likely to occur after the economy has escaped from the liquidity trap. At the same time, the positive wealth effect associated with the expected increase in labor productivity amplifies inflation while the economy is still in the liquidity trap. When the time to build is sufficiently long, this implies a larger fall in the long-term real interest rate compared with the case $\alpha = 0$, which also means that the multiplier is increasing in α.

Figure 3 also shows that the spending multiplier can be substantially large at the ZLB when a large fraction of public spending is invested in public capital and when the time-to-build delay is relatively long. With a time to build of 16 quarters, the multiplier associated with public investment is five times larger than in normal times and nearly twice as large as the multiplier associated with public consumption. In Appendix B, we show that this result holds robustly when we (*i*) focus on the present-value rather than the impact multiplier, (*ii*) take into account the gradual nature of public investment outlays, and (*iii*) allow for the possibility of endogenous exit from the liquidity trap.

4   Discussion: Productivity of Public Capital

Central to our results is the premise that public capital is productive. This hypothesis is certainly not uncontroversial, as public investment projects are sometimes regarded as being inefficient, public services are typically characterized by some degree of congestion, and public infrastructure often lacks adequate maintenance. The productivity of public capital has been empirically investigated by a voluminous literature using various samples and methodologies. In this section, we provide a rough typology of existing approaches and a brief summary of their findings; we then present some independent evidence on the productivity of public capital in the U.S.[13]

4.1   Overview of existing empirical work

Three different methodologies have been proposed to measure the output effect of public capital. The first — and arguably most prevalent — approach evaluates the elasticity of output with respect to public capital by estimating an aggregate production function in which the stock of public capital enters as an input. This approach was popularized by Aschauer (1989) who estimates an elasticity of private output with respect to nonmilitary public capital of 0.39 using U.S. data. Although the

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[12]These results are consistent with those reported by Coenen et al. (2013), who assume a time-to-build delay of one quarter and find that the multiplier associated with public investment is lower than that associated with public consumption when the nominal interest rate is kept constant for 2 years.

[13]For recent surveys of the empirical literature on the productivity of public capital see Romp & de Haan (2007), Pereira & Andraz (2013), and Bom & Ligthart (2014).

subsequent literature that followed this approach report a wide spectrum of estimates that range from large negative to large positive numbers, Bom & Ligthart (2014) note that most of available estimates are positive. These authors perform a meta-regression analysis based on a sample of 578 estimates collected from 68 published studies that estimate Cobb-Douglas production functions using either time-series or panel data. They find a meta-estimate of 0.082 when using a broad definition of public capital measured at the national level, and a larger estimate of 0.131 when focusing on core infrastructure (airports, railways, roads, utilities,...).

The second approach relies on cross-country growth regressions, allowing public investment to be one of the explanatory variables. This approach yields mixed results regarding the effect of public investment on growth. For instance, Easterly & Rebelo (1993) find that the share of public investment in transport and communication infrastructure has a positive effect on growth, whereas Devarajan et al. (1996) find a negative effect for developing countries.

The third approach uses vector autoregressions and/or vector error-correction models to estimate the dynamic output effects of exogenous and unanticipated changes in the stock of public capital or in the flow of public investment. These spending *shocks* are typically identified via recursive schemes whereby public capital/investment are assumed to be predetermined with respect to aggregate output. (e.g., Perotti (2004b), Kamps (2005), Leduc & Wilson (2013)). This literature generally finds that public investment has a positive and persistent effect on output.

An alternative — less common — line of research evaluates the effect of public capital on the productivity of private inputs. Using the production-function approach, Aschauer (1989) estimates a large positive effect of nonmilitary public capital on total factor productivity (TFP) in the U.S. Lynde & Richmond (1993) estimate a translog profit function for the U.S. and also conclude the ratio of public capital to labor is an important determinant of TFP. Finally, Fernald (1999) finds a causal effects of growth in roads (the largest component of U.S. public infrastructure) on the productivity of U.S. vehicle-intensive industries.

In light of these results, it is safe to conclude that existing empirical evidence generally supports the premise that public capital is productive, though probably not to the extent suggested by Aschauer (1989)'s work.

4.2   Some new evidence

We use cointegration techniques and annual U.S. data from 1960 to 2014 to empirically investigate the existence of a long-run relationship between the stock of public capital and TFP. Our approach is motivated by the availability of the adjusted TFP series constructed by Fernald (2014) for the U.S. business sector, which takes into account unobserved input variation and the possibility of non-constant returns to scale in the production function. Failing to account for factor utilization and/or incorrectly specifying the returns to scale when estimating aggregate production functions leads to mistaken inference about the productivity of public capital. The availability of the adjusted TFP

series allows us to measure the contribution of public capital to TFP without having to estimate a production function. In doing so, however, it is still important to account for the additional factors that may affect TFP in the long run. We restrict our attention to the two factors that figure prominently in the literature on the determinants of TFP, namely, technology and human capital. Our approach therefore consists in testing whether there is a cointegrating relationship between TFP, public capital, technology, and human capital. The last three variables are measured as follows.

Public capital is measured by the stock of government fixed assets (source: Bureau of Economic Analysis), expressed in real per capita terms. Technology is proxied by the real stock of research and development (R&D) spending, constructed from the real per capita flow of R&D spending (source: Federal Reserve Bank of Saint-Louis' FRED database) using the perpetual inventory method with a depreciation rate of 30 percent. This treatment follows Bloom & Reenen (2002), who argue that a stock measure of knowledge is more appropriate than a flow measure as the benefits from new discoveries are likely to persist over time. The same methodology is used to construct a proxy for the stock of human capital from the real per capita flow of personal spending on education (source: Federal Reserve Bank of Saint-Louis' FRED database).[14] Our constructed measures of TFP, public capital, technology and human capital are expressed in logarithm. The four series are depicted in Figure 10 in Appendix C.

We start by testing for the presence of a unit root in each of the series. We use the Augmented Dickey-Fuller and Phillips-Perron tests. A visual inspection of the time series depicted in Figure 10 suggests that the appropriate specification should include an intercept and a trend. The test results (shown in Table 3 in Appendix C) indicate that one cannot reject the null hypothesis that each of the series contains a unit root at the 5 percent level.[15] In addition, when the series are expressed in first differences, the null hypothesis (of a unit root) is strongly rejected at any conventional level of significance (not reported). Together, these results indicate that our series can be characterized as I(1) processes. Next, we test for the presence of cointegration among these series. We use the Engel-Granger and Johansen tests, and report the results in Table 4 in Appendix C. The Engel-Granger test rejects the null hypothesis of no cointegration at the 5 percent level. Johansen's Trace test indicates that there are 2 cointegrating vectors, whereas Johansen's Maximum Eignevalue test indicates that there is one cointegrating vector.

Having confirmed that there is at least one cointegrating relationship between TFP, the stock of public capital, the stock of R&D spending and the stock of personal spending on education, the final step of our analysis consists in estimating a cointegration equation in which the dependent variable is TFP and the regressors are the remaining variables. The equation is estimated using

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[14]To express variables in real terms, we have divided them by the GDP deflator. To express them in per capita terms, we have divided them by the civilian noninstitutional population age 16 and over. The GDP deflator and population series are taken from the Federal Reserve Bank of Saint-Louis' FRED database.

[15]For R&D spending, the null is rejected by the Phillips-Perron test at the 5 percent level but not by the ADF test, and the opposite is true for personal spending on education.

Dynamic Ordinary Least Squares (DOLS) and Fully Modified Ordinary Least Squares (FMOLS). The results are reported in Table 2 below. Note that because the variables are measured in log, and to the extent that the production technology can be described by a Cobb-Douglas function, the estimated coefficient of public capital can be interpreted as an estimate of the parameter b.

Starting with the DOLS estimates, the results indicate that all three determinants have positive and statistically significant effects on TFP. The estimated elasticity of TFP with respect to public capital is 0.065, which is fairly close to our calibrated value of 0.08. The FMOLS estimation yields a similar conclusion, although the estimated coefficient of public capital is somewhat higher (0.15). These results lend further support to the hypothesis that public capital enhances the productivity of private inputs.[16]

Table 2: Cointegration Equation Estimation Results (Dependent Variable: TFP)

| | Estimated Coefficient | |
Variable	Dynamic OLS	Fully Modified OLS
Public capital	0.0652 (0.0016)	0.1589 (0.0047)
R&D spending	0.2155 (0.0001)	0.1432 (0.0116)
Personal spending on education	0.2869 (0.0001)	0.1981 (0.0024)
Constant	-3.7637 (0.0001)	-3.6026 (0.0001)
Trend	-0.0045 (0.0021)	-0.0014 (0.4050)

Notes: RD spending is a proxy for technology. Personal spending on education is a proxy for human capital. P-values (between parentheses) are based on heteroskedasticity-autocorrelation robust standard errors computed using a pre-whitened Bartlett Kernel with a bandwidth of 4. The number of leads and lags used in the Dynamic OLS regression is selected based on the Schwarz criterion, which suggests to include one lead and no lags of the first difference of the regressors.

5 Public Investment and Time to Build in a Medium-Scale DSGE Model

Owing to its simplicity, the model presented in Section 2 has been useful to convey the main insights about the macroeconomic effects of public investment both in normal times and when the ZLB binds. A serious quantitative evaluation of these effects, however, requires a richer model that

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[16]We performed robustness checks by considering alternative proxies for technology and for human capital. For technology, we constructed a stock of issued patents using the same perpetual inventory method described above (source: The USPTO Historical Patent Data Files). For human capital, we used life expectancy at birth (source: Federal Reserve Bank of Saint-Louis' FRED database) and the school enrollment rate of the population age 20-21 and 22-24 (source: Census Bureau). The results are shown in Table 5 in Appendix C. In all of the cases considered, the estimated coefficient of public capital is found to be positive and highly statistically significant, ranging from 0.16 to 0.23.

incorporates empirically relevant features that can help account for business-cycle fluctuations, such as the accumulation of private capital, variable capacity of utilization, and labor-market frictions. There are by now a number of DSGE models that perform remarkably well in accounting for aggregate fluctuations. We choose to work with the model recently developed by Christiano et al. (2013), which we extend to allow for the accumulation of public capital and for time-to-build delays. We use the model to evaluate the multiplier associated with the spending component of the 2009's ARRA and to perform counterfactual experiments.

5.1 Overview of the model

We briefly summarize the model here and refer the reader to Christiano et al. (2013) for a complete exposition. The economy is populated by a representative household whose preferences exhibit habit formation with respect to consumption. The household accumulates capital subject to investment-adjustment costs and chooses the utilization rate when renting capital to firms. Working members of the household earn a real wage while the remaining members receive unemployment benefits.

The final good, which is used for consumption and investment purposes, is produced by a competitive representative firm using differentiated inputs produced by monopolistically competitive retailers. Retailers set their prices à la Calvo. Their production technology depends on private capital, public capital, and an intermediate good produced by wholesalers using labor. Formally, each retailer z has access to the following production technology:

$$X_t(z) = N_t(z)^a (u_t(z) K_t(z))^{1-a} K_{G,t}^b - \phi_t, \tag{23}$$

where $u_t(z)$ is the utilization rate, $N_t(z)$ is the quantity of intermediate good, and ϕ_t is a fixed cost.

To hire new workers, wholesale producers post vacancies at zero cost. At the beginning of the period, each worker engages in bilateral bargaining with a representative of the firm, and the equilibrium real wage is the outcome of an alternating offer bargaining process along the lines of Hall & Milgrom (2008). At the end of each period, a constant fraction of employed workers lose their jobs.

The government levies lump-sum taxes to finance its purchases and the unemployment benefits paid to unemployed workers. As in the simple model presented in Section 2, we assume that a fraction α of government spending on goods and services is allocated to investment goods, and that public investment projects are subject to time-to-build delays. In order to capture the fact that public investment outlays typically occur gradually over time after spending has been authorized, we specify the accumulation equation of public capital and the fiscal policy rule as, respectively

$$K_{G,t} = (1 - \delta) K_{G,t-1} + A_{t-T}^i,$$

and

$$A_t^i = A^i + \alpha (A_t - A),$$

17

where A_t^i and A_t are, respectively, investment and total spending authorized in period t. Authorized investment spending is converted into outlays according to

$$G_t^i = \sum_{j=0}^{T-1} \omega_j A_{t-j}^i,$$

where ω_j, $j = 0, 1..., T - 1$ are the spend-out rates, which satisfy $\sum_{j=0}^{T-1} \omega_j = 1$.[17]

Finally, the monetary authority follows a Taylor rule whereby deviations of the nominal interest rate from its steady-state value respond to deviations of inflation, output, and the lagged nominal interest rate from their respective steady-state values. Whenever this rule calls for a negative nominal interest rate, the latter is set to zero.

5.2   Calibration

To assign values to the model parameters, Christiano et al. (2013) fix a subset of them à priori and estimate the rest via a Bayesian minimum-distance strategy using U.S. data. We use their calibrated/estimated values. For the new parameters that we add to the model, namely, the elasticity of output with respect to public capital, b, the fraction of public investment in government spending, α, the length of time-to-build delay, T, and the spend-out rates, ω_i, we proceed as follows. For b, we choose the same value as in Section 3, that is, $b = 0.08$. For α, T, and ω_i, we turn to the composition of the spending component of the ARRA, the nature of public investment projects that it comprised, and their financing scheme.

The ARRA allocated roughly \$350 billion to non-transfer spending, i.e., spending on goods and services. Drautzburg & Uhlig (2013) estimate that roughly 40% of this amount was devoted to public investment. Accordingly, we set $\alpha = 0.4$. The bulk of public investment projects financed by the ARRA were infrastructure projects, including efficient and renewable energy projects (\$57 billion), highway construction (\$27.5 billion), transportation infrastructure (\$18 billion), and building government facilities (\$5.5 billion). Infrastructure projects typically involve long time-to-build delays. For instance, citing the Federal Highway Administration (FHWA), the Government Accountability Office (GAO) reports that most highway construction projects in the U.S. take between 4 and 6 years to complete, though the construction of major new highways may take up to 19 years from planning to completion.[18] Non-infrastructure projects may not entail such a long time to build, but they still require several quarters to complete.[19] Based on these arguments, we set $T = 16$

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[17] See Appendix B.2 for further details.

[18] Source: http://www.gao.gov/assets/110/109588.pdf

[19] Some studies attempted to estimate the time-to-build lags of private investment projects. For instance, Krainer (1968) finds that investment projects in the U.S. automobile industry take 2 to 3 years from approval to completion. Montgomery (1995) estimates that U.S. private non-residential construction projects have a mean time-to-build delay of 17 months. Koeva (1995) reports that the average construction lead time for new plants is around 2 years in most U.S. industries. While we are not aware of studies that estimate the time-to-build delays associated with public investment projects, it is widely believed that these projects take significantly longer to implement than those in the private sector.

quarters.[20]

According to the Congressional Budget Office, by the end of 2011, more than 90% of public spending under the ARRA was obligated. Outlays, on the other hand, spanned a longer period of time, as the awarded amounts were paid out to recipients gradually. To capture these features, we approximate the path of spending obligations by an AR(1) process with an autocorrelation coefficient of 0.8, and set the spend-out rates as follows. We set $\omega_0 = 0$ and $\omega_1 = \omega_2 = \omega_3 = 0.25/3$ for the first year, and $\omega_4 = \cdots = \omega_{15} = 0.75/12$ for the subsequent 3 years. This calibration of the spend-out rates assumes that obligations start to show up as outlays with a lag of one quarter, and that one fourth of the obligated amount is outlaid in each year.

5.3 Quantifying the spending multiplier of the ARRA

To compute the spending multiplier associated with the non-transfer component of the ARRA, we consider the following experiment. We assume that the economy is initially in steady state. Then, a sequence of positive shocks to the discount factor hit, driving the nominal interest rate down to zero. We choose the size and persistence of these shocks such that the resulting trough in output is 5.65% below the pre-shock level, exactly matching the observed decline in U.S. real per capita GDP between $2007Q4$ (peak) and $2009Q2$ (trough).[21] This generates a liquidity trap of 14 quarters. As soon as the ZLB binds, the government authorizes an increase in public spending equal to 1% of steady-state output. The composition and persistence of this stimulus mimic those of the non-transfer component of the ARRA. For simplicity, we henceforth refer to it as the ARRA.

Figure 4 depicts the dynamic response of the economy to the ARRA. The response of output is scaled by the initial shock so that it can be interpreted as a dynamic multiplier. The response of private consumption, private investment, and real marginal cost are expressed as percentage deviations from the benchmark scenario in which only the discount-factor shocks are in effect, while the response of inflation and the real interest rate are expressed as percentage point deviations from the benchmark scenario. The figure also shows the results of a counterfactual experiment in which the stimulus is exclusively composed of public consumption (i.e., $\alpha = 0$).

Starting with the latter case, the figure shows that the increase in public consumption generates a hump-shaped output response that reaches its peak 2 quarters after the shock. At the peak, the spending multiplier is equal to 1.21. The fact that the multiplier is larger than 1 echoes the by-now well-established result on the effects of public spending when the ZLB is binding. Because the nominal interest rate cannot adjust, the inflationary effect of higher public spending lowers the real interest rate, which crowds-in of private consumption and investment and further increases aggregate demand. The fact the multiplier is lower than those reported by earlier studies is due to two factors. First, studies that specify an AR(1) process for the spending shock usually assume

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[20]In their study of the effects of public infrastructure, Leduc & Wilson (2013) also assume a time-to-build delay of 4 years.

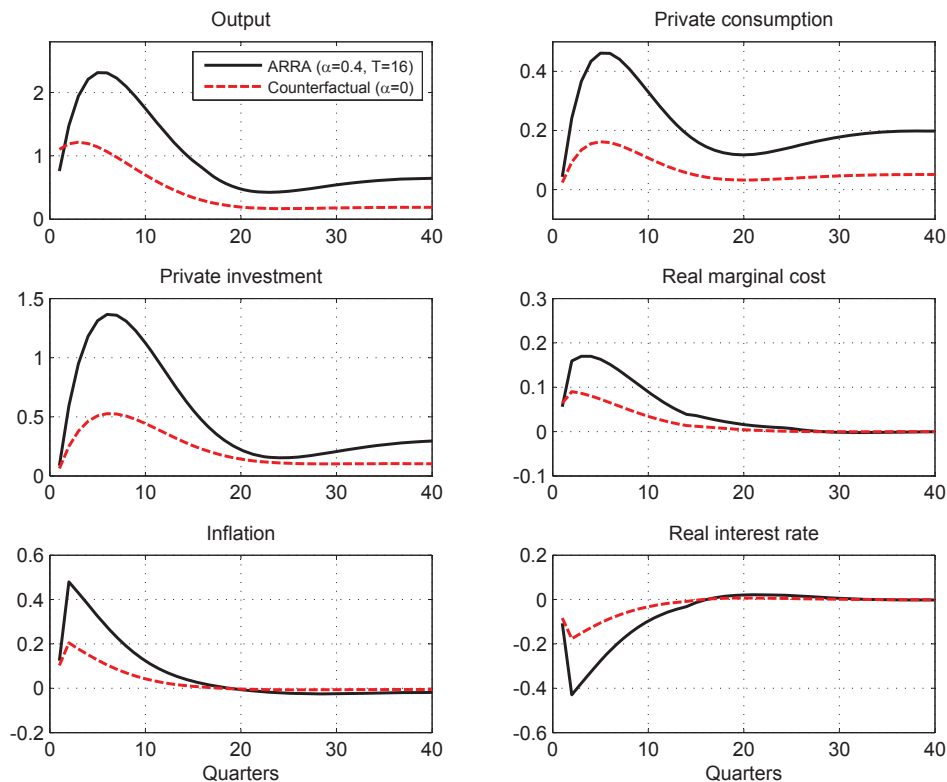[21]Source: http://research.stlouisfed.org/fred2/series/A939RX0Q048SBEA

Figure 4: Dynamic effects of the ARRA.

that the shock is highly persistent — an assumption that is hard to justify in the context of the ARRA. Second, the presence of real wage rigidity arising from the alternating offer bargaining process tends to dampen the response of inflation to demand shocks, thus lowering the size of the spending multiplier.

Next, consider the dynamic effects of the ARRA, which involves 40% of the authorized spending in the form of public investment and where public investment projects take 16 quarters to complete. The output response is larger in magnitude and exhibits a more pronounced hump compared with the case $\alpha = 0$. The peak multiplier is equal to 2.31, that is, nearly twice as large as the maximum multiplier associated with public consumption. The amplification effect of public investment and the time-to-build delay is even more dramatic on the present-value multiplier, which increases from 1.62 when all spending is assumed to be unproductive to 5.41 under the ARRA.[22] As explained in section 3.2, with long time-to-build delays, the disinflationary effects of public investment come into play far in the future, while the expected increase in the marginal productivity of private inputs

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[22]The present-value multiplier is computed for an horizon of 1000 quarters (see B.1 for details). The peak and present-value multipliers remain virtually unchanged when we consider $T = 20$ or $T = 24$ (results are available upon request).

raises households' expected wealth and contributes to increasing aggregate demand an inflation in the short run. Note that under our calibration, this additional inflation occurs while the ZLB is still binding, which amplifies the multiplier.

To summarize, our analysis suggests that the spending component of the ARRA had a large multiplier, whether measured at the peak or in present value, and that failing to account for the composition of the stimulus by counterfactually assuming that it was exclusively composed of public consumption spending would lead one to underestimate the multiplier by roughly 50%. Admittedly, our estimate of 2.31 might be subject to some important caveats, notably the fact that we abstract from public debt, transfers, and distortionary taxation. As such, our results are probably best viewed as illustrating the extent to which long-lasting public investment projects are a more potent tool than public consumption to stimulate an economy stuck in a liquidity trap.

5.4 Sensitivity analysis

In this section, we study the sensitivity of our results to changes in (i) the elasticity of output with respect to public capital, b, and (ii) the way in which public capital enters the production technology. For the latter case, we consider a CES production function and allow both for complementarity and substitutability between private and public capital. In all experiments, we adjust the size of the preference shock so as to obtain an output trough of 5.65% as in the baseline calibration.

5.4.1 Alternative values of the elasticity of output with respect to public capital

We consider two alternative values of b, 0.05 and 0.15, which are fairly close to the DOLS and FMOLS estimates we obtain in our empirical analysis discussed in Section 4.2. Figure 5 shows the spending multiplier associated with the ARRA obtained under the two alternative values of b. For ease of comparison, we also reproduce the results obtained using the benchmark value of b (0.08) and under the counterfactual scenario in which public capital remains constant ($\alpha = 0$).
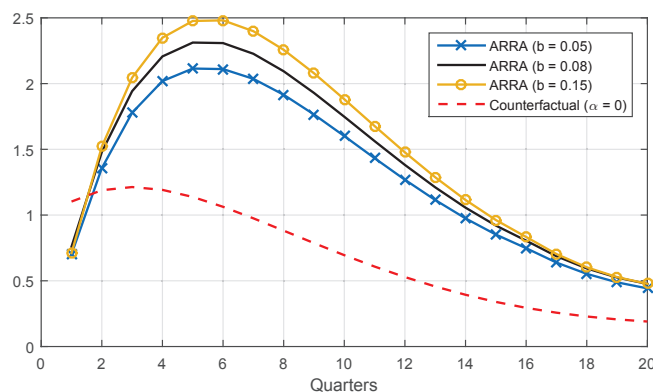

Figure 5: The spending multiplier with different values of b.

Figure 5 shows that the higher the elasticity of output with respect to public capital, the higher the spending multiplier. This result is very intuitive: a larger value of b means that the aggregate supply effects of public investment is amplified. Because the time-to-build delays imply that this effect occurs precisely when it is needed, i.e., once the economy has exited the liquidity trap, this translated into a larger multiplier. Note that even when b is as low as 0.05, the multiplier is still significantly larger than under the counterfactual scenario, reaching a peak of roughly 2.15.

5.4.2 CES production function

So far, we have worked with a Cobb-Douglas production technology that (abstracting from fixed costs) exhibits constant returns to scale with respect to private inputs, and which — by construction — implies a unitary elasticity of substitution between public capital and private inputs. We relax both assumptions by adopting the CES production function proposed by Coenen et al. (2013):

$$X_t(z) = N_t(z)^a \left\{ \varrho^{\frac{1}{\nu}} (u_t(z)K_t(z))^{\frac{\nu-1}{\nu}} + (1-\varrho)^{\frac{1}{\nu}} K_{G,t}^{\frac{\nu-1}{\nu}} \right\}^{\frac{\nu}{\nu-1}(1-a)} - \phi_t, \qquad (24)$$

where $0 \leq \varrho \leq 1$ is the share of private capital in the composite capital stock, and ν is the elasticity of substitution between private and public capital. As long as $\nu \neq 1$, the specification above implies that the elasticity of output with respect to public capital is not constant.

Coenen et al. (2013) fix $a = 0.7$ and $\varrho = 0.9$, and estimate $\nu = 0.84$, implying that there is a slight complementarity between private and public capital. In addition to this parametrization, we also consider the case in which private and public capital are substitutes by setting $\nu = 1.16$. The spending multiplier associated with the ARRA obtained in these two cases are reported in Figure 6. In each case, we also report the multiplier obtained under the counterfactual scenario ($\alpha = 0$).
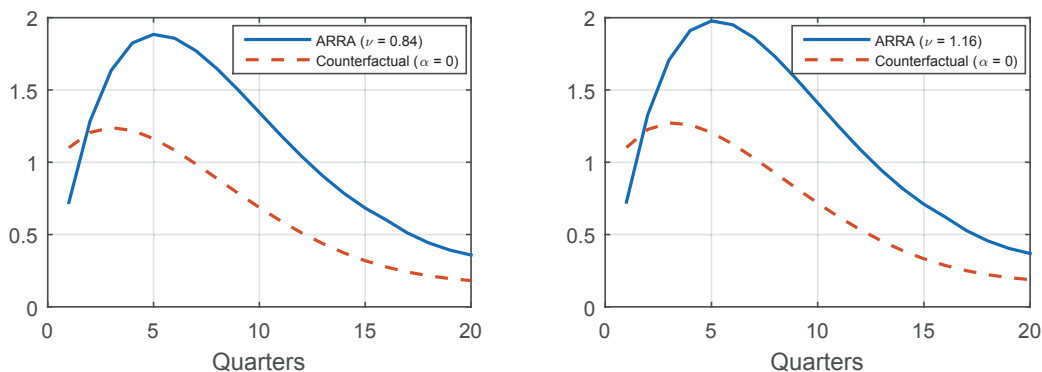

Figure 6: The spending multiplier with a CES production function and different values of ν.

Figure 6 shows that the value of the multiplier exhibits little sensitivity with respect to ν (within the chosen range). While the ARRA multiplier is slightly lower than that obtained in the baseline simulations, it remains large, peaking at roughly 1.9 when $\nu = 0.84$ and 2 when $\nu =$

1.16.[23] Importantly, these numbers are 50% larger than what one would obtain by counterfactually assuming that the stimulus plan was strictly composed of public consumption.

6 Conclusion

The main lesson from the literature on the effects of fiscal policy in a liquidity trap is that policies that stimulate aggregate demand can have substantially larger effects when the ZLB binds than in normal times. Conversely, policies that raise the natural level of output while the economy is in a liquidity trap may depress economic activity even further (Eggertsson (2011)). Yet, Fernández-Villaverde et al. (2011) and Eggertsson et al. (2014) show that supply side policies that take effect after the ZLB has ceased to bind are desirable because they generates a positive wealth effect that stimulates aggregate demand when the ZLB is still binding. In this paper, we have shown that when the time to build the stock of public capital is sufficiently long, public investment will raise both aggregate demand while the economy is in the liquidity trap, and the natural level of output once the economy has exited the liquidity trap, thereby further fueling aggregate demand in the short run. A corollary is that, in states where the ZLB binds, the spending multiplier will be an increasing function of the share of public investment in a stimulus plan. Because the time to build the stock of public capital is typically long — often spanning several years, our analysis suggests that public investment is a highly effective tool to stimulate the economy at the ZLB.[24]

Using an extended version of Christiano et al. (2013)'s medium-scale model to quantify the output effects of the 2009's ARRA, we found that the stimulus had a peak multiplier that exceeds 2. Our results also indicate that failing to account for the actual composition of the stimulus by overlooking its investment component would lead one to severely underestimate the associated multiplier.

Given our findings, one might be led to conclude that stimulus packages should be exclusively targeted towards public investment projects. This conclusion is of course unwarranted. A formal analysis of the normative aspects of fiscal policy — including the optimal allocation of public spending in and out of the ZLB — ought to be welfare-based. We are currently studying this issue in a companion paper (Bouakez et al. (2016)).

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[23]The result that the spending multiplier is larger in the case where public and private capital are substitutes than when they are complements is not general. It depends in a non-trivial way on the values of the parameters a and ϑ.

[24]Wieland (2013) finds little empirical support for the proposition that supply shocks have contractionary effects when the ZLB is binding. Wieland (2013), Kiley (2014) and Bundick (2014) propose different modifications of the standard new-Keynesian model to rationalize this evidence. Unless the time-to-build delays are implausibly short, our paper shows that public investment can stimulate output even if public capital becomes productive before the economy exits the liquidity trap. From this perspective, our results are consistent with the evidence reported by Wieland (2013).

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A Appendix A

A.1 Equilibrium conditions

A.1.1 First-order conditions

The household maximizes (1) subject to (2) and to no-Ponzi-game condition. The first order conditions for this problem are given by

$$W_t = -\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)}, \tag{A.1}$$

$$\frac{1}{1+i_t} = \beta_{t+1} E_t \left(\frac{U_C(C_{t+1}, N_{t+1})}{U_C(C_t, N_t)} \frac{P_t}{P_{t+1}} \right), \tag{A.2}$$

where $W_t = \frac{\mathcal{W}_t}{P_t}$ is the real wage rate and $U_X(C_t, N_t) = \partial U(C_t, N_t)/\partial X_t$. Note that the conditional-expectation operator, E_t, is not applied to β_{t+1} because the latter is known in period t.

The firms' first order condition for the optimal choice of P_t^o is given by

$$E_t \sum_{s=0}^{\infty} \phi^s Q_{t,t+s} X_{t,t+s} [P_t^o - \mu MC_{t,t+s}] = 0, \tag{A.3}$$

where $MC_{t,t+s} = \frac{(1-\tau)\mathcal{W}_{t+s}}{F_N(N_{t,t+s}, K_{G,t+s})}$ is the marginal cost of producing an additional unit of output in period $t+s$ if the price set at time t is still in effect, and $\mu = \theta/(\theta-1)$ is the desired steady-state markup over marginal cost.

Given the price setting mechanism just described, the price of the final good evolves according to

$$P_t^{1-\theta} = (1-\phi)(P_t^o)^{1-\theta} + \phi P_{t-1}^{1-\theta}. \tag{A.4}$$

A.1.2 Market clearing and equilibrium

Assume that the production function $F(.)$ satisfies

$$F(N_t(z), K_{G,t}) = J(N_t(z))H(K_{G,t}), \tag{A.5}$$

where the function $J(.)$ is such that $J(U_t V_t) = J(U_t)J(V_t)$.

Market clearing for each intermediate good z requires that

$$F(N_t(z), K_{G,t}) = J(N_t(z))H(K_{G,t}) = \left(\frac{P_t(z)}{P_t} \right)^{-\theta} Y_t,$$

which implies

$$N_t(z) = J^{-1} \left[\frac{Y_t}{H(K_{G,t})} \left(\frac{P_t(z)}{P_t} \right)^{-\theta} \right].$$

Aggregating across all intermediate-good producers and imposing labor market equilibrium, we obtain

$$N_t = J^{-1} \left(\frac{Y_t}{H(K_{G,t})} \right) \int J^{-1} \left[\left(\frac{P_t(z)}{P_t} \right)^{-\theta} \right] dz.$$

This yields

$$Y_t = \frac{F(N_t, K_{G,t})}{\Delta_t},$$

28

where $\Delta_t = J\left(\int J^{-1}\left[\left(\frac{P_t(z)}{P_t}\right)^{-\theta}\right]dz\right)$ is a measure of dispersion of relative prices.

Since households are identical, the net supply of bonds must be zero in equilibrium ($\mathcal{B}_t = 0$). Finally, the resource constraint is

$$Y_t = C_t + G_t. \tag{A.6}$$

An intertemporal equilibrium for this economy is a sequence of prices $\{P_t(z), P_t, W_t, i_t\}_{t=0}^{\infty}$ and quantities $\{X_t(z), N_t(z), N_t, Y_t, C_t, K_{G,t}, G_{t-T}^i\}_{t=0}^{\infty}$ such that, for a given sequence of exogenous variables $\{\beta_t, G_t\}_{t=0}^{\infty}$, households and firms solve their respective optimization problems, the accumulation equation of public capital holds, the spending and monetary rules hold, and all markets clear.

A.2 Summary of the model

The model equilibrium conditions are

$$
\begin{align}
Y_t &= C_t + G_t, \tag{A.7} \\
\Delta_t Y_t &= F(N_t, K_{G,t}), \tag{A.8} \\
\frac{1}{1+i_t} &= \beta_{t+1} E_t\left(\frac{U_C(C_{t+1}, N_{t+1})}{U_C(C_t, N_t)}\frac{P_t}{P_{t+1}}\right), \tag{A.9} \\
W_t &= -\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)}, \tag{A.10} \\
0 &= E_t\sum_{s=0}^{\infty}\phi^s Q_{t,t+s} X_{t,t+s}(P_t^o - \mu MC_{t,t+s}), \tag{A.11} \\
P_t^{1-\theta} &= (1-\phi)(P_t^o)^{1-\theta} + \phi P_{t-1}^{1-\theta}, \tag{A.12} \\
K_{G,t} &= (1-\delta)K_{G,t-1} + G_{t-T}^i, \tag{A.13} \\
G_t^i &= G^i + \alpha(G_t - G), \tag{A.14} \\
i_t &= \max\left(0; \ln\beta^{-1} + \phi_\pi\ln\frac{P_t}{P_{t-1}}\right), \tag{A.15}
\end{align}
$$

where

$$
\begin{align}
\Delta_t &= J\left(\int J^{-1}\left[\left(\frac{P_t(z)}{P_t}\right)^{-\theta}\right]dz\right), \\
Q_{t,t+s} &= d_{t+s}U_C(C_{t+s}, N_{t+s})/d_t U_C(C_t, N_t), \\
MC_{t,t+s} &= \frac{(1-\tau)\mathcal{W}_{t+s}}{F_N(N_{t,t+s}, K_{G,t+s})}.
\end{align}
$$

A.3 Steady state

Using equations (A.1) to (A.8) evaluated at steady state, we obtain a system of three equations that uniquely determine private consumption, hours worked and public capital in steady state. These quantities are all we need to compute the log-linearized version of the model.

$$
\begin{align}
C &= (1-\bar{g})F(N, K), \\
-\frac{U_N(C, 1-N)}{U_C(C, 1-N)} &= \frac{\theta-1}{(1-\tau)\theta}F_N(N, K), \\
K &= \frac{\bar{\alpha}}{\delta}\frac{\bar{g}}{1-\bar{g}}C.
\end{align}
$$

A.4 The linearized model

We linearize the model around a deterministic zero-inflation steady state. Variables without a time subscript denote steady-state values and variables in lowercase denote percentage deviations from steady state ($z_t = (Z_t - Z)/Z$), except for $g_t = (G_t - G)/Y$. Defining $r_{\beta,t} = \beta_{t+1}^{-1} - 1$ and $\bar{g} = G/Y$, the log-linearized model is given by

$$
\begin{aligned}
y_t &= (1 - \bar{g})\, c_t + g_t, \\
n_t &= \frac{F}{F_N N} y_t - \frac{F_K K_G}{F_N N} k_{G,t}, \\
c_t &= E_t c_{t+1} + \frac{U_C}{U_{CC} C} (i_t - E_t \pi_{t+1} - r_{\beta,t}) - \frac{U_{CN} N}{U_{CC} C} (n_t - E_t n_{t+1}), \\
w_t &= \left(\frac{U_{CN} C}{U_N} - \frac{U_{CC} C}{U_C} \right) c_t + \left(\frac{U_{NN} N}{U_N} - \frac{U_{CN} N}{U_C} \right) n_t, \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa \left(w_t - \frac{F_{NN} N}{F_N} n_t - \frac{F_{N K_G} K_G}{F_N} k_{G,t} \right), \\
k_{G,t} &= (1 - \delta)\, k_{G,t-1} + \alpha \tilde{\delta} g_{t-T}, \\
i_t &= \max\left(0; r_\beta + \phi_\pi \pi_t\right),
\end{aligned}
$$

where

$$
\begin{aligned}
\kappa &= \frac{(1-\phi)(1-\beta\phi)}{\phi} \frac{\frac{F_N N}{F}}{\frac{F_N N}{F} - \theta \frac{F_{NN} N}{F_N}} > 0, \\
\tilde{\delta} &= \frac{\delta}{\alpha \bar{g}} \geq 0,
\end{aligned}
$$

and where we have used the fact that $g_t^i = \alpha g_t$.

In compact form, the linearized model can be written as

$$
\begin{aligned}
c_t &= E_t c_{t+1} - \Phi_r (i_t - E_t \pi_{t+1} - r_{\beta,t}) + \Phi_g (g_t - E_t g_{t+1}) - \Phi_k (k_{G,t} - E_t k_{G,t+1}), & \text{(A.16)} \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa (\Theta_c c_t + \Theta_g g_t - \Theta_k k_{G,t}), & \text{(A.17)} \\
k_{G,t} &= (1 - \delta)\, k_{G,t-1} + \alpha \tilde{\delta} g_{t-T}, & \text{(A.18)} \\
i_t &= \max\left(0; r_\beta + \phi_\pi \pi_t\right), & \text{(A.19)}
\end{aligned}
$$

where

$$
\begin{aligned}
\Phi_r &= \frac{-\frac{U_C}{U_{CC} C}}{1 + (1 - \bar{g}) \frac{U_{CN} N}{U_{CC} C} \frac{F}{F_N N}} \qquad \Phi_g = \frac{F}{F_N N} \frac{U_{CN} N}{U_C} \Phi_r, \quad \Phi_k = \frac{F_{K_G} K_G}{F} \Phi_g, \\
\Theta_c &= \left(\frac{U_{CN} C}{U_N} - \frac{U_{CC} C}{U_C} \right) + (1 - \bar{g}) \Theta_g, \\
\Theta_g &= \left(\frac{U_{NN} N}{U_N} - \frac{U_{CN} N}{U_C} - \frac{F_{NN} N}{F_N} \right) \frac{F}{F_N N}, \quad \Theta_k = \frac{F_{N K_G} K_G}{F_N} + \frac{F_K K_G}{F} \Theta_g.
\end{aligned}
$$

A.5 Functional forms and the implied composite parameters

We consider the following production and utility functions:

$$
\begin{aligned}
U(C_t, N_t) &= \frac{\left(C_t^\gamma (1 - N_t)^{1-\gamma} \right)^{1-\sigma} - 1}{1 - \sigma} && \text{if } \sigma \neq 1 \\
&= \gamma \ln C_t + (1 - \gamma) \ln(1 - N_t) && \text{if } \sigma = 1,
\end{aligned}
$$

and

$$F\left(N_t, K_{G,t}\right) = N_t^a K_{G,t}^b.$$

These specifications imply the following first and second derivatives:

$$
\begin{aligned}
F_N &= a\frac{F\left(N, K_G\right)}{N}, \quad F_{K_G} = b\frac{F\left(N, K_G\right)}{K_G}, \\
F_{NN} &= (a-1)\frac{F_N}{N} \quad F_{K_G K_G} = (b-1)\frac{F_{K_G}}{K_G}, \quad F_{NK_G} = a\frac{F_{K_G}}{N}, \\
U_C &= \gamma C^{\gamma(1-\sigma)-1}\left(1-N\right)^{(1-\gamma)(1-\sigma)}, \\
U_{CC} &= -\left(1+\gamma\left(\sigma-1\right)\right)\frac{U_C}{C}, \\
U_{CN} &= (1-\gamma)\left(\sigma-1\right)\frac{U_C}{1-N}, \\
U_N &= -\left[(1-\gamma)\,C^{\gamma(1-\sigma)}\left(1-N\right)^{(1-\gamma)(1-\sigma)-1}\right], \\
U_{NN} &= -\left[\left(1+(1-\gamma)\left(\sigma-1\right)\right)(1-\gamma)\,C^{\gamma(1-\sigma)}\left(1-N\right)^{(1-\gamma)(1-\sigma)-2}\right].
\end{aligned}
$$

The steady-state values for C, N, and K_G are implicitly given by

$$
\begin{aligned}
C &= \left(1-\bar{g}\right) N^a K_G^b, \\
\frac{\theta-1}{(1-\tau)\theta}\frac{a}{1-\bar{g}} &= \frac{1-\gamma}{\gamma}\frac{N}{1-N}, \\
K_G &= \frac{\bar{\alpha}}{\delta}\frac{\bar{g}}{1-\bar{g}}C.
\end{aligned}
$$

The composite parameters Φ_r, Φ_g, Φ_k, Θ_g, Θ_k, and Θ_c, are given by

$$
\Phi_r = \frac{1}{1-\gamma\left(1-\sigma\right)+(1-\gamma)\left(1-\sigma\right)\frac{N}{1-N}\frac{1-g}{a}} = 1,
$$

$$
\Phi_g = \frac{1}{a}\left(1-\gamma\right)\left(\sigma-1\right)\frac{N}{1-N}\Phi_r = \frac{\gamma\left(\sigma-1\right)}{1-\bar{g}},
$$

$$
\Phi_k = b\Phi_g,
$$

$$
\Theta_g = -\frac{1}{a}\left((1-\gamma)\left(\sigma-1\right)\frac{N}{1-N}+a-1\right)
$$

$$
+\frac{1}{a}\frac{\left(1+(1-\gamma)\left(\sigma-1\right)\right)(1-\gamma)\,C^{\gamma(1-\sigma)}\left(1-N\right)^{(1-\gamma)(1-\sigma)-1}\frac{N}{1-N}}{(1-\gamma)\,C^{\gamma(1-\sigma)}\left(1-N\right)^{(1-\gamma)(1-\sigma)-1}} = \frac{1}{1-\bar{g}}\frac{\gamma}{1-\gamma},
$$

$$
\Theta_k = b\left(1+\Theta_g\right),
$$

$$
\Theta_c = 1+\left(1-\bar{g}\right)\Theta_g = \frac{1}{1-\gamma}.
$$

A.6 Analytical solution of the model with time to build $(T \geq 1)$

In this subsection, we explain how we solve the model with time to build both in normal times and when the ZLB binds. In both cases, we use the method of undetermined coefficients.

A.6.1 Normal times

Under the assumption that $\phi_\pi > 1$, the unique linear rational expectation solution is given by

$$c_t = \vartheta_k k_{G,t-1} + \sum_{\tau=0}^{T} \vartheta_g^\tau g_{t-\tau},$$

$$\pi_t = \zeta_k k_{G,t-1} + \sum_{\tau=0}^{T} \zeta_g^\tau g_{t-\tau}.$$

Substituting these two expressions into equations (14)-(17) (with $i_t = r_\beta + \phi_\pi \pi_t$).and equating the relevant coefficients, we obtain

$$\vartheta_k = (1-\delta) \frac{\kappa\left(\phi_\pi - (1-\delta)\right)\Phi_r\Theta_k - \delta\left(1-\beta\left(1-\delta\right)\right)\Phi_k}{\delta\left(1-\beta\left(1-\delta\right)\right) + \kappa\left(\phi_\pi - (1-\delta)\right)\Phi_r\Theta_c},$$

$$\zeta_k = -(1-\delta) \frac{\kappa\delta\left(\Theta_c\Phi_k + \Theta_k\right)}{\delta\left(1-\beta\left(1-\delta\right)\right) + \kappa\left(\phi_\pi - (1-\delta)\right)\Phi_r\Theta_c},$$

and

$$\begin{pmatrix} \vartheta_g^0 \\ \zeta_g^0 \end{pmatrix} = \alpha\tilde\delta\Lambda\mathbf{A}\begin{pmatrix} \vartheta_g^1 \\ \zeta_g^1 \end{pmatrix} + \begin{pmatrix} (1-\rho\beta)\left[\alpha\tilde\delta\Phi_k + (1-\rho)\Phi_g\right] - \kappa\left(\phi_\pi - \rho\right)\Phi_r\Theta_g \\ \kappa\left(1-\rho\right)\Theta_g + \kappa\left[\alpha\tilde\delta\Phi_k + (1-\rho)\Phi_g\right]\Theta_c \end{pmatrix},$$

$$\begin{pmatrix} \vartheta_g^\tau \\ \zeta_g^\tau \end{pmatrix} = \varpi\mathbf{B}\begin{pmatrix} \vartheta_g^{\tau+1} \\ \zeta_g^{\tau+1} \end{pmatrix}, \quad \tau = 1,...,T-2; \quad \text{for } T > 2,$$

$$\begin{pmatrix} \vartheta_g^{T-1} \\ \zeta_g^{T-1} \end{pmatrix} = \varpi\left[\mathbf{B}\begin{pmatrix} \vartheta_g^{T} \\ \zeta_g^{T} \end{pmatrix} + \alpha\tilde\delta\begin{pmatrix} \Phi_k \\ \kappa\Theta_c\Phi_k \end{pmatrix}\right], \quad \text{for } T > 1,$$

$$\begin{pmatrix} \vartheta_g^{T} \\ \zeta_g^{T} \end{pmatrix} = \alpha\tilde\delta\varpi\left[\mathbf{B}\begin{pmatrix} \vartheta_k \\ \zeta_k \end{pmatrix} + \begin{pmatrix} \kappa\phi_\pi\Phi_r\Theta_k - \delta\Phi_k \\ \kappa(\Theta_k + \delta\Theta_c\Phi_k) \end{pmatrix}\right], \quad \text{for } T \geq 1,$$

where

$$\Lambda = \left((1-\rho)(1-\beta\rho) + \kappa\left(\phi_\pi - \rho\right)\Phi_r\Theta_c\right)^{-1}, \qquad \varpi = \left(1 + \kappa\phi_\pi\Phi_r\Theta_c\right)^{-1},$$

and

$$\mathbf{A} = \begin{pmatrix} 1-\beta\rho & (1-\beta\phi_\pi)\Phi_r \\ \kappa\Theta_c & \kappa\Phi_r\Theta_c + \beta(1-\rho) \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 1 & (1-\beta\phi_\pi)\Phi_r \\ \kappa\Theta_c & \kappa\Phi_r\Theta_c + \beta \end{pmatrix}.$$

In practice, for any arbitrary T, ϑ_g^τ, and ζ_g^τ ($\tau = 0,...,T$) are computed backward recursively: Start by computing ϑ_g^T and ζ_g^T, which are only functions of the deep parameters. Once ϑ_g^T and ζ_g^T are known, compute ϑ_g^{T-1} and ζ_g^{T-1}, and so on up to ϑ_g^0 and ζ_g^0.

A.6.2 Liquidity trap

When the ZLB binds ($i_t = 0$), equations (14)-(17) imply

$$c_t^l = E_t c_{t+1} - \Phi_r\left(-E_t\pi_{t+1} - i_{\beta,t}\right) + \Phi_g\left(g_t^l - E_t g_{t+1}\right) - \Phi_k\left(\delta k_{G,t} - \alpha\tilde\delta E_t g_{t-(T-1)}\right),$$

$$\pi_t^l = \beta E_t\pi_{t+1} + \Theta_c c_t^l + \Theta_g g_t^l - \Theta_k k_{G,t}.$$

The unique linear rational expectation solution is given by

$$c_t^l = \vartheta_i^l r_\beta^l + \vartheta_k^l k_{G,t-1} + \sum_{\tau=0}^{T} \vartheta_g^{l,\tau} g_{t-\tau},$$

$$\pi_t^l = \zeta_i^l r_\beta^l + \zeta_k^l k_{G,t-1} + \sum_{\tau=0}^{T} \zeta_g^{l,\tau} g_{t-\tau}.$$

Unlike the simple case discussed in Section 3.2, whenever the ZLB ceases to bind, the economy does not immediately jump to the steady state, as the stock of public capital continues to adjust. Thus, the expected value of a given variable in $t+1$ is an average of its possible values in $t+1$ inside and outside the liquidity trap, weighted by the respective probabilities of being in these two states. Using the equilibrium paths of consumption and inflation inside and outside the ZLB state, we obtain

$$E_t c_{t+1} = p\left(\vartheta_i^l r_\beta^l + \vartheta_k^l k_{G,t-1} + \vartheta_g^{l,0} g_t + \sum_{\tau=1}^T \vartheta_g^{l,\tau} g_{t+1-\tau}\right) + (1-p)\left(\vartheta_k k_{G,t-1} + \sum_{\tau=1}^T \vartheta_g^\tau g_{t+1-\tau}\right),$$

$$E_t \pi_{t+1} = p\left(\zeta_i^l r_\beta^l + \zeta_k^l k_{G,t-1} + \zeta_g^{l,0} g_t + \sum_{\tau=1}^T \zeta_g^{l,\tau} g_{t+1-\tau}\right) + (1-p)\left(\zeta_k k_{G,t-1} + \sum_{\tau=1}^T \zeta_g^\tau g_{t+1-\tau}\right),$$

where we have used the fact that

$$g_{t+1} = g_t \text{ if the ZLB is still binding in } t+1,$$
$$= 0 \text{ otherwise.}$$

Equating the relevant coefficients yields

$$\vartheta_i^l = \frac{(1-\beta p)\Phi_r}{(1-p)(1-\beta p) - p\kappa\Phi_r\Theta_c},$$

$$\zeta_i^l = \frac{\Theta_c\Phi_r}{(1-p)(1-\beta p) - p\kappa\Phi_r\Theta_c},$$

$$\vartheta_k^l = \frac{(1-p)(1-\delta)[(1-p\beta(1-\delta))\vartheta_k + \Phi_r\zeta_k] - p\kappa(1-\delta)^2\Phi_r\Theta_k - (1-\delta)(1-p\beta(1-\delta))\delta\Phi_k}{(1-p(1-\delta))(1-\beta p(1-\delta)) - p\kappa(1-\delta)\Phi_r\Theta_c},$$

$$\zeta_k^l = \frac{(1-p)(1-\delta)\{\kappa\Theta_c\vartheta_k + [\beta(1-p(1-\delta)) + \kappa\Theta_c\Phi_r]\zeta_k\} - \kappa(1-\delta)[(1-p\beta(1-\delta))\Theta_k + \delta\Theta_c\Phi_k]}{(1-p(1-\delta))(1-\beta p(1-\delta)) - p\kappa(1-\delta)\Phi_r\Theta_c},$$

and

$$\begin{pmatrix} \vartheta_g^{l,0} \\ \zeta_g^{l,0} \end{pmatrix} = \alpha\tilde\delta\Lambda^l\mathbf{A}^l\left[p\begin{pmatrix} \vartheta_g^{l,1} \\ \zeta_g^{l,1} \end{pmatrix} + (1-p)\begin{pmatrix} \vartheta_g^1 \\ \zeta_g^1 \end{pmatrix}\right] + \Lambda^l\begin{pmatrix} (1-p\beta)(1-p)\Phi_g + p\kappa\Phi_r\Theta_g \\ \kappa(1-p)(\Theta_g + \Phi_g\Theta_c) \end{pmatrix},$$

$$\begin{pmatrix} \vartheta_g^{l,\tau} \\ \zeta_g^{l,\tau} \end{pmatrix} = \mathbf{B}^l\left[p\begin{pmatrix} \vartheta_g^{l,\tau+1} \\ \zeta_g^{l,\tau+1} \end{pmatrix} + (1-p)\begin{pmatrix} \vartheta_g^{\tau+1} \\ \zeta_g^{\tau+1} \end{pmatrix}\right], \quad \tau = 1,...,T-2, \quad \text{for } T > 2,$$

$$\begin{pmatrix} \vartheta_g^{l,T-1} \\ \zeta_g^{l,T-1} \end{pmatrix} = \mathbf{B}^l\left[p\begin{pmatrix} \vartheta_g^{l,T} \\ \zeta_g^{l,T} \end{pmatrix} + (1-p)\begin{pmatrix} \vartheta_g^T \\ \zeta_g^T \end{pmatrix}\right] + \alpha\tilde\delta\begin{pmatrix} \Phi_k \\ \kappa\Theta_c\Phi_k \end{pmatrix}, \quad \text{for } T > 1,$$

$$\begin{pmatrix} \vartheta_g^T \\ \zeta_g^T \end{pmatrix} = \alpha\tilde\delta\mathbf{B}^l\left[p\begin{pmatrix} \vartheta_k^l \\ \zeta_k^l \end{pmatrix} + (1-p)\begin{pmatrix} \vartheta_k \\ \zeta_k \end{pmatrix}\right] - \alpha\tilde\delta\begin{pmatrix} \delta\Phi_k \\ \kappa(\delta\Theta_c\Phi_k + \Theta_k) \end{pmatrix}, \quad \text{for } T \geq 1,$$

where

$$\Lambda^l = ((1-p)(1-\beta p) - p\kappa\Phi_r\Theta_c)^{-1},$$

and

$$\mathbf{A}^l = \begin{pmatrix} 1-p\beta & \Phi_r \\ \kappa\Theta_c & \kappa\Phi_r\Theta_c + \beta(1-p) \end{pmatrix}, \quad \mathbf{B}^l = \begin{pmatrix} 1 & \Phi_r \\ \kappa\Theta_c & \kappa\Phi_r\Theta_c + \beta \end{pmatrix}.$$

Again, the coefficients $\vartheta_g^{l,\tau}$ and $\zeta_g^{l,\tau}$ $(\tau = 0,...,T)$ are computed recursively starting from period T.

B Appendix B: Robustness Analysis

The results discussed in Section 3 are obtained under the specific assumptions that (*i*) the shock that makes the ZLB bind follows a two-state Markov process given by (20), (*ii*) public spending is higher than in steady state as long as the ZLB is binding, and (*iii*) the ZLB binds as long as the shock to the discount factor is in effect. While these assumptions allow us to obtain an analytical solution of the (linearized) model, they are however restrictive in the following ways. First, the random duration of the liquidity trap only allows one to compute the impact multiplier but not a dynamic or a cumulative multiplier. The latter two variants are particularly relevant in the context of our model given the inherent persistence brought about by the time-to-build requirement. Second, assumption (*ii*) is restrictive in that it prevents us from considering the realistic possibility that spending outlays occur gradually during the course of an investment project, which may exceed the expected duration of the liquidity trap. Finally, assumption (*iii*) precludes endogenous exit from the liquidity trap, that is, an outcome in which the ZLB ceases to bind as a result of higher public spending despite the fact that the shock that caused the ZLB to bind is still in effect. This outcome is more likely to occur the larger the fraction of public investment and the longer the time-to-build delays.

To study these issues, we relax the assumption that the discount factor and public spending follow a two-state Markov process and assume instead that both variables are governed by an AR(1) process. To solve the model under this alternative assumption, we use the piecewise linear algorithm developed by Guerrieri & Iacoviello (2014), which allows to solve dynamic stochastic general-equilibrium (DSGE) models with occasionally binding constraints by computing two linear decision rules, one for the regime in which the constraint binds, and one for the unconstrained regime.

B.1 Impact versus present-value multiplier

In addition to being subject to potentially significant time-to-build delays, public capital is a slow-moving variable that has a long lasting effect on economic activity. For this reason, the impact multiplier — on which we have focused so far — may not be a sufficient statistic to gauge the effects of public spending shocks on output. An alternative measure that better captures the dynamic nature of these effects is the present-value multiplier defined as

$$pvm = \frac{\sum_{t=0}^{h}(1+r_\beta)^{-1}\Delta y_t}{\sum_{t=0}^{h}(1+r_\beta)^{-1}\Delta g_t}, \tag{B.1}$$

where Δx_t is the difference between the response of variable x when the economy is hit both by a preference and a government spending shock and the response of x when there is only a preference shock. For simplicity, we assume that the dynamic responses of output and government spending are discounted at the constant rate r_β.

Figure 7 reports both the impact and the present-value multipliers at the ZLB as functions of α for $T = 1, 4, 16$. In order to maintain comparability with the results discussed in Section 3.2 regarding the role of public investment and the time to build, we choose the autocorrelation coefficients of the AR(1) processes and the size of the preference shock such that the impact multiplier associated with public consumption (i.e., $\alpha = 0$) at the ZLB is equal to that obtained in the analytical solution (that is, 2.3). We consider a small public spending shock (1 percent of steady-state output) such that there is no endogenous exit from the ZLB state for any given value of α and T. In computing the present-value multiplier, we set $h = 1000$.

The first observation that can be drawn from the left panel of Figure 7 is that the main findings based on the two-state Markov process for the shocks carry over to the specification with AR(1) processes: The multiplier increases monotonically with α when the time to build is relatively long, and becomes substantially large when a significant fraction of public spending is allocated to investment. Qualitatively, the only difference with respect to the results shown in Figure 3 is that the spending multiplier becomes increasing in α even when the time to build is 1 quarter. Quantitatively, the impact multiplier is larger under the autoregressive structure of the shocks than under the Markov process for any given value of α and T.

The right panel of Figure 7 shows that the present-value multiplier at the ZLB displays the same pattern as the impact multiplier, taking substantially larger values when all public spending is in the form of investment and the time to build is long than when public spending is entirely wasteful. For instance, when $T = 16$, the present-value multiplier associated with public investment is more than 3 times larger than the multiplier associated with public consumption.
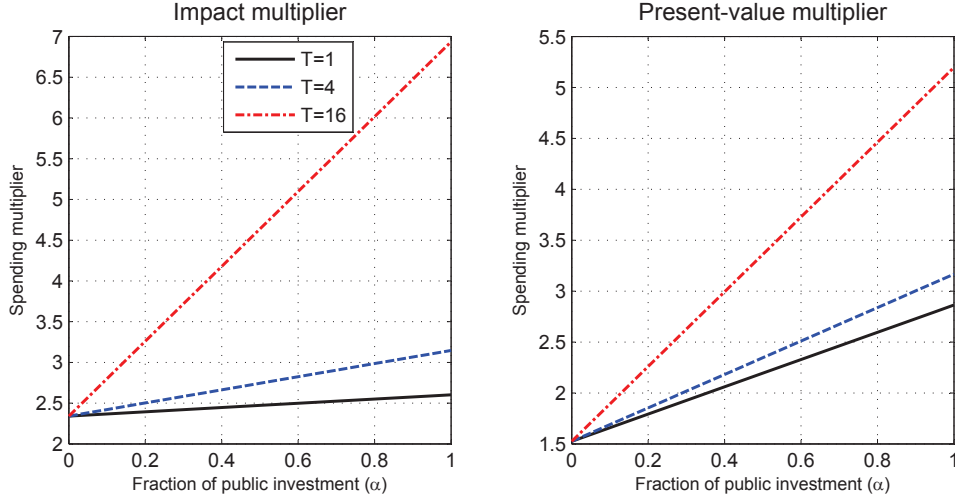
Figure 7: Impact versus present-value spending multiplier in a liquidity trap.

B.2   Timing of public investment outlays

Typically, public investment outlays occur gradually over time after the spending has been authorized.[25] To capture this feature of the data, we amend the model presented in Section 2 as follows. Let A_t be the amount authorized in period t and assume, as before, that a fraction α of that amount is allocated to investment projects while the remaining fraction is devoted to public consumption. Assume further that only investment outlays occur gradually, and denote by ω_j, $j = 0, 1..., T-1$, the spend-out rates, i.e., the rates at which the authorizations are converted into outlays.

In the linearized version of this extended model, the accumulation equation of public capital becomes

$$k_{G,t} = (1 - \delta) k_{G,t-1} + \alpha \tilde{\delta} a_{t-T},$$

where a_t (the deviation of A_t from its steady-state value, as a percentage of steady-state output) is exogenously given and follows the AR(1) process

$$a_t = \rho_a a_{t-1} + \epsilon_t^a.$$

For national accounting purposes, government spending is now defined as

$$g_t = (1 - \alpha) a_t + \alpha \sum_{j=0}^{T-1} \omega_j a_{t-j},$$

where $\sum_{j=0}^{T-1} \omega_j = 1$ and where the second expression on the right hand side of the equation indicates that public investment is the sum of current and past outlays.

In this model, the relevant (and economically meaningful) multiplier is the dollar change in output that results from a dollar change in the authorization, i.e., $dY_t/dA_t = y_t/a_t$.[26] To compute the spending multiplier, we set ρ_a to the same value as ρ_g in the version of the model with outright outlays, and calibrate the spend-out rates as follows. For $T = 1$, we set $\omega_0 = 1$; for $T = 4$, we set $\omega_0 = 0$ and $\omega_1 = \omega_1 = \omega_3 = 1/3$;

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[25]See Leeper et al. (2010) and Leduc & Wilson (2013) for a detailed discussion of the legislative process governing public investment decisions in the U.S.

[26]Indeed, assuming $\alpha = 1$ and $\omega_0 = 0$ implies that the impact multiplier with respect to current public spending is infinite.

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and for $T = 16$, we set $\omega_0 = 0$, $\omega_1 = \omega_2 = \omega_3 = 0.25/3$, and $\omega_4 = ... = \omega_{15} = 0.75/12$. In other words, when $T = 4$ or $T = 16$, authorizations start to show up as outlays with a lag of one quarter (as in Leeper et al. (2010)). When $T = 16$, the authorized amount is spread equally over the 4 years. Note that this version of the model nests the one presented in Section 2 as a special case in which $\omega_0 = 1$ and $\omega_1 = ... = \omega_{15} = 0$.

Figure 8 shows the impact and present-value multipliers when allowing for gradual investment outlays. For $T = 1$, the results are obviously identical to those depicted in Figure 7. For $T = 4$ and $T = 16$, on the other hand, the impact multiplier is lower than that depicted in Figure 7 for any strictly positive value of α. Intuitively, when the outlays occur gradually, the inflationary effect of public spending is spread over time, implying a lower impact multiplier at the ZLB compared with a scenario in which the funds are disbursed immediately after being authorized. When $T = 4$, because public capital becomes productive rather rapidly, the gradual nature of outlays plays a relatively large role in mitigating short-term inflation, actually to the point that the impact multiplier becomes a decreasing function of α. When $T = 16$, on the other hand, the impact multiplier continues to be an increasing function of α and to take substantially large values when α is large.

The right panel of Figure 8 depicts the results for the present-value multiplier. These results are virtually identical to those shown in the corresponding panel of Figure 7. This similarity is not surprising given that we define the multiplier with respect to authorizations rather than to current spending.
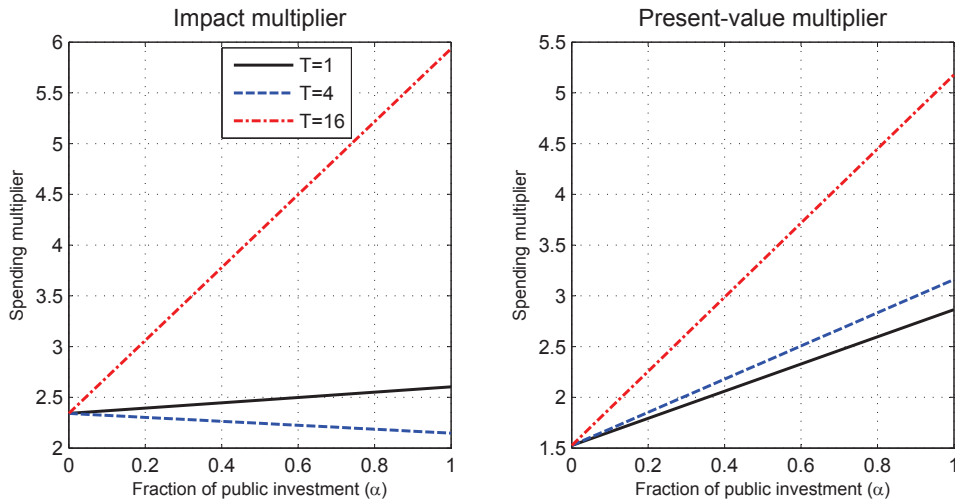

Figure 8: Spending multiplier with gradual investment outlays in a liquidity trap.

B.3 Endogenous exit

So far, the magnitude of the public spending shock that we have considered was small enough not to affect the duration of the ZLB spell, that is, we did not allow for endogenous exit from the liquidity trap. An important result established by the literature on the effects of public spending at the ZLB, however, is that fiscal stimuli that lower the duration of the liquidity trap will be characterized by relatively small multipliers (e.g., Fernández-Villaverde et al. (2012) and Erceg & Lindé (2014)). This is because the portion of the stimulus that takes place outside the ZLB generates inflation once the ZLB ceases to bind, which mitigates the fall in the long-term real interest rate and leads to a smaller increase in private spending while the economy is still at the ZLB.

To allow for the possibility of endogenous exit, we consider public spending shocks of larger magnitudes: 2% and 4% of steady-state output. None of these magnitudes generates endogenous exit when the time to build is 1 or 4 quarters. Therefore, we only report the results for $T = 16$, which are shown in Figure 9. Starting with the impact multiplier, the left panel of the figure reveals that exit occurs at decreasing values of

α as the shock becomes larger. When the increase in public spending is equal to 2% of steady-state output, the economy leaves the liquidity trap one quarter earlier for $\alpha \geq 0.4$. When the shock is twice as large, the economy exits the ZLB one quarter earlier for $0.05 \leq \alpha < 0.75$ and two quarters earlier for $\alpha \geq 0.75$. In conformity with the result discussed above, endogenous exit reduces the value of the impact multiplier, *ceteris paribus*. Nonetheless, even with a shock that is abnormally large (4% of steady-state output), the reduction in the size of the spending multiplier is limited and the latter remains quantitatively large relative to the case of wasteful spending and relative to normal times when a significant fraction of public spending is in the form of investment. The same observation applies to the present-value multiplier, shown in the right panel of Figure 9.
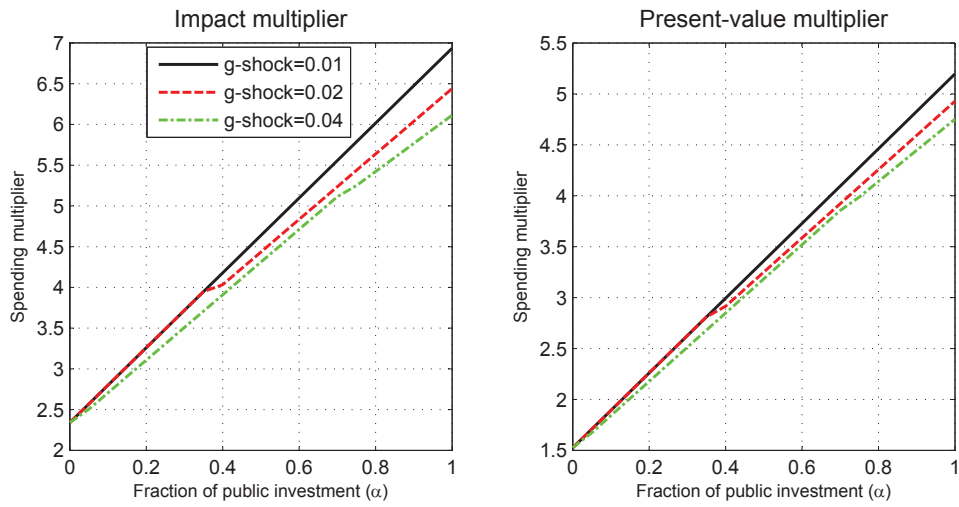

Figure 9: Spending multiplier when there is endogenous exit from the liquidity trap ($T = 16$).
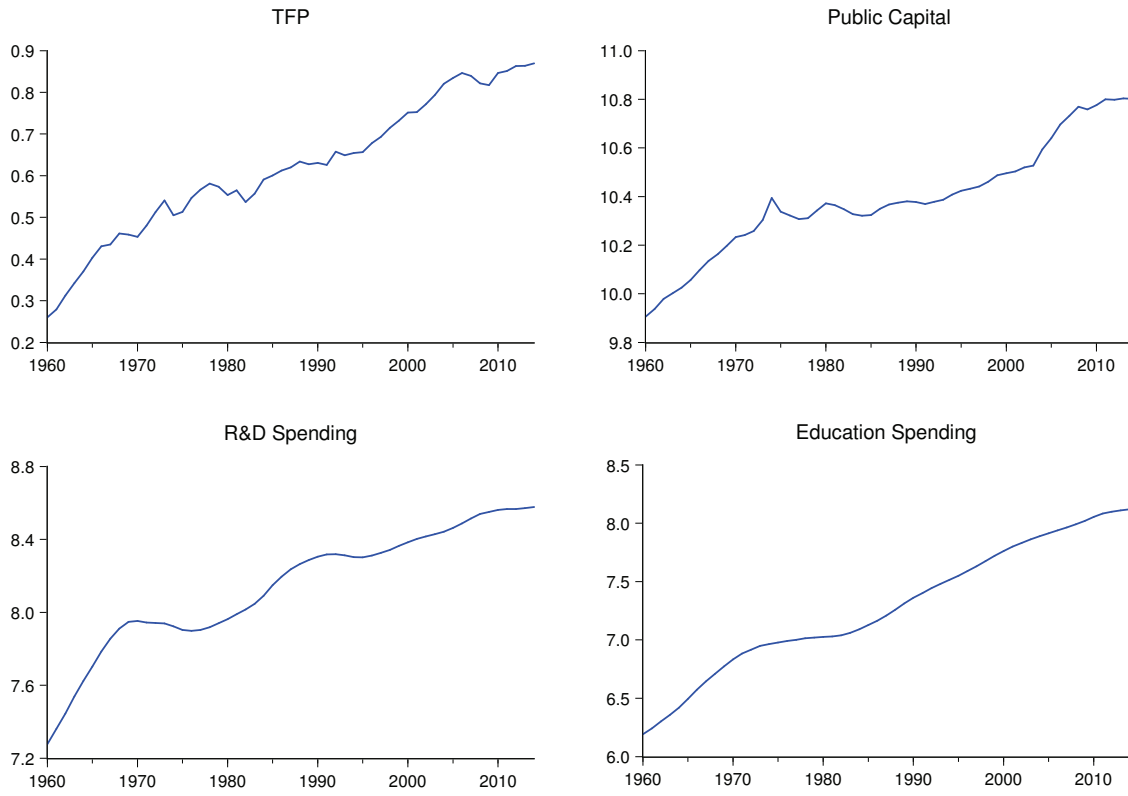
C   Appendix C: Empirical Analysis

Figure 10: U.S. Data.

Note: See Section 4.2 for the definition of the time series shown in the figure and their construction.

Table 3: Unit-Root Test Results

Series	ADF Test	Phillips-Perron Test
TFP	-3.2078 (0.0938)	-3.2002 (0.0952)
Public capital	-1.7435 (0.7179)	-1.8531 (0.6644)
R&D Spending	-3.4867 (0.0516)	-3.6909 (0.0315)
Personal spending on education	-5.7794 (0.0001)	-2.4051 (0.3729)

Notes: Figures between parentheses are p-values. The tests are based an a specification that includes an intercept and a linear trend. P-values are based on Mckinnon (1996) critical values.

Table 4: Cointegration Test Results

Null Hypothesis	Alternative Hypothesis	Engle-Granger Test z Statistic	Johansen Test Trace Statistic	Max-Eig. Statistic
$r = 0$	$r = 1$	-35.4251 (0.0167)	82.1805 (0.0007)	35.1473 (0.0206)
$r \leq 1$	$r = 2$	$-$	47.0332 (0.0183)	21.2729 (0.1782)
$r \leq 2$	$r = 3$	$-$	25.7603 (0.0516)	15.5038 (0.1678)
$r \leq 3$	$r = 4$	$-$	10.2564 (0.1160)	10.2564 (0.1160)

Notes: Figures between parentheses are p-values. r denotes the number of cointegrating vectors. The tests are based an a specification that includes an interecept and a linear trend. P-values for the Engel-Granger tests are based on Mckinnon (1996) critical values. Johansen's tests are based on a vector error-correction model with two lags. P-values for Johansen tests are based on MacKinnon-Haug-Michelis (1999) critical values.

Table 5: Robustness of Cointegration Equation Estimation Results

Scenario	Estimated Coef. of Public Capital Dynamic OLS	Fully Modified OLS
Stock of issued patents as a proxy for technology	0.2296 (0.0180)	0.1669 (0.0142)
Life expectancy as a proxy for human capital	0.1659 (0.0595)	0.2283 (0.0017)
School enrollment rate as a proxy for human capital	0.1896 (0.0166)	0.2269 (0.0020)

Notes: P-values (between parentheses) are based on heteroskedasticity-autocorrelation robust standard errors computed using a pre-whitened Bartlett Kernel with a bandwidth of 4. The number of leads and lags used in the Dynamic OLS regression is selected based on the Schwarz criterion.