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The effect of recycling over a mining oligopoly: competition for market shares, collusion for market power within a Cournot-Stackelberg model

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Abstract

In this paper we show that the mining oligopoly face a trade-off between market share and market power. Its dominant position can still holds with a higher number of firms, but at a cost of a lower market power. Unexpectedly, we also found conditions under which the latter can be greater independently of strategic considerations. Assuming a better recycling efficiency and availability of scrap in the long run, the firms might undertake a vertical or horizontal integration to keep providing inputs to the downstream industry. Regarding recyclers, a technology threshold is required to enter the market. In terms of competition, we show that a very high level of technology and availability of scrap have to be simultaneously reached to hope for dominating the material supply.

Keywords: oligopoly, market power, recycling, raw materials
JEL Classification: L72, D43

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1 Introduction

In view of switching from a linear to a circular economy, recycling plays a fundamental role through three main aspects[1]. First, energy transition and digital economy increase the demand in materials and strengthen the need to adress the scarcity issue. Second, environmental impacts occurring in the mining activity are tremendous in terms of emissions of pollutants, biodiversity losses and land use. This is even worse by including externalities in the downstream industrial process and end-use waste management. Last but not least, countries with fewer resources can be highly dependant of others countries sufficiently endowed in virgin materials, hence, implementing a recycling sector at home can be positive in terms of balance trade and for local employment. Dussaux and Glachant (2018) showed that a 10% rise in recycling leads to a 2% decrease of virgin materials imports. However, as recycling yields a substitute to the virgin material, prior extraction is potentially source of later competition between the mining firms and a fringe of competitive recyclers. The formers have the advantage to determine both what remains to be extracted and what could be recycled in the next period. In addition to the fact that recycling presents environmental benefits and has an effect on the dynamic of resources since it indirectly increases the resource stock [2], an IO-related aspect of recycling arises.

In this paper, we investigate the effect of recycling on the supply of materials, in terms of market shares and market power. Our preliminary findings based on a leader(s)-followers model, show how much a rise in the number of mining firms, helps them to keep a dominant position against recyclers, but at a cost of a lower market power. More widely, our contribution highlights the role of each variable that might affect the magnitude of competition: the technology of recycling, the availability of scrap, the growth of demand, the cost of mining and the number of mining firms. We extend the theoretical framework because of the following new evidences which were not considered in the previous literature.

To better reflect current market conditions, we take into account the existence of a mining oligopoly instead of a monopoly. It does one reflect new perspectives *vis-à-vis* the recycling sector, and also implies potential strategies among the mining firms [3]. As far as we know for most virgin resources, the rise in demand and in the international trade of commodities throughout the second half of the twentieth century, as well as antitrust regulations and privatisation of state owned mining firms helped the mining sector to gain attractivity and made it moving from a monopoly to a worldwide oligopoly. Nevertheless, the need to cover important fixed costs and the large scale of destination markets make the mining sector controlled by only few companies and prevent it from being perfectly competitive (Kesler and Simon, 2015).

Second, we assume that in the long run, the firms might likely face rising costs due to the depletion of high quality ore deposit (e.g. the average ore grade of Chilean copper

[1] For further details, see the communication COM/2015/0614 from the European Commission called ≪Closing the loop - An EU action plan for the Circular Economy ≫

[2] The question of in which extent recycling can stop or slow resource scarcity is out of the frame of this paper. For further details, see e.g. Weinstein and Zeckhauser (1974), Andre and Cerda (2006), Ba and Mahenc (2018)

[3] This market feature has been showed empirically by Wan and Boyce (2014). Salo and Tahvonen (2001) also mentionned in their theoretical paper that ≪many natural resource industries cannot be described using a pure monopoly or competitive models ≫.

mines, has decreased approximately 28.8% in just ten years (Calvo and all., 2015)) and environmental regulations (e.g. CO2 emission tax, water pollutant emissions standards, land restoration policy). Then, the issues of virgin materials scarcity and environmental externalities lead us to consider a non linear marginal cost of mining. We are implicitly assume that the extraction technology exhibits decreasing returns to scale.

Third, the magnitude of recycling differs a lot among materials. For those used in electronic devices such as lithium, silicium or most of the Rare Earth Elements (REE), recycling only exists at a very low level. This brings us to pay attention on how recycling can enter the market.

Fourth, in addition to have a look on the market power of the mining firm(s), we focus on the market shares of the resource supply and analyse how recycling can be improved through public policies [4]. As far as we know, these four aspects are not considered in the non abundant related literature about the influence of recycling over a non competitive mining sector, while it appears to be at stake.

Most of this related literature started with the famous Alcoa antitrust case. In 1945, the Aluminum Company of America (Alcoa) was found in a monopolistic position with around 90% of the market share, in violation with the Sherman Antitrust Act. To support its decision, the US Justice Department disregarded the recycling sector from the relevant market, by arguing this was also controlled by Alcoa's strategic behavior. If the competitive supply of recycled aluminum inexorably drives the price toward the competitive level, the court was wrong in its findings. Based on this case, Gaskins (1974) is the first to work on the pro-competitive effect of the recycling sector over a mining firm. His results show that the existence of the secondhand market makes things worse in the short run. To limit the future competition with recyclers, the mining firm might strategically decrease its output which implies a rise in the price. Besides, the dominance of the virgin materials producer in the long run, relies on the steady rate of the demand growth. Since product demand was increasing over time, he concluded that Alcoa would have considerable market power in the long run [5]. The sensitivity of the results to the rate of demand growth was criticized by Swan (1980). Although his model predicted that long run price was close to the predicted monopoly price in the absence of recycling, he also found theoretical conditions under which the price of aluminium would be driven down to the marginal cost of virgin aluminium ingot production. Martin (1982) considered various forms of vertical integration by the monopolist. His results confirmed the Judge Hand's decision, since ≪long run price will be strictly greater than the marginal cost of virgin production, as long as any depreciation occurs in scrap recovery ≫. From this, Martin inferred that any improvement in the technology of scrap recovery or scrap conversion will lower monopoly rent and that any leakage of scrap into export markets will raise monopoly rent and lower industry output. Grant (1999) also stated that ≪the market power of the dominant firm will continually erode as the amount of resources available for recycling increases over time ≫. Gaudet and Van Long (2003) dropped the assumption that the virgin producer does not benefit from a leadership role anymore and

[4]Modelling public policies is out of the scope of our paper and is subject to a further work.

[5]His empirical findings show that the initial price practiced by the monopoly with a recycling sector is 6% higher than without the recycling and 3.5 times higher than the competitive price. In the long run, the price is estimated 14% lower with the recycling sector but sill 2.8 times higher than the competitive level. The simulation also indicates that the secondary sector entails a progressive decrease of the price, but 100 years would be necessary to see long run equilibrium value reduced by 5%.

face one or several recyclors simultaneously on the market (*i.e* à la Cournot). They also show that the market power of the virgin producer measured by the Lerner Index is a decreasing function of the time delay between virgin production and recycling when the marginal cost of the virgin producers is increasing.

Our paper is organized as follows. Section 2 shows the conditions under which recyclers can enter the market as a new supply for the downstream industry. From then we modelize the competition holding with the oligopoly in Section 3 and analyse the effect of recycling in terms of market shares. Based on this Cournot-Stackelberg model, Section 4 focuses on the effect on the market power. Section 5 discusses on mining strategies while a section 6 analyses implications in terms of public policies. Section 7 concludes.

2 The recycling activity

For convenience, we assume in our whole analysis that both virgin and recycled materials are perfectly substituables [6]. We define a recycling function $r(z)$, where z is the recycling cost per unit of scrap. It includes expenses in collecting, sorting, crushing and processing scrap in order to deliver a recycled material. Higher are the expenses in recycling, better recycling is. $0 < r(z) < 1$ shows that scrap recovery can never be greater than scrap stock and also illustrates a phenomenon of depreciation (*i.e.* leaks of materials related to complex alloys and/or low technology) observed in the recycling process. We also consider a parameter θ representing the proportion of scrap that is available for recycling in the next period [7]. So we have $0 < \theta < 1$ and $(1 - \theta)$ shows that a proportion of materials is definitely lost or hold in products for a too long period to be recycled.

We define the whole production of materials as $Q_t = Q_t^Y + S_t$, which is the result of a mining output Q_t^Y and a secondary output $S_t = r(z_t)\theta Q_{t-1}$ made through recycling. The profit function for the recyclers is determined by $\Pi_t^S = (p_t r(z_t) - z_t)\ \theta Q_{t-1}$ and the FOC is:

$$\Pi_t^{S'} = p_t r'(z_t) - 1 = 0 \Leftrightarrow r'(z_t) = \frac{1}{p(Q_t)}$$

Like Swan (1980) and Martin (1982), we assume $r(z)$ as concave such as $r(0) = 0$, $r'(z) > 0$ and $r''(z) < 0$. The diminishing returns reflect the increasing difficulty to recycle despite of higher expenses per ton of scrap.

We assume now $r(z_t) = 1 - e^{-kz_t}$(Swan (1980)) with the exogenous parameter k measuring the efficiency of the recycling technology. A high level of technology can help recyclers to separate materials more easily, and then less rely on the quality of the composition of waste. This functional form of $r(z)$ allows us to find the optimal solution which we consider as the marginal cost of recycling $\hat{z}_t = \frac{lnk + lnp_t}{k}$. A high price works has an incentive to spend more per unit of scrap, but might be offset by the level of

[6]This might appear as a strong assumption but for regarding the steelmaking industry for instance, it is technically equivalent to produce one ton of crude steel with iron ore than with scrap. Besides, we consider that if the recycled material is not pure enough to be competitive, the issue lies in a low level of technology to make the material more pure.

[7]The one time period corresponds theoretically to one life cycle of product, so that by definition, virgin sales equal total sales minus the secondary supply generated by the previous period's production. Hence, here we assume to only take into consideration the flow of short lifetime products.

technology. $\hat{z}_t$ verifies that $\Pi_t^{S'} = 0$ and gives the following optimal level of recycling :

$$r(\hat{z}_t) = 1 - \frac{1}{kp(Q_t)} \tag{1}$$

To allow recyclers to enter the market (*i.e.* $r(z_t) > 0$) and assuming for convenience a linear inverse demand function such as $p(Q_t) = 1 - Q_t$, the technology of recycling has to reach the threshold $\tilde{k} = \frac{1}{1-Q_t}$ which relies on the level of output before the arrival of recycling, so the threshold becomes $\tilde{k} = \frac{1}{1-Q_t^Y}$. It means that in addition to determine what is going to be recycled in the next period, the level of mining output is also a determinant to the minimum level of technology needed for recyclers to enter the market. The extractor can exploit its first mover advantage by influencing the recycler's entry decision.

To make it as simple as possible, let recycling be only embodied by θ. The oligopoly takes into account the future competition in its profit maximisation which leads to the following mining output equilibrium $Q^{Y*} = (n - nc)(1 + \frac{n}{1-\theta} + \frac{\delta\theta}{1-\delta\theta})^{-1}$. First it confirms a lower mining output with the presence of recycling (*Appendix A*). Second, it allows us to determine the technology threshold as follows:

$$\tilde{k} = \frac{1+n}{1+n(\theta+c+c\theta)} \tag{2}$$

While we can easily observe that $\frac{\partial\tilde{k}}{\partial\theta} < 0$ and $\frac{\partial\tilde{k}}{\partial c} < 0$, the sign of $\frac{\partial\tilde{k}}{\partial n}$ is more ambiguous. The effect is negative if $\theta + c + c\theta > 1$ and positive if $0 < \theta + c + c\theta < 1$. Regarding the latter case, it means that recyclers should benefit from a greater technology threshold to enter the market with a more competitive mining sector, because of the low deposit of scrap and the low cost of extraction.

Proposition 1:*There is a minimum level of recycling efficiency (i.e. a technology threshold) that allows recyclers to enter the market and compete with the mining firm(s). This threshold decreases with the marginal cost of the mining activity and with scrap availability. However, the effect of a more competitive mining sector on the technology threshold relies on the level of these latter factors.*

While $k < \tilde{k}$ means there is no recycling at all, $k > \tilde{k}$ does not necessary mean recyclers can compete with the mining firm(s) since a minimum level of available scrap has also to be reached. This latter situation refers for instance to materials used in electronic devices such as lithium, silicium or most of the Rare Earth Elements (REE), where we observe a very marginal level of recycled material compared to the mining output.

3 Recycling *vs* mining oligopoly: a Cournot-Stackelberg model

3.1 A basic two periods approach

We assume now that a sufficient level of recycling efficiency is reached. It allows recyclers to enter the market with $0 < r(z) < 1$, so that we can focus on the effect on the competition in the materials supply. Since the mining output determines the quantities recycled in the next period, we consider that the oligopoly has a temporal and informational advantage over recyclers. In our model, it makes them leader against recyclers. The mining firms choose their optimal output by considering the existence of a competitive secondary supply as given. Hence, they face a residual demand resulting from the total demand reduced by the secondary supply. Thereafter, the competitive recyclers equate their marginal cost to the given price. We assume that the n leader firms $i, j, (...)$ are symetrics with the same size and the same cost structure. Hence, a competition *á la Cournot* holds while this dominating position *vis-à-vis* the recycling sector leads us to modelize a Stackelberg game.

To illustrate our approach, let first consider a two periods model where the mining firm i is confronted to a competition with an other mining firm j and a fringe of competitive recyclers that provide an input S. In period 1, the stock of scrap is nil so that recyclers cannot enter the market. We have $S_1 = 0$ and $Q_1 = q_1^i + q_1^j$.

In period 2, given the optimal level of recycling in (1), we can express the secondary output according to the output of i and j as follows (*Appendix B*) :

$$S_2^*(q_1^i; q_1^j; q_2^i; q_2^j) = \left[q_1^i(\theta k - \theta q_2^i - \theta q_2^j) + q_1^j(\theta k - \theta q_2^i - \theta q_2^j)\right] \times \frac{1}{k + \theta q_1^i + \theta q_1^j} \tag{3}$$

The optimal level S_2^* here illustrates the best response function of recyclers. It first relies on the level of the mining output in period 1 which determines the capacity constraint such as $S_2 \leq \theta Q_1$. Second, this best response function relies on the mining output in period 2, resulting from the first mover position of i and j. It allows us to see the twofold effect of the mining decision on the secondary supply such as:

$$\frac{\partial S_2^*}{\partial q_1^i} = \left(\frac{\theta k - \theta(q_2^i + q_2^j)}{k + \theta(q_1^i + q_1^j)}\right) - \left(\frac{\theta(q_1^i + q_1^j)[\theta k - \theta(q_2^i + q_2^j)]}{[k + \theta(q_1^i + q_1^j)]^2}\right)$$

$$\frac{\partial S_2^*}{\partial q_1^i} = \left(\frac{\theta k - \theta(q_2^i + q_2^j)}{k + \theta(q_1^i + q_1^j)}\right)\left(1 - \frac{\theta(q_1^i + q_1^j)}{k + \theta(q_1^i + q_1^j)}\right) \tag{4}$$

And

$$\frac{\partial S_2^*}{\partial q_2^i} = -\frac{\theta(q_1^i + q_1^j)}{k + \theta(q_1^i + q_1^j)} \tag{5}$$

The first partial derivative turns out to be positive since we assume $\tilde{k} > Q_2^Y$ to allow recyclers entering the market, and that $0 < \frac{\theta(q_1^i + q_1^j)}{k + \theta(q_1^i + q_1^j)} < 1$. Second, we observe a negative effect between the output of i in period 2 and the secondary supply, like we can expect in a standard Stackelberg model, where i is the first mover.

With (4) and (5), a traditionnal Cournot competition between i and j holds where S_2^* is given. Hence, i maximize its profit such as:

$$\max_{q_1^i; q_2^i} \Pi^i = p(q_1^i; q_1^j)q_1^i - c(q_1^i) + p(q_2^i; q_2^j; S_2^*)q_2^i - c(q_2^i) \tag{6}$$

Let assume that there is no resource constraints for mining (*i.e.* the stock is not exhausted over the two periods)and δ being the discount factor assumed to be equal to 1 for convenience. This gives the following FOCs:

$$\begin{cases} \dfrac{\partial p(Q_1)}{\partial q_1^i} q_1^i + p(Q_1) + \dfrac{\partial p(Q_2)}{\partial q_1^i} q_2^i = c'(q_1^i) \\ \dfrac{\partial p(Q_2)}{\partial q_2^i} q_2^i + p(Q_2) = c'(q_2^i) \end{cases}$$

Breaking down the effect of $q_{1;2}^i$ on the price in period 2 gives:

$$\begin{cases} p_1' q_1^i + p(Q_1) + \overbrace{\dfrac{\partial S_2^*}{\partial q_1^i}}^{(+)} \times \overbrace{\dfrac{\partial p(Q_2)}{\partial S_2^*}}^{(-)} q_2^i = c'(q_1^i) \\ \underbrace{\dfrac{\partial S_2^*}{\partial q_2^i}}_{(-)} \times \underbrace{\dfrac{\partial p(Q_2)}{\partial S_2^*}}_{(-)} q_2^i + p(Q_2) = c'(q_2^i) \end{cases} \tag{7}$$

Here we highlight how the secondary sector affects the mining firm's profit. In period 1, its marginal revenue is not directly affected since recylers are not yet on the market. However, its decision to produce in this first period has to take into account the effect in the next period. Here, i defines a part of the capacity constraint of the secondary sector. The other part is subject to the produciton of j in period 1.

In period 2, the mining firm faces a residual demand whose the magnitude depends on the capacity constraint defined in the previous period, a paramater of availability of scrap, a level of technology and the current price. The latter relies on the second decision of i (and j). Here the quantity of i (and j) determine the incentive for the secondary sector to use the whole stock of scrap in the recycling process, as illustrated by (1).

Solving the model gives q^{i*} and q^{j*} that depend on the value of θ and k, as well as the price elasticity of demand. Nevertheless, although this two periods game stated here allows a good first understanding of the mining firms' dillemma, it cannot stands in reality for at least two reasons. First the mining sector might be composed of more than two firms in the oligopoly. It arises a twofold strategy issue regarding the definition of the capacity constraint for recyclers, and the incentive through the price (*c.f.* section 5). Second, the stock of scrap intially fed by the mining firms, also grows with the output of recyclers as long as a third period is taken into account. In other words, the secondary sector also produces its future input. This makes any mining strategy less efficient, since there is a degree of inertia caused by the recycling loop. Of course, as recycling is efficient this degree of inertia turns out to be longer.

3.2 A generalized model with infinite horizon

Let modelize our Cournot-Stackelberg competition in infinite horizon, according to several parameters that might affect the magnitude of competition: the technology of recycling, the availability of scrap, the growth of demand, the cost of mining and the number of mining firms.

Let the intertemporal profit function of the firm i be:

$$\Pi^i = [p(Q_t)q_t^i - c(q_t^i)] + \sum_{\tau=1}^{\infty} \delta^\tau [p(Q_{t+\tau})q_{t+\tau}^i - c(q_{t+\tau}^i]$$

By considering $Q_t = Q_t^Y + S_t$ and since we denote the mining output as $Q_t^Y = \sum_{i=1}^n q_t^i$, we have the following FOC :

$$p_t'(Q_t)Q_t^Y \left(1 + \overbrace{\frac{\partial S_t}{\partial Q_t}}^{price\ effect}\right) + np_t(Q_t) - nc'(Q_t^Y) + \sum_{\tau=1}^{\infty} \delta^\tau p_{t+\tau}'(Q_{t+\tau})Q_{t+\tau}^Y \overbrace{\frac{\partial Q_{t+\tau}}{\partial Q_t}}^{stock\ effect} c'(Q_{t+\tau}^Y) = 0$$

Here we observe the two effects already mentionned in the two periods case. With $\frac{\partial S_t}{\partial Q_t}$ expected to be negative, we highlight the link between mining output and the recycling supply in period t through the immediate price effect. In addition, the component $\frac{\partial Q_{t+\tau}}{\partial Q_t}$ expected to be positive, captures the fact that recycling creates a loop and rises the resource productivity over time such as a rise of production in t is back on the market in the future $t+\tau$. As it has already been shown in a monopoly case, recycling leads to a lower virgin output. Our interest here is to assess the magnitude of this decrease. We consider that this magnitude relies on the efficiency of recycling, the cost of mining and the number of mining firms (≪internal factors ≫), as well as on the availability of scrap and the demand growth (≪external factors ≫).

With the secondary materials supply $S_t = r(z_t)\theta Q_{t-1}$, the whole materials supply is $Q_t = q_t^i + Q_t^{Y-i} + r(z_t)\theta Q_{t-1}$. The FOC becomes (*Appendix C*):

$$p_t'(Q_t)Q_t^Y \left(1 + \theta Q_{t-1}\frac{\partial r(z_t)}{\partial Q_t}\right) + np(Q_t) - nc'(q_t^i) + \sum_{\tau=1}^{\infty} \delta^\tau p_{t+\tau}'(Q_{t+\tau})Q_{t+\tau}^Y \frac{\partial Q_{t+\tau}}{\partial Q_t} = 0 \quad (8)$$

We define the inverse demand function as $p(Q_t) = \frac{Ag^t}{Q_t{}^\alpha}$ where g^t and α capture a rate of growth and a price elasticity of demand, respectively. For any t and by using (1) we have:

$$\begin{aligned} p_t'(Q_t)Q_t^Y &= p_t'(Q_t) \times Q_t \left(1 - \theta\frac{r(z_t)}{g}\right) \\ &= -\alpha p(Q_t)\left(1 - \frac{\theta}{g} + \frac{\theta}{kgp(Q_t)}\right) \end{aligned} \quad (9)$$

where g is defined as $g = \frac{g^t}{g^{t-1}}$.

Besides, since we have:

$$\frac{\partial r(z_t)}{\partial Q_t} = -\frac{\alpha}{kQ_tp(Q_t)}$$

Hence,

$$1 + \theta Q_{t-1}\frac{\partial r(z_t)}{\partial Q_t} = 1 - \frac{\alpha\theta}{kgp(Q_t)} \quad (10)$$

We define at steady state $z_t = z^*$, $Q_t^Y = Q^{Y*}$, $Q_t = Q^*$ and $e_{t,\tau} = \frac{\partial Q_{t+\tau}}{\partial Q_t} = e_\tau$. The latter is such that:

$$\begin{aligned} e_\tau &= g\frac{\theta(kp(Q^*)-1)}{kp(Q^*)g+\alpha\theta} \times e_{\tau-1} \\ &= g^\tau \left[\frac{\theta(kp(Q^*)-1)}{kp(Q^*)g+\alpha\theta}\right]^\tau \end{aligned} \tag{11}$$

From (11), we can see that except in a closed-loop system (*i.e.* both θ and $r(z)$ equal to 1), where the effect of a marginal rise of the total output will become constant and equal to 1, higher is τ (i.e. a given period), lower will be e_τ. This captures the marginal resource productivity (*i.e.* the marginal productivity of recycling) over time since the output in a period t becomes the input for recyclers allowing them to produce for more than one period (i.e. $Q_{t+\tau}$). With the cumulated depreciation, this marginal rise in production decreases over time until going to 0. Better is recycling, closer to 1 the resource productivity will be, before reaching 0.

We also assume a convex cost function such that $c(q_t^i) = c\frac{(q_t^i)^2}{2}$. This illustrates the rising expenses of the mining firms since we expect in the long run a lower grade of ore and environmental cost to internalize. Hence we have:

$$nc'(q_t^i) = cQ^* \left(1 - \frac{\theta}{g} + \frac{\theta}{kgp(Q^*)}\right) \tag{12}$$

Combining (9), (10), (11), (12) within the FOC, it implies :

$$\begin{aligned} &-\alpha p(Q^*)\left(1-\frac{\theta}{g}+\frac{\theta}{kgp(Q^*)}\right)\left(1-\frac{\alpha\theta}{kgp(Q^*)}\right)\left(\sum_{\tau=1}^{\infty}\delta^\tau g^\tau\left[\frac{\theta(kp(Q^*)-1)}{kp(Q^*)g+\alpha\theta}\right]^\tau\right) \\ &= cQ^*\left(1-\frac{\theta}{g}+\frac{\theta}{kgp(Q^*)}\right) - np(Q^*) \end{aligned} \tag{13}$$

Using (13) and since $Q^{Y*} = Q^*\left(1 - \frac{\theta}{g} + \frac{\theta}{kgp(Q^*)}\right)$ we are able to find the optimal mining output Q^{Y*} and the market shares between the primary and secondary sectors. This also allows us to observe the magnitude of the market power exerted against the downstream industry. Like for the mining output, we assume this rely on the level of recycling.

3.3 Numerical application

(i) The effect of a more competitive mining sector

Our assumption to deal with an oligopoly leads us to see what is the effect of a higher number of mining firms on the market. If we expect a positive relation between n and Q^{Y*}, we might wonder how the secondary supply interferes in this relation according to a given level of k and θ. For convenience, we assume in the following simulations $\delta = 1$, a constant demand growth rate with $g = 1$, a marginal cost $c = 0.3$ and a price elasticity equals to 0.3 [8]. While the positive relation between Q^{Y*} and n is confirmed on the left hand side of the Figure 1, we also observe that except for a high level of secondary supply

[8] The simulations conducted with different level of mining cost and elasticity do not change the trend of the results that are presented in this subsection.

(*i.e.* both θ and k are at a high level), the optimal mining output follows the same path with a more competitive market structure.

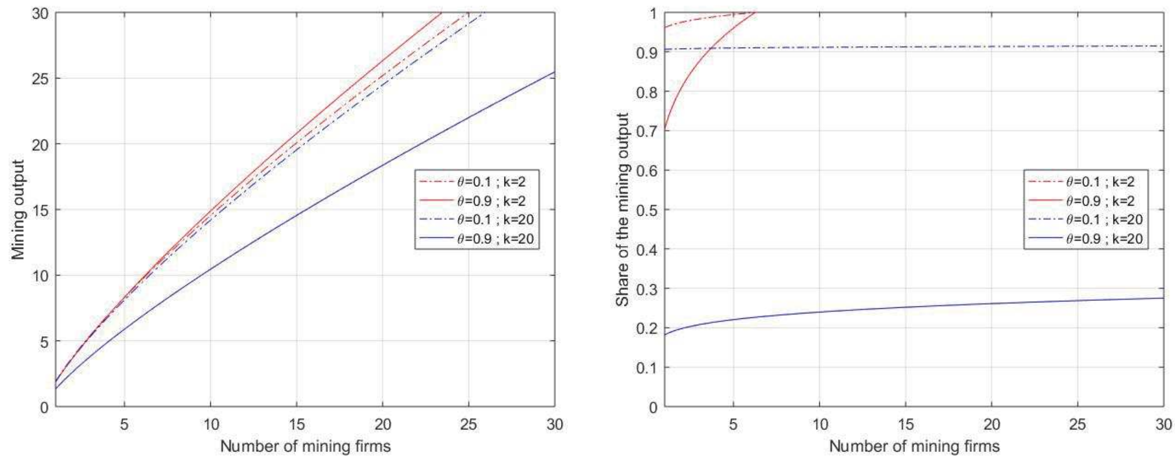

Figure 1: Mining output (left) and market share (right) of the oligopoly with respect n

On the right hand side of the Figure 1, the blue solid and dashed lines show how important the recycling technology is. No matter the level of competition, the market share of the oligopoly does not vary that much since the decreasing price coming from a more competitive sector is offset by a high level of k. The latter helps recyclers to less rely on the market conditions such as price variations.

In addition, like it exists a threshold $\tilde{k}$ for recyclers to enter the market, we argue that a more competitive mining industry might lead to push recyclers out of the market. We assume the existence of an optimal n^* where $Q^{Y*} = Q^*$. Considering a very low level of recycling (*i.e.* $k = 2$), here the optimal number of mining firms is $n^* = 6$ [9].

We now narrow the effect of a more competitive mining sector to the secondary supply. This allows us to assume that a higher number of firms implies a ≪price effect≫and a ≪stock effect≫. The former discourages recyclers to rise their expenses for a better recycling rate. The latter is related to the rising available stock of scrap that would help recyclers to increase their supply with a constant recycling rate.

As already noticed, a high level of k offsets the decreasing price induced by a more competitive mining sector. Hence, the stock effect is greater the the price effect so that the secondary supply rises. It is also worth noticing the hump shaped of the solid and dashed red curves. This means that for a certain level of competition, the price effect becomes too strong. Despite of a greater stock of scrap, the share available for recyclers is too low and the price is not incentive enough to invest in collecting and processing the higher deposit.

Proposition 2: *A higher number of mining firms helps the virgin sector to keep and even increase its dominant position against recyclers. Alike, this implies a greater secondary supply since except for a very low level of recycling, the stock effect exceeds the price effect.*

[9] A higher mining cost encourages the sector to be more competitive if it wants to push recyclers out of the market.

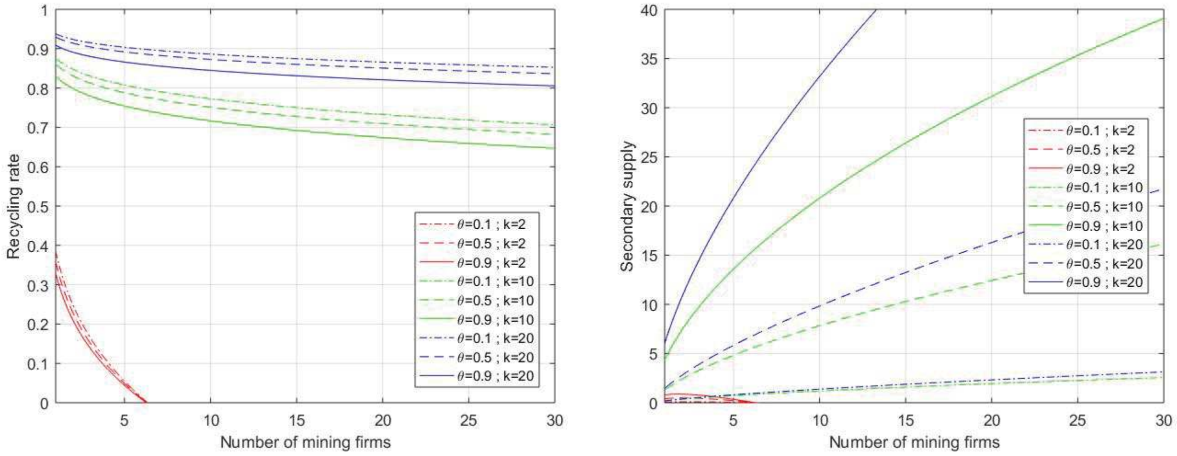

Figure 2: Evolution of the recycling rate (left) and the secondary supply (right) with respect to n

Remark 1: In addition, the rise of n prevents from the entrance of new recyclers as it pushes the threshold $\tilde{k}$ up (c.f. *Proposition 1*).

Remark 2: However, if this rise in the number of firms seems to benefit to the virgin producers by limiting the influence of recyclers, it might also reduces the market power through a decreasing price.

(ii) The role of the demand

Like stated in the previous literature, the growth of demand benefit to the mining sector in the long run (Gaskins, 1974). By taking into account a constant number of mining firms assumes to be $n = 3$, we observe the evolution of the market shares with respect to a constant rate of demand growth.

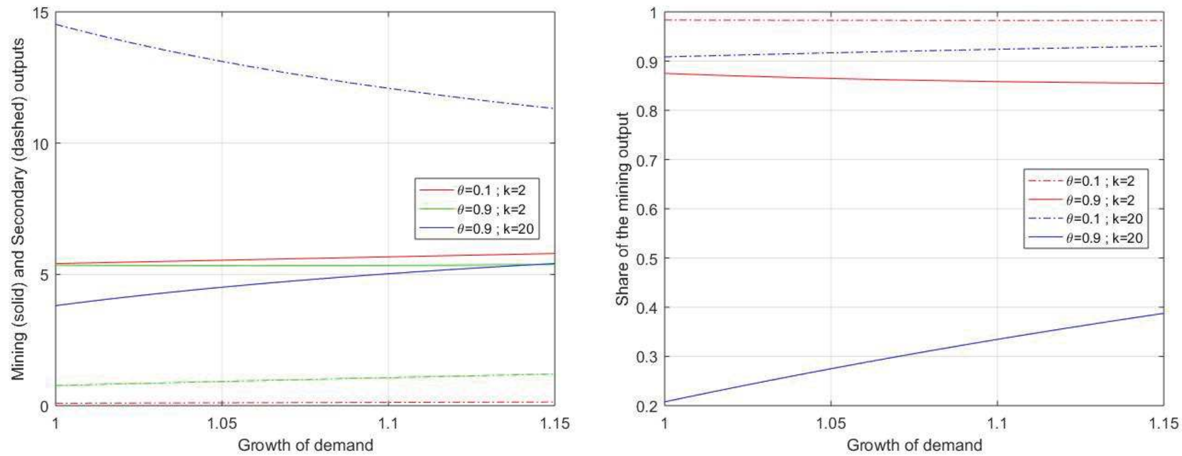

Figure 3: Mining and Secondary output (left) and market share (right) of the oligopoly with respect to the growth rate of demand

The positive effect of the demand growth on the mining sector is confirmed on the Figure 3 with a greater market share on the right-hand part. This seems to be more significant when the secondary sector is high, otherwise the market share remains constant. Using the former case and $g = 1$, the mining output in the total supply of materials is

20.8% while it goes to 27.5% with a 5% constant rate of growth of demand. This rise results from a 9.6% decrease of the secondary supply and a 18.7% increase of the mining output. Hence the better market shares for the mining sector comes mostly from a rise of its mining output (solid blue line) than from a decrease of the secondary output (blue dashed line).

4 The effect on the market power

The fact that recyclers rise their expenses with the price (*i.e.* with (1) we have $\frac{\partial r(\hat{z}_t)}{\partial p(Q_t)} > 0$) implies that they implicitly benefit from the market power hold by the mining firms, but as price-taker, recyclers cannot individually influence this non competitive market price. While the both material suppliers compete in terms of market share, they are in a situation of collusion regarding the market power.

Gaudet and Van Long (2003) showed for a monopoly that $Q^{Y*} < Q^{O*} \leq Q^*$, where Q^{O*} is the optimal output without recycling. This illustrates the fact that the secondary supply equals or more than offset the mining output decrease. Hence, the former case gives $p(Q^*) = p(Q^{O*})$, while we have with the latter case $p(Q^*) < p(Q^{O*})$ meaning that the market price goes to the direction of its competitive level. With a constant elasticity price of demand and marginal cost, this gives a lower market power such as the following mark-up shows:

$$\frac{p(Q^*) - c}{p(Q^*)} < \frac{p(Q^{O*}) - c}{p(Q^{O*})} \tag{14}$$

Since in our case we assume a convex cost function in the mining sector, the mark-up becomes:

$$\frac{p(Q^*) - cQ^{Y*}}{p(Q^*)} = 1 - \frac{cQ^{Y*}}{p(Q^*)} \tag{15}$$

Hence, the effect of recycling on the market power becomes more ambiguous. While we showed that a more competitive mining sector implies a greater level of output, the induced effect on the price strenghtens the decreasing trend of the mark-up. But with (15), we also wonder how the magnitude of the secondary supply affects the market power. This depends on the level of the availability of scrap and the recycling technology on both the mining output and the market price.

Since we can fairly assume that $\frac{\partial S}{\partial \theta} > 0$ and $\frac{\partial S}{\partial k} > 0$, that $\frac{\partial p(Q)}{\partial S} < 0$, we consider that any increase of θ or k pushes the price down to the benefit of the downstream industry (Figure 4).

More unexpected is the effect of recycling on the mark-up when we use the formula expressed in (15). While a greater availability of scrap implies a lower mark-up it turns out that a more efficient technology of recycling allows the mining firms to benefit from a greater mark-up (Figure 5).

For instance, assuming a recycling rate of 17% (red line on the left-handed graph), an improve of the availability of scrap from 10% to 20% implies a decrease of the mark-up of 2.6%. If 50% of scrap is available for recycling, the decrease of the mark-up is about 10.5%. This lower market power means that the availability of scrap has a greater effect on the price than on the mining output (*i.e.* on the marginal cost of mining). Indeed, rising θ from 0.1 to 0.5 implies a decrease of the price and the mining output of 3% and

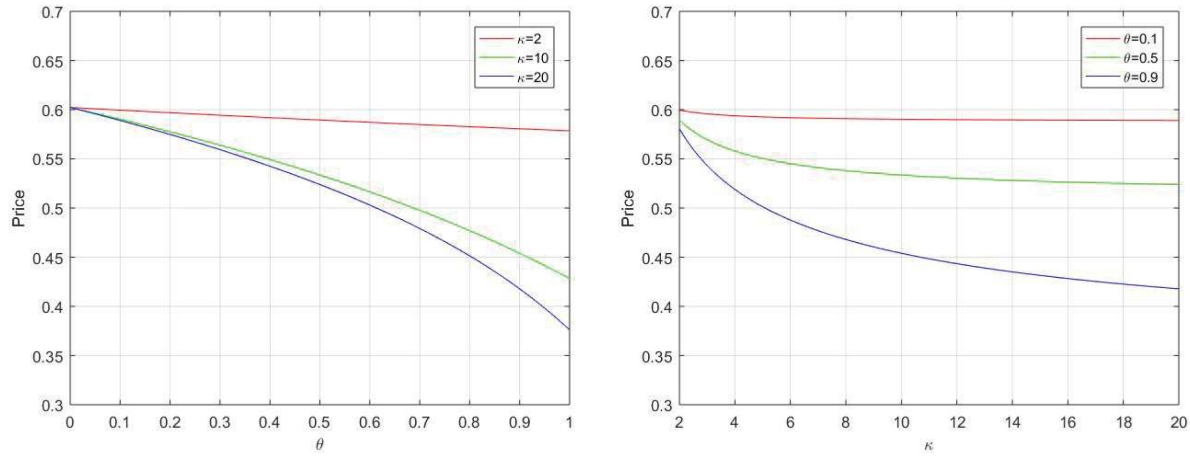

Figure 4: Evolution of the price with respect to the availability of scrap (left) and the technology of recycling (right)

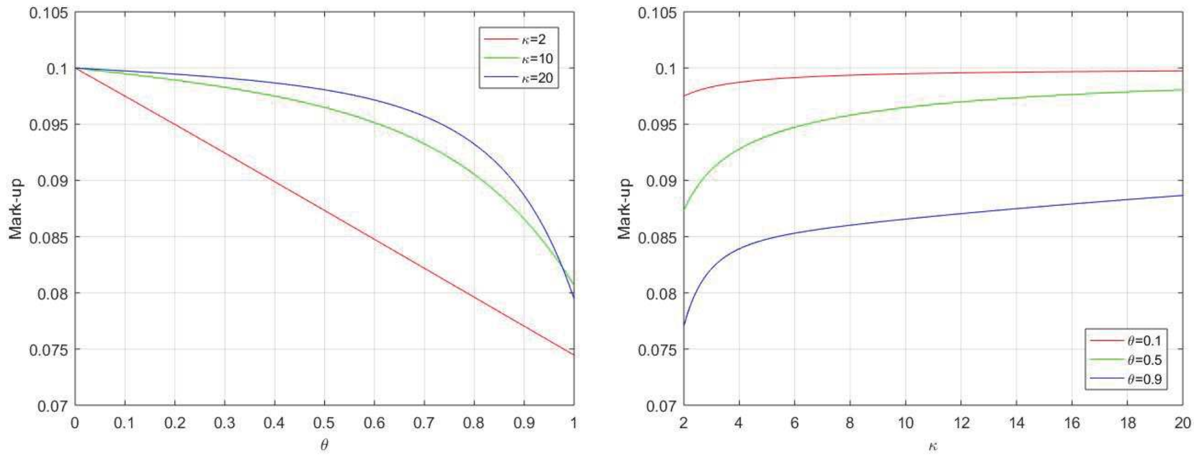

Figure 5: Evolution of the markup with respect to the availability of scrap (left) and the technology of recycling (right)

2% respectively. Formally, with a constant unit cost it gives:

$$\frac{\partial c Q^Y}{\partial \theta} < \frac{\partial p(Q)}{\partial \theta} \Leftrightarrow \frac{\partial Q^Y}{\partial \theta} < \frac{\partial p(Q)}{\partial \theta} \tag{16}$$

Conversely, it seems that the technology of recycling has a greater effect on the mining output than on the price, so that the mark-up can rise. If we assume a recycling rate that goes from 20% to 40% and that 50% of scrap goes to recycling, we can expect a rise of the mark-up of 4.9%. In this case the decrease of the mining output is of 2.8% compared to 2.4% for the price. Formally, with a constant unit cost it gives:

$$\frac{\partial c Q^Y}{\partial k} > \frac{\partial p(Q)}{\partial k} \Leftrightarrow \frac{\partial Q^Y}{\partial k} > \frac{\partial p(Q)}{\partial k} \tag{17}$$

Proposition 3: *While improving the recycling technology and the availability of srap implies a lower price, the effect on the market power differs. Unlike the latter way to improve the secondary supply, a greater technology for recycling might lead to a greater* *market power for the mining firms.*

This result is unexpected since both availability of scrap and technology aim at improving the secondary supply. Such difference might be explained by a different effect of each of these two parameters over the secondary supply, and then, over the mining output and the price.

The related literature already pointed out the possibility of a greater market power with the existence of recycling, but only by taking into consideration potential strategies. Like stated by Tirole (1988) in his textbook or Gaskins (1974) and Martin (1982) in their respective paper, a mining monopolist might aim at restraining the output in order to limit the stock of scrap and prevent from the future competition with recyclers. Considering such a possibility of strategy and a convex cost function, we would have $cQ_t^Y < cQ_t^O$ and $p(Q_t^Y) > p(Q_t^O)$ which implies that recycling can make things worse in the short run.

5 Mining strategies

Without recycling, a mining monopolistic firm only faces the demand features on a given market [10]. Then, either $k < \tilde{k}$ or $\theta \to 0$ so that the firm fully benefit from its market power (MP) and can easily restrain its output without loosing market shares (MS). Within an oligopolistic market, the competition comes from the other mining firms. A cooperative strategy will lead the oligopoly to fully benefit from a MP while the profit will be divided according to their respective MS. In the present case, the mining firms have to take into consideration the presence of recycling as a new competition since $k > \tilde{k}$ and $\theta > 0$. The previous literature with a monopoly showed that the firm might restrain its output to limit future competition with recyclers. It aims at least at maintaining MS and it also rises MP in the short run. The presence of a higher number of mining firms pressures each one of them into also looking at the situation on the virgin market. Their respective dilemma lies on the fact whether the competition with the secondary sector worth setting up a strategy, or paying attention to the competition within the oligopoly turns out to be more relevant. We have used here the simplest case where it exists a duopoly. For instance, assuming symmetric firms, a decision to reduce the output and anticipate the entry of recyclers would not benefit to the first moving firm, unless it knows that the other firm does it too. A cooperation arises since the unilateral move is unexpected at any time. The case in which the assumption of symmetric mining firms is relaxed might worth looking at. This would lead to a Stackelberg-Stackelberg model where in a duopoly i would also be dominant against j and against recyclers.

However, the decreasing output strategy introduced in the literature leads to a greater price in the short run. This creates an incentive to rise expenses in recycling so that a high level of θ can jeopardize such strategy in terms of MP and MS. As alternative and still assuming a cooperation, the mining firms can rise their output which would aim at restraining the entry of recyclers because of the lower price, especially if they have high fixed costs. The induced greater deposit of available scrap seems to make this strategy risky in the long run. Besides, since in realistic conditions the mining output is homogeneous, a product differentiation strategy does not seem to be possible, as well

[10] For convenience, we assume a competitive market structure in the downstream industry, although for some materials like iron ore, the concentration of demand might lead to a thwarted monopsony and reduce the market power of the mining firms (Sourisseau, 2018)

as a potential pricing strategy *à la Bertand*, so that the mining firms can only use a quantity-based strategy.

In the long run, assuming a rise in θ and k, it seems that any of these strategies cannot prevent the mining firm(s), either in a monopoly or in an oligopoly, from loosing MS and a part of its/their MP. This longer-term perspective might push the firms to undertake an integration strategy to contain the erosion of their initial dominant position. In this context, an horizontal integration over the recycling activity would strengthen their position in the upstream industrial process, while a vertical integration over the downstream industry would allow them to expand the potential of foreclosure strategies.

6 Public policies for recycling

We consider recycling as welfare improving so that public policies which aim at increasing the share of recycling in the material supply is of interest.

6.1 Availability of scrap and technology for recycling

As the previous sections highlights, the magnitude of the effect of recycling over the oligopoly relies on the number of mining firms (c.f. *Proposition 2*), the technology of recycling and the proportion of available scrap. Here we estimate how a change in the secondary supply affects the oligopoly's output. Using our model setup at steady state in the previous section, we apply different values of parameters θ and k to see how it affects the virgin and secondary production.

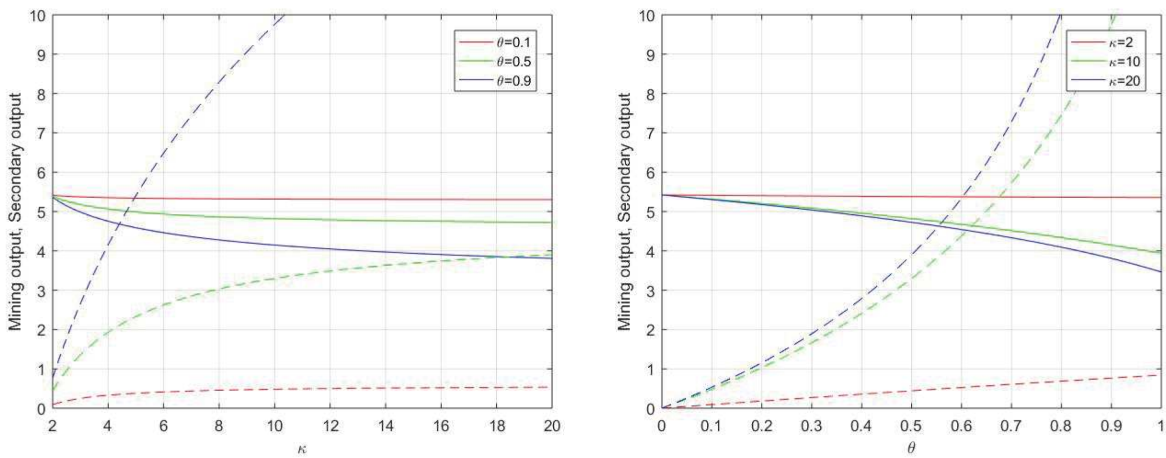

Figure 6: Mining and Secondary ouput with respect to the level of technology (left) and the availability of scrap (right)

The analysis of the determinants of the secondary supply sheds light on the need for high levels in both the availability of scrap and recycling efficiency. When only 10% of scrap is available for recycling, the level of the secondary supply (red dashed line on the left-hand graph), and so as the level of the mining output (red solid line on the left-hand graph), remains constant even if the recycling rate is improved through a better technology. Here the stock constraint captured by a very low θ appears too high for a rise of the secondary supply and a significant change in the market supply structure. Assuming that 50% of scrap is available for recycling, even a recycling rate of 90% (*i.e.* corresponding to $k = 20$) does not allow recyclers to dominate the market. This corresponds to the case where the green dashed line is lower than the green solid line on the left-hand graph, and the blue dashed line is also lower than the blue solid line on the right-hand graph with $\theta = 0.5$.

In terms of public policy, any incentives to encourage the secondary supply must therefore be taken in the light of the initial values of recycling and availability of scrap. For instance, a subsidy to recycling might be good to help recyclers entering the market but useless to push them dominating the market if scrap is not available enough.

Proposition 4: *As long as a sufficient stock of scrap is not reached, the share of the secondary supply in the market will be maintained low. In this case, any public policy that aim at fostering the secondary supply turns out to be efficient if it is based on the availability of scrap rather than the recycling rate.*

6.2 Empirical discussion

We assume two type of materials that have to be distinguished, base metals on one hand [11], and strategic metals on an other hand [12]. Besides, a distinction must be made regarding the country in which a secondary sector should be improved. Differences in price, availability of scrap and technology make the secondary supply rather heterogeneous between emerging and developping countries. For instance, the use of recycled material compared to virgin iron ore in the steelmaking industry is more than 65% in the United States while it is less than 7% in China.

In the emerging countries, no matter the type of material, most of them are immobilized in infrastructures and other durable goods. The metal use per capita is still low compared to industrialized countries so that the parameter θ is very low. The rise in the availability of scrap grows along with the shift of the economy based on consumption instead of investment. Demand for metals is still high and favors primary metal production activities. A material supply dominated by the recyclers appears to be difficult. Therefore it seems better to stimulate the technological efficiency embodied by our parameter k, through various type of investments in research. In addition that the technology threshold k is high, economies of scale in recycling are also low since this activity is not very capital intensive and remains in the artisanal state. In longer term, both θ and k are therefore expected to be greater.

In industrialized countries with low growth, base metals can be mobilized with a higher θ than in the previous case. Public policies in European Union have put in place the principle of Extended Producer Responsibility (EPR) for 25 years to mobilize the fraction of recyclable materials contained in the household and similar waste. It aims to increase the proportion θ available for recycling. This instrument is associated with measures aimed at raising technological efficiency (research tax credit, eco-design). Thus, the technology threshold $\tilde{k}$ is low for this type of metals. Demand for base metals has slowed

[11] It corresponds to ferrous and non ferrous base metals. The mains are aluminium, copper, iron, lead, manganese, nickel, tin and zinc.

[12] Materials that are important to a company/sector/country's strategic plan and supply chain management. The European Commission regularly update a list of Critical Raw Materials (CRM). The mains are for instance cobalt, gallium, platinum, rare earth elements, tungsten.

since the late 2000s and recyclers are well established. On the other hand, for strategic metals, recyclers are not yet present. We are seeing a compensated effect: China has a monopoly over 90% of the rare earths extraction and we should see a significant number of recyclers of these critical metals. But the production cost of extraction is relatively low because of social and environmental reasons. Thus, the technology threshold $\tilde{k}$ is high for this type of metals. Demand for these metals is growing rapidly worldwide and policies need to focus on raising both θ and k. The urban mine makes it possible to mobilize these resources for the high-tech industry.

7 Conclusion

In this paper, we investigate the effect of recycling on the supply of materials through an IO approach. More specifically, our Cournot-Stackelberg model aims at analysing this effect in terms of market shares and market power, according to various parameters that affect its magnitude. We first point out that to allow recyclers to enter the market and compete with the mining firms, a technology threshold must be reached. This threshold is lower when the cost of mining and/or the availability of scrap is high.

As far as we know, the prevous literature only focused on a monopoly, while most of the market structures in this sector seems to be oligopolistic. The effect of a higher number of mining firms is threefold. First, it modifies the technology threshold which allows recyclers to enter the market. Second, it helps the virgin sector to keep and even increase its dominant position against recyclers. Meanwhile, this would also benefit to the secondary sector which seems to be more sensitive to the induced rise of the stock, than the decrease of the price (*i.e.* the stock effect exceeds the price effect). However, a higher market share for the virgin producers appears to be done at a cost of a lower market power. Third, the entry of recycling and the induced trade-off between market share and market power implies the existence of potential strategies, and the more competitive the mining sector, the more difficult is the possibility of setting them up. In the long run though, it seems that any strategy cannot prevent the mining firm(s) from loosing market shares, because demand will become lower, recycling more efficient and availability of scrap greater.

Also, we point out how recyclers benefit from the non-competitive price while at the same time, any rise in the proportion of scrap available for recycling or any improvement of technology push this price down. Assuming a convex cost function, it turns out that the latter way of improving the secondary supply leads to a greater market power since technology has a lower effect on the price decreasing, than the availability of scrap. In terms of competition, we shed light on the challenge of gathering high level of both recycling technology and availability of scrap. This can be reached through public policies which would adress the environmental and scarcity issues of materials supply by improving recycling. However, according to materials and countries, various constraints arise and push policies to be heterogeneous in the issue recycling *vs* mining.

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Appendix A: Proof of Proposition 1

To facilitate the computation and analysis of $\tilde{k}$, we assume that the firms consider $r(z) = 1$ so that the level of recycling is embodied by θ and also depends on Q_{t-1}. We determine the mining output equilibrium resulting from a Cournot competition holding between the n firms from the oligopoly. Considering the same cost structure with c as the constant unit cost, the intertemporal and simplest profit function of firm i is:

$$\Pi^i = \sum_{\tau=0}^{\infty} \delta^{t+\tau}(1 - Q_{t+\tau} - c)q^i_{t+\tau} \tag{18}$$

with δ as discount factor. From the FOC and by aggregating the n firms, we have:

$$Q^Y_t = n - nQ^Y_t - n\theta Q_{t-1} - nc - \sum_{\tau=1}^{T}(\delta\theta)^\tau Q^Y_{t+\tau} \tag{19}$$

At steady state, the equilibrium becomes:

$$Q^{Y*} = (n - nc)(1 + \frac{n}{1-\theta} + \frac{\delta\theta}{1-\delta\theta})^{-1} \tag{20}$$

Since with $\theta = 0$ we have the Cournot equilibrium generalized with n firms such as:

$$Q^{Y*}\big|_{\theta=0} = \frac{n(1-c)}{n+1} \tag{21}$$

With recycling it gives:

$$Q^{Y*}\big|_{\theta=0} < Q^{Y*}\big|_{\theta>0} \tag{22}$$

Assuming $\delta = 1$, we have $Q^{Y*} = \frac{(n-nc)(1-\theta)}{1+n}$ that we can include in the technology threshold. It gives:

$$\tilde{k} = \frac{1}{1 - \left(\frac{(n-nc)(1-\theta)}{1+n}\right)}$$

$$\tilde{k} = \frac{1+n}{1 + n(\theta + c + c\theta)} \tag{23}$$

From (23), we can easily observe that $\frac{\partial\tilde{k}}{\partial\theta} < 0$ and $\frac{\partial\tilde{k}}{\partial c} < 0$. The sign of $\frac{\partial\tilde{k}}{\partial n}$ is positive if: $0 < \theta + c + c\theta < 1$.

Otherwise if $(\theta + c + c\theta) > 1$ the effect of a more competitive mining sector on the threshold is negative.

Appendix B: The optimal secondary supply in t=2

With (1) we have in period 2:

$$S_2 = r(\hat{z})\theta Q_1 = \frac{\theta Q_1 kp(Q_2) - \theta Q_1}{kp(Q_2)} \tag{24}$$

$$\theta Q_1 kp(Q_2) - S_2 kp(Q_2) = \theta Q_1 \tag{25}$$

Let $p(Q) = \frac{1}{Q^\alpha}$ be the inverse demand function with α representing the price elasticity of demand that we assume equal to 1. In order to have $r(\hat{z}) > 0$ in period 2, it implies a technology threshold $\tilde{k} > Q_2$, then we assume $k > Q_2$. It gives:

$$\theta Q_1 = \frac{S_2 k}{k - Q_2} \tag{26}$$

$$S_2 k = \theta k Q_1 - \theta Q_1 Q_2^Y - \theta Q_1 S_2 \tag{27}$$

$$S_2(k + \theta Q_1) = \theta k Q_1 - \theta Q_1 Q_2^Y \tag{28}$$

$$S_2^* = \frac{\theta k Q_1 - \theta Q_1 Q_2^Y}{k + \theta Q_1} \tag{29}$$

Hence, we define:

$$S_2^*(q_1^i; q_1^j; q_2^i; q_2^j) = \left[q_1^i(\theta k - \theta q_2^i - \theta q_2^j) + q_1^j(\theta k - \theta q_2^i - \theta q_2^j)\right] \times \frac{1}{k + \theta q_1^i + \theta q_1^j} \tag{30}$$

Appendix C: The optimal mining output at steady state

Based on the intertemporal profit function of mining and under the case where the output of the recycling sector is defined as $r(\hat{z}_t)\theta Q_{t-1}$, , we have the following FOC:

$$p_t'(Q_t)Q_t^Y \left(1 + \theta Q_{t-1}\frac{\partial r(z_t)}{\partial Q_t}\right) + np(Q_t) - nc'(q_t^i) + \sum_{\tau=1}^{\infty} \delta^\tau p_{t+\tau}'(Q_{t+\tau})Q_{t+\tau}^Y \frac{\partial Q_{t+\tau}}{\partial Q_t} = 0 \tag{31}$$

(i) Equation (9)

Consider the balance growth path, where:

$$Q_t = Q_0 g^t, \\ Q_t^Y = Q_0^Y g^t$$

for some $g_q > 1$. Since

$$Q_t = Q_t^Y + \theta r(z_t)Q_{t-1},$$

one has

$$\begin{aligned} Q_t^Y &= Q_t - \theta r(z_t)Q_{t-1} \\ &= Q_0 g^t - \theta r(z_t)Q_0 g^{t-1} \\ &= Q_0 g^t \left(1 - \theta\frac{r(z_t)}{g}\right) \\ &= Q_t \left(1 - \theta\frac{r(z_t)}{g}\right). \end{aligned}$$

With $p(Q_t) = \frac{Ag^t}{Q_t{}^\alpha}$, observe that for any Q_t, we have:

$$p'(Q_t)Q_t = -\alpha p(Q_t)$$

Hence:

$$\begin{aligned} p_t'(Q_t)Q_t^Y &= p_t'(Q_t) \times Q_t \left(1 - \theta \frac{r(z_t)}{g}\right) \\ &= -\alpha p(Q_t) \left(1 - \frac{\theta}{g} + \frac{\theta}{kgp(Q_t)}\right) \end{aligned}$$

(ii) Equation (10)

For any t, since we have $r(z_t) = 1 - \frac{1}{kp(Q_t)}$, then:

$$\frac{\partial r(z_t)}{\partial Q_t} = -\frac{\alpha}{kQ_t p(Q_t)}$$

Hence,

$$1 + \theta Q_{t-1} \frac{\partial r(z_t)}{\partial Q_t} = 1 - \frac{\alpha\theta}{kgp(Q_t)}$$

(iii) Equation (11)

As we know that a part of the total output of the industry in t is back on the market in $t+1$ through the recycling process, we observe that for any $\tau > 0$ we have:

$$\frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_t} = \frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_{t+\tau}} \times \frac{\partial Q_{t+\tau}}{\partial Q_t}$$

Since we know that $Q_{t+\tau} = Q_{t+\tau}^Y + \theta r(z_{t+\tau})Q_{t+\tau-1}$, this implies:

$$\frac{\partial Q_{t+\tau}}{\partial Q_t} = \theta r(z_{t+\tau}) \frac{\partial Q_{t+\tau-1}}{\partial Q_t} + \theta Q_{t+\tau-1} \frac{\partial r(z_{t+\tau})}{\partial Q_t},$$

Hence

$$\begin{aligned} \frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_t} &= \frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_{t+\tau}} \times \frac{\partial Q_{t+\tau}}{\partial Q_t} \\ &= \frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_{t+\tau}} \times \theta r(\hat{z}_{t+\tau}) \frac{\partial Q_{t+\tau-1}}{\partial Q_t} + \frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_{t+\tau}} \times \theta Q_{t+\tau-1} \frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_t}. \end{aligned}$$

This implies

$$\left(1 - \frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_{t+\tau}} \times \theta Q_{t+\tau-1}\right) \frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_t} = \frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_{t+\tau}} \times \theta r(\hat{z}_{t+\tau}) \frac{\partial Q_{t+\tau-1}}{\partial Q_t}.$$

Hence

$$\frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_t} = \frac{\frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_{t+\tau}} \times \theta r(\hat{z}_{t+\tau})}{1 - \frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_{t+\tau}} \times \theta Q_{t+\tau-1}} \times \frac{\partial Q_{t+\tau-1}}{\partial Q_t}.$$

And we finally have:

$$
\begin{aligned}
\frac{\partial Q_{t+\tau}}{\partial Q_t} &= \theta r(\hat{z}_{t+\tau})\frac{\partial Q_{t+\tau-1}}{\partial Q_t} + \theta Q_{t+\tau-1}\frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_t} \\
&= \theta r(\hat{z}_{t+\tau})\frac{\partial Q_{t+\tau-1}}{\partial Q_t} + \theta Q_{t+\tau-1} \times \frac{\frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_{t+\tau}} \times \theta r(\hat{z}_{t+\tau})}{1 - \frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_{t+\tau}} \times \theta Q_{t+\tau-1}} \times \frac{\partial Q_{t+\tau-1}}{\partial Q_t} \\
&= \theta r(\hat{z}_{t+\tau}) \left(1 + \frac{\frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_{t+\tau}} \times \theta Q_{t+\tau-1}}{1 - \frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_{t+\tau}} \times \theta Q_{t+\tau-1}}\right) \frac{\partial Q_{t+\tau-1}}{\partial Q_t} \\
e_{t,\tau} &= \frac{\theta r(\hat{z}_{t+\tau})}{1 - \frac{\partial r(\hat{z}_{t+\tau})}{\partial Q_{t+\tau}} \times \theta Q_{t+\tau-1}} \frac{\partial Q_{t+\tau-1}}{\partial Q_t}
\end{aligned}
$$

At steady state we have for $\tau \geq 1$:

$$
\begin{aligned}
e_\tau &= g\frac{\theta r(z^*)}{1 - d^* \times \theta Q^*} \times \frac{\partial Q_{t+\tau-1}}{\partial Q_t} \\
&= g\frac{\theta r(\hat{z})}{1 - d^* \times \theta Q^*} e_{\tau-1}.
\end{aligned}
$$

where $d^* = \frac{\partial r(z^*)}{\partial Q^t}$. Observe that when $\tau = 0$, $e_\tau = e_0 = 1$. This implies for any $\tau \geq 1$ we have:

$$
\begin{aligned}
e_\tau &= \left(g\frac{\theta r(\hat{z})}{1 - d^* \times \theta Q^*}\right)^2 e_{\tau-2} \\
&= \cdots \\
&= \left(g\frac{\theta r(\hat{z})}{1 - d^* \times \theta Q^*}\right)^\tau \\
&= g^\tau \left[\frac{\theta(kp(Q^*) - 1)}{kp(Q^*)g + \alpha\theta}\right]^\tau
\end{aligned}
$$