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EPEE

CENTRE D'ÉTUDE DES POLITIQUES ÉCONOMIQUES DE L'UNIVERSITÉ D'EVRY

**Is There Enough RD Spending ?
A Reexamination of Romer's (1990) Model**

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The Level of R&D Spending in the Variety-Based Endogenous Growth Model

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Abstract

We show that equilibrium endogenous growth may be excessive in the variety-based endogenous growth model *à la* Romer (1990). This result is obtained by relaxing the assumption on the constant elasticity of the demand function for intermediate goods.

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1. Introduction

Romer (1990) has developed a model of growth in which the advancement of technology is based on the creation of new varieties of intermediate goods. The core of the so-called “expanding product variety” approach is to combine non-convexity in the technology, due to the non-rivalrous nature of ideas embodied in new varieties, with imperfect competition, which allows to remunerate the innovators with monopoly profits. Because of *both* knowledge spill-over and monopoly pricing, Romer argues that the decentralized economy does not allocate enough resources to R&D investment. Consequently, equilibrium growth is below optimum at all times.

The robustness of this conclusion has been questioned by Benassy (1998) who shows that, by modifying the assumptions governing the “returns to specialization” in the final good sector, “Romerian” model may in fact lead to excessive equilibrium growth. The present article points to a crucial assumption made by

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Romer (1990) and systematically used in the literature (*e.g.* Barro and Sala-I-Martin (1995) : the constant elasticity form of the demand function for intermediate goods. Indeed, by allowing market power in the monopolistic sector to vary with the level of demand, we show that monopoly pricing may lead to over-compensation of innovations and thus result in excessive R&D spending. This effect may conflict with the knowledge spill-over, which is a source of underinvestment, so that the net effect on endogenous equilibrium growth is ambiguous.

We use the original framework of Romer (1990), except for the specification of the final good, which is assumed to be non-durable. The introduction of physical capital is also analyzed in an appendix. Following Jones and Williams (1998), the result is proven by comparing the private rate of return to R&D with the social rate of return along the Equilibrium Growth Path (EGP). It is hoped that an additional contribution of this article is to provide an intuitive way to present Romer's model.

The remainder of this paper is organized as follows. Section 2 describes the market economy. Section 3 characterizes the static allocation prevailing along an EGP. Section 4 fully determines the EGP by studying the dynamic allocation. Section 5 concentrates on a comparison between the private and social rates of return to R&D, and shows how overinvestment may arise. Section 6 concludes. In the appendix A, we show that our assumption concerning the absence of physical capital is not crucial to prove our point.

2. The market economy

This section describes the structure of the market economy and provides a definition of an Equilibrium Growth Path (EGP).

2.1. Technology and preferences

The production side is a traditional three sectors structure, including a final good sector, an intermediate goods sector and a R&D sector.

Time is continuous. At any time, the final good firm produces a flow Y of final good by combining a collection $\{x(i), i \in [0, A]\}$ of intermediate goods with an amount L_Y of labor. A captures the number of types of intermediate good invented at that time. The production function is given by:

$$Y = L_Y \int_0^A f\left(\frac{x(i)}{L_Y}\right) di, \quad (2.1)$$

where $f(\cdot)$ is continuous, increasing, differentiable, strictly concave and satisfies $f(0) = 0$. It includes the standard case of a power function $f(x) = x^\alpha$, $\alpha < 1$, used by Romer (1990) or Barro and Sala-I-Martin (1995).

The intermediate goods sector is a continuum of producers indexed by $i \in [0, A]$. The producer i rents the patent needed to produce the intermediate good

i , from households and is the monopolistic supplier of that type of good to the producer of final good. It is assumed that an amount $x(i)$ of i is obtained by transforming $x(i)$ units of final good at no additional cost. Intermediate goods are non-durable and the flow $x(i)$ is destroyed in the process of the production of the final good. This no-physical capital assumption facilitates the exposition of the model without affecting the main results. It will be relaxed in the appendix A. We note $K = \int_0^A x(i)di$ as the quantity of final good used as input in the intermediate goods sector.

Creation of blueprints to produce a new variety of intermediate good is performed in the R&D sector. It is assumed to depend on the amount of labor employed, L_A , and on the current stock of “ideas”, captured by the range of existing varieties, A , as follows:

$$\dot{A} = \gamma AL_A, \tag{2.2}$$

where γ is a time-invariant productivity parameter. The level of L_A is chosen by individual R&D firms while A acts as an externality. The initial level of A is exogenous and noted $A(0)$.

The economy is populated by an infinitely lived representative household endowed with the following preferences:

$$\int_0^\infty e^{-\rho t} (C^{1-\sigma} - 1)/(1 - \sigma) dt, \tag{2.3}$$

where $\rho > 0$ is the rate of time preference, and $\sigma > 0$, the inverse of the elasticity of intertemporal substitution. C is the amount of final good allocated to consumption. There is no population growth and the total labor supply is L . The household owns the stock of infinitely lived patents corresponding to the existing varieties of intermediate goods and firms (which, as we detail below, earn zero profits at equilibrium).

2.2. The Equilibrium Growth Path (EGP)

In the market economy, we need to analyze what percentage of the total labor supply L is allocated to R&D, and how final output is shared between consumption and input for producing intermediate goods. Ultimately, this will determine the Equilibrium Growth Path and the endogenous growth rate of the economy.

Final good firms, R&D firms and the representative household are price takers on all markets. This includes markets for labor, final good, patents, renting patents, and intermediate goods. By contrast, intermediate goods firms act as monopolists on the intermediate goods markets and charge a markup over marginal cost.

Concerning the intermediate goods, it is not restrictive to focus on *symmetric* equilibria, for which prices and quantities produced are the same for all goods

$i \in [0, A]$. A consequence of this symmetry is that infinitely lived patents, each corresponding to one variety, are perfect substitutes.

In what follows, the notation \tilde{x} is used to indicate the value of a variable x along the EGP, while $\tilde{x}(\cdot)$ is the associated continuous time path. The price paths include $\tilde{p}_A(\cdot)$, the price of patents, $\tilde{r}_A(\cdot)$, the rental rate of patents, $\tilde{w}(\cdot)$, the wage rate and, $\tilde{p}(\cdot)$, the price of intermediate goods. The final good is used as numeraire.

An EGP ($\tilde{A}(\cdot), \tilde{C}(\cdot)$) starting from the initial condition $A(0)$ must satisfy the following:

1. The household takes as given the path $(\tilde{r}_A(\cdot), \tilde{p}_A(\cdot), \tilde{w}(\cdot))$ and chooses the consumption and patents path $(\tilde{A}(\cdot), \tilde{C}(\cdot))$ maximizing (2.3) under a no-Ponzi game condition.
2. The R&D firms take as given $(\tilde{p}_A(\cdot), \tilde{w}(\cdot))$, and the path of externality $\tilde{A}(\cdot)$, and choose the path of labor $L_A(\cdot)$ maximizing, at any time, its instantaneous profit $\tilde{p}_A \gamma \tilde{A} L_A - \tilde{w} L_A$. These profits are equal to zero at equilibrium.
3. The final good firm takes as given $(\tilde{p}(\cdot), \tilde{w}(\cdot))$ and the path of intermediate goods range $\tilde{A}(\cdot)$, and chooses the path of labor $L_Y(\cdot)$ and of intermediate goods $x(\cdot)$, maximizing, at any time, its instantaneous profit $\tilde{A} L_Y f(x/L_Y) - \tilde{w} L_Y - \tilde{p} \tilde{A} x$. These profits are equal to zero at equilibrium.
4. The intermediate goods firms choose the path of intermediate price $\tilde{p}(\cdot)$, by maximizing, at any time, its instantaneous monopoly rent $(p - 1)\tilde{x}(p)$, where $\tilde{x}(p)$ is the demand for intermediate good formulated by the final good firm. The maximized rent is noted $\tilde{\pi}_A$.
5. At any time, the following resources constraints hold:

$$\tilde{L}_A + \tilde{L}_Y = L, \quad (2.4)$$

$$\tilde{C} + \tilde{K} = \tilde{Y}. \quad (2.5)$$

where \tilde{Y} is the equilibrium output of final good, and $\tilde{K} = \tilde{A}\tilde{x}$ is the level of final output transformed in intermediate goods.

6. At any time, *free entry* into the intermediate goods sector implies that the monopoly rent is transferred to patent holders (household) as royalties, *i.e.*, $\tilde{r}_A(\cdot) = \tilde{\pi}_A(\cdot)$.

This last condition completes the definition of the EGP starting from $A(0)$. An EGP is said to be *balanced*, when the sharing of labor force between the final good sector and the R&D sector is constant through time. Then, all variables $(C, A, Y\dots)$ are growing at a common constant endogenous growth rate, noted \tilde{g} .

The EGP will be characterized in two steps. First, for a given pair (L_Y, A) , we focus on the static allocation, which describes the interplay between the final good sector and the intermediate goods one and indicates how income generated by “physical” sectors is shared between workers and patent holders. Second, the dynamic allocation permits to endogenize the dynamics of (L_Y, A) through time.

3. The static allocation

3.1. Algebra

At one arbitrary instant in time, we consider (L_Y, A) as given. Focusing on symmetric allocations over i ($x(i) = x, \forall i \in [0, A]$), the flow of final good Y is a function of $K = Ax, L_Y$ and A such that:

$$Y = L_Y A f\left(\frac{K}{AL_Y}\right) = L_Y A f(k), \quad k \equiv K/(AL_Y). \quad (3.1)$$

Net output devoted to consumption is $C = Y - K = [f(k) - k]AL_Y \equiv \Omega(k)AL_Y$. $\Omega(k)$ denotes the “intensive net output” and is maximized when k takes the value k^* , satisfying¹ $f'(k^*) = 1$.

The market level of k is determined as follows. Maximizing its profit, the final good firm’s demand for intermediate goods satisfies $p = f'(k)$, with p the price of intermediate goods. This implies that the elasticity of demand is $\epsilon = -f'(k)/(kf''(k))$, which is constant at the level $1/(1 - \alpha)$ when $f(\cdot)$ is a power function, but which may vary with the level of demand in the general case.

In the intermediate goods sector, monopolists charge a mark-up $\mu = (1 - 1/\epsilon)^{-1}$ over the marginal cost, which is one. Hence, \tilde{k} , the equilibrium level of k , solves:

$$p = \mu \times 1 = f'(k) = 1 - kf''(k). \quad (3.2)$$

The function $f(\cdot)$ is constrained such that (3.2) has a unique interior solution. Then, free entry in the intermediate goods sector implies:

$$\tilde{r}_A = (\mu - 1)x = (f'(\tilde{k}) - 1)\tilde{k}L_Y = -\tilde{k}^2 f''(\tilde{k})L_Y. \quad (3.3)$$

The equilibrium net output is $Y - K = \Omega(\tilde{k})AL_Y \equiv \tilde{\Omega}AL_Y$, hence equilibrium consumption satisfies:

$$\tilde{C} = \tilde{\Omega}AL_Y. \quad (3.4)$$

The corresponding income is shared among wages and royalties according to:

$$\tilde{\Omega}AL_Y = \tilde{r}_A A + \tilde{w}L_Y, \quad (3.5)$$

where $\tilde{w} = A [f(\tilde{k}) - \tilde{k}f'(\tilde{k})]$ is the equilibrium wage. It is noticed that \tilde{k} and \tilde{w}/A do not depend on (L_Y, A) , which means that these ratios remain constant along the EGP.

¹ $f(\cdot)$ is restricted such as k^* is interior.

3.2. A graphical characterization

As shown in figure 3.1 the static dimension of the model is similar to a standard monopoly problem. Curve (C) defines the demand schedule $p = f'(k)$. The horizontal line at level 1 is the marginal cost. The area which lies between these two curves is equal to the intensive net output $\Omega(z)$. It is maximized for $k = k^*$, while the equilibrium level \tilde{k} maximizes the monopoly rent, *i.e.* the area of the rectangle $(ABCD)$. Obviously, $\tilde{k} < k^*$. The loss of net output arising from monopoly pricing is represented by the area (BDE) .

The figure indicates how output $\tilde{\Omega}$ is shared between royalties and wages according to (3.5). Monopoly pricing affects the *structure* of the remuneration scheme. Indeed, while varieties and labor are symmetric arguments to the production function (3.1), there is no reason why monopoly rent should be equal to total wages paid by the final sector firm. This means that the structure of remuneration is distorted whenever the respective shares of royalties and wages are not 50%-50%. A measure of this distortion is provided by the constant $\tilde{\phi}$, which is:

$$\tilde{\phi} = \frac{\tilde{r}_A A}{\tilde{w} L_Y} = \frac{\tilde{k} [f'(\tilde{k}) - 1]}{[f(\tilde{k}) - \tilde{k} f'(\tilde{k})]}. \quad (3.6)$$

This distortion is crucial as we turn to the dynamic part of the model where the supplies A and L_Y become elastic.

4. The dynamic allocation

We are now able to fully determine the EGP by endogeneizing the evolution of (L_Y, A) .

4.1. Consumption smoothing and the demand for patents

In order to smooth the consumption along its life-cycle, the household purchases infinitely lived patents from R&D firms and rents them to producers of intermediate goods.

The competitive price of patents satisfies $\tilde{p}_A = \tilde{w}/(\gamma A)$, which is $[f(\tilde{k}) - \tilde{k} f'(\tilde{k})]/\gamma$ as soon as \tilde{L}_Y and \tilde{L}_A are strictly positive. The equilibrium rental rate \tilde{r}_A is given by (3.3). Since \tilde{p}_A is time invariant along an EGP, there are no capital gains and the *private* rate of return on R&D investment (PROR) may be expressed as:

$$\tilde{r} = \frac{\tilde{r}_A}{\tilde{p}_A} = \frac{\tilde{k} [f'(\tilde{k}) - 1]}{f(\tilde{k}) - \tilde{k} f'(\tilde{k})} \gamma L_Y = \tilde{\phi} \gamma L_Y. \quad (4.1)$$

The PROR measures the intertemporal equilibrium price faced by the house-

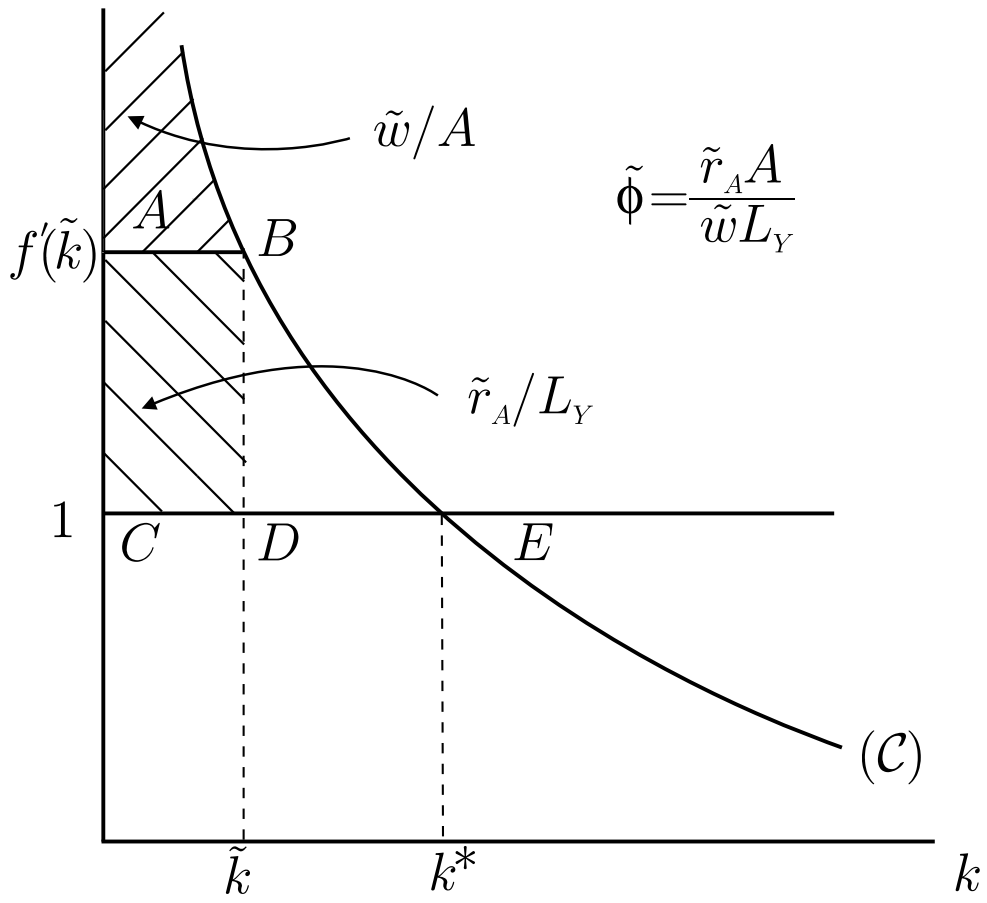


Figure 3.1: The static allocation

hold. Then, according to the Ramsey's rule, consumption grows at the rate:

$$\frac{\dot{\tilde{C}}}{\tilde{C}} = (\tilde{r} - \rho)/\sigma. \quad (4.2)$$

4.2. The equilibrium dynamics

The EGP ($\tilde{C}(\cdot), \tilde{A}(\cdot)$) is entirely described by a two-dimensional differential system defined by (4.2) and (2.2), in which we include equations (4.1) and (3.4), and the labor constraint (2.4). Defining $\tilde{\chi} \equiv \tilde{C}/\tilde{A}$, it follows that the evolution of this ratio is governed along an EGP by:

$$\frac{\dot{\tilde{\chi}}}{\tilde{\chi}} = \frac{\dot{\tilde{C}}}{\tilde{C}} - \frac{\dot{\tilde{A}}}{\tilde{A}} = \frac{(\tilde{\phi} + \sigma)\gamma}{\sigma\tilde{\Omega}}\tilde{\chi} - \left(\gamma L + \frac{\rho}{\sigma}\right). \quad (4.3)$$

The saddle path solution of this equation indicates that the market economy jumps instantaneously to a balanced EGP, characterized by a constant ratio \tilde{C}/\tilde{A} , which is:

$$\tilde{\chi} = \frac{\sigma\gamma L + \rho}{(\tilde{\phi} + \sigma)\gamma}\tilde{\Omega}. \quad (4.4)$$

The constant endogenous growth is then given by:

$$\tilde{g} = (\tilde{\phi}\gamma L - \rho) / (\tilde{\phi} + \sigma). \quad (4.5)$$

The situation may be understood in terms of rate of growth and rate of return. The supply side of the model provides a decreasing relationship between the rate of return to R&D and the endogenous growth rate \tilde{g} :

$$\tilde{r} = \tilde{\phi}[\gamma L - \tilde{g}]. \quad (4.6)$$

It is obtained from (4.1), (2.2) and (2.4). More growth (in varieties) implies lower labor input in final good sector, and less private return to R&D investment. On the demand side, Ramsey's rule (4.2) shows that more growth (in consumption) has to be compensated by a higher rate of return to saving.

In the figure 4.1, the supply side relationship (4.6) is represented by the line (D) while the line (D') represents the demand side equation (4.2). The equilibrium level (\tilde{g}, \tilde{r}) is at the intersection between (D) and (D'). It is interesting to note that the extent of the distortion in the remuneration scheme, measured by $\tilde{\phi}$, is a crucial parameter in determining the endogenous growth rate.

5. Under or overinvestment in R&D along the EGP

Does the market economy allocate enough resources to R&D investment? This section provides an answer to this question by comparing, as proposed by Jones and Williams (1998), the private rate of return to R&D with its social counterpart.

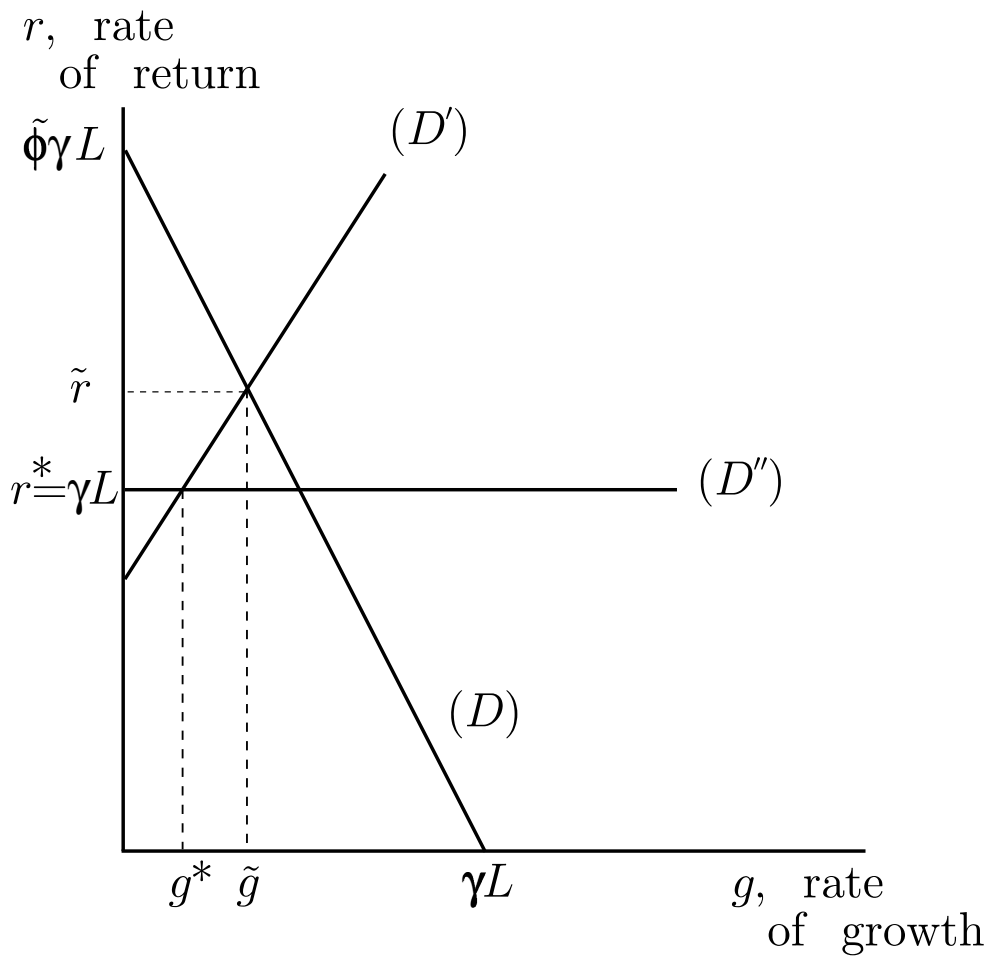


Figure 4.1: Endogenous growth path

5.1. Private and Social Rates of Return to R&D

The social rate of return (SROR) associated to the EGP is determined as in Jones and Williams (1998). Assume that, along the EGP, a social planner reallocates one unit of output from consumption to R&D, and then allocates the proceeds to future consumption, leaving the subsequent path unchanged. This change is made possible through a reallocation of labor from the final good sector to research. Along the EGP, one unit of consumption yields $\gamma / [f(\tilde{k}) - \tilde{k}f'(\tilde{k})]$ additional units of new varieties. Each new variety directly raises net output by $[f(\tilde{k}) - \tilde{k}f'(\tilde{k})] \tilde{L}_Y$. In addition, these varieties induce a spill-over effect which increases labor productivity in the R&D sector. This allows for a reallocation of labor to the final good sector, which in turn increases – indirectly – net output by an amount of $\gamma \tilde{L}_A$.

The SROR is a measure of the total gain from this variational change, in terms of consumption. Its expression is:

$$r^* = \gamma L. \tag{5.1}$$

We are now in a position to compare the value of the private rate of return with the social one. Equations (4.1) and (5.1) show that the wedge between \tilde{r} and r^* results from two distinct distortions. First, the term $\tilde{\phi}$ in (4.1) accounts for monopoly pricing. In this perspective, the PROR is distorted if the pattern of remuneration is not symmetric between variety and labor—*i.e.*, if the remuneration of these factors is not strictly 50%-50%. Second, this wedge also originates from the knowledge spill-over, which is represented only by L_Y in equation (4.1) – instead of L as in equation (5.1).

The critical point is that these two effects may conflict with one another. Indeed, while the knowledge spill-over will tend to bring the PROR below the social one, monopoly pricing may compensate this effect if $\tilde{\phi}$ is higher than L/L_Y . This would be the case if $\tilde{\phi}$ exceeds unity.

5.2. An Overinvestment Case

The value of $\tilde{\phi}$ depends on the exact form of the demand function for intermediate goods. For a power function $f(k) = k^\alpha$, $\alpha < 1$, it is easy to verify that the ratio $\tilde{\phi}$ is equal to α and is always less than unity. In that case, the social rate is higher than the private one due to monopoly pricing only. This implies that the market economy tends to under-invests in R&D and grows at a rate below the optimal level. This corresponds to the situation described by Romer (1990) or Barro (1995).

Unfortunately, this conclusion is not robust to a change in the specification of the demand function. Assume that the demand function is linear as a result

of a quadratic $f(\cdot)$:

$$f(k) = \begin{cases} -\frac{b}{2}k^2 + ak, & \forall k \in [0, a/b] \\ a^2/2b, & \forall k \geq a/b, a > 1, b > 0. \end{cases}$$

The elasticity of demand decreases with demand. It is easy to verify that $\tilde{\phi} = 2$. Rents captured by the monopolists are twice the labor income generated by the final sector firm. Monopoly pricing therefore induces a negative wedge between r^* and \tilde{r} , which may give rise to overinvestment in R&D. The problem is now to examine whether this effect can dominate the knowledge spillover effect along the EGP.

In figure 4.1, we plot the horizontal line (D'') at level γL , which represents the SROR. Graphically, overinvestment in R&D and excessive growth ($\tilde{r} < r^*$) occur as soon as the line (D') crosses (D) above the horizontal line (D'') This situation is possible when both $\tilde{\phi}$ is higher than unity and the economy is a fast-growing one (ρ and σ high). In such a situation, monopoly pricing creates overinvestment in R&D – which offsets underinvestment due to knowledge spillover. Consequently, the equilibrium growth rate is excessive.

Alternatively, the figure allows for a comparison of the equilibrium growth rate, \tilde{g} , with the optimal growth rate, g^* . The optimal path is obtained by considering a planner which would maximize the intertemporal welfare (2.3) under dynamic resources constraints. The optimal allocation is represented by the intersection of (D') and (D''). The conclusion is similar. We can not rule out the case $\tilde{g} > g^*$.

5.3. On the empirical relevance of overinvestment in R&D

In an expanding varieties growth model with labor in the R&D sector, depending on the precise form of the demand function for intermediate goods, monopoly pricing may generate excessive monopoly rent, which may lead to overinvestment in R&D and excessive endogenous growth. The appendix A shows that this result holds in the long run when there is physical capital. This shows that a widely accepted property of a variety-based growth model *à la* Romer (1990) depends crucially on the behavior of the third-order derivative of the final good production function $f(\cdot)$. Such behavior is difficult to test based on the existing empirical evidence.

However, our approach provides an observable necessary condition for the equilibrium growth rate to be excessive due to overinvestment in R&D: the sum of monopoly profits earned by patent holders should be higher than the wage bill paid by the final good sector. Basic evidence indicates that it is hardly the case. Does it imply that our point is empirically irrelevant? The answer is in the negative because the technological side of the model is obviously oversimplified.

Recall that the necessary condition for overinvestment comes directly from the fact that A and L_Y enter as *symmetric* arguments in the final good sector

reduced form production function (3.1). This symmetry is required to ensure the existence of a balanced growth path, whatever the precise form of the function $f(\cdot)$ is. This is because A , the range of available varieties, acts as an Harrod neutral technological change in (3.1). Can we circumvent this constraint without altering our result?

A solution is to consider a multi-sectoral structure for producing final goods. For instance, we may think that the production function (2.1) describes the technology of a high-tech capital good sector. Its production is used by the intermediate goods sector and a final good sector, which combines high-tech capital with labor and others inputs in a Cobb-Douglas production function. In such a case, unbalanced growth may occur and our necessary condition for excessive growth concerns only the distribution of income in the high-tech sector. It seems empirically more plausible.

6. Conclusion

This paper illustrates the lack of robustness of one of the main results of Romer (1990). In a model of growth with expanding product variety, monopoly pricing and knowledge spillover do not imply that the market economy underinvests in R&D. On one hand, a knowledge spillover always generates a positive wedge between private and social rates of return on R&D. On the other hand, monopoly pricing may lead to an overcompensation of patents – depending on the shape of the demand function for intermediate goods. In the general case, the net effect is ambiguous. In contrast with Benassy (1998), this result is not obtained through an external effect of the variety index on production.

Obviously, this does imply that overinvestment in R&D should be expected in the real economy. Empirical evidence surveyed in Camero (1996) shows that the private rate of return to R&D is less than the social optimum and Jones and Williams (1998) relate the empirical literature to a model with expanding product variety. The point made in the article is that, among the sources of discrepancy between social and private rates, monopoly pricing has an uncertain influence even in models with expanding product variety.

A. Relaxing the no-physical capital assumption

This appendix shows that formulae (4.1) and (4.1) remain valid along the *balanced* Equilibrium Growth Path as soon as the no-physical capital assumption is relaxed. It implies that the figure 4.1 describes the market economy in the *long run*. The analysis of an over- or underinvestment in R&D during the transition toward the balanced path is beyond the scope of the article.

Assume that the final good is durable so that the intermediate good i is a capital good. Let r_K be the competitive rental rate for physical capital paid by the intermediate good firms. This rental rate is the marginal cost on which the intermediate goods firms charge a markup at the rate μ to determine its supply price. Using our notation, it is not difficult to see that the equilibrium level of k solves:

$$r_K = f'(\tilde{k}) + \tilde{k}f''(\tilde{k}). \quad (\text{A.1})$$

Then free entry in the intermediate goods sector implies that the equilibrium rental rate for a patent is:

$$\tilde{r}_A = r_K \times (\mu - 1)x = -\tilde{k}^2 f''(\tilde{k})L_Y. \quad (\text{A.2})$$

Compared to the no-physical capital case (equation (3.3)), the formula is unchanged. More generally, the pattern of remuneration (including the expression of the wage rate) is not modified. Consequently, the competitive price of patent remains $\tilde{p}_A = [f(\tilde{k}) - \tilde{k}f'(\tilde{k})]/\gamma$.

A difference with the no-physical capital case comes from the fact that \tilde{p}_A may evolve over time as a result of the adjustments of r_K and \tilde{k} toward their long-run values. This means that capital gains or losses on the patent asset arise during the adjustment. Consequently, the formula (4.1) holds only in the long run, when \tilde{k} has reached its stationary value. The equilibrium in the household's portfolio implies the PROR is equal to the rental rate r_K .

The determination of the SROR to R&D along the Equilibrium Growth Path is obtained as in subsection 5.1. As for the PROR, the difference may come from the “social” capital gain or loss obtained when the economy is not in a long-run state. To avoid this issue, we focus only on long-run path for which formula (5.1) applies.

In figure 4.1, the (long-run) supply side is always described by the two lines (D) and (D''), while the Ramsey's rule holds and is depicted in the long-run by the line (D'). The balanced endogenous growth path always corresponds to the intersection of (D) and (D').

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