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Liquidity Constraint, Increasing Returns and Endogenous Fluctuations

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Abstract

In this paper we show that local indeterminacy, endogenous fluctuations, periodic and quasi-periodic orbits may emerge in a one-sector infinite-horizon competitive economy where (i) at the end of each period agents must hold a share of their wealth in the form of money and (ii) technology exhibits increasing returns to scale. In contrast to other contributions on this subject, we find that such phenomena occur when consumption is intertemporally substitutable and labour is supplied inelastically. The scope for indeterminacy depends basically on the fact that, in view of the financial constraint, total returns on investment represent a weighted average of capital marginal productivity and deflation, and the latter is positively related to the rate of growth of capital.

Keywords: Financial constraint; Intertemporal substitution; Indeterminacy

JEL classification: D90; E32; E50

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1 Introduction

The occurrence of indeterminacy in growth models is receiving increasing attention from macroeconomists in view of the new insights it may offer for the explanation of asymmetric fluctuations observed in countries with similar fundamentals and analogous public policies. When equilibria are indeterminate, there can indeed be rational expectations equilibria in which prices and quantities exhibit repetitive and persistent fluctuations, even though no extrinsic uncertainty affects fundamentals. Fluctuations of this type occur in response to random events (the sunspot process) and represent self-fulfilling revisions of agents' expectations.

According to many recent contributions, in one-sector infinite-horizon competitive economies the source of indeterminacy may be mainly viewed as arising either from the presence of positive external effects in production not mediated by markets (Howitt and McAfee (1988), Farmer and Guo (1994), Benhabib and Farmer (1994), (1996), (1997), Cazzavillan (1996)), or from the introduction of liquidity constraints preventing agents from exercising their stabilizing smoothing behaviour (Woodford (1986), Grandmont *et al.* (1998), Bloise *et al.* (1999)), or from both (Cazzavillan *et al.* (1998)). In order to generate indeterminacy, however, these market imperfections very often need to be coupled either with elastic labour supply¹ or with important income effects in intertemporal consumption².

In order to improve the understanding of the role of market imperfections in generating indeterminacy and of the extent to which it may really matter, in this paper we study a one-sector infinite-horizon economy with spillover effects in production and financial constraint, according to which at the end of each period agents must hold a given share of their wealth in the form of money. This financial constraint is similar to that considered by Tirole (1985) and Crettez *et al.* (1998) in a Diamond *OLG* model, and its amplitude captures, roughly, the inverse of the degree of financial intermediation of the economy³. In this context, we show that local indeterminacy and

¹In particular, Boldrin and Rustichini (1994) show that in one-sector models with positive externalities indeterminacy generically may not arise when labour is supplied inelastically. An interesting exception is provided in Cazzavillan (1996) in a model displaying long-run growth and in which public spending enters production function and utility.

²This is true, e.g., in models with cash-in-advance constraint on consumption purchases, as in Bloise *et al.* (1999).

³Alternatively, it can be viewed as the degree of a reserve requirement.

sunspot equilibria, as well as deterministic periodic and quasi-periodic orbits, may emerge when consumption is intertemporally substitutable and labour is supplied inelastically, provided that the elasticity of inputs substitution and/or the externality contribution to production are sufficiently high to entail an interest rate slightly increasing in capital intensity. It is important to stress that both market imperfections, namely externalities and liquidity constraint, are necessary for the emergence of indeterminacy.

More in detail, when the elasticity of intertemporal substitution in consumption is continuously increased, local indeterminacy arises through a Hopf bifurcation and eventually disappears through a flip one. In any case, the range generating indeterminacy includes high elasticities of intertemporal substitution in consumption and is very large with respect to a wide calibration of the relevant parameters. It is worth noticing, however, that it shrinks monotonically, and finally vanishes, when the rate of money growth increases continuously. However, in order to reduce indeterminacy significantly, the rate of money growth should be sustained too much to be realistically implementable. Therefore, it would be more appropriate to follow a fiscal stabilization policy entailing a progressive income tax, sufficiently elastic to offset the externality effect and to restore a return on physical investment decreasing in capital intensity, which would completely rule out indeterminacy.

The scope for indeterminacy depends basically on the fact that, in view of the liquidity constraint, total returns on investment represent a weighted average of capital marginal productivity (the yields of physical investment) and the deflation factor (the return on money balances). To see how indeterminacy comes about, let us consider, starting from the steady state equilibrium, that agents expect that an alternative equilibrium path exists with, say, higher returns on investment. If consumption is highly substitutable, agents acting on this belief react by diverting current *GDP* from consumption to investment. The increase in capital, in view of the positive externalities, will cause its marginal productivity to augment. However, as agents increase consumption in the following period, physical investment falls. This contraction causes an increase in inflation to maintain equilibrium in the money market (inflation is indeed inversely related to the rate of growth of physical investment). But higher inflation partially offsets the higher returns on capital and allows the Euler condition to be satisfied, while consumption and capital move back towards their initial positions.

We also identify conditions for the existence of the stationary solution as

well as for its uniqueness, and analyze the effects upon the welfare of the agents of changes in the share of wealth to be held liquid and in the rate of money growth. In particular, when the steady state is indeterminate a more severe financial constraint as well as a higher rate of money growth may drive the economy towards Pareto-superior steady states (money is therefore not superneutral). It follows that an expansive monetary policy may entail a twofold positive effect: it makes indeterminacy less likely to occur and increases welfare at the steady state. In addition, when the externality contribution to production increases continuously and goes through the critical value in correspondence to which the interest rate changes the sign of its slope, a transcritical bifurcation generically occurs and there is a change in stability between two nearby steady states.

The two-dimensional feature of the system describing the equilibrium dynamics of the economy allows us to perform a simple geometrical analysis adopted, among others, by Grandmont *et al.* (1998) and Cazzavillan *et al.* (1998), in order to fully characterize the dynamics around fixed points and along bifurcations. In addition, in view of the presence of exactly one predetermined variable in each period, it could also be exploited to prove the existence of stochastic sunspot equilibria around an indeterminate steady state as well as along supercritical flip and Hopf bifurcations.

2 The model

We consider a one-sector economy with a continuum of identical households and identical firms whose sizes are normalized to unity, two assets (money and capital) and a single non-perishable good that can be either consumed or invested. Markets are competitive and prices fully flexible so that equilibrium prevails in each period. The representative household maximizes the discounted stream of utilities $\sum_{t=0}^{\infty} \beta^t u(c_t)$, where $0 < \beta < 1$ is the discount factor and u the instantaneous utility of consumption satisfying the following standard assumption.

Assumption 1. *u is C^n for $c > 0$ and n high enough, with $u' > 0$, $u'' < 0$. We denote $\varepsilon(c) \equiv |u'(c) / [u''(c)c]|$ the elasticity of intertemporal substitution in consumption which belongs to $(0, +\infty)$.*

Households in each period t inelastically supply one unit of labour, choose how much to consume c_t , invest in capital k_{t+1} and in nominal balances M_{t+1} ,

subject to the budget constraint $p_t c_t + p_t [k_{t+1} - (1 - \delta) k_t] + M_{t+1} = p_t r_t k_t + p_t w_t + M_t + \tau_t$, where p is the price of the good, r the capital real rental price, w the real wage, τ the monetary lump-sum transfers issued by government and $\delta \in [0, 1]$ the constant depreciation rate of capital. In addition, at the end of each period t agents must hold at least a share $\mu \in (0, 1)$ of real balances $m_{t+1} \equiv M_{t+1}/p_t$ in total wealth $a_{t+1} \equiv m_{t+1} + k_{t+1}$, i.e. they are subject to the liquidity constraint

$$M_{t+1} \geq q p_t k_{t+1} \quad (1)$$

where $q \equiv \mu / (1 - \mu)$ and belongs to $(0, +\infty)$. The amplitude q of constraint (1) captures, roughly, the inverse of the degree of financial intermediation of the economy. If we assume capital to dominate money in terms of returns, i.e.

$$1 - \delta + r_{t+1} > p_t / p_{t+1},$$

the financial constraint (1) is binding and the Euler equation for the consumer has the form

$$u'(c_t) = [\beta / (1 + q)] (1 - \delta + r_{t+1} + q p_t / p_{t+1}) u'(c_{t+1}). \quad (2)$$

The arbitrage equation (2) reflects the fact that if one wishes to decrease one unit of consumption in period t , the investment in capital (in correspondence to which one earns the gross real interest rate $1 - \delta + r_{t+1}$) is $1 / (1 + q)$ and that in real balances (whose gross rate of return corresponds to the deflation factor p_t / p_{t+1}) is $q / (1 + q)$. The transversality condition

$$\lim_{t \rightarrow +\infty} \beta^t u'(c_t) (m_{t+1} + k_{t+1}) = 0 \quad (3)$$

completes the solution of the individual optimization problem. We assume, in addition, that government follows a simple monetary rule according to which *fiat* money is injected into the economy at the constant growth rate γ by means of nominal lump-sum transfers to the households, i.e. $\tau_t = \gamma M_t$.

Firms produce the good by renting capital and labour and according to a technology exhibiting constant returns to scale at the private level. There are, however, positive externalities in aggregate capital and labour which entail increasing social returns to scale compatible with perfect competition. For a single productive unit the quantity of good produced depends indeed on the quantity of inputs employed as well as on the average quantity of labour \bar{l} and capital \bar{k} available in the economy. Normalizing to one the inelastic

labour supply, in correspondence to a stock of capital k the effective services of capital and labour amount to, respectively, $A\psi(\bar{k})k$ and $A\psi(\bar{k})$, where $A > 0$ is a scaling parameter which will allow the calibration of the model. Since firms are all identical, at symmetric equilibrium one has $\bar{k} = k$ and production is given by $y = A\psi(k)F(k, 1)$, where F is the private production function which is homogeneous of degree one. The intensive production function $f(k) \equiv F(k, 1)$ satisfies the following assumption.

Assumption 2. $f(k)$ is continuous for $k \geq 0$, C^n for $k > 0$ and n high enough, with $f' > 0$, $f'' < 0$. Therefore $\rho(k) \equiv f'(k)$ is decreasing in k , meanwhile $\omega(k) \equiv f(k) - kf'(k)$ is increasing. The externality function $\psi(k)$ is continuous on R_+ , C^n on R_{++} for n high enough, with $\psi'(k) > 0$. We denote with $s(k) \equiv f'(k)k/f(k)$ and with $\varepsilon_\psi(k) \equiv \psi'(k)k/\psi(k)$, respectively, the share of capital in total income and the elasticity of $\psi(k)$.

Profit maximization of the firms implies that the real interest rate is given by $r(k) = A\psi(k)\rho(k)$ and the real wage by $w(k) = A\psi(k)\omega(k)$. In the money market prices adjust in order to satisfy the demand for real balances according to $p_t/p_{t+1} = m_{t+2}/[m_{t+1}(1 + \gamma)]$ which, in the light of constraint (1), can be rewritten as

$$p_t/p_{t+1} = k_{t+2}/[k_{t+1}(1 + \gamma)]. \quad (4)$$

Finally, by Walras' law the goods market clears according to

$$k_{t+1} = g(k_t) + (1 - \delta)k_t - c_t \quad (5)$$

where $g(k) \equiv A\psi(k)f(k)$. Solving (5) for c_t and (5) lagged once for c_{t+1} , we can substitute the respective expressions in (2). Analogously, we can replace the deflation factor appearing in (2) with its expression given in (4). A perfect foresight equilibrium corresponding to a binding constraint (1) consists then in a deterministic sequence $\{k_t\}_{t=0}^\infty$, $k_t > 0$ for every t , satisfying the second order difference equation

$$\begin{aligned} K(k_t, k_{t+1}, k_{t+2}) &\equiv u'(g(k_t) + (1 - \delta)k_t - k_{t+1}) \\ &\quad - [\beta/(1 + q)][1 - \delta + r(k_{t+1}) + qk_{t+2}/[k_{t+1}(1 + \gamma)]] \\ &\quad u'(g(k_{t+1}) + (1 - \delta)k_{t+1} - k_{t+2}) = 0 \end{aligned} \quad (6)$$

subject to the initial endowment of capital k_0 and the transversality condition (3). One readily verifies that constraint (1) binds at each stationary solution

if and only if the discount factor β is lower than the gross rate $1 + \gamma$ of money growth. Under this condition, system (6) is consistent with equilibrium in a small neighbourhood of each steady state.

3 Steady state analysis

In this section we first identify conditions for the existence and uniqueness of the stationary solution of system (6) and show that when the magnitude of the externality changes continuously one generically obtains a transcritical bifurcation. We then analyze the effects of changes in the parameters q and γ on the welfare of agents evaluated at the steady state.

Capital stock, consumption and real balances are constant at the steady state. If we drop the time index in equation (6), we find that the capital steady state values are obtained by solving equation

$$r(k) = \alpha \tag{7}$$

where⁴ $\alpha \equiv 1/\beta - (1 - \delta) + q[1/\beta - 1/(1 + \gamma)]$. For each solution k^* of equation (7), it is easy to verify that the corresponding stationary level of consumption is given by $c^* = [\alpha/s(k^*) - \delta]k^*$ and that of real balances by $m^* = qk^*$. Conversely, the price level need not be constant at the steady state; the stationary inflation rate corresponds indeed to $(p_{t+1} - p_t)/p_t = \gamma$ and is zero if and only if the government prints no new money. The following proposition, which is immediately verifiable, shows that it is possible to calibrate the scaling parameter A in order to obtain $k = 1$ as an interior stationary solution of system (6).

Proposition 1 (*Existence of the steady state*). $k = 1$ is an interior stationary solution of system (6) if and only if the scaling parameter A is chosen as $A = \alpha/\psi(1) f'(1)$.

For the sequel of our analysis, it is more informative to write all the expressions in terms of the capital-labour elasticity of substitution $\sigma(k) \in (0, +\infty)$ which satisfies by definition $1/\sigma(k) = \varepsilon_\omega(k) - \varepsilon_\rho(k)$, where $\varepsilon_\omega(k)$ and $\varepsilon_\rho(k)$ denote the elasticities of, respectively, $\omega(k)$ and $\rho(k)$. In particular, one has the useful expression $|\varepsilon_\rho(k)| = [1 - s(k)]/\sigma(k)$. When $r(k)$

⁴Notice that α is positive when the financial constraint binds.

is monotonic in $k > 0$, i.e. the sign of its derivative is constant, equation (7) gets at most one solution. Since one has $r'(k) = [A\psi(k)f'(k)/k] \{\varepsilon_\psi(k) - [1 - s(k)]/\sigma(k)\}$, this is true when externalities are uniformly large or uniformly small, as is stated in the following proposition, which is easy to verify.

Proposition 2 (*Uniqueness of the steady state*). *There exists at most one stationary solution of system (6) if one of the following conditions is satisfied:*

- (i) $[1 - s(k)]/\sigma(k) - \varepsilon_\psi(k) < 0$ for every $k > 0$;
- (ii) $[1 - s(k)]/\sigma(k) - \varepsilon_\psi(k) > 0$ for every $k > 0$.

Under the calibration of proposition 1, $k = 1$ is then the unique interior stationary solution of (6). In the special case in which s , σ and ε_ψ are constant, either condition (i) or (ii) is satisfied except when $(1 - s)/\sigma = \varepsilon_\psi$, in which case under the calibration of proposition 1 there is a continuum of stationary solutions.

By direct inspection of condition (ii) in proposition 2, it is possible to verify that, as one would expect, the stationary equilibrium is in particular unique when there are no externalities at all ($\varepsilon_\psi(k) \equiv 0$). When conditions (i) and (ii) of proposition 2 are violated, system (6) may conversely possess multiple fixed points⁵. In particular, if we increase ε_ψ continuously, keeping s and σ constant at the persistent stationary solution $k = 1$, when ε_ψ goes through $(1 - s)/\sigma$ we generically get a transcritical bifurcation; accordingly, one would expect a change in stability between two nearby steady states.

We may now reasonably wonder what could be the effects of changing q or γ on the welfare, evaluated at a given steady state, of the representative agent. The rate of growth γ of money supply, in particular, represents a policy parameter which is chosen by the government. In order to study such an issue, for a given q and a given γ , let k^* be a fixed point of (6), i.e. a solution of (7). Let now us increase slightly q or γ so that α becomes higher. By the continuity of $r(k)$, there will generically exist a new stationary solution k^{**} close to k^* which will be higher (lower) if and only if the interest

⁵In particular, it can be shown that under appropriate boundary conditions there exist at most two solutions of equation (7) when the left-hand sides of conditions (i), (ii) in proposition 2 are either increasing or decreasing in $k > 0$. For an in-depth study of the occurrence of multiple fixed points in a similar context and an application to CES economies, see Cazzavillan *et al.* (1998).

rate $r(k)$ is increasing (decreasing) in a neighbourhood of k^* . Observing that the stationary level of consumption is increasing⁶ in k^* , one finds that in this case agents will be better off (worse off) at the new steady state. If we take into account the fact that $r(k)$ is increasing (decreasing) in k in a neighbourhood of k^* if and only if condition (i) ((ii)) of proposition 2 holds at k^* , the following proposition can be easily verified.

Proposition 3 *Let k^* be a stationary solution of system (6) corresponding to a given q and a given γ . A slight increase in q or γ is Pareto-improving (worsening) if and only if condition (i) ((ii)) of proposition 2 holds at k^* .*

Proposition 3 suggests in particular that money is not superneutral and that the two market imperfections, the aggregate externalities and the liquidity constraint, can compensate each other in terms of welfare. However, as soon as the externalities are small, an increase in q or γ is Pareto-worsening and welfare would be maximal for q equal to zero or by setting γ equal to $\beta - 1$ in accordance with the "Chicago rule". Indeed, in both these cases, the inflation tax would not distort the allocation between savings and investment and the economy would attain the stationary solution of the unconstrained model. Conversely, the presence of relevant externalities can reverse such results and a small increase in q or γ can promote investment and therefore welfare.

4 Local stability and bifurcations

In order to study the dynamics of the economy, let us set the vector $Z_t \equiv (k_{t+1}, k_t)^T$ and the map

$$G(Z_t, Z_{t+1}) \equiv \begin{bmatrix} K(k_{t+2}, k_{t+1}, k_t) \\ k_{t+1} - k_t \end{bmatrix}. \quad (8)$$

Since for every $Z^* = (k^*, k^*)^T$, where k^* is a fixed point of equation (6), $G(Z^*, Z^*) = 0$ and the determinant of $D_2G(Z^*, Z^*)$ does not vanish⁷, by the implicit function theorem there exist open neighbourhoods I, I' of Z^*

⁶Indeed $dc^*/dk^* = [\varepsilon_\psi(k^*) + s(k^*)][\alpha/s(k^*)] - \delta > 0$.

⁷ $D_iG(Z_1, Z_2)$, with $i = 1, 2$, denotes the matrix of the derivatives of G in respect to Z_i .

and a C^n map $H : I \rightarrow I'$ such that $G(Z, H(Z)) = 0$ for every Z belonging to I . In addition, and again in view of the implicit function theorem, one has $DH(Z) = -[D_2G(Z, H(Z))]^{-1} D_1G(Z, H(Z))$, where $DH(Z)$ denotes the Jacobian of $H(Z)$. Finally, it is immediately verifiable that Z^* represents a fixed point for H . The dynamics of capital accumulation in a neighbourhood of the steady state can then be described by the difference equation $Z_{t+1} = H(Z_t)$. If the Jacobian $DH(Z^*)$ evaluated at the stationary solution under study does not admit eigenvalues on the unitary circle, the dynamics of system (8) is locally topologically equivalent to that of the linear system $Z'_{t+1} = DH(Z^*) Z'_t$ where $Z' = Z - Z^*$ represents the deviation of Z from its stationary value Z^* . The eigenvalues of $DH(Z^*)$ are given by the roots of the characteristic polynomial $Q(\xi) = \xi^2 - T\xi + D$, where T and D are, respectively, the trace and the determinant of $DH(Z^*)$ and correspond, respectively, to the sum and the product of the characteristic roots. Straightforward computations yield the following expressions for T and D which depend exclusively on the parameters of the model and not on endogenous variables (to simplify notation, in the following we omit the arguments of the elasticities):

$$\begin{cases} T = 1 + D - \Lambda \\ D = \frac{1 - \delta + \alpha(1 + \varepsilon_\psi/s)}{1 + \varepsilon[\beta/(1 + \gamma)][q/(1 + q)](\alpha/s - \delta)} \end{cases} \quad (9)$$

where

$$\Lambda \equiv \frac{\varepsilon\alpha\beta(\alpha/s - \delta)[\varepsilon_\psi - (1 - s)/\sigma]/(1 + q)}{1 + \varepsilon[\beta/(1 + \gamma)][q/(1 + q)](\alpha/s - \delta)} \quad (10)$$

represents the deviation of the point (T, D) from the line $T = 1 + D$. Since the initial condition $Z_0 = (k_1, k_0)^T$ of system (8) contains only one predetermined component, the initial endowment k_0 of capital, a stationary solution Z^* will be locally indeterminate if and only if the dimension of the stable manifold of $DH(Z^*)$ is two, i.e. if and only if both eigenvalues lie inside the unit circle and the stationary solution is stable. In this case, for every initial stock of capital close to the steady state, there would indeed exist a one-dimensional space in which it is possible to collocate the initial condition Z_0 so that $\{Z_t\}_{t=0}^{+\infty}$ converges to the stationary solution and respects the transversality condition (3). As is well known, a bifurcation analysis can provide important additional implications in terms of indeterminacy⁸. For

⁸See Grandmont *et al.* (1998).

example, along a supercritical Hopf bifurcation there exists generically a subset including a limit cycle arising around the unstable steady state so that any trajectory starting nearby will converge to the cycle, so that equilibrium will be indeterminate. Similarly, in correspondence to a supercritical flip bifurcation along which the steady state from sink becomes saddle, a stable two-period cycle will generically arise around the determinate steady state; again, any trajectory starting near the cycle will converge to it⁹.

The simple form of expressions (9) and (10) for the trace and the determinant makes it easy to study qualitatively the stability of the characteristic roots and their bifurcations with the help of a simple geometrical methodology¹⁰ consisting in locating the point (T, D) in the plane and studying how it varies when some parameter changes continuously.

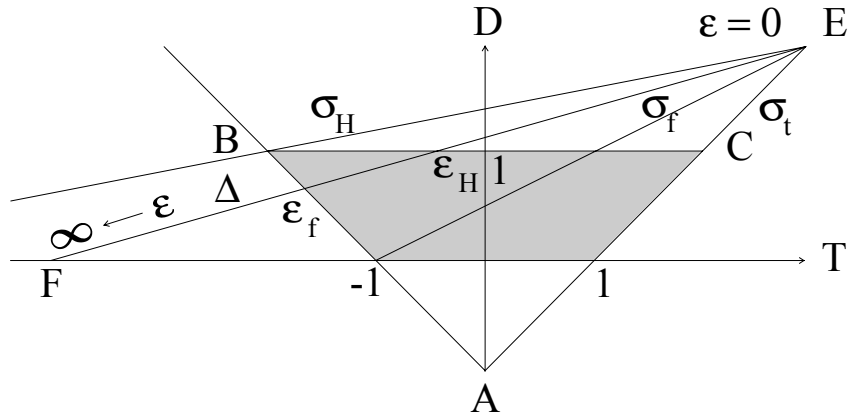


Figure 1. Local dynamics and bifurcations.

In particular, if T and D lie in the interior of the triangle ABC depicted in figure 1, the stationary solution is a sink (locally indeterminate), otherwise it is locally determinate; either saddle when $|T| > |1 + D|$, or source. In addition, when the point (T, D) crosses either the interior of the segment BC , or the line $T = -1 - D$ or the line $T = 1 + D$, one generically gets, respectively, a Hopf, a flip and a transcritical bifurcation. Our program now consists, for each $\sigma \in (0, +\infty)$ and with the other parameters of the model

⁹In order to specify the direction of bifurcations, one should dispose of additional information concerning higher order derivatives of the map and this would require some specification for the fundamentals.

¹⁰See, e.g., Grandmont *et al.* (1998) and Cazzavillan *et al.* (1998).

(i.e. β , δ , s , q , γ and ε_ψ) fixed, in locating in the plane the trace and determinant when the elasticity ε of intertemporal substitution in consumption changes continuously. From expressions (9) and (10) one readily verifies that T and D are fractions of first degree polynomials in ε with the same denominator. When ε varies from zero to infinite one then obtains a parametrized curve $\{(T(\varepsilon), D(\varepsilon))\}$ describing part of a half-line Δ starting, when $\varepsilon = 0$, from the point E which lies on the line $T = 1 + D$ and has ordinate $D = 1 - \delta + \alpha(1 + \varepsilon_\psi/s) > 1$ (which does not depend upon σ). Since D is decreasing in ε and tends to zero as ε diverges to infinite, the point (T, D) moves downward along Δ as ε increases continuously and finally, when ε diverges to infinite, approaches the point F which lies on the horizontal axis at $T = 1 - \alpha(1 + \gamma)[\varepsilon_\psi - (1 - s)/\sigma]/q$. The slope of the half-line Δ is given by

$$S_\Delta = \left[1 + \frac{\alpha(1 + \gamma)(\varepsilon_\psi - (1 - s)/\sigma)}{q[1 - \delta + \alpha(1 + \varepsilon_\psi/s)]} \right]^{-1}, \quad (11)$$

it is one when $\sigma = \sigma_t$, where

$$\sigma_t \equiv (1 - s)/\varepsilon_\psi, \quad (12)$$

and tends to $\left(1 + \frac{\alpha(1 + \gamma)\varepsilon_\psi}{q[1 - \delta + \alpha(1 + \varepsilon_\psi/s)]}\right)^{-1} \in (0, 1)$ when $\sigma \rightarrow +\infty$. From (10) and (11), it is easily seen that when σ goes through σ_t (or, if we fix σ , when ε_ψ goes through $(1 - s)/\sigma$), the point (T, D) crosses the line $T = 1 + D$ and, as we have already mentioned, a transcritical bifurcation generically occurs.

With the help of figure 1, one may now readily verify that local indeterminacy is possible (i.e. the curve $\{(T(\varepsilon), D(\varepsilon))\}$ intersects the interior of the triangle ABC) only if the slope S_Δ of the half-line Δ is positive and lower than one (i.e. only if Λ is positive). In view of (10), this requires $\sigma > \sigma_t$ which, in the light of (12), can be satisfied if and only if there are externalities ($\varepsilon_\psi > 0$). An important piece of information consists in locating the half-line Δ when $\sigma = +\infty$; indeed, if it intersects the interior of the triangle ABC , since S_Δ is monotonically decreasing in σ (the half-line Δ rotates in a clockwise sense with σ around the point E), it will intersect it (and indeterminacy will occur) for every $\sigma > \sigma_t$, otherwise for all $\sigma_t < \sigma < \sigma_H$, where σ_H is such that Δ goes through the point $B = (-2, 1)$, as is shown in figure 1. Straightforward computation yields

$$\sigma_H = (1 - s) / \left[\varepsilon_\psi - \frac{4q}{\alpha(1 + \gamma)} \left(1 + \frac{1}{\alpha(1 + \varepsilon_\psi/s) - \delta} \right) \right] \quad (13)$$

and therefore the existence of such σ_H depends crucially on the structural parameters, as we will analyze later in more depth¹¹. It is also immediately verifiable that indeterminacy arises always through a Hopf bifurcation, when ε goes through $\varepsilon = \varepsilon_H$ whose expression is

$$\varepsilon_H = \frac{1 + \gamma}{\beta} \frac{1 + q}{q} \frac{\alpha(1 + \varepsilon_\psi/s) - \delta}{\alpha/s - \delta}. \quad (14)$$

A further important step for the purpose of our analysis consists in verifying whether the point F lies to the left of $(-1, 0)$ when $\sigma = +\infty$. In the affirmative case, there will exist σ_f in correspondence to which the half-line Δ goes through $(-1, 0)$ and therefore for all $\sigma > \sigma_f$ it will intersect the line $T = -D - 1$ at $\varepsilon = \varepsilon_f$ and a flip bifurcation will occur¹². The corresponding expression of σ_f is

$$\sigma_f = (1 - s) / \left[\varepsilon_\psi - \frac{2q}{\alpha(1 + \gamma)} \right] \quad (15)$$

and that of ε_f

$$\varepsilon_f = \frac{2(1 + q)[2 + \alpha(1 + \varepsilon_\psi/s) - \delta]}{\beta(\alpha/s - \delta) \{ \alpha[\varepsilon_\psi - (1 - s)/\sigma] - 2q/(1 + \gamma) \}}. \quad (16)$$

In view of the above considerations and again with the help of figure 1, one may now easily verify the following proposition which fully characterizes the local dynamics and bifurcations of system (6). Notice that we consider the general case in which σ_H (and therefore σ_f) exists. If only σ_f existed, regime (iv) would be ruled out and σ_H would be replaced by $+\infty$. If also σ_f did not exist, it would be replaced by $+\infty$ and only regimes (i) and (ii) would actually be operating.

Proposition 4 (*Local dynamics and bifurcations*). *According to the magnitude of σ , there may be four relevant regimes for the local dynamics and bifurcations of system (6).*

- (i) $0 < \sigma < \sigma_t$. *The steady state is a saddle (locally determinate).*

¹¹Notice that σ_H exists when its expression given in (13) is positive.

¹²Also the existence of σ_f depends on the structural parameters and corresponds to a positive value of its expression (15). In particular, σ_f exists when σ_H does. For more details, see below.

(ii) $\sigma_t < \sigma \leq \sigma_f$. The steady state is a source (locally determinate) when $0 < \varepsilon < \varepsilon_H$ and a sink (locally indeterminate) when $\varepsilon > \varepsilon_H$. A Hopf bifurcation occurs at $\varepsilon = \varepsilon_H$.

(iii) $\sigma_f < \sigma < \sigma_H$. The steady state is a source (locally determinate) when $0 < \varepsilon < \varepsilon_H$, a sink (locally indeterminate) when $\varepsilon_H < \varepsilon < \varepsilon_f$ and a saddle (locally determinate) when $\varepsilon > \varepsilon_f$. A Hopf bifurcation occurs at $\varepsilon = \varepsilon_H$ and a flip bifurcation at $\varepsilon = \varepsilon_f$.

(iv) $\sigma \geq \sigma_H$. The steady state is a source (locally determinate) when $0 < \varepsilon < \varepsilon_f$ and a saddle (locally determinate) when $\varepsilon > \varepsilon_f$. A flip bifurcation occurs at $\varepsilon = \varepsilon_f$.

It is clear then that local indeterminacy occurs in correspondence to important substitution effects in intertemporal consumption (namely high ε), as is also confirmed below in our numerical example. This result is somewhat original once one recognizes that under alternative specifications of the financial constraint, for example under cash-in-advance constraint on consumption purchases, the scope for indeterminacy requires sufficiently high income effects¹³.

Following Grandmont *et al.* (1998), it would be possible to prove the existence of stochastic sunspot equilibria near an indeterminate steady state as well as along supercritical Hopf and flip bifurcations¹⁴. Indeed, system (8) shares all the relevant features with the one studied in Grandmont *et al.* (1998), namely the two-dimensional setting and the presence of exactly one pre-determined variable (initial capital) in each period.

In view of expressions (14), (16), one may improve the characterization of local indeterminacy by studying how the range of the elasticity ε of intertemporal substitution in consumption (which captures preferences) generating local indeterminacy is modified when either one of the technological parameters σ , ε_ψ or one of the financial ones q , γ , changes continuously. In order to facilitate this aim, we refer to figures 2 – 5 which, although corresponding to a specific calibration, are rather robust from a qualitative as well as quantitative point of view and in which the shaded area represents the indeterminacy region. Let us start by progressively increasing σ . When σ goes through σ_t , it gives rise to a transcritical bifurcation. Then, as soon as σ belongs to

¹³See Bloise *et al.* (1999).

¹⁴In the case of supercritical Hopf and flip bifurcations, stochastic sunspot equilibria remain in a compact set containing in its interior, respectively, the stable invariant curve and the stable two-period cycle.

(σ_t, σ_f) , the range for local indeterminacy corresponds to $\varepsilon > \varepsilon_H$ and does not undergo any modification, since ε_H does not depend upon σ . When conversely σ increases over σ_f , indeterminacy occurs in correspondence to the open interval $(\varepsilon_H, \varepsilon_f)$ which shrinks monotonically and finally, whenever σ_H exists, vanishes (figure 2 refers to the case in which neither σ_H nor σ_f exists). Indeed, from a simple inspection of (16), it is possible to see that when ε_f is positive, it is decreasing in σ and eventually becomes lower than ε_H , so ruling out indeterminacy. Let us now conversely fix σ and increase ε_ψ from $(1-s)/\sigma$ (in correspondence to which a transcritical bifurcation occurs) to infinite. The scope for indeterminacy, for values of ε_ψ close to $(1-s)/\sigma$, requires $\varepsilon > \varepsilon_H$ (see figure 3) and successively $\varepsilon \in (\varepsilon_H, \varepsilon_f)$. Since, from some ε_ψ on, ε_H and ε_f , respectively, increase and decrease monotonically with ε_ψ , the interval $(\varepsilon_H, \varepsilon_f)$ shrinks monotonically and finally vanishes when ε_ψ is high enough. Conversely, when q increases from zero to infinite, the range of ε generating indeterminacy depends crucially upon the graph of ε_f which can assume two shapes, according to the magnitude of the externalities (meanwhile ε_H decreases monotonically from $+\infty$ ($q \rightarrow 0^+$) to some finite value ($q \rightarrow +\infty$)). As a matter of fact, when externalities are relatively low, the range generating indeterminacy is empty when q is very low, coincides with an increasing interval $(\varepsilon_H, \varepsilon_f)$ for moderate values of q , and corresponds finally to $\varepsilon > \varepsilon_H$ (figure 4 refers substantially to this case). If externalities are conversely relatively important, both ε_H and ε_f are decreasing in q and intersect twice so that indeterminacy occurs in correspondence to q belonging to a bounded open interval. Finally, the effects on $(\varepsilon_H, \varepsilon_f)$ observed when γ moves away from $\beta - 1$ (figure 5) are similar to those generated by increasing ε_ψ . At least for σ not very larger than σ_t , indeterminacy indeed occurs first for all $\varepsilon > \varepsilon_H$ and successively, as γ becomes larger, in correspondence to the interval $(\varepsilon_H, \varepsilon_f)$ which shrinks continuously and finally vanishes for some finite, albeit very high, γ .

As is well known, the occurrence of indeterminacy and sunspot fluctuations may call for the implementation of stabilization policies in order to remove the associated losses in welfare. As we have seen above, indeterminacy could be made less likely to occur by simply setting the money growth rate γ sufficiently high. This, as is shown in the previous section, may have the additional positive effect of improving agents' welfare at the steady state. However, a significant reduction of the possibility of indeterminacy would require, as is suggested by our sensitivity analysis (see figure 5), implausible high monetary rules. Conversely, an income tax exhibiting a progressive fea-

ture and sufficiently elastic to contrast externalities and ensure a return on physical investment decreasing in capital intensity would be not merely more reliable but would also rule out indeterminacy completely .

4.1 A numerical example

We now provide a numerical example in order to test the robustness of the results obtained and to show that indeterminacy occurs in correspondence to a wide range for the relevant parameters. Throughout we chose the standard values $\beta = 0.95$, $\delta = 0.1$ and $s = 0.33$. The values of the other parameters, when they are not made to vary, correspond¹⁵ to $\sigma = 3$, $\varepsilon_\psi = 0.3$, $q = 0.2$ and $\gamma = 0.03$. In figure 2 we have represented the range of ε generating indeterminacy in correspondence to each $\sigma > \sigma_t \approx 2.23$. Since ε_H does not depend upon σ and under the adopted calibration σ_f does not exist, local indeterminacy occurs in correspondence to all $\sigma > \sigma_t$ and $\varepsilon > \varepsilon_H \approx 3.4$. In figure 3 we have depicted the indeterminacy region as a correspondence of ε_ψ and it is possible to see that indeterminacy occurs for a whole range including low (and high) values of ε_ψ (approximately between 0.22 and 2.5) for all $\varepsilon > \varepsilon_H$. Subsequently it occurs in correspondence to the interval $(\varepsilon_H, \varepsilon_f)$ which shrinks continuously and vanishes when $\varepsilon_\psi \approx 6.3$. Figure 4 shows the modification undergone by the interval $(\varepsilon_H, \varepsilon_f)$ generating indeterminacy when q increases from zero to infinite. In particular, it is possible to see that for q greater than 0.006 indeterminacy emerges for all $\varepsilon > \varepsilon_H$. Finally, figure 5 illustrates the link between γ and indeterminacy. One easily sees that in correspondence to a rate of money growth between $\beta - 1 = -0.05$ and approximately 13.5 indeterminacy occurs for all $\varepsilon > \varepsilon_H$. Subsequently, it occurs for $\varepsilon \in (\varepsilon_H, \varepsilon_f)$ up to $\gamma \approx 77$. Finally, when γ becomes even larger,

¹⁵It is worth noticing that we could obtain analogous features even if we set externalities smaller.

the phenomenon vanishes completely.

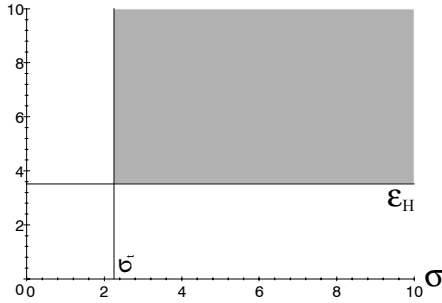


Figure 2 : $(\varepsilon_H, \varepsilon_f)(\sigma)$.

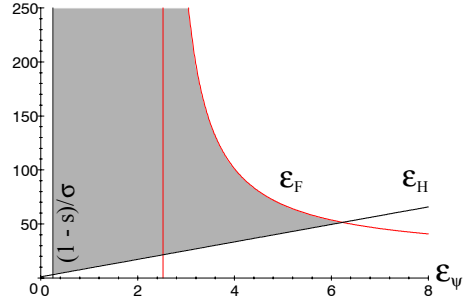


Figure 3 : $(\varepsilon_H, \varepsilon_f)(\varepsilon_\psi)$.

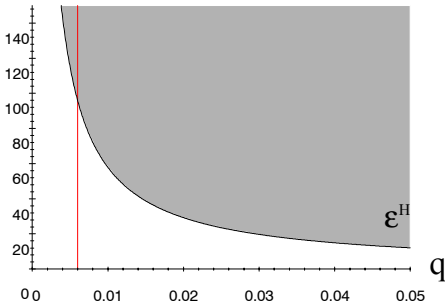


Figure 4 : $(\varepsilon_H, \varepsilon_f)(q)$.

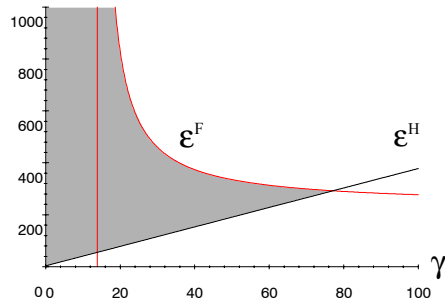


Figure 5 : $(\varepsilon_H, \varepsilon_f)(\gamma)$.

5 Conclusion

In this paper we have presented a model in which at the end of each period agents must hold a share of their wealth in the form of money, and technology exhibits increasing returns to scale. In contrast to other contributions on this subject, we have shown that local indeterminacy, sunspot equilibria, deterministic periodic and quasi-periodic orbits may emerge when consumption is intertemporally substitutable and labour is supplied inelastically. Indeterminacy and sunspot equilibria require in particular a marginal productivity of capital slightly increasing in capital intensity and an amplitude of the liquidity constraint not very close to zero. Stabilization policies in order to reduce sunspot fluctuations can be implemented by an expansive monetary policy or, more reliably, by an income tax exhibiting a progressive feature. The contextual presence of increasing returns to scale and liquidity constraint also entails important consequences in terms of welfare properties of the steady

states. In particular, when the steady state is indeterminate a higher rate of money creation can be (locally) Pareto-improving. Compared to alternative frameworks, our results suggest that the qualitative features of an economy depend crucially on the specification of the liquidity constraints and the way they interact with other market imperfections, such as externalities.

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