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Educating Children: A Look at Household Behavior in Côte d'Ivoire.

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Abstract

We consider a dynamic model of investment in human capital where one decision maker (the head of the household) builds up human capital stocks of his/her children. In a stochastic framework, tradeoffs between investments in (physical) assets and investments in human capital achieved by different children within the family are modelled as resulting from risk diversification and differing opportunity costs of investments. We show how to specify the structural model as a system of simultaneous discrete choice equations so that it is estimable using panel or cross section data on children' school attendance. We estimate structural parameters using data from the Côte d'Ivoire in 1985/1986 using standard and simulated maximum likelihood techniques. Results indicate that returns to learning by doing are much larger than returns to school at least at young ages, that the estimated intertemporal substitution elasticity is quite large and that the elasticity of substitution between financial and human capital assets is negative.

Keywords: Intertemporal choices, human capital formation, discrete simultaneous equations, simulated ML, economic development.

JEL Classification: D91, J24, I21, C35, C15, 015

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1. Introduction[1]

In less developed countries, demand for education seems to respond elastically to economic conditions (Schultz, 1988, Alessie, Baker, Blundell, Heady and Meghir, 1992, Jacoby and Skoufias, 1997). The design of public policies for education is then particularly important and the choice of instruments quite diversified: allocating subsidies to primary or secondary schools, to vocational or formal schooling, regulating demand by different kinds of incentives like fees or grants, or other regulating policies like in many developed countries, where schooling is compulsory until a certain age. Indirect policies such as better access to credit and insurance markets can also be recommended (Jacoby and Skoufias, 1997). Policy design should rely on an evaluation of the social value of education including externalities, the engine for endogenous growth. It should also rely on careful studies about the way economic agents invest in human capital in order to understand the determinants of such investments. This paper deals with households' demand for education, focussing on the issue of the allocation of investments in financial assets and in human capital among children within the family.

The topic of individual demands for education has been largely explored in the human capital literature where individual private investments are modelled as the result of comparing individual costs and benefits (Becker, 1975). It seems however worthwhile to consider that human capital investments are decisions taken at the level of the household, at least as far as primary or secondary education are concerned. Apart from the impact of child's characteristics (age, health...) on education demand, it is important to understand whether and how the level of human capital of a child is influenced by that of his/her siblings and by household characteristics,

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such as education of the parents and household wealth. The main line of arguments concerning this point was developed by Becker and Tomes (1976) and Berhman, Pollack and Taubman (1982) in a static framework. It first states that the allocation of human capital investments in the household can be thought of as obeying the rule of individual comparative advantage for education and that in such a case, completed levels of schooling can be unequal. Although education is regarded as a family investment, another conclusion of these models is that human capital investments of the children of the family will be determined independently from each other, as long as the household does not face credit constraints. Investments in human capital are undertaken until the rate of return is equal to the interest rate.

It seems to us that this static model is not best suited to study education demand in developing countries. First, the static framework is difficult to use in applied analysis because it requires for instance, for the child under study and for all his/her siblings, the knowledge of completed levels of schooling and, at the same time, of determining variables from when the child was young, variables which often are unobserved. Second, a static model cannot describe changes in the technology or in the environment that could take place during the household's life-cycle. Returns to human capital investments are uncertain and liquidity constraints can bind at some periods. With uncertainty and dynamics, the separation between different types of investments that holds in the static model, will only hold if markets are complete, an assumption which seems to be rejected in many instances in less developed countries (see, in the case of education investments and for other references to this quickly growing literature, Jacoby and Skoufias, 1997 and in the case of Côte d'Ivoire, Deaton, 1992 and Grimard, 1997). Finally, this static model cannot account for some stylized facts commonly observed in less developed countries, and in particular in Côte d'Ivoire, such as delayed school enrollment and frequent cases of children repeating classes (Jacoby, 1994, Bommier and Lambert, 1998).

In this paper, a stochastic dynamic model is developed to describe the allocation, at each period, of human capital investments within the household. We extend the usual framework of the human capital literature to the case of one decision maker (the household's head) and several recipients (the children). It is akin to a portfolio choice model and explicitly accounts for possible trade-offs between investments in financial assets and human capital stocks of different children. The household diversifies risk as in Kotlikoff and Spivak (1981) or Rosenzweig (1988), because insurance and/or financial markets are incomplete. Three types of assets are introduced in the model: financial assets, general human capital acquired through schooling, and specific human capital, such as skills in family self-employed activities (farming or working in the informal sector). The distinction between these two types of activities permits to explain short term tradeoffs for children between working and going to school in the case where unexpected income shocks occur as shown in Alessie *et al.* (1992). In our model, this tradeoff is dynamic and is used as an argument to explain delays in school enrollment, as in Bommier and Lambert (1998), because children wages or implicit wages depend on these two types of investments. This is a quite different view from the one developed in Jacoby (1994), where delays are explained by the existence of liquidity constraints and from Glewwe and Jacoby (1995), where the rôle of early childhood malnutrition is put forward. The model we develop also differs somewhat from Jacoby and Skoufias (1997) although we believe superficially. It is mainly a matter of definition of what human capital is – the expected capitalized value of wages in their setting and the number of years of schooling in ours. The difference is in the motivation for their interest is to test for the completeness of markets, while we try to estimate structural parameters such as the intertemporal substitution elasticity and the elasticity of substitution between financial assets and human capital, under the alternative assumption that markets are incomplete.

The empirical analysis is guided by this structural model. The econometric model is developed in order to follow the economic model as closely as possible, so that a structural interpretation can be given to the estimated coefficients. Using a particular specification for preferences, we are able to derive structural equations that are estimable using panel and cross section data. We use data from a short rolling panel survey from Côte d'Ivoire in 1985 and 1986 that is one of the Living Standard Surveys conducted around the world with the help of the World Bank. The only reliable information that we can use on current human capital investments is whether children go to school or not. In the case of panel data, the structural model describing school attendance that is estimated is a system of standard discrete choice equations. In the case of cross section data, the estimated structural model is a system of simultaneous discrete choice equations, where right hand side variables include truncated latent variables. This model is an extension to discrete/continuous models of consumer demand (Hanemann, 1984). Standard Probit and Simulated Maximum Likelihood methods are used to estimate the model using panel and cross section data respectively. We also pay particular attention to the endogeneity issues that are treated by adapting instrumental variables methods to our setting.

The structural equations describe the within period tradeoffs of the family head between consuming and investing in human capital stocks of the children. Although these equations depend on dynamic tradeoffs, they do not depend on the specification of the Euler equation for consumption. In this sense, they are robust to liquidity constraints that households might experience. The price we pay is that the intertemporal elasticity of substitution is not identified, or more precisely only a lower bound for this parameter can be identified in some cases. Bounds for the elasticity of substitution between financial and human capital assets can also be obtained. The main results indicate that returns to specific human capital are higher than to general human capital, that the intertemporal substitution elasticity is large and that the elasticity of substitution between financial and human capital assets is significantly different from zero.

Section 2 briefly describes the educational system in Cote d'Ivoire and the most important stylized facts appearing in the data. Section 3 builds up the structural model which leads to its estimable counterpart presented in section 4. Estimation methods are developed in section 5 and results are given in section 6. Section 7 concludes.

2. An informal look at the data.

Our study, as well as a number of other studies on the Côte d'Ivoire, is based on the Côte d'Ivoire Living Standard Survey (CILSS). This is a nation-wide survey of about 1600 households (15000 people), located in 100 clusters, that has been conducted by the Direction de la Statistique in Côte d'Ivoire in collaboration with the World Bank. Each round of the survey covers a 12 months period. The CILSS provides detailed information about the schooling of each household member, as well as data on sources of household income, expenditures and assets holdings (Ainsworth and Muñoz, 1986). Moreover, for rural clusters, the household questionnaire is supplemented with a community questionnaire that, in particular, provides information on the supply of schools. In this study we used two rounds of this survey: 1985 and 1986.

Over the last 25 years, the average level of education in Côte d'Ivoire has notably increased (see, for instance, Vijverberg 1993 or World Bank 1996). Nevertheless the average education level remains low, at least by developed countries standards, and marked differences can be noticed between sexes and regions of residence (boys and children living in urban areas receiving much more education than girls and children living in rural areas), thus showing that much progress remains to be accomplished for large segments of the population.

In Côte d'Ivoire school starts normally at five. Children stay in primary school for a total of six years then move to junior secondary school (4 years) and to upper secondary school (3 years). Entrance to junior secondary school is permitted upon success to the "Certificat d'Etudes Primaires" examination and access to upper-secondary school is controlled by the "Brevet d'Etudes du Premier Cycle". At the end of secondary school, each student has to take the "Baccalauréat" examination before going to university.

As can be shown in figure 1, a large number of children delay their entry into school. Even though the normal age for first enrollment in school is seven, in the data only about 60% of the children enter school at this age or earlier and by thirteen only 72% have ever gone to school. Most children enroll in school between 5 and 7, with attendance peaking at 10 years of age. Numbers of completed years of education are low when children are leaving school. Using CILSS, De Vreyer (1996) estimates demand for education in Côte d'Ivoire using an ordered probit on the completed years of education. According to his results, at birth, the average boy can expect to complete about 6.5 years of education, but this falls to only 4.6 years of education for the average girl.

Another fact that characterizes demand for education in the Côte d'Ivoire is the inequality that can be observed, within families, in the school attendance of children. This can be seen in figure 2 which shows, for our sample, the distribution of families according to the proportion of their children of schooling age, that are currently or have been going to school. As can be seen from this figure, about 16% of the sampled families do not send any of their children to school[2], whereas in 25% of them all children have received some education. This leaves us with about 60% of the families in which *some* children, but not all, are going or have been going to school. It is clear that delayed primary school enrollment can explain part of this result. However this proportion is still as high as 46%, if one reduces the sample to children between 10 and 25 years old.

Finally, one can also notice the large proportion of children that are repeating classes. The CILSS survey provides repeated observations in two consecutive years on a sub-sample of children. Among them a total of 347 children were going to school in 1985 and 1986. On this total 121, that is 34.9%, have repeated their class.

Thus delayed primary school enrolment, inequalities within families in the educational investment of children and frequent repetition of classes characterize the demand for education in Côte d'Ivoire and are part of the explanation to the low levels of education that are observed in this country. Our goal is to provide and test a model that can explain these facts.

One of the reasons why demand for education remains low in developing countries

[2] On a total of 1098 families with at least two children of schooling age.

is the lack of schools. However this is probably not the only explanation in Côte d'Ivoire. In our data, there is a primary school in about 90% of the communities that have been surveyed and the average distance to a primary school is less than 500 meters[3]. Secondary schools however are much farther. In rural areas, secondary schools are located about 25 kilometers away on average. Lavy (1992), using data from Ghana, finds results suggesting that the distance to secondary schools is significantly detrimental to the attendance to primary schools. This result makes sense in Ghana, where the returns to primary schools in terms of cognitive achievement are found extremely low (Glewwe 1992). Parents have then a strong incentive to maintain their children at school at least until junior secondary school is completed, and attendance to primary school should be responsive to the supply constraints on secondary schools. In Côte d'Ivoire however, Van der Gaag and Vijverberg (1987) and Vijverberg (1991, 1993) have found positive returns to primary education on the labour market, so that supply constraints on secondary schools should have only a minor impact on primary school attendance.

There remains the possibility that even with primary schools located in the community, the number of places in the schools is not sufficient to satisfy the demand. In such a case it is likely that children would have to wait to be accepted in schools and this could explain the high incidence of delayed primary school enrollment. In rural areas the CILSS community questionnaire asked what was the main schooling problem in the village. Only about 15% of the respondent answered that is was the lack of space in schools. Moreover, our data shows that about one fifth of the children between 5 and 18 that had been going to school but were not currently going in 1985 intended to return to school in the future. This is not the kind of behaviour one would expect if the number of places in schools were rationed. If it were the case indeed, once a child has been accepted in school, there would be a high incentive to maintain him/her there, at least until the primary cycle has been completed.

[3] The farthest primary school is located at 8 kilometers.

3. The theoretical framework.

In this section, a stochastic dynamic model of family human capital investment decisions is set up. Consider a family with n children of schooling age. Each child shares his/her available time between school and work. While at school, children acquire a "general" human capital that is to be distinguished from the human capital acquired through participation **in** family's activities. The former type of human capital will thereafter also be termed "education" while the latter may be called "specific human capital" or "work experience". Both types of human capital, imperfectly substitutable, increase children' productivity on the labour market. Schultz (1975) and later Rosenzweig and Wolpin (1985) and Rosenzweig (1988) have emphasized the importance of specific experience in the investment decisions and labour arrangements of farm households. Tradeoffs between working and going to school for children in LDCs is an argument that is used to explain temporary withdrawal from school when there are adverse income shocks as demonstrated in Alessie et al (1992). It is the argument that we put forward to explain why children are delayed in their entry into school. If returns to experience are higher than returns to formal education, it is optimal to wait until children have been trained in family activities. Returns to experience are likely to decrease as learning goes by and then it becomes profitable for children to acquire formal education. This insight is made precise by the theoretical framework.

The household's head decides about smoothing the consumption of family members in a dynastic way. It is achieved by accumulating or decumulating financial assets and by accumulating human capital stocks of the children. Consumption smoothing depends on the distributions of future returns to all assets. We first deal with the equations describing the accumulation of assets and then turn to the decision program.

3.1. Stocks accumulation equations.

Time spent at school is the unique input of the general human capital production function. During period t, child i dedicates a share e_t^i of his/her non-leisure time to schooling. The remaining time, $1 - e_t^i$, is devoted to working. At date $t + 1$, human capital stock, S_{t+1}^i, is supposed to be given by:

$$S_{t+1}^i = S_t^i + e_t^i$$

where we assume away any non linearity in the accumulation of human capital as well as any random element. Linearity in the accumulation equation is one point where we depart from the framework of Jacoby and Skoufias (1997). As the wage equation is constant in their setting and not in ours however, we do not believe that the two approaches are inconsistent since they mainly differ in the implicit definition of what human capital is – the expected annualized value of wages in their setting and the number of years of education in ours.

In the same way, work experience at date $t + 1$, L_{t+1}^i, is given by the following equation:

$$L_{t+1}^i = L_t^i + 1 - e_t^i$$

Writing the law of motion of the different human capital stocks as above supposes that each unit of time dedicated to schooling or to work is associated to a fixed amount of leisure. Leisure decisions are not modelled here. This hypothesis implies that education and work experience are related by the following constraint:

$$S_t^i + L_t^i = \tau_t^i \tag{3.1}$$

where τ_t^i is child's age at date t. Experience can therefore be measured by the difference between age and the number of years of educational attainment.

When working, a child i receives a wage that depends on education, experience and ability (z^i):

$$\overline{\omega}(S_t^i, L_t^i, z^i)$$

The wage function $\overline{\omega}$ is supposed to be non decreasing in its arguments. It is a measure of the child's productivity, both in family activities and on the labour market. For what follows, it is convenient to express the wage as a function of age rather than experience. We obtain this by reporting equation (3.1) in the wage function, which leads to:

$$\overline{\omega}(S_t^i, L_t^i, z^i) = \overline{\omega}(S_t^i, \tau_t^i - S_t^i, z^i) = \omega(S_t^i, \tau_t^i, z^i) \tag{3.2}$$

Denoting parents' income y_t^0, family's income at period t can then be written as:

$$y_t = y_t^0 + \sum_{i=1}^{n} \omega(S_t^i, \tau_t^i, z^i)(1 - e_t^i)$$

In period t, household's consumption is equal to c_t. Denote A_t the household's stock of physical assets at the beginning of this period. Interest rates are given by $r_t(A_t)$, where $r_t()$ is a decreasing and convex function for negative levels of assets so as to capture liquidity constraints. The family's budget constraint can then be written as:

$$A_{t+1} = (1 + r_t(A_t)).A_t + y_t - c_t$$

We assume that households face uncertain returns for the different types of assets they can invest in. Wages of all members and the interest rate function are subject to random shocks at each date t. Uncertainty is resolved at the beginning of each period, so that at time t households know the interest rate function $r_t(A_t)$, wages ω_t^i and parental income y_t^0.

3.2. Household's decision programme.

The household maximizes the expected present value of the sum of each period utility and a terminal value function:

$$E_0(\sum_{t=0}^{T} \phi^t.U(c_t) + \phi^{T+1} V_{T+1}(A_{T+1}, S_{T+1}^1, ., S_{T+1}^n))$$

The within period utility (U) is assumed to depend only on the level of aggregate consumption c_t. E_t is the expectation operator at date t when expectations rely on the information set available at date t. ϕ is a discount factor and T is the time horizon. It could either correspond to the life expectancy of the household's head, or to the end of the human capital investment period, or to any other point arbitrarily chosen in the future. The way the household's head values human capital stocks of his/her children is given by the value function at date $T+1$: $V_{T+1}(A_{T+1}, S^1_{T+1}, .., S^n_{T+1})$. This transversality condition is left unspecified[4] and no anonymity restrictions on children' identities are imposed. Each human capital stock might not be given the same importance and differences can indeed reflect preferences for a given child or for a given group of children or specialization (e.g. according to their gender or rank).

Households have to decide on aggregate consumption c_t and on investments in human capital $(e_t)_{i=1,...,n}$, that they want to achieve at each future period $t = 1, ..., T$. The state variables are the stocks already accumulated: A_t, $(S^i_t)_{i=1,...,n}$. They summarize past actions and determine what can be undertaken in the present. Note that only state variables related to the stocks of physical assets and of education are considered because at a given date, children' age being fixed, experience is uniquely determined as a function of education.

Denote $V_t(A_t, S^1_t, ..., S^n_t)$ the value function at time t. According to the Bellman principle, this sequence of functions verifies the following property:

$$\begin{array}{rrl} V_t(A_t, S^1_t, ..., S^n_t) = & \max\limits_{\{c_t, e^1_t, ..., e^n_t\}} & \left\{U(c_t) + \phi.E_t V_{t+1}(A_{t+1}, S^1_{t+1}, ..., S^n_{t+1})\right\} \\ & subject\ to & A_{t+1} = A_t.(1 + r(A_t)) + y_t - c_t \\ & & y_t = y^0_t + \sum\limits_{i=1}^{n} \omega(S^i_t, \tau^i_t, z^i)(1 - e^i_t) \\ & & S^i_{t+1} = S^i_t + e^i_t \qquad \forall i \in \{1, ..., n\} \\ & & e^i_t \in [0,1] \qquad \forall i \in \{1, ..., n\} \end{array} \tag{3.3}$$

This program captures the most important arguments that we put forward to explain family aspects of human capital investments. First, schooling is a family's investment and it does not directly generate utility for the members of the family. It clearly is an important identifying assumption. Children' consumption is included in aggregate consumption c_t and is therefore treated by the parents as being equally valuable **as** their own consumption. Other forms of altruism can be introduced in the model through the transversality condition, V_{T+1}, that expresses the value of

[4] Only in very peculiar cases can the planning horizon be known (see *e.g.* Rust 1995). In the general case, the tranversality condition can be left unspecified and we treat it as a function to estimate in the empirical part.

capital stocks at date $T+1$. Second, the diversification of risks between assets is taken into account through the future value function. For instance, a simple portfolio argument implies that, if this function belongs to the CARA family, there should be a trade-off between mean returns and variances in the levels of the different assets detained. As a result, investment in one child's human capital can depend upon other family's investments, including human capital of siblings and physical assets. Third, the model captures the trade-off between schooling and working, which we think partly commands human capital investments in less developed countries.

In order to be able to characterize optimal decisions, assumptions on the primitives of the problem are imposed so as to make it globally concave. It is summed up by the following proposition.

Proposition 3.1. *Assume that the utility function is strictly concave and twice differentiable and that, at each period t, $t \leq T$, the value function V_t is strictly concave and twice differentiable and some additional technical assumptions stated in the proof. Then there is a unique solution to the decision program. The consumption function and the investment functions are differentiable with respect to the stocks of physical and human capital, except on points located on borders between regimes, defined according to whether constraints on human capital investment are binding or not ($e_t^i = 0$ or 1) . The Lebesgue measure of this set is zero.*

Proof. See appendix.

The assumption that the value functions at any period are concave and twice differentiable could be justified by "deeper" assumptions on the tranversality condition and other primitives of the problem. However, because the Euler equations of this dynamic problem are non linear, the proof of such a result is tedious. Since the empirical application presented in this paper concentrates on the estimation of such a model on short panels and cross-sections, we shall not pursue this route here.

3.3. First order conditions.

The previous proposition ensures that first order conditions are necessary and sufficient to characterize optimal decisions. However, corner solutions, no education or no work, are likely to occur. We start by studying interior solutions for which interpretation is easier and then characterize corner solutions.

3.3.1. Interior solutions.

For simplicity sake, the derivatives of the various functions will be denoted with indices that correspond to the variable relative to which they are taken. For example, $V_{A_{t+1}} = \frac{\partial V_{t+1}}{\partial A_{t+1}}$. When all solutions are interior ($i.e.$ $\forall i, e_t^i \in]0,1[$), the first order conditions are:

$$U_{c_t} = \phi . EV_{A_{t+1}} \tag{3.4}$$

$$\frac{EV_{S_{t+1}^i}}{EV_{A_{t+1}}} = \omega_t^i \quad \forall i \in \{1, ..., n\} \tag{3.5}$$

The first condition expresses the trade-off between consumption and savings. The marginal utility of consumption is equal to the discounted expected marginal value of assets, i.e. the future shadow price of assets. The second condition reads as the trade-off between investing in general human capital and working. The wage is equal to the return of one unit of investment in general human capital relative to that of one unit of investment in physical assets: $p_t^i(e_t^i) = \frac{EV_{S_{t+1}^i}}{EV_{A_{t+1}}}$. It can be interpreted as the relative shadow price of general human capital. This return depends on decisions related to all investment and consumption decisions, as well as on state variables, but we leave them aside and make it explicitly depend on child's i investment, e_t^i only. The justification for this choice of specification shall become clear in the following section.

The demand for education is a function of the capital stock A as well as of the education and experience acquired by the child, through the remuneration of these various types of assets on the capital and labor markets. It also depends on the stock of human capital accumulated by other children in the family, insofar as the value of a marginal increase in the child's stock of general human capital is a function of the stock he/she already accumulated, relative to the stocks accumulated by other children, as induced by portfolio diversification. It should be noticed that if there were existing contingent securities for all the states of the world, then our model would predict independence between the investments in the human capital of the children in the family (Jacoby and Skoufias, 1997). As predicted by Behrman, Pollak and Taubman (1982) these would be determined entirely by parental preferences and by each child's ability according to the rule of comparative advantage. In our model, interdependence results from the incompleteness of security markets.

3.3.2. Corner solutions.

For a given child, a corner solution exists either because the marginal productivity of the child on the labour market is too high, relative to the returns on general human capital ($e_t^i = 0$ because for all e_t^i: $\omega_t^i > p_t^i(e_t^i)$), or because this marginal productivity is too low ($e_t^i = 1$ because for all e_t^i: $\omega_t^i < p_t^i(e_t^i)$). Since $p_t^i(e_t^i)$ is a decreasing function of e_t^i (see proof of proposition 3.1), we can write:

$$\begin{cases} e_t^i = 0 & if \quad \omega_t^i > p_t^i(0) \\ e_t^i = 1 & if \quad \omega_t^i < p_t^i(1) \end{cases}$$

The first case corresponds to that of developing countries where most households are running family businesses and where children are often involved in productive activities. The child might be kept out of school because his/her initial marginal productivity is large and because returns to experience are higher than those to formal schooling. The second case corresponds to developed countries where children' marginal productivity in family's activities is close to zero. In this case, schooling takes up all of the child's available time.

When an interior solution exists, the child spends only a fraction of his/her time at school. General human capital is then accumulated at a slower pace than normal which might justify the need for repeating classes. Jacoby (1994) explains part time schooling by credit rationing: in case of a tight credit constraint, children are gradually withdrawn from school in order to allow household's consumption smoothing. In the present model, part time schooling can take place even if credit is not constrained. It stems simply from the fact that the child's productivity increases with experience.

Altogether the model can explain a wide variety of behaviour ranging from no schooling at all to full-time schooling, encompassing as well part-time schooling. It can also explain why some children delay their entry in school. Enrollment will be delayed if, initially, returns to experience are higher than returns to schooling ($\omega_t^i > p_t^i(0)$) and if, while specific human capital is accumulated, $p_t^i(0)$ increases faster than ω_t^i does. In such a case, the inequality $\omega_t^i > p_t^i(0)$ might reverse itself when some years of experience have been accumulated.

4. The econometric framework

In our sample, schooling investments are observed as discrete decisions: children are attending school or not. The exact amount of time dedicated to schooling is not known. Therefore, the econometric model deals with the simultaneous dichotomic decisions of whether to send each child to school or not. These decisions are determined by conditions describing corner solutions, from which we can derive the latent variables of the model.

A child does not attend school if $e_t^i = 0$ and is involved in schooling, at least part-time, when $e_t^i > 0$. That is:

$$\begin{cases} e_t^i = 0 & if \quad \omega_t^i \geq p_t^i(0) \\ e_t^i > 0 & if \quad \omega^i < p_t^i(0) \end{cases} \tag{4.1}$$

Let y_t^i be the dummy variable taking the value of 1 if the child attends school and 0 otherwise. Denoting $y_t^{i*} = \ln p_t^i(0) - \ln \omega_t^i$, the estimable model using equations (4.1) can be written as functions of y_t^i and y_t^{i*}:

$$\begin{cases} y_t^i = 1 & if \quad y_t^{i*} > 0 \\ y_t^i = 0 & if \quad y_t^{i*} \leq 0 \end{cases} \tag{4.2}$$

At this point, two approaches can be considered. The first one consists in choosing a precise functional form for all the primitives of the problem and to test the fit of this form to the data. This approach is the most demanding one, but it permits to give a structural interpretation to the coefficients of the econometric equation. The second one takes the reverse path and consists in choosing the functional form that fits the data best. In this case, it is not possible to give any structural interpretation to the coefficients. In this paper we concentrate on the structural approach and do not pursue the other one.

A simple specification of the primitives of this problem is chosen in order to allow the derivation of a log-linear structural model that is estimable with short panel data (two periods). We also derive a structural model suited to cross section data and its associated reduced form. As shown below, given that the econometric model is a system of equations for binary variables where endogeneity of right hand side variables is an important issue, the simple form that we use is the only specification that we found that permits a sensible treatment of all the issues that are involved.

4.1. Specifying the primitives

To carry out the estimation of this model, it is necessary to specify the functional forms of the utility function, the future value function and the wage function. We first assume that the current period utility and the expectation taken at date t of the value function at date $t+1$, can be written in the following manner:

$$U(c_t) = \frac{1}{1-\delta}.(c_t^{1-\delta} - 1) \tag{4.3}$$

$$E_t V_{t+1}(A_{t+1}, S_{t+1}^1, ..., S_{t+1}^n) = \frac{1}{1-\gamma}(\exp \Psi(A_{t+1}, S_{t+1}^1, ..., S_{t+1}^n) - 1) \tag{4.4}$$

where:

$$\Psi = (1-\gamma)(\ln(A_{t+1} + a_0) + \sum_{i=1}^{n} \lambda_i \ln(S_{t+1}^i + s_0))$$

$\delta, \gamma, \lambda_i, a_0$ and s_0 are unknown parameters which may depend on time, demographics and unobserved heterogeneity[5]. The restrictions on the values that can be taken by these parameters and their interpretation will be precised below.

[5]Note that if $\delta = 1$ or $\gamma = 1$ these functions are well defined by taking limits.

Equation (4.4) reflects that $E_t V_{t+1}$ is a function of its known (at time t) arguments $A_{t+1}, \{S^i_{t+1}\}_{i=1,.,n}$. The choice of a functional form for the expected value function at date $t+1$ is justified by the fact that we have information neither on the planning horizon nor on the transversality condition. When the planning horizon is finite, a programme such as (3.3) is usually solved by backward induction starting from period T. Given the transversality condition and the functional form of the utility function, it is then possible to obtain the value function. Nevertheless, when the tranversality condition and the planning horizon are not known, solving the problem requires to choose arbitrarily T and the functional form of the value function at date T . In this case, it is equivalent to impose a functional form for the value function at date $t+1$. Parameters entering this functional form should however be considered as semi-structural parameters, in contrast with other parameters entering the instantaneous utility function or the wage function. Namely, these parameters depend on deeper structural parameters such as those of the utility function, but also on the specification of expectations which is left implicit here. This is also why there are no structural restrictions relating parameters entering (4.3) and (4.4).

Furthermore, to verify the requirement of proposition 3.1, the following result offers sufficient conditions.

Lemma 4.1. *If the parameters are such that:*

$\delta > 0, \lambda_i > 0, \gamma > \frac{\sum_{i=1}^n \lambda_i}{1+\sum_{i=1}^n \lambda_i}$

then the utility function and the value function are increasing and concave in their arguments.

Proof. See appendix

The interpretation is now straighforward. δ is the standard relative risk aversion of the household with respect to a consumption risk. Its reciprocal is the intertemporal substitution elasticity. Parameter γ, accordingly, is the relative risk aversion of the household with respect to a risk in financial assets, in an hypothetical world where there would be an additional risk in assets. λ_i can be interpreted as the substitutability between financial wealth and human capital assets as shown in appendix 2. The larger λ_i is, the more substitutable with financial assets human capital stock S_i is.

The wage function is the last primitive of the problem. We selected the following specification:

$$\ln \omega_t^i = \mu_{is}.\ln(S_t^i + s_0) + \mu_{i0} \tag{4.5}$$

where μ_{i0}, μ_{is} may be functions of demographics (e.g. age). It should be noticed that, in a model where the wage is expressed as a function of education and age, μ_{is} sums up effects of general and specific human capital on the current wage. Thus it could be either positive or negative. A negative μ_{is} will be found when returns to specific human capital (experience) are sufficiently high relative to returns to general human capital (education) as can be seen by differentiating (3.2):

$$\frac{1}{\omega_t^i}\frac{\partial \omega_t^i}{\partial S_t^i} = \frac{\mu_{is}}{S_t^i + s_0} = \frac{1}{\bar{\omega}_t^i}\left[\frac{\partial \bar{\omega}_t^i}{\partial S_t^i} - \frac{\partial \bar{\omega}_t^i}{\partial L_t^i}\right]$$

4.2. The latent variables

The return to a child's investment in human capital can now be written:

$$p_t^i(e_t^i) = \frac{EV_{S_{t+1}^i}}{EV_{A_{t+1}}} = \lambda_i \frac{A_{t+1} + a_0}{S_{t+1}^i + s_0} \tag{4.6}$$

The latent variable of the model (4.2) can therefore be written using $S_{t+1}^i = S_t^i$ if $e_t^i = 0$:

$$\begin{aligned} y_t^{i*} &= \ln p_t^i(0) - \ln \omega_t^i \\ &= \ln \lambda_i + \ln(A_{t+1}^{(i)} + a_0) - \ln(S_t^i + s_0) - \ln \omega_t^i \end{aligned} \tag{4.7}$$

where $A_{t+1}^{(i)}$ is the value of assets corresponding to $e_t^i = 0$, all other decisions being set at their optimal values. In our sample, information on assets is of poor quality and computing $A_{t+1}^{(i)}$ seems untractable. However, consumption is a sufficient statistic for assets, as in the usual labour supply model (Blundell and Walker, 1986). This can be seen as follows. Using the first order condition for consumption and equations (4.3) and (4.4), we obtain:

$$c_t^{-\delta} = U_c = \phi E V_A = \phi \frac{\exp(\Psi)}{A_{t+1} + a_0}$$

It is possible to express the future stock of physical assets, A_{t+1}, as a function of current consumption and human capital stocks:

$$\ln(A_{t+1} + a_0) = \frac{1}{\gamma} \left[\ln(\phi) + \delta \ln c_t + (1-\gamma) \sum_{j=1}^{n} \lambda_j \ln(S_{t+1}^j + s_0) \right] \tag{4.8}$$

Now, setting current decisions c_t and $\{e_t^j\}_{j \neq i}$ at their optimal values and $e_t^i = 0$, we obtain:

$$\ln(A_{t+1}^{(i)} + a_0) = \frac{1}{\gamma} \left[\ln(\phi) + \delta \ln c_t + (1-\gamma)\lambda_i \ln(S_t^i + s_0) + (1-\gamma) \sum_{j \neq i} \lambda_j \ln(S_{t+1}^j + s_0) \right] \tag{4.9}$$

Using (4.9), (4.7) and (4.5), the latent variable of the model becomes:

$$\begin{aligned} y_t^{i*} &= C^i + \frac{\delta}{\gamma} \ln c_t + (\frac{1-\gamma}{\gamma}\lambda_i - 1 - \mu_{is}) \ln(S_t^i + s_0) \\ &\quad + \frac{1-\gamma}{\gamma} \sum_{j \neq i} \lambda_j \ln(S_{t+1}^i + s_0) \end{aligned}$$

where $C^i = \ln \lambda_i + \frac{1}{\gamma} \ln(\phi) - \mu_{i0}$

In this equation, γ is not identifiable. The reason is that these equations are describing within-period demands for education. A between-period equation – the Euler equation for consumption for instance – is needed to recover the intertemporal substitution elasticity δ^{-1} and therefore γ (see e.g. Blundell, Browning and Meghir, 1994, for a similar point). We re-parametrize the model by denoting: $\frac{\delta}{\gamma} \equiv \delta^*$, $-(1 + \mu_{is}) \equiv \mu_{is}^*$ and $\frac{\lambda_j(1-\gamma)}{\gamma} \equiv \lambda_j^*$. Lemma 4.1 implies $\delta^* > 0$ and $\sum \lambda_j^* < 1$. The latent variable becomes:

$$y_t^{i*} = C^i + \delta^* \ln c_t + (\mu_{is}^* + \lambda_i^*) \ln(S_t^i + s_0) + \sum_{j \neq i} \lambda_j^* \ln(S_{t+1}^j + s_0) \tag{4.10}$$

This equation leads to predictions on coefficients signs. The coefficient of current consumption δ^* can be interpreted as the income effect in demands for education.

The larger δ (and therefore δ^*) is, the larger the income effect is. This is because consumption is less volatile *vis à vis* income shocks if δ increases, as its reciprocal is the intertemporal substitution elasticity. That is to say that the same changes in consumption signal larger underlying income shocks if δ is larger. It has therefore a larger effect on investments in human capital. On the other hand, the larger γ is, the smaller the income effect is. Parameter γ affects the marginal propensity to consume in assets (see equation 4.8). The same changes in consumption signal smaller underlying asset changes if γ is larger and, as substitutability between financial and human capital assets is constant, smaller changes in investments in human capital.

The sign of one of the coefficients affecting the child's general human capital stock, $\mu_{is}^* = -(1+\mu_{is})$, will depend upon the relative returns to general and specific human capital. In particular μ_{is}^* will be positive if μ_{is} is negative and large enough in absolute value, in which case returns to specific human capital are larger than those to general human capital. The last effects concerning human capital stocks of other children are more complicated. Note that they do not appear in equation 4.7 when we only condition on assets. Their effects come from the conditioning on current consumption instead of assets. If (relative) risk aversion vis à vis asset risks is important ($\gamma > 1$) the effect of the stock is negative. Namely, by fixing consumption c, an increase in human capital stocks means that financial assets are smaller (see equation 4.8). As investments in human capital and financial assets are substitutes, the investment in the child's human capital is therefore smaller. On the other hand, if (relative) risk aversion *vis à vis* asset risks is moderate ($\gamma < 1$) the effect of the stock is positive for the opposite reason.

To be completely general, the value function might not only depend on the general human capital stocks accumulated by the children of the family, but also on the age of each of the children. Indeed, the only state variables of the problem are savings and education stocks, but this is due to the fact that experience is uniquely determined as a function of age and education. This is why age might be a determinant of C^i, μ_{is}^* and λ_i^*. Other demographics can enter the various parameters. Individual characteristics, such as gender and rank, can affect λ_i^*. Household characteristics, such as family size, geographic location, education of the father and the mother, can affect any of the structural parameters.

Last but not least, unobserved heterogeneity is assumed to influence the intercept of the estimable equation, C^i, only. To understand the endogeneity issues that arise, consider first that there exists some current income shocks that lead to a temporary adjustment in consumption and simultaneously to an adjustment of human capital investments. Consumption is clearly endogenous in this case. Such temporal shocks also render endogenous the stocks of human capital of other children, since these are computed at date $t+1$ and therefore depend on investments made between t and $t+1$. Secondly, if an individual fixed effect, such as ability that is not controlled for, determines both the number of years of schooling and the current school attendance, then the child's general human capital stock is endogenous. Finally, an uncontrolled family fixed effect makes education stocks and consumption endogenous. These issues are tackled with, by using instrumental variable methods that will be exposed in the next section.

The estimable equation derived above can only be used when panel data are available. In cross-section, variables describing the future capital stocks of the other children on the RHS are unobserved. These variables however are related to the other latent variables. In the empirical part, we will not only estimate the model using panel data but also using cross-section. In order to achieve this latter goal, we now need to derive the relevant estimable equation. The procedure is described in the following section.

4.3. Deriving the current period structural equation

The system of equations (4.10) for the different children in the family comprises variables on the RHS – period $t+1$ education stocks of the other children – that depend on the latent variables in the other equations. After some algebraic manipulations, the structural form of the education demand system in a cross section is given by[6]:

Proposition 4.2.

$$\begin{aligned} y_i^* = \; & C_i + \delta^* \ln(c) + \mu_{is}^* \ln(S_i + s_0) \\ & + \textstyle\sum_{j=1}^n \lambda_j^* \ln(S_j + s_0) + \sum_{j \neq i} \frac{\lambda_j^*}{1-\lambda_j^*} y_j^* \mathbf{1}\{y_j^* \geq 0\} \end{aligned} \tag{4.11}$$

Proof. See appendix

This system of simultaneous equations is composed of continuous and truncated endogeneous variables. The difficulty with estimating this system stems from the truncation of variables and is reminiscent of simultaneous equations for discrete variables, as developed in Heckman (1978). It raises issues of coherency of the system (Gouriéroux, Monfort and Laffont, 1980, Van Soest, Kapteyn and Kooreman, 1993). Note also that the present model does not belong to the class of models developed in Blundell and Smith (1994) and conditional likelihood methods do not apply.

The reduced form of this system is difficult to derive. However, as the endogeneous variables are not discrete but truncated only, there is a change in variables that yields a equivalent expression for the structural form that can be easily transformed into a reduced form amenable to estimation.

Proposition 4.3. *The structural model is equivalent to:*

$$\begin{cases} y_i = 1 & if \quad z_i^* \geq 0 \\ y_i = 0 & if \quad z_i^* < 0 \end{cases} \tag{4.12}$$

where

$$\begin{aligned} z_i^* = \; & C^i + \delta^*. \ln(c) + \mu_{is}^*. \ln(S_i + s_0) \\ & + \textstyle\sum_{j=1}^n \lambda_j^* \ln(S_j + s_0) + \sum_{j=1}^n \lambda_j^* z_j^* \mathbf{1}\{z_j^* \geq 0\} \end{aligned} \tag{4.13}$$

Proof. See appendix

[6] We dropped index t as it is a static system of equations describing demands for education conditional on expenditures and human capital stocks at time t

4.4. The reduced form

For a family size equal to n, denote the vector of latent variables $Z^{*\prime} = (z_1^*, ., z_n^*)$, and denote D, the vector composed of the elements:

$$C_i + \delta^* \ln(c) + \mu_{is}^*. \ln(S_i + s_0) + \sum_{i=1}^{n} \lambda_j^* \ln(S_j + s_0)$$

Define the row vector

$$\Lambda_y = \begin{pmatrix} \lambda_1^* y_1 & . & \lambda_n^* y_n \end{pmatrix}$$

where y_i are the observed variables and, if I_n is the identity matrix and j_n is the n-vector which elements are all equal to 1 :

$$M = I_n - \Lambda_y \otimes j_n$$

The structural form is:

$$MZ^* = D$$

As structural restrictions in lemma 4.1 imply that $\Lambda_y j_n = \sum \lambda_j^* < 1$, it is straightforward that:

$$M^{-1} = I_n + \frac{1}{1 - \Lambda_y j_n} \Lambda_y \otimes j_n$$

and the reduced form is given by:

$$Z^* = M^{-1} D \tag{4.14}$$

In this case, the system is coherent under the structural restrictions and ML methods can be used. It is important to note that showing that M is invertible is not sufficient to prove the existence of the reduced form, since M depends on the observables. The following proposition states a sufficient condition only. A necessary and sufficient condition seems difficult to derive in this case (Van Soest, Kapteyn and Kooreman, 1993)

Proposition 4.4. *If the conditions of lemma 4.1 are verified, there is a one-to-one mapping between D and Z^**

Proof. See appendix

5. Estimation methods

The sample consists of children between 5 and 18 years old, of whom one parent is the head of the household, and whose records on schooling and family background are complete. The CILSS provides information on the education of all children of the household members, whether they were sharing the same housing or had been fostered away in other households. Both kinds of children have been included in the cross-section sample. Children that had been fostered in the household are not included. Overall, we kept 3424 sampled children in 1066 households in 1985 and 3399 children in 1069 households in 1986.

Estimation can also be based on a panel data set built from the two waves of the survey. However, one limitation of the CILSS is that it does not allow to track the education level of children that had been fostered away one year or the other. For this reason we could not include fostered children in the panel sample. This data set includes 1224 observations in 428 families.

Our estimation strategy is twofold. We first use panel data to estimate equation (4.10) since it includes variables dated t and $t+1$. Leaving out endogeneity issues that we shall tackle later, (4.10) is a system of dichotomic dependent variables that can be estimated using Probit methods. However, correlation between heterogeneity terms among children of the same family is likely and the previous methods, although consistent, are not efficient and standard errors need to be corrected. One possibility to improve efficiency would be to use multivariate Probit methods with simulation (Gouriéroux and Monfort, 1995, Keane, 1994). However, the estimation using panel data from CILSS could be criticized on three grounds. First, the panel dimension is short and the number of observations is much smaller than in the cross section dimension. Second, measurement errors on future education stocks in (4.10) are likely to be sizeable and are likely to be of a much higher order of magnitude in the panel dimension than in the cross section dimension. Third, fostered children are absent from the panel data set, which could be criticized on the ground that investments in the human capital stocks of these children should enter in the household's portfolio. These arguments indicate that using panel data could lead to very imprecisely estimated coefficients and this is indeed what happens. This is why we have more confidence in estimating the reduced form of the structural equation using cross-section data (4.14) and use results in the panel dimension as indicative.

Heterogeneity terms in the structural model (4.13) are assumed to be normally distributed with constant variance and a constant coefficient of correlation between heterogeneity terms within the family – that is to say a one-factor structure. The likelihood function cannot be computed very easily because the variance-covariance matrix of the heterogeneity terms in the reduced form (4.14) is a complicated function of the structural parameters. This is why we rely on simulated maximum likelihood to estimate the coefficients. We use a GHK simulator as it seems the one that performs best (Hajivassiliou, McFadden and Ruud, 1996).

Given the potential endogeneity of several of the explanatory variables, we use instrumental variables methods in order to correct for biases that might result in the estimates. An adaptation of Smith and Blundell (1986) conditioning technique can be explained as follows. Consider the basic structure of our model as:

$$\begin{cases} z^* = \alpha c + x_0 \gamma + u \\ c = x\beta + v \end{cases}$$

where u and v are possibly correlated and where identifying restrictions are supposed to hold. Assume that:

$$u = \rho v + \epsilon$$

where v and ϵ are independent, conditional on (x, x_0) and where ϵ is supposed to be normally distributed.

A consistent procedure is to estimate the auxiliary regression of c on x, compute residuals $\hat{v}$ and plug them in the augmented equation

$$z^* = \alpha c + x_0 \gamma + \rho \hat{v} + \epsilon$$

Estimates of α are consistent and the test that $\rho = 0$ is an exogeneity test. The crucial hypothesis is that ϵ and v are independent which makes ϵ independent of any function of c. No distributional assumption is made for v and therefore the estimation procedure is a sequence of a pseudo-maximum likelihood step – the instrumental equations – and a conditional maximum likelihood step. If exogeneity is rejected, standard errors should therefore be corrected for the two steps nature of the procedure in a M-estimation framework (see Duncan, 1987). We use this procedure for all potentially endogenous variables in the models. Tests for the validity of instruments as well as for overidentifying restrictions are performed.

6. Results

The data we use were already described in section 2 and are extracted from the CILSS rolling panel. They can be used as either short panel data (two years) or two cross sections. Descriptive statistics on variables that are used in the analysis are given in tables 1a to 1c according to the cross-sectional or panel dimension. Almost half of the children aged between 5 and 18 go to school in the samples. The number of children of the household's head lies between 6 and 7. Sampled clusters are spread across the country and survey periods are spread evenly between January and December.

6.1. Explanatory variables

The list of explanatory variables includes the logarithm of aggregate household consumption (see Johnson *et al.,* 1989), as well as the logarithm of the child's education plus 0.1 and a spline function of his/her age.[7] The general human capital stock of other children in the family is added in the regression as the sum of the logarithms of each other children's education (plus 0.1). In the estimation using panel data, the child's own education is observed in 1985 and the education of his/her siblings is observed in 1986. Child's gender and age is added to control for observable heterogeneity. We also introduce a series of dummies indicating the month in which the household has been interviewed, in order to control for events that do not modify

[7] That is: the value of s_0 is assumed equal to 0.1. This value was obtained after a grid search as the one that maximizes the cross section likelihood. In 1985 and 1986, the same estimate was obtained. We consider in the rest of the paper that this is a fixed value.

aggregate consumption but that can induce temporary withdrawal from school (such as harvesting periods in rural areas). Finally, we add the distance to the nearest primary school and a series of dummies indicating the region in which the household lives, as controls for inequality in the availability of schooling services.

6.2. Exclusion restrictions

Potentially endogeneous variables are of two types. Household's consumption is an aggregate variable while human capital stocks are individual variables. We use instruments accordingly. Consumption is instrumented using household-level variables only, while human capital stocks are instrumented by the same household-level variables but also by individual variables. Our selection is based upon economic and statistical criteria. The main identifying assumption when instrumenting aggregate consumption is that asset variables (household working assets or housing characteristics) though measured with errors are excluded from the equations of interest (4.13), as it is predicted by the structural model. It will be true provided that measurement errors in consumption and assets are independent and that unobserved household effects in (4.13) are independent of assets. The main identifying assumption with respect to instrumenting human capital stocks is that past shocks on supplies of education services (e.g. new schools or changing conditions in existing schools) were different across the different regions. These past shocks are supposed to have affected children' school attendance differently according to their age, but do not affect the current investment decisions provided regional dummy variables are included. Thus we use the interaction of the child's age with his/her region of residence as instrument. Those two subgroups of variables provide the benchmark instruments. Other subgroups of instruments are also considered. In order to determine the final set of instrumental variables we tested for the validity of each sub-group separately. Table 2 sums up the results. A sub-group was accepted as valid if, first, it was significant in the corresponding instrumental regressions (Fisher test, columns 3 and 4 of table 2) and if it did not contribute to the probability of going to school when added to the list of explanatory variables (Wald test, column 2). The latter test procedure is performed under the assumption that endogenous variables were instrumented using other sub-groups as instruments. If this subgroup did not pass the second requirement, we added the corresponding variables in the equation of interest. This procedure is adopted for instrumenting equations in all the estimations.[8]

6.3. Estimation results

When estimating equation (4.13) in cross section (see below), care should be taken over the coherency conditions as developed in proposition 4.4 since ML estimation is only valid under these conditions. These conditions are that the coefficient of log-expenditure is positive, that all λ_i^* should have the same sign and that $\sum \lambda_i^* < 1$. We performed the estimations assuming that the λ_i^* and μ_{is}^* are not heterogeneous, that is: $\lambda_i^* = \lambda^*$, $\mu_{is}^* = \mu_s^*$ for all i. The only restriction on λ^* is then that $n\lambda^*$ should be less than 1, with n the number of children in the household. One difficulty stems from the fact that households differ by their size. The only way we have found to circumvent this problem is to estimate a model where $\lambda_i^* \equiv \frac{\lambda^*}{n}$ for all i. Therefore λ^* should be interpreted as the total weight that the household puts on human capital stocks of all its children.

6.3.1. Panel data estimation

The structural equation (4.10) is written as:

$$y_t^{i*} = C^i + \delta^* \ln c_t + \mu_s^* \ln(S_t^i + s_0) + \lambda^* B \tag{6.1}$$

where:

$$B = \frac{1}{n}(\ln(S_t^i + s_0) + \sum_{j \neq i} \ln(S_{t+1}^j + s_0))$$

Estimation results are presented in table 3. In column 1 no variable is instrumented, whereas in column 2 aggregate consumption, as well as all human capital stocks, are instrumented. Standard errors were not corrected for the two stage nature

[8] The full results of the estimation of the instrumental equations are available upon request to the authors.

of the estimation, but attempts in previous versions indicated that the correction was not sizeable. As cross section estimates in next section are much more precisely estimated, we did not think that this correction was a real issue. Tests of exogeneity are anyhow valid.

Without any instrumentation, the estimation shows a positive relationship between the probability of going to school and consumption or the number of completed years of education. Examination of the results presented in column 2 of table 3 shows that the level of the own human capital stock of a child is endogeneous, while the exogeneity of aggregate consumption and the family human capital variable B cannot be rejected. The coefficients of aggregate consumption and other children' human capital stocks are non significantly different from zero. Although unprecisely estimated, the effect of the residual of aggregate consumption is consistent with the hypothesis that children' labour is employed to adjust family consumption when an adverse external shock occurs (Alessie *et al.*, 1992, Jacoby and Skoufias, 1997). The positive coefficient of the schooling variable, μ_s^*, is of particular interest. It shows that, in the reduced form of labor market gains that we used, where wage depends on education and age, the returns to education on the labor market are negative (remember that $\mu_s^* = -(1 + \mu_s)$). This means that the returns to specific human capital are higher than those to general human capital. This is a major source of explanation for the relatively low level of education observed for children who completed schooling, as well as for the delays in school enrollment and the high occurrence of classes repeated. However, the coefficient of this variable seems to be very large indeed. That is why we turn to cross section estimation in order to check the robustness of these results and to improve their precision because of larger sample sizes.

6.3.2. Cross section estimation

Tables 4a and 4b report results for cross section data in 1985 and 1986 estimating equation (4.13) by simulated maximum likelihood. The number of simulations used is equal to 50 and increasing the number of simulations does not significantly modify results. When computed, the first order asymptotic bias is very small (Lee, 1995). Robust standard errors are computed using Duncan (1987), since simulated ML estimation can be interpreted as pseudo ML estimation (Lee, 1995) and therefore as M-estimation. Another technical point is that although the variance of heterogeneity terms is theoretically identified, it has huge standard errors in the empirical procedures, especially in 1986. The likelihood functions are almost flat in this parameter. That is why we report estimates of the parameters under the standard assumption in the discrete choice literature that the variance of heterogeneity (σ) equals 1. "Standardized" results are very similar when other values are used ($\sigma = 3, \sigma = 10$). We also experimented with specifications where parameter λ depends on observed heterogeneity, but interactions never came out as significant.

Results are broadly similar to what we obtained previously but they are much more precise. The coefficient of aggregate expenditures (δ^*) is positive and significant both in 1985 and 1986. Exogeneity of this variable cannot be rejected however, and the estimated coefficient of correlation between expenditures and schooling is negative. It agrees well with the idea that households are heterogeneous in terms of preferences towards the future and consumption smoothing. The larger are expenditures today because of unobserved heterogeneity, the smaller are human capital investments. It could also agree with the presence of measurement errors. The coefficient of the education variable (μ_s^*) is positive and strongly significant. The conclusion we had, using panel data, that returns to experience tend to dominate returns to formal education is confirmed. Estimates of the coefficient of this variable's residual show that exogeneity is not rejected for both years. If it were rejected, the sign of this coefficient might indicate that unobserved heterogeneity affecting the demand today is correlated to unobserved heterogeneity having influenced the accumulation of human capital in the past. Last of the structural parameters, the estimate of λ^* is found positive and significantly different from zero in 1985, but not in 1986. Exogeneity of the corresponding variable is not rejected by the data. The correlation coefficient between the error term of two observations belonging to the same family is found significantly positive both years, supporting the idea of a strong family effect in education decisions.

These results can be used to infer estimates of the underlying "deeper" parameters, the relative risk aversion in consumption – or the intertemporal substitution elasticity – and the elasticity of substitution between financial and human capital assets. As we do not estimate an Euler equation for consumption, we cannot identify parameter γ in the structural model. Since parameter $\lambda^* = \dfrac{\lambda(1-\gamma)}{\gamma}$ is found to be significantly positive in 1985, it could be inferred that the relative risk aversion parameter with respect to a risk in assets, γ, is less than one at least that year[9]. The test for this condition is equivalent to the test that λ^* is greater than zero, since parameter λ is positive if the value function is increasing with human capital stocks. This condition ($\gamma < 1$) implies in turn that bounds on the relative risk aversion $\delta = \gamma\delta^*$ can be found, $\delta \in [0, \delta^*]$. As the intertemporal substitution elasticity (ISE) is the reciprocal of this parameter, bounds for this parameter are therefore $[\frac{1}{\delta^*}, +\infty[$. The same kind of argument permits to derive bounds for the elasticity of substitution between financial assets and human capital. Table 5 gives estimates of these two parameters under alternative assumptions about the value of $\gamma \in]0, 1[$.

First, using results in table 5, the elasticity of substitution between financial assets and human capital is estimated to be between 0.0012 ($\gamma = 0.01$) and 0.922 ($\gamma = 0.99$). The range of estimates is quite large, but indicates that this elasticity is significantly positive in 1985.

Estimates of the ISE are well out of the range of those found in the literature using microdata in developed countries and even farther from what is found using macrodata, but the latter difference could be explained by aggregation biases (Attanasio and Browning, 1995, Blundell, Browning and Meghir, 1994). We do not find this result too discomforting however. The argument is that our estimation procedure relies on a very different method than the usual one through the Euler equation by using tradeoffs between investments in different assets. Note in particular that in the

[9] Remember from section 4 that parameter γ is a semi-structural parameter which summarizes preferences and expectations.

traditional Euler equation setting, the ISE is directly estimated while we estimate its reciprocal. In the case where unobserved individual effects are not completely controled for by lack of appropriate instruments, attenuation biases could explain why the ISE is downward biased in the traditional Euler setting and upward biased in our setting. That argument is close to the argument that is developed to reconcile evidence about ISE using micro or macrodata (Blundell, Browning and Meghir, 1994).

If longer panel data were available, it could be possible to estimate an Euler equation and design estimation procedures that could help to reconcile the two approaches – the Euler equation and our estimation procedure. Using two years of data only and given doubts about the validity of specifying an Euler equation accounting for liquidity constraints make us leave this point for further research.

That the presence of unobserved heterogeneity could lead to upward biases in the estimate of the ISE is obviously a criticism of our procedure. Results in tables 4 show that unobserved household heterogeneity seems to be important. The coefficient of correlation between individuals within households is quite large (around 0.4). If we return to the structural model in equation (4.10), this correlation can be explained either by a common unobserved household effect in prices of children labour, or by a household effect in the discount rate ϕ. Namely, if structural and semi-structural parameters, γ, δ and λ are supposed to be homogeneous across households, the only source of heterogeneity in household preferences is ϕ. In the case where the discount rate is heterogeneous, the quality of our main instruments (household assets)– or more correctly our identifying assumption – is doubtful because heterogeneous discount rates obviously alter the asset accumulation process. By contradiction, that is to say that one of our maintained identifying assumption is that household preferences are homogeneous – conditional on observed characteristics – and that unobserved effects are supposed to affect prices of child labour only.

Using the same line of arguments, differences arising between years 1985 and 1986 could be explained by changing economic conditions (Berthélemy and Bourguignon, 1995). 1985 was a boom year before the crisis of 1986. Results show that parameter $\delta^* - \lambda^*$ as well – is smaller in 1986. They are related to the reciprocal of parameter γ. This decrease could be explained by the fact that the relative risk aversion with respect to assets γ could be larger in 1986 than in 1985, since expectations were no doubt much more buoyant in 1985 than in 1986. As this parameter is a semi-structural parameter which value is affected by the expectation of the future value function, it is sensitive to aggregate or specific shocks or more generally new information.

Finally, estimates of the effect of other variables are close to what is found in the literature. Investments in human capital for boys are larger than for girls, decrease with age but less strongly as age increases. The cost of education services as proxied by the distance to primary schools has the expected negative impact on investments. Finally, the rank of the child never came out as a significant variable in the various estimations we performed. If liquidity constraints were the main reason for delaying entry into school, we would expect this variable to be significant.

7. Conclusion.

The theoretical model presented in this paper permits to consider simultaneously several lines of argument to explain family demand for education, that have not been taken into account so far in the existing literature or have only been considered in isolation. The model brings an explanation for stylized facts regarding education often observed in developing countries, such as frequent repetition of classes, delayed school enrollment and inequalities within families. According to this model, the leading force in the decision to educate children or not resides in a trade-off between education and work experience. If the marginal returns to experience are higher than those to education, schooling is postponed until the situation is reversed. Thus, the low levels of education and the high occurrence of delayed school enrollment that are currently observed in developing countries would appear to be linked to the productive role attributed to children and to the high opportunity costs this induces.

Estimation of the model shows that the returns to specific experience indeed dominate those of schooling. This has important policy implications, since encouraging earlier school enrollment and a higher level of educational investment altogether, cannot be achieved by simply trying to increase the supply of schooling facilities or facilitating access to credit and insurance (Jacoby and Skoufias, 1997). It will only result from deep changes in the economic opportunities open to educated individuals.

The main rationale we put forward in this paper to explain delayed school enrollment and slow completion of schooling due to repeated classes is not exclusive of other determinants of education demand, playing an important part in explaning the levels of education actually achieved by children. Other explanations could be consistent, in particular, with the negative estimated impact of the distance to school. The existence of liquidity constraints, for example, could contribute to such a result. Nevertheless, what is shown in this paper is that a trade-off between experience and schooling actually exists and is justified by the high relative returns to experience.

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Appendix

A. Proof of proposition 3.1

Suppose that:

1.
   - **Assumption 1:** *The functions* $\{V_t\}_{t=1,.,T+1}$ *and* U *are strictly increasing and concave and twice differentiable in their arguments.*
   - **Assumption 2:** *Inada condition*

$$\lim_{c \to 0^+} \frac{\partial U(c)}{\partial c} = +\infty$$

   - **Assumption 3:** *The measure of probability of the random shocks is absolutely continuous with respect to the Lebesgue measure.*

The first assumption insures the global concavity of the program. The Inada conditions insure that aggregate consumption is never equal to zero, eliminating thereby corner solutions of the type $c_t = 0$.

We first demonstrate the global concavity of the program. It is written as follow:

$$\begin{gathered} \max_{\{c_t, e_t^1, \ldots, e_t^n\}} \{U(c_t) + \\ \phi . E_t V_{t+1}((1 + r(A_t))A_t + y_t^0 + \sum_{i=1}^{n} \omega(S_t^i, \tau_t^i, z^i)(1 - e_t^i) - c_t, S_t^1 + e_t^1, \ldots, S_t^n + e_t^n)\} \\ s.c. \quad e_t^i \in [0, 1] \quad \forall i \in \{1, \ldots, n\} \\ = \max_{\{c_t, e_t^1, \ldots, e_t^n\}} \psi(c_t, e_t^1, \ldots, e_t^n) \end{gathered}$$

ψ is a concave function, twice differentiable in each of its arguments as V_{t+1} is strictly concave and the arguments in the functions are linear in c_t, e_t^i. The optimal solution is therefore unique and the first order conditions are necessary and sufficient to characterize them.

In order to show the differentiability of the consumption and investment function, we use the first order conditions of the program. The following lemma will be useful.

Lemma A.1. $p_t^i(e) = \frac{EV_{S_i}}{EV_A}$ *is a differentiable and decreasing function of* e.

Proof As the programme is globally concave, the second order condition implies that $\frac{\partial p_t^i(e)}{\partial e} < 0$. Differentiability stems from the second order differentiability of the value function ■

We can now demonstrate the second part of the proposition. Since $p_t^i(e)$ is a differentiable and monotonous function, equation (3.5) can be inverted so as to express education demand, e_t^i, as a function of assets and educations. The human capital investment function is differentiable, except for the set of points located on the border between regimes. The Lebesgue measure for this set is equal to zero because of assumption 3. The same argument can be applied to the consumption function. ■

B. Concavity restrictions: proof of lemma 4.1

It is straightforward that the utility function is increasing and concave if and only if $\delta > 0$. Dropping index $t+1$, the first derivatives of the value function are given by:

$$\begin{cases} EV_A = \exp(\Psi - \ln(A + a_0)) \\ EV_{S_i} = \lambda_i \exp(\Psi - \ln(S^i + s_0)) \end{cases}$$

and EV_{S_i} is positive if $\lambda_i > 0$. Furthermore, the second derivatives of the value function are given by:

$$\begin{cases} EV_{AA} = -\gamma \exp(\Psi - 2\ln(A + a_0)) \\ EV_{AS_i} = (1-\gamma)\lambda_i \exp(\Psi - \ln(A + a_0) - \ln(S_i + s_0)) \\ EV_{S_iS_j} = (1-\gamma)\lambda_i\lambda_j \exp(\Psi - \ln(S_j + s_0) - \ln(S_i + s_0)) \\ EV_{S_iS_i} = \lambda_i((1-\gamma)\lambda_i - 1)\exp(\Psi - 2\ln(S_i + s_0)) \end{cases}$$

It is straightforward to show that the value function is strictly concave if the symmetric matrix H_n of size $n+1$ is definite negative with:

$$H_n = \begin{bmatrix} \lambda_0(\lambda_0(1-\gamma)-1) & \cdots & (1-\gamma)\lambda_0\lambda_i & \cdots & (1-\gamma)\lambda_0\lambda_n \\ & \ddots & \vdots & & \vdots \\ & & \lambda_i(\lambda_i(1-\gamma)-1) & \cdots & (1-\gamma)\lambda_i\lambda_n \\ & & & \ddots & \vdots \\ & & & & \lambda_n((1-\gamma)\lambda_n - 1) \end{bmatrix}$$

introducing $\lambda_0 = 1$ for reasons of symmetry. Postmultiplying by the diagonal matrix D, which elements are $(\frac{1}{\lambda_i})$ yields:

$$H_nD = \begin{bmatrix} \lambda_0(1-\gamma)-1 & \cdots & (1-\gamma)\lambda_0 & \cdots & (1-\gamma)\lambda_0 \\ \vdots & \ddots & \vdots & & \vdots \\ (1-\gamma)\lambda_i & \cdots & \lambda_i(1-\gamma)-1 & \cdots & (1-\gamma)\lambda_i \\ \vdots & & \vdots & \ddots & \vdots \\ (1-\gamma)\lambda_n & \cdots & (1-\gamma)\lambda_n & \cdots & (1-\gamma)\lambda_n - 1 \end{bmatrix}$$

One of the eigenvalues of this matrix is $(1-\gamma)(\sum_{i=0}^{n}\lambda_i) - 1$ associated to the eigenvector $(\lambda_i)_{i=1,..,n}$ and the other eigenvectors lie in the space whic basis are vectors containing one element equal to 1, another equal to -1, all other being equal to zero. Associated eigenvalues are equal to -1. The determinant of H_nD can be computed and therefore the determinant of H_n is equal to:

$$\det H = (-1)^{n+1}(1 - (1-\gamma)(\sum_{i=0}^{n}\lambda_i))\prod_{i=0}^{n}\lambda_i$$

Therefore, H_n is definite negative if for all subsets $J \subset \{0,.,n\}$:

$$(1 - (1-\gamma)(\sum_{i\in J}\lambda_i))\prod_{i\in J}\lambda_i > 0$$

Replacing $\lambda_0 = 1$, using $\lambda_i > 0$, and considering the minimum and maximum of this expression yields the equivalent condition:

$$(\gamma - (1-\gamma)(\sum_{i=1}^{n}\lambda_i)) > 0 \text{ and } \gamma > 0$$

It is also straighforward to show that λ_i is an index of substitutability between any type of assets: Let p_{S_i} be the implicit value of education i.e. $p_{S_i} = \frac{EV_{S_i}}{EV_A}$. Then:

$$\frac{d\ln(A+a_0)}{d\ln p_{S_i}}|_{EV=cste} = \frac{\lambda_i}{1+\sum_{i=1}^{n}\lambda_i}$$

C. The cross section structural equations: proof of propositions 4.2 and 4.3

The estimable equation is a function of future period stocks S_{t+1}^j, which can be written as functions of the latent variables in the following way. Consider (4.6) and (4.9) and take differences between the values of the function at the optimal solution $e = e_t^j$ and at $e = 0$.

$$\ln(p_t^j(e_t^j)) - \ln(p_t^j(0)) = (-1 + \lambda_j \frac{1-\gamma}{\gamma})(\ln(S_{t+1}^j + s_0) - \ln(S_t^j + s_0))$$

Hence using $y_t^{j*} = \ln(p_t^j(0)) - \ln(w_t^j)$ and $\lambda_j^* = \lambda_j \frac{1-\gamma}{\gamma}$:

$$\ln(S_{t+1}^j + s_0) = \ln(S_t^j + s_0) + \frac{1}{1-\lambda_j^*}(y_t^{j*} + \ln w_t^j - \ln p_t^j(e_t^j))$$

There are two cases[10].

- either $e_t^j = 0$ and $y_t^{j*} < 0$; then:

$$S_{t+1}^j = S_t^j$$

- or $e_t^j > 0$ and $y_t^{j*} \geq 0$; then the first order solution is characterized by $\ln(w_t^j) = \ln(p_t^j(e_t^j))$ and:

$$\ln(S_{t+1}^j + s_0) = \ln(S_t^j + s_0) + \frac{1}{1-\lambda_j^*}y_t^{j*}$$

Replacing in (4.10) yields the structural form of the system of demands for education given in proposition 4.2 ■

For proving proposition 4.3, define:

$$z_i^* = \frac{1}{1-\lambda_i^*}y_i^*\mathbf{1}\{y_i^* \geq 0\} + y_i^*\mathbf{1}\{y_i^* < 0\} \tag{C.1}$$

As $\mathbf{1}\{z_j^* \geq 0\} = \mathbf{1}\{y_j^* \geq 0\}$, it is straightforward that the relationship between observed and latent variables can still be written as:

$$\begin{array}{lll} y_i = 1 & if & z_i^* \geq 0 \\ y_i = 0 & if & z_i^* < 0 \end{array}$$

and by adding to (4.11) the term $\frac{\lambda_i^*}{1-\lambda_i^*}y_i^*\mathbf{1}\{y_i^* \geq 0\}$ yields:

[10] The second case of corner solutions, $e_t^j = 1$, is neglected which we think is a reasonable approximation in our data. Taking it into account would seriously complicate the structure of the model.

$$z_i^* = \quad C^i + \delta^* \log(c) + \mu_{is}^* \log(S_i + s_0)$$
$$+ \sum \lambda_j^* \log(S_j + s_0) + \sum_{j=1}^n \frac{\lambda_j^*}{1 - \lambda_j^*} y_j^* \mathbf{1}\{y_j^* \geq 0\} \tag{C.2}$$

As (C.1) yields:

$$y_i^* \mathbf{1}\{y_i^* \geq 0\} = (1 - \lambda_i^*) z_i^* \mathbf{1}\{z_i^* \geq 0\}$$

(C.2) is equivalent to:

$$z_i^* = \quad C^i + \delta^*. \log(c_t) + \mu_{is}^* \log(S_i + s_0)$$
$$+ \sum_{j=1}^n \lambda_j^* \log(S_j + s_0) + \sum_{j=1}^n \lambda_j^* z_j^* \mathbf{1}\{z_j^* \geq 0\}$$

■

D. The coherency issue: proof of proposition 4.4

Let the structural form be:

$$z_i^* = \quad D_i + \sum_{j=1}^n \lambda_j^* z_j^* \mathbf{1}\{z_j^* \geq 0\}$$

We shall prove that there is a one to one mapping between $Z^* = (z_1^*, ., z_n^*)$ and $D = (D_1, ., D_n)$. It is obvious that given Z^*, there is one and only one D that corresponds to it. Now set D to a particular value. What shall be shown is that there is one and only one possible value for Z^*.

The reduced form is given by (4.14). Let write this equation as:

$$Z^* = D + a_f(K).e$$

where e is the size n vector which elements are equal to 1, and:

$$a_f(K) = \frac{1}{1 - \sum_{i \in I} \lambda_i^*} \sum_{i \in I} \lambda_i^* D_i \tag{D.1}$$

where I is the set of indices of children going to school composed by K elements:

$$K = \sum_{j=1}^n \mathbf{1}\{z_j^* \geq 0\}$$

Then:

$$z_i^* = D_i + a_f(K)$$

First, children are going to school in the order of the D_is. If k is such that $z_k^* > 0$ then $\forall i$ such that $D_i > D_k$; $z_i^* > 0$ because $z_i^* = z_k^* + (D_i - D_k)$.It also means that conditional on K, there is a one-to-one mapping between D and Z^*. By reshuffling indices, assume that D is ordered as:

$$D_1 \geq D_2 \geq ... \geq D_{k_0} > 0 > D_{k_0+1} \geq ... \geq D_n$$

where $k_0 \in \{0, 1, ., n\}$. Consider the sequence for all $k \in \{0, 1, ., n\}$, $a_f(k)$ defined following (D.1), as:

$$\begin{aligned}
a_f(k+1) &= \frac{1}{1 - \sum_{i \leq k+1} \lambda_i^*} \sum_{i \leq k+1} \lambda_i^* D_i \\
&= \frac{1}{1 - \sum_{i \leq k+1} \lambda_i^*} (\sum_{i \leq k} \lambda_i^* D_i + \lambda_{k+1}^* D_{k+1}) \\
&= \frac{1 - \sum_{i \leq k} \lambda_i^*}{1 - \sum_{i \leq k+1} \lambda_i^*} a_f(k) + \frac{1}{1 - \sum_{i \leq k+1} \lambda_i^*} \lambda_{k+1}^* D_{k+1} \\
&= a_f(k) + \frac{1}{1 - \sum_{i \leq k+1} \lambda_i^*} \lambda_{k+1}^* (D_{k+1} + a_f(k)) \\
a_f(k+1) &= a_f(k) + \zeta_k (D_{k+1} + a_f(k))
\end{aligned}$$

with $a_f(0) = 0$.Note that the structural restrictions (Lemma 4.1), imply that either $(\forall k;\ \zeta_k > 0)$ or $(\forall k;\ \zeta_k < 0)$. We shall tackle the first case only, $(\forall k;\ \zeta_k > 0)$,the second one being very similar.

There are two cases:

- If $k_0 = 0$ then $\forall k; D_k < 0 \Rightarrow \forall k; a_f(k) < 0 \Rightarrow K = 0$ et $\forall i; z_i^* < 0$. No children goes to school.
- If $k_0 > 0$: then $\forall k \leq K_0; a_f(k) > 0$ hence $z_k^* = D_k + a_f(K_0) > 0$. The number of children going to school is such that $K \geq k_0$.

Suppose $k_0 > 0$; the algorithm that we consider is the following:

- if $D_{k_0+1} + a_f(k_0) < 0$ then $a_f(k_0 + 1) < a_f(k_0)$ and more generally $\forall k > k_0; a_f(k+1) < a_f(k)$; the solution $K = k_0$ is the only possible one.
- if $D_{k_0+1} + a_f(k_0) > 0$ then $a_f(k_0 + 1) > a_f(k_0)$ and child $k_0 + 1$ goes to school. The number of children going to school is $K \geq k_0 + 1$. Replace k_0 by $k_0 + 1$ and start up the previous step of the algorithm until it stops or $K = n$.

This algorithm might also be used for policy simulation purposes.

Table 1a: Descriptive statistics of variables used in the analysis, 1985.

Variables	Mean	Std	Min	Max	Valid
School attendance (*1 = yes*)	0.48	-	0.00	1.00	3424
ln(Education + s_0)	0.88	0.86	-2.30	2.71	3424
lnExpenditures (*lnCFA*)	12.24	0.63	10.63	14.39	3424
Sex (*1 = male*)	0.53	-	0.00	1.00	3424
Age	10.50	3.90	5.00	18.00	3424
Rank	5.08	3.34	1.00	23.00	3424
Number of children	6.59	3.52	1.00	19.00	3424
Father's education	2.53	4.14	0.00	21.00	3424
Male family head (*1 = yes*)	0.97	-	0.00	1.00	3424
Mother's education	0.78	2.42	0.00	21.00	3424
Distance to primary school (*km*)	0.26	1.04	0.00	8.00	3424
East forest (*1 = yes*)	0.16	-	0.00	1.00	3424
West forest	0.19	-	0.00	1.00	3424
Savanna	0.11	-	0.00	1.00	3424
Rural coast	0.04	-	0.00	1.00	3424
Kassou Lake	0.07	-	0.00	1.00	3424
Abidjan	0.20	-	0.00	1.00	3424
Other urban	0.23	-	0.00	1.00	3424
January	0.05	-	0.00	1.00	3424
February	0.09	-	0.00	1.00	3424
March	0.03	-	0.00	1.00	3424
April	0.09	-	0.00	1.00	3424
May	0.12	-	0.00	1.00	3424
June	0.09	-	0.00	1.00	3424
July	0.10	-	0.00	1.00	3424
August	0.11	-	0.00	1.00	3424
September	0.10	-	0.00	1.00	3424
October	0.10	-	0.00	1.00	3424
November	0.02	-	0.00	1.00	3424
December	0.10	-	0.00	1.00	3424

Notes : Variables "East forest" to "Other urban" refer to a set of geographical dummies. Variables "January" to "December" refer to a set of monthly dummies corresponding to the survey month.

Table 1b: Descriptive statistics of variables used in the analysis, 1986.

Variables	Mean	Std	Min	Max	Valid
School attendance (*1 = yes*)	0.44	-	0.00	1.00	3399
ln(Education $+ s_0$)	0.88	0.86	-2.30	2.71	3399
lnExpenditures (*lnCFA*)	12.24	0.67	10.46	14.85	3399
Sex (*1 = male*)	0.53	-	0.00	1.00	3399
Age	10.60	3.80	5.00	18.00	3399
Rank	5.50	3.58	1.00	26.00	3399
Number of children	6.92	3.70	1.00	21.00	3399
Father's education	2.62	4.28	0.00	21.00	3399
Male family head (*1 = yes*)	0.96	-	0.00	1.00	3399
Mother's education	0.96	2.67	0.00	21.00	3399
Distance to primary school (*km*)	0.28	1.24	0.00	10.00	3399
East forest (*1 = yes*)	0.16	-	0.00	1.00	3399
West forest	0.19	-	0.00	1.00	3399
Savanna	0.12	-	0.00	1.00	3399
Rural coast	0.04	-	0.00	1.00	3399
Kassou Lake	0.07	-	0.00	1.00	3399
Abidjan	0.18	-	0.00	1.00	3399
Other urban	0.24	-	0.00	1.00	3399
January	0.05	-	0.00	1.00	3399
February	0.06	-	0.00	1.00	3399
March	0.14	-	0.00	1.00	3399
April	0.01	-	0.00	1.00	3399
May	0.14	-	0.00	1.00	3399
June	0.06	-	0.00	1.00	3399
July	0.10	-	0.00	1.00	3399
August	0.11	-	0.00	1.00	3399
September	0.10	-	0.00	1.00	3399
October	0.10	-	0.00	1.00	3399
November	0.05	-	0.00	1.00	3399
December	0.08	-	0.00	1.00	3399

Notes: See table 1a.

Table 1c: Descriptive statistics of variables used in the analysis, panel 1985-1986

Variables	Mean	Std	Min	Max	Valid
School attendance (*1 = yes*)	0.48	-	0.00	1.00	1224
ln(Education $+ s_0$)	0.87	0.84	-2.30	2.56	1224
lnExpenditures (*lnCFA*)	12.26	0.65	10.72	14.39	1224
Sex (*1 = male*)	0.54	-	0.00	1.00	1224
Age	10.33	3.73	5.00	18.00	1224
Rank	5.16	3.36	1.00	18.00	1224
Number of children	4.40	2.36	1.00	11.00	1224
Father's education	2.25	3.81	0.00	18.00	1224
Male family head (*1 = yes*)	0.97	-	0.00	1.00	1224
Mother's education	0.65	2.10	0.00	16.00	1224
Distance to primary school (*km*)	0.37	1.30	0.00	8.00	1224
East forest (*1 = yes*)	0.13	-	0.00	1.00	1224
West forest	0.23	-	0.00	1.00	1224
Savanna	0.08	-	0.00	1.00	1224
Rural coast	0.05	-	0.00	1.00	1224
Kassou Lake	0.08	-	0.00	1.00	1224
Abidjan	0.19	-	0.00	1.00	1224
Other urban	0.24	-	0.00	1.00	1224
January	0.08	-	0.00	1.00	1224
February	0.08	-	0.00	1.00	1224
March	0.005	-	0.00	1.00	1224
April	0.09	-	0.00	1.00	1224
May	0.11	-	0.00	1.00	1224
June	0.08	-	0.00	1.00	1224
July	0.09	-	0.00	1.00	1224
August	0.12	-	0.00	1.00	1224
September	0.12	-	0.00	1.00	1224
October	0.12	-	0.00	1.00	1224
November	0.005	-	0.00	1.00	1224
December	0.10	-	0.00	1.00	1224

Notes: See table 1a.

Table 2 : Tests of overidentifying restrictions

Variables group	Degrees of freedom	Wald test	Fisher test (instrumental equation)		Valid/non valid instruments
			Consumption	Education stock	
1985					
1. Household's head education and age	4	8.74	7.5*	15.7*	Valid
Household's head occupation	4	8.12	11.7*	2.4	Valid
Housing characteristics	12	10.1	34.5*	5.1*	Valid
Household working assets	18	25.3	14.9*	4.3*	Valid
(Child's age)*(Head's occupation)	4	11.8*	-	7.8*	Non valid
(Child's age)*(Region of residence)	5	3.25	-	16.1*	Valid
Child's anthropometrics, rank, mother's education & size of household	9	8.83	-	2.24*	Valid
1986					
1. Household's head education and age	6	10.3	19.1*	17.3*	Valid
2. Household's head occupation	4	11.7*	18.2*	2.7*	Non valid
3. Housing characteristics	16	27.6*	49.2*	5.8*	Non valid
4. Household working assets	18	8.56	15.4*	3.9*	Valid
5. (Child's age)*(Head's occupation)	4	16.7*	-	5.2*	Non valid
6. (Child's age)*(Region of residence)	5	2.69	-	9.6*	Valid
7. Child's anthropometrics, rank, mother's education & size of household	9	23.4*	-	7.87*	Non valid

Table 3: Panel data estimation of the structural model, 1985-1986.

	(1)		(2)	
Variables	estimates	t-stat.	estimates	t-stat.
LnExpenditures (δ*)	0.46	3.83	0.31	1.35
Residual			0.14	0.52
$\ln(\text{Education} + s_{0.})$ (μ_s^*)	2.72	15.1	3.54	8.23
Residual			-0.89	2.17
Stock of family human capital (λ^*)	0.04	1.33	0.03	0.75
Residual			-0.05	0.63
Gender (β)	0.10	0.91	-0.03	0.23
Age	2.85	6.79	2.64	6.14
Age squared	-0.29	7.25	-0.29	7.25
Cube of age	0.01	10.0	0.01	10.5
Rank	0.04	2.00	0.03	1.50
Father's education	0.05	2.50	0.03	1.50
Christian	-0.11	0.73	-0.10	0.67
Muslim	0.16	1.00	0.41	2.05
Male household head	0.38	1.19	0.47	1.47
Distance to primary school	-0.15	2.50	-0.11	1.57
Number of observations	1224		1224	
LogLikelihood	-362.81		-360.28	

Notes : These estimations also included monthly and geographical dummies that are not reported here.
The variable related to family human capital is defined by equation (6.1)

Table 4a : Cross-section estimation of the structural model, 1985

(SML-GHK, 50 simulations, σ=1)

Mean log-likelihood -1.98068
Number of observations 3424

Variables	Estimates	t-stat.
Intercept	-5.071	-2.48
lnExpenditures (δ*)	0.547	3.71
Residual	-0.245	-1.46
ln own human capital stock (μ_s*)	2.09	13.0
Residual	-0.139	-0.81
Stock of family human capital (λ*)	0.119	-1.83
Residual	0.270	1.64
Gender	0.191	3.08
Age	-1.45	-3.38
(Less than 11 years old)*age	-3.16	-5.40
(Between 11 and 15 years old)*age	2.79	3.88
Distance to primary school.	-0.096	-1.85
ρ	0.327	7.53

* Robust standard errors (Duncan correction) are used

Note : This estimation also included geographical and monthly dummies, as well as the set of variables that could not be retained as instruments (see table 2)

Table 4b : Cross-section estimation of the structural model, 1986 (SML-GHK, 50 simulations, σ=1)

Mean log-likelihood -2.09179
Number of observations 3399

Variable	**Estimates**	**Est./s.e.**
Intercept	-1.32	-0.94
ln Expenditures (δ*)	0.280	2.85
Residual	-0.151	-1.21
In own human capital stock (μ_s*)	1.85	11.5
Residual	-0.259	-1.54
Stock of family human capital (λ*)	-0.053	-0.67
Residual	0.120	0.70
Gender	0.136	2.28
Age	-1.64	-4.05
(Less than 11 years old)*age	-2.20	-4.29
(Between 11 and 15 years old)*age	1.03	1.53
Distance to primary school.	-0.145	-2.32
ρ	0.472	10.6

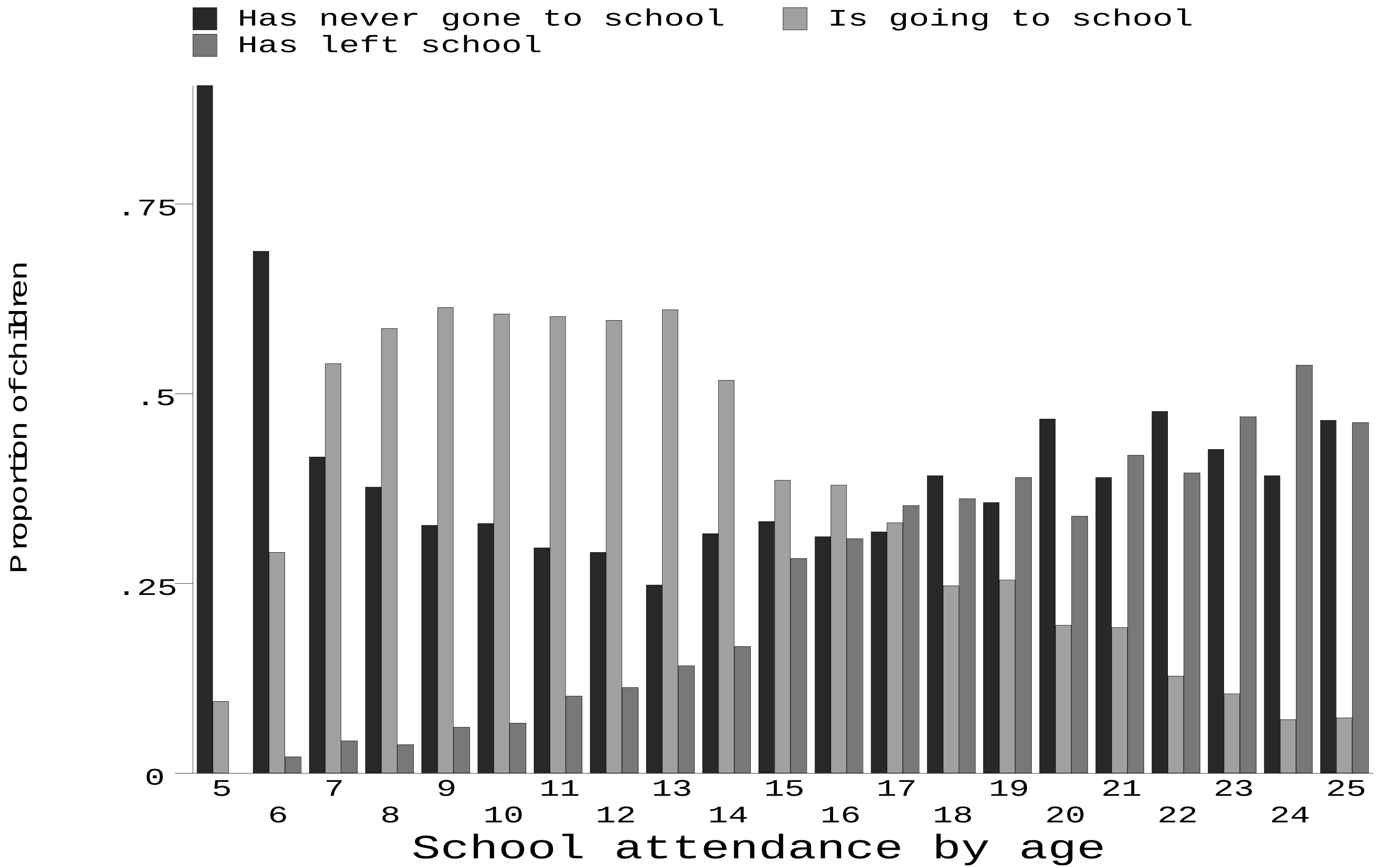
* Robust standard errors (Duncan correction) are used

Note : This estimation also included geographical and monthly dummies, as well as the set of variables that could not be retained as instruments (see table 2)

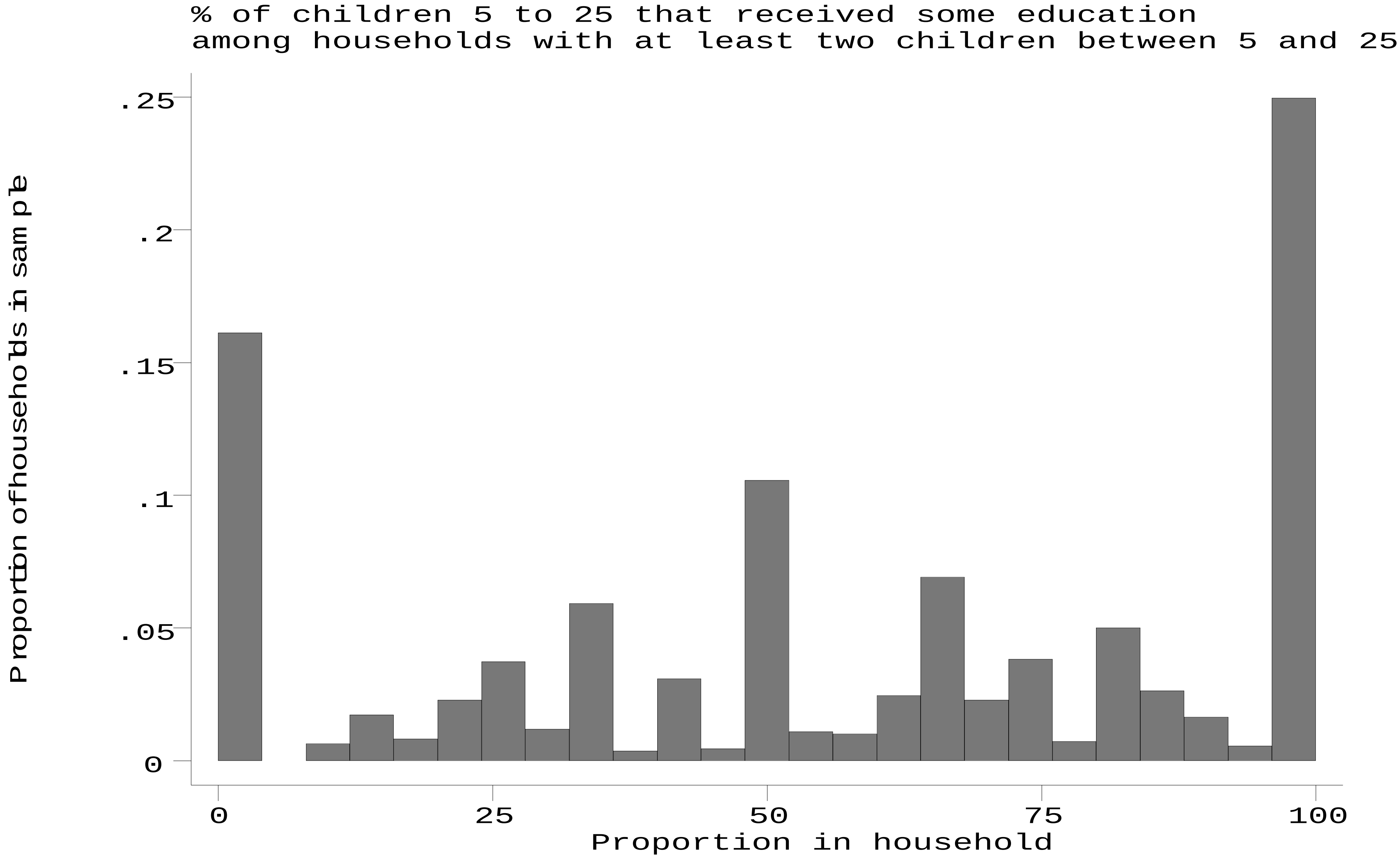
Relative risk aversion (γ)	ISE ($\frac{1}{\delta}$)	$\dfrac{\lambda}{1+\lambda}$
0.01	182.8 (3.71)	0.0012 (1.83)
0.1	18.3 (3.71)	0.013 (1.85)
0.5	3.65 (3.71)	0.106 (2.05)
0.9	2.03 (3.71)	0.517 (3.79)
0.99	1.85 (3.71)	0.922 (23.4)

Notes: $\frac{1}{\delta}$ is the intertemporal substitution elasticity. $\dfrac{\lambda}{1+\lambda}$ is the elasticity of substitution between financial assets and human capital (see appendix 4.1). Student t statistics are between parentheses.

Table 5: Structural parameters according to values of γ in 1985



Has never gone to school
Is going to school
Has left school
Proportion of children
.75
.5
.25
0
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
School attendance by age



% of children 5 to 25 that received some education
among households with at least two children between 5 and 25
Proportion of households in sample
.25
.2
.15
.1
.05
0
0
25
50
75
100
Proportion in household

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