EPEE
UNIVERSITE D'EVRY
VAL D'ESSONNE

DOCUMENT DE RECHERCHE

EPEE

CENTRE D'ETUDE DES POLITIQUES ECONOMIQUES DE L'UNIVERSITE D'EVRY

Endogenous Business Cycles and Liquidity Constraint

Stefano BOSI & Francesco MAGRIS

99 – 18 R

UNIVERSITE D'EVRY – VAL D'ESSONNE, 4 BD. FRANÇOIS MITTERRAND, 91025 EVRY CEDEX

Site Web : http://www.univ-evry.fr/PagesHtml/laboratoires/Epee/EPEE.html

ENDOGENOUS BUSINESS CYCLES AND LIQUIDITY CONSTRAINT

Stefano Bosi[y] — EPEE, Université d'Evry
Francesco Magris — DAE, University of Cambridge

December 6, 2000

Abstract. It is well known from the literature that the introduction of liquidity constraints in infinite-horizon economies may be responsible for the occurrence of local indeterminacy and sunspot fluctuations. Yet, the question of the robustness of such phenomena when the constraints are progressively relaxed, and the possibility of intertemporal arbitrage on the part of the agents increases, remains open. In this paper we study such an issue by departing from the Cazzavillan et al. (1998) framework with heterogeneous agents, financial constraint on wage income, and positive externalities in aggregate capital and labor. We observe that local indeterminacy and sunspot fluctuations persist for a wide range of amplitudes of the liquidity constraint, although they finally disappear when the "degree" of liquidity of the economy is made arbitrarily large.

JEL Classification: C61; E32.
Keywords: Liquidity constraint; Indeterminacy; Sunspot equilibria.

[*]We would like to thank Jean-Michel Grandmont for precious suggestions and constant encouragement. We also wish to thank Claude d'Aspremont, Philippe Liegeois, Colin Rowat, Alain Venditti and two anonymous referees for very helpful comments. Any remaining errors are our own.

[y]Corresponding author. EPEE, Département d'Economie, Université d'Evry - Val d'Essonne, 4 bd. F. Mitterrand, 91025 Evry cedex, France. Tel: (++ 33) 1 69 47 70 47; fax: (++ 33) 1 69 47 70 50. E-mail address: Stefano.Bosi@eco.univ-evry.fr

1. Introduction

There is widespread agreement among economists that modern economies under laissez faire conditions exhibit fluctuations which, although irregular, appear to be quite repetitive and persistent. An approach which is gaining ground suggests that a significant cause of such phenomena is to be found in the more or less volatile agents' expectations, which would affect output and prices through a mechanism of self-fulfilling prophecies, even in the absence of any shock on economic fundamentals. Such expectation-driven business cycles, often labeled in literature sunspot fluctuations, have been proved to be perfectly compatible with the rational expectations assumption, individual optimization and competitive market clearing (see, among the others, the contributions of Grandmont, 1985, and Woodford, 1986a,b). General studies on these issues, as in Woodford (1986a) and, more recently, Grandmont et al. (1998), on the other hand have successfully provided formal support to the initial conjecture that sunspot equilibria exist in economies displaying local indeterminacy, i.e. economies in which there is a robust continuum of non-explosive deterministic equilibria. Yet, a solid body of literature has made it clear that one needs some capital market imperfections in order to observe such features in infinite-horizon competitive economies[1]. In the opposite case, indeed, they would be generically ruled out, since agents endowed with convex preferences would find it preferable to smooth consumption across periods and would be able to coordinate themselves over some particular equilibrium path.

Yet, a question that remains open to discussion concerns the robustness and the determination of how "fast" these phenomena disappear when the market imperfections are progressively relaxed and the economies approach their usual configurations. Specifically, it would be interesting to understand the evolution of the qualitative features of the economy as well as their sensitivity to the magnitude of the capital market imperfections accounted for. In this paper we aim at studying such an issue by considering a discrete-time model with heterogeneous agents, "impatient" workers and "patient" capitalists, and financial constraint on workers' wage income. This model was presented initially in Woodford (1986b), amended by Grandmont et al. (1998) in order to account for capital-labor substitution, and extended by Cazzavillan et al. (1998) by incorporating positive externalities in aggregate capital and labor. In our study, we depart from Cazzavillan et al. (1998) and introduce a variable financial constraint $\pi \in [0; 1)$; in the spirit of Grandmont and Younès (1972), whose amplitude

[1] According to the existing literature on this subject, in infinite-horizon competitive economies indeterminacy seems to principally come about in view of the presence of increasing returns to scale in production (e.g., Matsuyama, 1991, Farmer and Guo, 1994, Benhabib and Farmer, 1994, 1996a, 1997; Cazzavillan, 1996) or frictions in the financial markets due to the introduction of money (e.g., Woodford, 1986b, 1994; Benhabib and Farmer, 1996b, Grandmont et al., 1998, or from both (Cazzavillan et al., 1998).

captures the share of current labor income which workers may dispose of and to which we will refer to as the degree of liquidity of the economy. By relaxing ¼ continuously from zero to one, we analyze the consequences for the local dynamics of the economy and for the change of the conditions for indeterminacy.

When the …nancial constraint is applied to the whole workers' current labor income (¼ = 0; cash-in-advance), as assumed in the above mentioned contributions, one …nds that liquidity constrained workers behave like short-lived agents in OLG models, although the length of the periods in which actions take place can be considered shorter, e.g. a quarter, or a year. This leads to theoretical results very similar to those found in OLG models, although possible ‡uctuations now occur at frequencies close to those of actual business cycles. In the case without spillover e¤ects in production studied in Grandmont et al. (1998), the emergence of indeterminacy and sunspot ‡uctuations requires highly complementary productive inputs. By assuming increasing returns to scale in production, Cazzavillan et al. (1998) improve considerably the scope for such phenomena which reappear, under very mild assumptions, as the elasticity of input substitution is made arbitrarily large.

The results we obtain by relaxing ¼ continuously from zero to one suggest that local indeterminacy and sunspot ‡uctuations represent phenomena which are rather robust. Indeed they persist for a wide range of degrees of liquidity although they disappear in a continuous way when workers have access to larger shares of their current wage income. The progressive evanescence of indeterminacy is due to the easily interpretable fact that the augmented possibility of intertemporal arbitrage by workers, made possible by the relaxation of the …nancial constraint, favors their consumption smoothing behavior and prevents the equilibrium dynamics from ‡uctuating in response to revisions in agents' expectations. This is true in spite of the externalities' persistence which, although amplifying considerably the eventuality of sunspot ‡uctuations, seem not to represent, in the model under study, an autonomous source of instability. Actually, according to our numerical investigations, local indeterminacy and sunspot ‡uctuations may occur up to degrees of liquidity very close to one (¼ 0:999) when externalities fall within plausible ranges. The importance of externalities in terms of indeterminacy is con…rmed by the observation that, in their absence, the phenomenon vanishes more rapidly.

Our analysis is carried out with the help of a geometrical method adopted in Grandmont et al. (1998) and Cazzavillan et al. (1998). This permits us to easily study the dynamics of the economy around a given steady state as well as along bifurcations (changes in stability), as a function of the solely structural parameters of the model, such as the degree of liquidity, the elasticity of input substitution and of labor supply. In addition, the presence of exactly one predetermined variable in each period, the inherited capital, could make it possible to extend the applicability of this geometrical method to the characterization of the existence of sunspot equilibria, following analogous lines to those in Grandmont et al. (1998).

The combined effects of positive externalities and variable liquidity constraints have important consequences in terms of uniqueness, multiplicity and optimality of the stationary solutions. As a matter of fact, we confirm the results, already found in Cazzavillan et al. (1998), that the stationary solution is unique when externalities are uniformly large or uniformly small, whereas multiple Pareto-ranked stationary equilibria typically occur otherwise. The relaxation of the financial constraint plays, by contrast, an important role on agents' welfare, which is more likely to improve when externalities are small and/or firms' optimal utilization of productive inputs is not very sensitive to their relative prices. When these conditions are not met, one observes, conversely, that either both agents are worse off or some redistributive effect whereby one type of agent is better off, the other being worse off.

The remainder of the paper is organized as follows. In section 2 we describe the economy and the behavior of the agents. The intertemporal equilibrium with perfect foresight is derived in section 3. Meanwhile, section 4 is devoted to the steady state analysis. Section 5 focuses on the study of the local dynamics of the model; in section 6 we report the results obtained from the sensitivity analysis. Section 7 concludes the paper.

2. Agents and intertemporal optimization

We consider an economy with two types of infinite-lived agents, "patient" capitalists and "impatient" workers (the latter discount the future more), two assets, money and capital, and a single produced good that can be either consumed or invested. Workers consume, supply labor and are subject to a liquidity (financial or borrowing) constraint that allows them to use only a share $\pi \in [0;1)$ of their current wage income to buy the good: we will refer equally to π as to the degree of liquidity of the economy. This constraint is due to the assumption of incomplete financial markets preventing workers from borrowing against future labor income. By contrast, capitalists consume but do not work. Under these hypotheses, in a neighborhood of each deterministic monetary steady state, capitalists end up holding the whole capital stock and no money (they finance consumption and investment exclusively out of capital income) whereas workers are forced to save the share $(1 - \pi)$ of their wage bill in the form of money balances. Firms produce the good by renting capital and labor according to a technology exhibiting constant returns to scale at the private level. However, there are positive externalities in aggregate capital and labor entailing increasing social returns to scale compatible with perfect competition. Our main aim is to study how the regime of local stability and bifurcations of the economy evolves when, starting from the case $\pi = 0$ (cash-in-advance), we progressively relax the financial constraint and let π converge to 1 (workers are thus financially unconstrained).

2.1. Workers. Workers choose in each period t how much to consume (c_t^w), work (l_t), invest in capital (k_t^w) and hold in nominal balances (M_t^w), in order to maximize the utility function $\sum_{t=1}^{\infty} \gamma^t \left[u\left(c_t^w/B\right) - v\left(l_t\right)\right]$, where $0 < \gamma < 1$ is their common discount factor, $u\left(c/B\right)$ is the per-period utility of consumption and $v\left(l\right)$ is the per-period disutility of labor, whereas B is a scaling parameter[2]. They are subject to the budget constraint $p_t\left[c_t^w + \left(k_t^w - (1-\delta)\, k_{t-1}^w\right)\right] + M_t^w = M_{t-1}^w + r_t k_{t-1}^w + w_t l_t$, where p is the price of the good, r the nominal interest rate, w the nominal wage and $\delta \in [0, 1]$ the depreciation rate of capital. Workers face in addition the liquidity constraint $p_t\left[c_t^w + \left(k_t^w - (1-\delta)\, k_{t-1}^w\right)\right] \leq M_{t-1}^w + r_t k_{t-1}^w + \pi w_t l_t$ of "degree" $\pi \in [0, 1)$. The per-period utility functions u and v satisfy the following properties.

Assumption 1. The functions u and v are C^r, with r large enough, for, respectively, $c > 0$ and $0 \leq l < l^*$, where $l^* > 0$ is the (possibly infinite) workers' endowment of labor, and satisfy $u'(c) > 0$, $u''(c) < 0$, $v'(l) > 0$, $v''(l) > 0$ and $\lim_{l \to l^*} v'(l) = +\infty$. Consumption and leisure are assumed to be gross substitutes, i.e. $u'(x) + xu''(x) > 0$ with $x \equiv c/B$.

We shall focus in the following on the case where

$$(1-\delta)\, p_{t+1} + r_{t+1} > p_t \tag{1}$$

and

$$u'\left(c_t^w/B\right) > \gamma u'\left(c_{t+1}^w/B\right)\left[(1-\delta)\, p_{t+1} + r_{t+1}\right]/p_{t+1} \tag{2}$$

hold at all dates. Then workers choose not to hold capital $(k_t^w = 0)$ and are forced to hold money $(M_t^w = (1-\pi)\, w_t l_t)$, the financial constraint being binding. The first order condition of workers' program can be therefore written in the form

$$V\left(l_t\right) = \pi U\left(c_t^w\right) w_t l_t / \left(p_t c_t^w\right) + (1-\pi)\, \gamma U\left(c_{t+1}^w\right) w_t l_t / \left(p_{t+1} c_{t+1}^w\right) \tag{3}$$

where $V(l) \equiv l v'(l)$ and $U(c) \equiv c U'(c/B)/B$.

2.2. Capitalists. Capitalists do not work and maximize the utility function $\sum_{t=1}^{\infty} \beta^t \ln c_t^c$, where c^c denotes their consumption and β their discount factor which satisfies $\gamma < \beta < 1$. They are subject only to the budget constraint $p_t\left\{c_t^c + [k_t^c - (1-\delta) k_{t-1}^c]\right\} + M_t^c = M_{t-1}^c + r_t k_{t-1}^c$, where M^c and k^c denote, respectively, money balances and capital. Since, under condition (1), the gross rate of return on capital is higher than the profitability of money holding, capitalists hold only capital and the solution of their optimization problem is, for all $t \geq 1$, $k_t^c = \beta\left[(1-\delta) + r_t/p_t\right] k_{t-1}^c$, $c_t^c = (1-\beta)\left[(1-\delta) + r_t/p_t\right] k_{t-1}^c$ and $M_t^c = 0$.

[2] We follow here the idea of exploiting B as a scaling parameter to ensure the persistence of a stationary solution. For more details, see Cazzavillan et al. (1998).

2.3. Production with externalities. Firms produce in each period t the good y_t by combining labor l_t and capital stock k_{t-1} inherited from the previous period. The presence of positive externalities[3] in aggregate capital and labor means, for a single firm, that production depends on the quantity of both inputs employed as well as on their average level $\bar{l}$ and $\bar{k}$ in the economy. In particular, for a quantity of capital k and labor l, the effective input services amount to, respectively, $A\left(\bar{k}, \bar{l}\right)k$ and $A\left(\bar{k}, \bar{l}\right)l$, where $A\left(\bar{k}, \bar{l}\right)$ is the externalities' contribution to production. Since firms are identical, at the symmetric equilibrium one has $l = \bar{l}$ and $k = \bar{k}$. The gross production function $F(k, l)$ is homogeneous of degree one and the quantity of goods produced is then given by

$$y = F(A(k, l)k, A(k, l)l) = A(k, l)F(k, l) = A(k, l)lf(a), \tag{4}$$

where $a_t \equiv k_{t-1}/l_t$ is the capital intensity and $f(a)$ the reduced production function. The aggregate technology satisfies the following properties.

Assumption 2. The reduced production function f is continuous for $a \geq 0$, C^r for $a > 0$ and r large enough, and satisfies $f' > 0$ and $f'' < 0$. Therefore $\rho(a) \equiv f'(a)$ is decreasing in a, whereas $\omega(a) \equiv f(a) - af'(a)$ is increasing. The externality contribution $A(k, l)$ on production is continuous on R^2_+, C^r on R^2_{++} for r large enough, homogeneous of degree γ and can be therefore written as $A(k, l) = Al^\gamma \psi(a)$, where $\psi(a)$ is increasing in a and $A > 0$ is a scaling factor. The contributions of capital and labor to the externalities are, respectively, $\varepsilon_\psi(a) > 0$ and $\gamma - \varepsilon_\psi(a) > 0$, where $\varepsilon_\psi(a) \equiv a\psi'(a)/\psi(a)$.

In view of (4) and Assumption 2, the real wage is given by $\Omega(k, a) = A(k/a)^\gamma \psi(a)\omega(a)$ and the real gross return on capital by $R(k, a) = (1 - \delta) + A(k/a)^\gamma \psi(a)\rho(a)$.

3. Intertemporal equilibrium

Equations of the model are the first order conditions of the producers $w_t/p_t = \Omega(k_{t-1}, a_t)$ and $r_t/p_t = R(k_{t-1}, a_t) - (1 - \delta)$, the first order condition (3) of the workers, the program of the capitalists and equilibrium conditions in money, labor and good market. Let $M > 0$ be the fixed quantity of money exogenously given. Since workers hold the share $(1 - \pi)$ of their labor income in money balances, equilibrium in the money market requires $(1 - \pi)w_t l_t = M$ for every t. Labor market

[3]Externalities can be due to effects like learning spillovers, public knowledge or learning by doing, particular market configurations (e.g. thick markets), or mechanisms leading to fast matching between workers and firms (see, inter alia, Romer, 1986, King et al., 1988, Benhabib and Farmer, 1994, Farmer and Guo, 1994).

equilibrium is implied by using the same notation for labor demand and labor supply. The goods market clears automatically by Walras' law, i.e. $c_t^w + c_t^c + k_t^c = (1 - \delta) k_{t-1} + A l_t^{1+\gamma} \psi(a_t) f(a_t)$. From the money market equilibrium and the workers' budget constraint one has $p_{t+1} c_{t+1}^w = M + \pi w_{t+1} l_{t+1}$ and, since $w_t l_t$ is constant over time, $p_{t+1} c_{t+1}^w = w_t l_t = w_{t+1} l_{t+1}$. Therefore the workers' first order condition (3) can be rewritten as

$$V(l_t) = \pi U(c_t^w) + (1 - \pi) \gamma U\left(c_{t+1}^w\right). \tag{5}$$

We can now introduce the intertemporal equilibrium with perfect foresight in terms of k and a where k_{t-1}, in each period t, is a predetermined variable (in order to simplify notation, we will set $c = c^w$ and $k = k^c$) and $k_0 > 0$ is the stock of physical equipment available in period zero.

Definition 1. An intertemporal equilibrium with perfect foresight is a sequence $(k_{t-1}, a_t) > 0$ for $t \geq 1$ that satisfies the following system:

$$\begin{cases} k_t = \beta R(k_{t-1}, a_t) k_{t-1} \\ V(k_{t-1}/a_t) = \pi U((k_{t-1}/a_t)\Omega(k_{t-1}, a_t)) + (1 - \pi)\gamma U((k_t/a_{t+1})\Omega(k_t, a_{t+1})). \end{cases} \tag{6}$$

It is easy to see that our formulation is consistent, i.e. that conditions (1) and (2) hold at each stationary solution of the system defined by (6). Indeed, at each steady state, one has $R(k, a) = 1/\beta > 1$ which corresponds to condition (1). By continuity, (1) holds along intertemporal equilibria near the steady state. On the other hand, condition (2) also holds at a steady state since workers' consumption is constant and $\gamma < \beta$. Thus, by continuity, (2) is satisfied too in a neighborhood of each stationary solution. It follows that system (6) describes the dynamics of the economy near each fixed point.

4. Existence of the steady state and welfare analysis

In this section our first objective is to ensure the existence of a stationary solution of system (6). We will then study the influence of relaxing π upon the welfare, evaluated at the steady state, of both types of agents.

Definition 2. An interior steady state equilibrium is a stationary sequence $(k_{t-1}, a_t) = (k^*, a^*) > 0$ that satisfies, for all $t \geq 1$, the two-dimensional system (6).

A steady state equilibrium must therefore be a solution of the stationary system in terms of l and a

$$\begin{cases} A l^\gamma \psi(a) \rho(a) = 1/\beta - (1 - \delta) \\ V(l) = [\pi + (1 - \pi)\gamma] U(A l^{\gamma+1} \psi(a) \omega(a)). \end{cases} \tag{7}$$

4.1. Existence of the steady state. Following the scaling procedure adopted in Cazzavillan et al. (1998), it is possible to obtain a normalized steady state by selecting appropriately the two parameters A and B. By direct inspection one sees that the following holds.

Proposition 3. Under the Assumptions 1 and 2 and the boundary condition[4] $\lim_{c\to 0} U(c) < V(1) < \lim_{c\to+\infty} U(c)\gamma$; $(a, l) = (1, 1)$ is a solution of (7) if and only if the scaling parameters A and B solve $A = [1/\beta - (1-\delta)]/[l\gamma\psi(1)\rho(1)]$ and $Bv'(1) = [\pi + (1-\pi)\gamma]\, A\psi(1)\omega(1)\, u'(A\psi(1)\omega(1)/B)$.

It follows that under the assumptions in Proposition 3 the steady state level of capital is also normalized to unity. In view of (7), it is easy to verify that the number of stationary solutions of system (6), as well as the associated optimality features, are not affected by π. Therefore the results[5] found in Cazzavillan et al. (1998) can be immediately applied to our case. By contrast, relaxation of the liquidity constraint may entail important consequences in terms of agents' welfare, evaluated at the steady state. In order to study such an issue as well as, in the next section, the local dynamics of system (6), it is useful to introduce here expressions for the elasticity of input substitution $\sigma(a) \in (0, +\infty)$; the share of capital in total income $s(a) \equiv a\rho(a)/f(a) \in (0, 1)$; and the local elasticity $\varepsilon(l) \equiv [lV'(l)/cU'(c)]/[\pi + (1-\pi)\gamma]$ of the local workers' offer curve[6] $c = \lambda(l) \equiv U^{-1}(V(l)/[\pi + (1-\pi)\gamma])$ belonging to $(1, +\infty)$; all evaluated at the steady state solution (a, l). By definition $1/\sigma(a)$ is the elasticity with respect to a of the ratio of the rental prices of capital and labor and satisfies $1/\sigma(a) = \varepsilon_\omega(a) - \varepsilon_\rho(a)$, where $\varepsilon_\omega(a) \equiv \omega'(a)a/\omega(a)$ and $\varepsilon_\rho(a) \equiv \rho'(a)a/\rho(a)$ represent the elasticities of, respectively, $\omega(a)$ and $\rho(a)$. In addition, it is easy to prove that $\varepsilon(l)$ is inversely related with the local elasticity $e(l) \equiv (dl/l)/(d\omega/\omega) \in (0, +\infty)$ of the workers' labor supply and satisfies $\varepsilon(l) = e(l)^{-1} + 1$.

4.2. Liquidity and welfare. Starting from a stationary equilibrium corresponding to a given π and a given configuration of the scaling parameters A and B; it is

[4] Notice that under this condition one has $\lim_{c\to 0}[\pi + (1-\pi)\gamma]U(c) < V(1) < \lim_{c\to+\infty}[\pi + (1-\pi)\gamma]U(c)$ for every $\pi \in [0, 1)$.

[5] In Cazzavillan et al. (1998) it is shown that there is generically a unique stationary equilibrium either when externalities are uniformly large or when they are uniformly small. When these conditions are not met, multiple Pareto-ranked stationary equilibria may easily coexist, maybe in a small neighborhood. Cazzavillan et al. (1998) then apply the general results to the family of CES economies and observe that, generically, either the steady state is unique or there are exactly two stationary solutions. It also emerges that the Cobb-Douglas case is very peculiar and structurally unstable.

[6] Notice that, under Assumption 1; consumption and labor are gross substitutes and therefore U is invertible. In addition, in a neighbourhood of each steady state, U^{-1} is defined for all values that $V(l)/[\pi + (1-\pi)\gamma]$ takes.

possible to investigate how a slight relaxation of π changes the welfare, evaluated at the steady state, of both types of agents. To this purpose, let us first observe that in view of the presence in the economy of spillover effects in production, the relaxation of π does not necessarily produce a Pareto-improving movement. Indeed, the effects we observe on the welfare of the agents by increasing the liquidity of the economy are ambiguous: in some cases the welfare of both workers and capitalists improves, in others is made worse, but there may even be some redistributive effects such that one type of agent ends up being better off, the other being worse off. In any case, one could reasonably expect that the lower the level of distortion due to the externalities, the more likely it is that the relaxation of π is Pareto-improving.

Loosely, the mechanism at work can be illustrated as follows. When π increases, the incentive to work is higher, thus workers increase their supply of labor. This means that the higher the wages at the new stationary solution are, the more likely it is that workers benefit. On the other hand, capitalists are better off if and only if at the new steady state they consume more, i.e. the amount of capital is greater. Now, let us observe that when the externalities are sufficiently mild to entail an interest rate decreasing in the capital intensity a; in response to an increase in the amount of the labor supplied a must also increase in order to re-establish the first equation in system (7) (Modified Golden Rule). But higher levels of l and a imply in turn a higher capital stock as well as a higher wage income, and then make both types of agents better off. Conversely, if externalities are rather large (and the interest rate is increasing in a), the response of the first equation in (7) to an increase in the labor supply is likely to require a sharp decline of a; and therefore of capital, making capitalists worse off. At the same time, this will also produce a fall in the wage, and possibly a decline of workers' income, making them worse off too. The next proposition fully characterizes the different possible effects on the welfare of the agents induced by a relaxation of π.

Proposition 4. Let $(a^*; l^*)$ be a stationary solution of system (6) corresponding to the liquidity constraint π^*. Then, generically, the steady state $(a^*; l^*)$ is locally unique and is a C^r function $(a^*(\pi); l^*(\pi))$ of π near π^*. Let us set $s_1 \equiv -(l^*/a^*)$; $s_2 \equiv -\{[\varepsilon_\psi(a^*) + s(a^*)/\sigma(a^*)]\, l^*/a^*\}/[\nu + (1-\pi^*)(1-\gamma)]$ and $s_3 \equiv l^*/\{a^*\sigma(a^*)[\varepsilon(l^*)-1]\}$. Then, when π increases slightly from π^*, both agents are better off at the new steady state if and only if $\{[1-s(a^*)]/\sigma(a^*) - \varepsilon_\psi(a^*)\}\, l^*/(a^{*\prime}) \in (-\infty; \min\{s_1; s_2\}) \cup (s_3; +\infty)$; worse off if and only if it belongs to $(\max\{s_1; s_2\}; s_3)$; whereas workers (capitalists) are better off and capitalists (workers) worse off if and only if it is included in the interval $(s_1; s_2)$ $((s_2; s_1))$.

Proof. See the appendix ∎

In view of Proposition 4, one can immediately verify that in the special case in which there are no externalities at all ($\nu = \varepsilon_{\psi}(a) = 0$ for all $a > 0$) a relaxation of π is always Pareto-improving. Indeed, in this case the financial constraint represents the sole market imperfection and the welfare-improving effects of its relaxation cannot be offset by any "perverse" substitution in the use of productive inputs.

5. Local stability and bifurcations analysis

In this section we analyze the local dynamics of system (6) around one of its interior stationary solutions as well as along bifurcations. Our principal aim is to study how it evolves when the financial constraint is continuously relaxed, i.e. as π increases from zero to one. When $\pi = 0$; we know from Cazzavillan et al. (1998) that local indeterminacy and sunspot fluctuations occur for sufficiently low and sufficiently large elasticities of input substitution. As soon as π is relaxed, the increased possibility of intertemporal arbitrage by the workers should reduce the scope for indeterminacy. In other words, the range of parameter values generating it should shrink and finally, when π is large enough, vanish. Yet, it would be interesting to see how "fast" local indeterminacy and sunspot fluctuations disappear and how persistent such phenomena are with respect to the degree of liquidity of the system.

According to the usual procedure, we study the linear map associated with the Jacobian matrix (provided it is invertible and with no eigenvalues on the unit circle[7]) evaluated at the fixed point under study. Let $(k^*; a^*)$ be an interior stationary solution of system (6) and let $\varepsilon;\ \varepsilon_{R;k};\ \varepsilon_{R;a};\ \varepsilon_{\Omega;k};\ \varepsilon_{\Omega;a};\ \varepsilon_{\psi}$ be the elasticities, respectively, of the (local) offer curve $\lambda(l) \equiv U^{-1}(V(l)/[\pi + (1-\pi)\gamma])$, the functions $R(k;a)$; $\Omega(k;a)$ (where the derivatives are taken with respect to k and a) and $\psi(a)$, all evaluated at the given steady state. The linearized dynamics for the deviations $dk = k - k^*$ and $da = a - a^*$ is then determined by the two dimensional map

$$\begin{bmatrix} dk_t \\ da_{t+1} \end{bmatrix} = J^* \begin{bmatrix} dk_{t-1} \\ da_t \end{bmatrix} \tag{8}$$

where J^* is the Jacobian[8] of system (6) evaluated at the steady state under study and whose expression is given by:

$$\begin{bmatrix} 1+\varepsilon_{R;k} & l\,\varepsilon_{R;a} \\ \frac{1}{l}\frac{\varepsilon[\pi+(1-\pi)\gamma]-\pi\left(1+\varepsilon_{\Omega;k}\right)-(1-\pi)\gamma\left(1+\varepsilon_{R;k}\right)\left(1+\varepsilon_{\Omega;k}\right)}{(1-\pi)\gamma\left(\varepsilon_{\Omega;a}-1\right)} & \frac{-\varepsilon[\pi+(1-\pi)\gamma]-\pi\left(\varepsilon_{\Omega;a}-1\right)-(1-\pi)\gamma\,\varepsilon_{R;a}\left(1+\varepsilon_{\Omega;k}\right)}{(1-\pi)\gamma\left(\varepsilon_{\Omega;a}-1\right)} \end{bmatrix}:$$

[7] Hartman-Großman Theorem: see, e.g., Guckenheimer and Holmes (1983) and Grandmont (1988).

[8] Condition $\varepsilon_{\omega}(a^*) + \varepsilon_{\psi}(a^*) \neq 1 + \nu$ is necessary to avoid that the Jacobian evaluated at the steady state vanishes.

To ensure that a steady state exists for the whole range of parameter values we will consider, we assume that it has been normalized at $(k^*, a^*) = (1, 1)$ through the scaling procedure illustrated in Proposition 3. The trace T and determinant D of J^* correspond, respectively, to the sum and the product of the roots of the characteristic polynomial $P(\lambda) = \lambda^2 - T\lambda + D$. Straightforward computations yield the following expressions:

$$T = T_1 - (\varepsilon - 1)\frac{\pi + (1-\pi)\gamma}{(1-\pi)\gamma\left(\varepsilon_{\Omega,a} - 1\right)}, \quad D = D_1 + (\varepsilon - 1)\frac{[\pi + (1-\pi)\gamma]\left(|\varepsilon_{R,a}| - \varepsilon_{R,k} - 1\right)}{(1-\pi)\gamma\left(e_{\Omega,a} - 1\right)}$$

where

$$T_1 = 1 + D_1 + \frac{[\pi + (1-\pi)\gamma]\left(\varepsilon_{R,k}\varepsilon_{\Omega,a} + |\varepsilon_{R,a}|\varepsilon_{\Omega,k}\right)}{(1-\pi)\gamma\left(\varepsilon_{\Omega,a} - 1\right)},$$
$$D_1 = \frac{-\pi\left(\varepsilon_{\Omega,a} + |\varepsilon_{R,a}|\varepsilon_{\Omega,k} + \varepsilon_{R,k}\varepsilon_{\Omega,a}\right) + (1-\pi)\gamma\left(|\varepsilon_{R,a}| - \varepsilon_{R,k} - 1\right)}{(1-\pi)\gamma\left(\varepsilon_{\Omega,a} - 1\right)}$$

In view of the complicated form of the above expressions, it may seem that the study of the local dynamics of system (6) requires long and tedious computations. However, by applying the geometrical method[9] adopted in Grandmont et al. (1998) and Cazzavillan et al. (1998), it is possible to analyze qualitatively the (in)stability of the characteristic roots of J^* and their bifurcations (changes in stability) by locating the point (T, D) in the plane and studying how (T, D) varies when the value of some parameter changes continuously. If T and D lie in the interior of the triangle ABC depicted in figures $1 - 4$, the stationary solution is a sink, hence locally indeterminate. In the opposite case, it is locally determinate: it is either a saddle when $|T| > |1 + D|$, or a source. If we fix all the parameters of the model with the exception of the elasticity ε of the workers' offer curve that we let vary from 1 to $+\infty$, we obtain a parametrized curve $\{T(\varepsilon), D(\varepsilon)\}$ that describes a half-line Δ starting from the point (T_1, D_1) when ε is close to one. The linearity of such a locus follows by direct inspection of the expressions of T and D. This geometrical method makes it possible at the same time to characterize the different bifurcations that may arise when ε moves from 1 to $+\infty$. In particular, as shown in figure 1, when the half-line Δ intersects the line AC (at $\varepsilon = \varepsilon_T$), one eigenvalue goes through one and a transcritical bifurcation generically occurs; accordingly, we should expect an exchange of stability between

[9] When $\pi = 1$, the geometrical approach used throughout the paper does not work any longer as the dynamical system becomes one-dimensional. Specifically, in this case workers act as if they were solving a static problem, consuming in each period an amount which is exactly their wage bill; current labor supply (and therefore the capital-labor ratio) then depends exclusively on the predetermined variable, current capital. The equilibrium dynamics is then described by a first-order difference equation in capital. The immediate implication of this feature is that local indeterminacy is ruled out, although it is possible to show that deterministic cycles are still possible through a flip bifurcation. The case $\pi = 1$ therefore produces results consistent with those found when π is close to one. For more details, see Bosi and Magris (1997).

two nearby steady states[10]. When Δ goes through the line AB (at $" = "_F$), one eigenvalue is equal to $¡1$ and we expect a ‡ip bifurcation. Finally, when Δ intersects the interior of the segment BC (at $" = "_H$), the modulus of the complex conjugate eigenvalues is one and the system undergoes, generically, a Hopf bifurcation.

Following Grandmont et al. (1998), this analysis is also strong enough to characterize the occurrence of sunspot equilibria around an indeterminate stationary solution of system (6) as well as along supercritical ‡ip and Hopf bifurcations[11]. Indeed, as in Grandmont et al. (1998), system (6) has in each period t one predetermined variable, the inherited capital $k_{t¡1}$, and one which is not, the workers' current labor supply l_t (which …xes in turn the current capital intensity a_t), chosen on the basis of workers' "state of expectations" about future prices. Therefore, the geometrical characterization presented by Grandmont et al. (1998) can be easily applied to our case.

In order to carry out our investigation, it is convenient to convert the expressions of the various elasticities given above into expressions in terms of the structural parameters of the models such as $"_Ã$; º; s; ¾ and µ: The expressions for these elasticities are the following: $"_{R;k} = µº$; $j"_{R;a}j = µ[(1 ¡ s)=¾ + º ¡ "_Ã]$; $"_{\Omega;k} = º$; $"_{\Omega;a} = s=¾ ¡ º + "_Ã$; where $µ ´ 1 ¡ ¯(1 ¡ ±) > 0$: Therefore, the trace and the determinant of the Jacobian can be rewritten as

$$T = T_1 ¡ (" ¡ 1)\frac{[¼+(1¡¼)°]¾}{(1¡¼)°\left[s¡¾\left(1+º¡"_Ã\right)\right]};\ D = D_1 + (" ¡ 1)\frac{[¼+(1¡¼)°]\left[µ(1¡s)¡¾\left(1+µ"_Ã\right)\right]}{(1¡¼)°\left[s¡¾\left(1+º¡"_Ã\right)\right]} \tag{9}$$

with

$$T_1 = 1 + D_1 + \frac{[¼+(1¡¼)°]µº}{(1¡¼)°\left[s¡¾\left(1+º¡"_Ã\right)\right]};D_1 = \frac{¡¼\left[s+µº¡¾\left(º¡"_Ã\right)\right]+(1¡¼)°\left[µ(1¡s)¡¾\left(1+µ"_Ã\right)\right]}{(1¡¼)°\left[s¡¾\left(1+º¡"_Ã\right)\right]}: \tag{10}$$

The structural parameters which capture the main features of the economy, and that we will let vary in the following, are ¼; ¾ and ": As we have seen, ¼ re‡ects the degree of liquidity of the system, ¾ summarizes the main properties of the technology and " those of the preferences of the workers. By adapting the geometrical approach developed by Grandmont et al. (1998) and Cazzavillan et al. (1998), we shall focus on locating the half-line Δ in the $(T;D)$ plane as a function of the degree of liquidity ¼, i.e. in determining its origin $(T_1;D_1)$; its slope and its position when ¼ is made to increase from zero to one, with the other parameters of the model, i.e. ¯; ±; °; $"_Ã$; º; s and ¾; …xed. By repeating this procedure with di¤erent values of the elasticity

[10] The fact that we get a transcritical bifurcation generically is a consequence of the persistence of one steady state due to the scaling procedure illustrated in Proposition 3.

[11] In the case of supercritical Hopf bifurcation and ‡ip bifurcation, sunspot equilibria remain in a compact set containing in its interior, respectively, the stable invariant curve and the stable two-period cycle.

σ of input substitution, we will be able to appraise the whole evolution of the local dynamics and bifurcations obtained by relaxing π. As we will see, by adopting this strategy we obtain the useful feature that only the origin (T_1, D_1) of Δ depends upon π, both its slope and position being influenced exclusively by σ. The main implication of this picture is that it makes it possible to study the effects on Δ of relaxing π from zero to one by simply shifting in the (T, D) plane each half-line Δ corresponding to $\pi = 0$ (see figures $1-4$).

In order to locate the origin (T_1, D_1) of Δ, let us observe, since T_1 and D_1 are fractions of first degree polynomials in π with the same denominator, that the locus (T_1, D_1) as a function of π describes a part of a line Δ_1 whose slope S_{Δ_1} is given by $(\partial D_1/\partial\pi)/(\partial T_1/\partial\pi)$ and is independent of π. Straightforward computations yield the following expression for S_{Δ_1}:

$$S_{\Delta_1} = 1 + \theta\nu/[s - \sigma(\nu - \varepsilon_\psi)]. \tag{11}$$

One can immediately verify that $S_{\Delta_1} > 1$ when $0 < \sigma < \sigma_6 \equiv s/(\nu - \varepsilon_\psi)$ and diverges to $+\infty$ when σ tends to σ_6 from below. When σ moves from σ_6 to $+\infty$, S_{Δ_1} increases from $-\infty$ to one; in particular, it is zero when $\sigma = \sigma_7 \equiv (\theta\nu + s)/(\nu - \varepsilon_\psi) > \sigma_6$. Another important piece of information for the application of our geometrical analysis consists in establishing how the origin $(T_1, D_1)_{\pi=0}$ evaluated at $\pi = 0$ varies when σ moves from zero to $+\infty$. Indeed, once we locate it for a given σ, one can appraise the variation of (T_1, D_1) obtained by relaxing π from zero to one, since (T_1, D_1) will move continuously along the line Δ_1. In view of (10), it is possible to see that for $\sigma = 0$ one has $T_{1;\pi=0} = 1 + \theta(1 - s + \nu)/s$ and $D_{1;\pi=0} = \theta(1 - s)/s$, and that for $\sigma = +\infty$ they are both equal to $(1 + \theta\varepsilon_\psi)/(1 + \nu - \varepsilon_\psi)$. Throughout we will assume that $T_{1;\pi=0}$ and $D_{1;\pi=0}$ are lower than, respectively, two and one[12] when $\sigma = 0$ and when $\sigma = +\infty$, and that both are decreasing in σ. It is not difficult to prove that, when θ is small enough (this is what one should indeed expect by looking at reasonable parameter values), these conditions are equivalent to the inequalities given below in Assumption 3.

Assumption 3. The structural parameters of the model satisfy the double inequality

$$\theta(1 - s + \nu)/s < (1 + \theta\varepsilon_\psi)/(1 + \nu - \varepsilon_\psi) < 1. \tag{12}$$

As it is shown in the following lemma, inequalities in (12) put an upper bound ν^* on the externality parameter ν.

[12] As we will see below, this implies that the half-lines Δ corresponding to $\sigma = 0$ and $\sigma = +\infty$ intersect the interior of the stability region ABC.

Lemma 5. Under Assumption 3; the externality contribution º satis…es º < º¤ < $[$s ¡ µ$(1$ ¡ s$)]$ =µ; where º¤ is the unique positive root of µº^2 $+$µ$(2$ ¡ s$)$ º $+$µ$(1$ ¡ s$)$ ¡ s $= 0$:

Proof. See the appendix ■

In view of Assumption 3; as ¾ increases from zero, both T$_{1;¼=0}$ and D$_{1;¼=0}$ decrease and tend to ¡ 1 when ¾ tends to ¾$_4$ ´ s=$(1 +$ º ¡ "$_{\tilde{A}}$) from below. When ¾ moves from ¾$_4$ to $+$ 1 , T$_{1;¼=0}$ as well as D$_{1;¼=0}$ decrease from $+$ 1 to $(1 +$ µ"$_{\tilde{A}}$) = $(1 +$ º ¡ "$_{\tilde{A}}$) : From the expressions in (9), one also gets all the necessary information in order to characterize the behavior of the slope of Δ whose expression is S$_\Delta$ $= 1 +$ µ"$_{\tilde{A}}$ ¡ µ$(1$ ¡ s$)$ =¾ and does not depend upon ¼: this implies that Δ undergoes parallel shifts in the $($T; D$)$ plane as ¼ is relaxed from zero to one. In addition, Δ rotates counterclockwise as ¾ increases from 0 (S$_\Delta$ $=$ ¡ 1) to $+$ 1 (S$_\Delta$ $= 1 +$ µ"$_{\tilde{A}}$) : What still remains to be checked is the position of Δ with respect to Δ_1: For the purpose of our analysis, it is su¢cient to observe, in view of (9), that T decreases with " when ¾ < ¾$_4$ and that both T and D are increasing in " when ¾ > ¾$_4$:

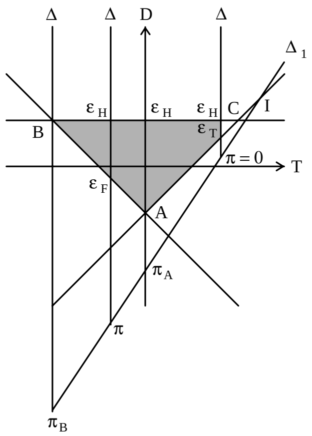


Figure 1 : ¾ $= 0$:

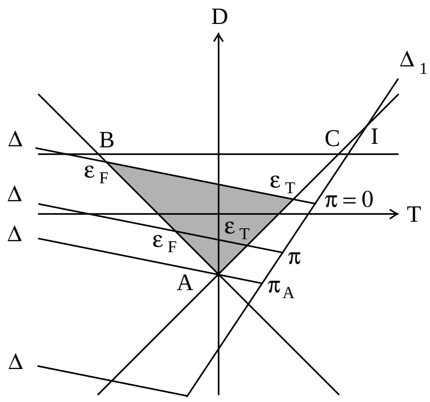


Figure 2 : ¾$_2$ < ¾ < ¾$_3$:

We now apply the geometrical approach for low elasticities of input substitution, namely 0 · ¾ < ¾$_4$; since the evolution of the local dynamics obtained by relaxing ¼ shows in this case some interesting regularities. As anticipated, our program consists of …xing ¾ and then in locating in the $($T; D$)$ plane the half-line Δ corresponding to each ¼ 2 $[0; 1)$: This procedure requires one in particular to draw the line Δ_1 of the origins, to locate correctly the half-line Δ corresponding to ¼ $= 0$ and, …nally, to make it shift along Δ_1:

A useful benchmark is represented by the case ¾ $= 0$; to which …gure 1 refers. In this case, the line Δ_1 has a slope greater than one and intersects the line AC at the point I whose ordinate is greater than one[13]. At the same time, the half-line Δ lies above Δ_1, has a slope equal to $-\infty$ and, when $\pi = 0$, its origin has coordinates $T_{1,\pi=0} = 1 + \theta(1 - s + \nu)/s$ and $D_{1,\pi=0} = \theta(1 - s)/s$, which are both positive and, under Assumption 3, lower than, respectively, two and one. All this is sufficient to ensure that Δ intersects the interior of the triangle ABC when $\pi = 0$. When π is made to increase from zero, both D_1 and T_1 decrease and tend to $-\infty$ as π tends to one. Therefore, as it shown in figure 1, Δ shifts to the left and intersects the interior of the triangle ABC for all $0 \le \pi < \pi_{B,\sigma=0}$ where $\pi_{B,\sigma=0} = [\theta(1 - s + \nu) + 3s]/[\theta(1 - s + \nu) + 3s + s/\gamma]$ is the value of π such that Δ goes through the point B. The immediate implication of this configuration is that local indeterminacy occurs for $0 \le \pi < \pi_{B,\sigma=0}$ and elastic labor supplies (low ε), whereas the steady state is bound to be locally determinate for all $\pi_{B,\sigma=0} \le \pi < 1$. Figure 1 allows one also to prove that local indeterminacy always disappears through a Hopf bifurcation[14] (at $\varepsilon = \varepsilon_H$), whereas it arises through a transcritical bifurcation (at $\varepsilon = \varepsilon_T$) when $0 \le \pi < \pi_{A,\sigma=0}$, where $\pi_{A,\sigma=0}$ is the value of π such that Δ lies on the vertical axis, and through a flip bifurcation (at $\varepsilon = \varepsilon_F$) when $\pi_{A,\sigma=0} < \pi < \pi_{B,\sigma=0}$. As will be shown in the next section, where we shall perform a sensitivity analysis, the range of ε displaying local indeterminacy shrinks monotonically with π before vanishing at $\pi = \pi_{B,\sigma=0}$. When σ is set slightly larger than zero, the picture in figure 1 does not undergo any relevant modification: the main difference is that the slope of Δ is higher, and the ordinate D_1 of its origin, in correspondence to each π, lower. Local indeterminacy will then occur for degrees of liquidity low enough to ensure that the half-line Δ passes to the right of the point B. More interesting changes in the regime of local dynamics occur, conversely, when σ becomes first larger than $\sigma_1 \equiv \theta(1 - s)/(2 + \theta\varepsilon_\psi)$, where σ_1 is the value of σ such that the slope of Δ is equal to -1, and then larger than σ_2, while σ_2 is that value such that Δ evaluated at $\pi = 0$ goes through point B. In the case $\sigma_1 < \sigma < \sigma_2$ local indeterminacy emerges (through a transcritical bifurcation at $\varepsilon = \varepsilon_T$) in correspondence to degrees of liquidity lower than $\pi_{A,\sigma_1<\sigma<\sigma_2}$, where $\pi_{A,\sigma_1<\sigma<\sigma_2}$ is the value of π such that the half-line Δ goes through point A. In addition it disappears through a Hopf bifurcation when $0 \le \pi < \pi_{B,\sigma_1<\sigma<\sigma_2}$, where $\pi_{B,\sigma_1<\sigma<\sigma_2}$ is that π such that the half-line Δ goes through point B, and then, when $\pi_{B,\sigma_1<\sigma<\sigma_2} < \pi < \pi_{A,\sigma_1<\sigma<\sigma_2}$, through a flip bifurcation. By contrast, as shown in figure 2, when σ becomes slightly larger than σ_2, local indeterminacy disappears always through a flip bifurcation and there is no more room for Hopf bifurcations. Notice that, since $D_{1,\pi=0}$ decreases with σ while

[13] The ordinate of the point I is $1 + \theta(1 - s + \nu - \sigma\varepsilon_\psi)/[s - \sigma(1 + \nu - \varepsilon_\psi)]$. It is then greater than one when $\sigma = 0$ and diverges to $+\infty$ when σ tends to σ_4^-. This implies, in particular, that I lies above the point C for every $\sigma < \sigma_4$. When σ increases from σ_4 to $+\infty$, the ordinate of I increases from $-\infty$ and tends to $1 + \theta\varepsilon_\psi/(1 + \nu - \varepsilon_\psi) > 1$ when σ tends to $+\infty$.

[14] The expressions of ε_T, ε_F, and ε_H are given in the appendix.

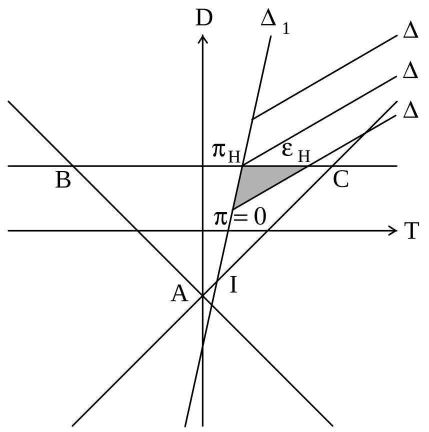
the slope of Δ increases, there exists, by continuity, a $\sigma_3 > \sigma_2$ such that the half-line Δ goes through the point A at $\pi = 0$. Therefore, if we take into account the fact that D_1 decreases with π, we may easily state that local indeterminacy is completely ruled out when $\sigma_3 \leq \sigma < \sigma_4$. On the basis of the same observations, we may claim that for π greater than $\pi_{B,\sigma=0}$, the steady state is bound to be locally determinate whatever σ low.

In the case with no externalities ($\gamma = \varepsilon_\psi = 0$) studied by Grandmont et al. (1998), the locus Δ_1 of the origins (T_1, D_1) is located on the line AC, as one can verify by direct inspection of (10). For low elasticities of input substitution, the pictures depicted in figures 1 and 2 do not change qualitatively[15]: local indeterminacy occurs for low degrees of liquidity and disappears for sufficiently high ones. As σ increases from s to $+\infty$ (indeed, in this case $\sigma_4 = s$), $D_{1,\pi=0}$ decreases from $+\infty$ to one. Moreover, for each fixed $\sigma > s$, D is increasing in both ε and π. The immediate geometrical implication of these facts is that, for high elasticities of input substitution, the half-line Δ lies outside the stability region ABC, and therefore local indeterminacy and sunspot fluctuations cannot emerge, whatever the degree of financial intermediation.

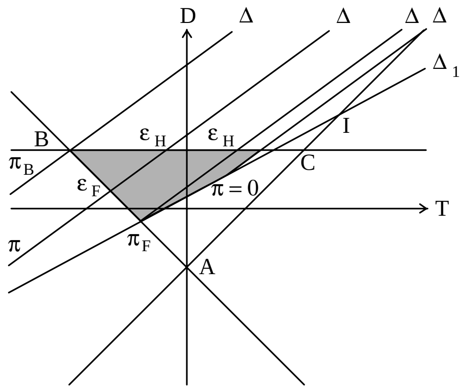
As shown by Cazzavillan et al. (1998), the presence of even mild positive externalities may reverse these results and make local indeterminacy and sunspot fluctuations reemerge for high σ's. As a matter of fact, what we are now going to study is the evolution of local dynamics resulting by relaxing π, when $\sigma > \sigma_4$. To this end, let us first observe, in view of expressions in (10) and under Assumption 3, that when σ increases from σ_4 to $+\infty$, $D_{1,\pi=0}$ decreases monotonically from $+\infty$ to $(1+\theta\varepsilon_\psi)/(1+\gamma-\varepsilon_\psi)$ which, again under Assumption 3, is lower than one. The ordinate $D_{1,\pi=0}$ is in particular equal to one when $\sigma = \sigma_5 \equiv [s-\theta(1-s)]/[\gamma-(1+\theta)\varepsilon_\psi] > \sigma_4$. Therefore, once we observe that, when $\sigma_4 < \sigma < \sigma_5$, D is increasing in both ε and π, we may conclude that, for the mentioned range of values of σ, it is larger than one, whatever π is. The modulus of at least one characteristic root lies then outside the unit circle and the steady state is bound to be locally indeterminate. By contrast, if we take into account the facts that, in view of (10) and Assumption 3, $T_{1,\pi=0}$ and its deviation from the line $T = 1 + D$ are, respectively, positive and negative when σ is large enough, we can conclude that for all $\sigma > \sigma_5$ the origin $(T_1, D_1)_{\pi=0}$ lies in the interior of the stability region ABC (hence the steady state is locally indeterminate), specifically in the intersection with the positive orthant (figures 3 and 4 refer to this configuration). Actually, these are the results reached by Cazzavillan et al. (1998),

[15] The only relevant exception is that no transcritical bifurcation occurs.

i.e. when ¼ $= 0$ in the present context.



F igure 3 : ¾$_5$ < ¾ < ¾$_6$:



F igure 4 : ¾ > ¾$_7$:

For the application of our geometrical analysis when ¾ is large, it would be useful to establish the value that S_Δ assumes at ¾ $=$ ¾$_5$: In order to ...x ideas, we assume that it is lower than one (which is actually true for a wide range of parameter values). Since the slope of Δ is equal to one at ¾ $=$ ¾$_8$ ´ $(1$ ¡ s$)$="$_Ã$; we will ensure this condition by imposing the inequality in Assumption 4.

Assumption 4. The slope of the half-line Δ for ¾ $=$ ¾$_5$ is less than one, i.e. "$_Ã$ < $(1$ ¡ s$)$ º:

In view of Assumption 4; one can verify that the critical elasticities of input substitution introduced above satisfy the order ¾$_5$ < ¾$_6$ < ¾$_7$ < ¾$_8$: Moreover, since S_Δ is increasing in ¾; Assumption 4 ensures that it is lower than one for all ¾ < ¾$_8$ (and a transcritical bifurcation generically occurs when Δ intersects the line AC; see ...gures 3 and 4) and larger than one for all ¾ > ¾$_8$ (no transcritical bifurcation occurs for all " > 1). It is also possible to show analytically that the half-line Δ crosses the interior of the segment BC; and therefore a Hopf bifurcation generically emerges, for all ¾ > ¾$_5$: When ¾$_5$ < ¾ < ¾$_6$; as shown in ...gure 3; the slope of Δ_1 is greater than one and D_1 tends to $+$ 1 as ¼ tends to one. On the basis of the preceding discussion and with the help of ...gure 3; one concludes that Δ intersects the interior of ABC; and there are, therefore, sunspot equilibria, for all ¼ < ¼$_H$; where ¼$_H$ veri...es $D_1 = 1$; and in correspondence of elastic labor supplies (" close to one). It is also easy to see that local indeterminacy disappears through a Hopf bifurcation at " $=$ "$_H$:

Similar con...gurations arise when ¾$_6$ < ¾ < ¾$_7$; with the only relevant di¤erence that the line Δ_1 is downward sloping and T_1 then decreases with ¼: The case ¾$_7$ <

¾ < ¾$_8$ is illustrated in ...gure 4: The slope of the line Δ_1 is again positive, although it is now lower than one and the ordinate of the intersection I of Δ_1 with the line AC is greater than D$_{1;¼=0}$ (see footnote 13). At the same time, $(T_1; D_1)$ moves downward along Δ_1 and both T$_1$ and D$_1$ tend to ¡ 1 when ¼ tends to one. The direct implication of these features is that the half-line Δ will intersect the interior of ABC (and local indeterminacy will occur) for all ¼ < ¼$_B$; where ¼$_B$ is the value of ¼ such that Δ goes through the point B: With the help of ...gure 4; one may also conclude, on the one hand, that local indeterminacy always disappears through a Hopf bifurcation at " $=$ "$_H$; and, on the other one, that it emerges through a ‡ip bifurcation at " $=$ "$_F$ when ¼$_F$ < ¼ < ¼$_B$; where ¼$_F$ is the value of ¼ such that $(T_1; D_1)_{¼=0}$ lies on the intersection of Δ_1 with the line AB: Finally, a very similar picture is obtained in the case ¾ > ¾$_8$; with the only qualitative di¤erence that now the slope of Δ is greater than one and therefore does not intersect the line AC (as we have already seen, this rules out the transcritical bifurcation). This is true also in the limit case ¾ $= +$ 1 in correspondence to which the line Δ_1 coincides with the line AC and, as ¼ increases from zero to one, Δ undergoes a downward shift along AC as D$_1$ decreases from $(1 +$ µ"$_Ã$)=$(1 +$ ° ¡ "$_Ã$) < 1 to ¡ 1:

6. Sensitivity Analysis

We now present the results of the numerical investigations performed in order to provide some quantitative insight of the extent to which the theoretical results reached in the previous section apply. The pictures we obtain suggest that local indeterminacy and sunspot ‡uctuations represent rather robust phenomena and disappear quite slowly when the liquidity of the system is increased. This is in particular true when the elasticity of input substitution is large and the degrees of liquidity are very close to one. By contrast, the persistence of endogenous ‡uctuations is observed to be somewhat lower when technology does not provide incentives for a rapid substitution in the productive inputs, and includes degrees of liquidity lower than approximately $0{:}75$:

In our numerical investigations, we will assume the period of the model to be short (e.g. one year) and accordingly impose a crude calibration for the parameters based on annual data. Thus, we will set the numerical speci...cation for the capital depreciation rate ± $= 0{:}1$; the value for the share of capital in total income s $= 1{=}3$; and we will identify “impatient” workers and “patient” capitalists with the discount rates given, respectively, by ° $= 0{:}9$ and ¯ $= 0{:}99$: Accordingly, the parameter µ will assume the numerical value $0{:}109$: By contrast, the calibration of the externality contributions to production is controversial[16]. If we are willing to admit relatively large externalities,

[16] For estimates of the increasing returns parameters (although the structure of the externalities is not the same as in our model) see, e.g., the time series estimate of the production function in Baxter and King (1991) and the cross-section studies in Caballero and Lyons (1992). More recent

we can set the value $\gamma = 0.2$ for the overall externality contribution and $\varepsilon_{\psi} = 0.04$ for the specific contribution of capital. The corresponding critical values of the elasticity of input substitution introduced in the previous section are reported in the following table:

σ_1	σ_2	σ_3	σ_4	σ_5	σ_6	σ_7	σ_8
0.036	0.056	0.17	0.29	1.67	2.08	2.22	16.67

Throughout our simulations, we will carry out the stability analysis from a different perspective with respect to that adopted in the previous section. Indeed, for a given σ, if we map the (T, D) plane in the (π, ε) one, the sides of the triangle ABC become hyperbola and the stability region a connected set, delimited by the hyperbola, in the extended plane (π, ε). Figures $5-8$ correspond to figures $1-4$ and represent the progressive sections associated, respectively, with the magnitudes 0, 0.1, 2 and 4 of the elasticity of input substitution. The curves denoted with ε_T, ε_H and ε_F correspond, in the (T, D) plane, respectively, to the curves $D = T - 1$, $D = 1$ and $D = -T - 1$ and represent the bifurcation values of ε (for sake of precision, the Hopf bifurcation occurs in correspondence of those ε_H lying on the boundary of the stability region). Therefore, the shaded area represents the pairs (π, ε) generating local indeterminacy. Figure 5 refers to the case $\sigma = 0$. The stability region is delimited from above by the curve ε_H, which increases with π, approximately, from 4.59 (π close to zero) to 5.58 when π approaches 0.75 and, as figure 5 shows, local indeterminacy is ruled out. On the other hand, the stability region is delimited from below by the curve ε_T when $\pi < 0.54$, and by the curve ε_F when $0.54 < \pi < 0.75$. One may immediately see that ε_T does not depend upon π and is equal to 1.3, while ε_F increases very fast and intersects ε_H at $\pi = 0.75$. If we slightly increase σ and set it equal to 0.1, we get the picture shown in figure 6. The stability region is non-empty for degrees of liquidity lower than $\pi = 0.32$ and lies between the curves ε_T and ε_F (the Hopf bifurcation in this configuration is ruled out). More precisely, ε_T is constant and equal to 1.3, while ε_F decreases from 4.14 $(\pi = 0)$, intersecting ε_T at $\pi = 0.32$. In view of the results discussed in the previous section, we know that if there were no externalities, figures 5 and 6 would not change substantially, the only qualitative difference being the absence of the curves ε_T delimiting the stability region from below. In addition, one could show that local indeterminacy would be ruled out for σ's arbitrarily large and that the highest degree of liquidity compatible with sunspot fluctuations would be, approximately, 0.74. By contrast, as we have seen, the presence of externalities makes local indeterminacy reemerge for elasticities of input substitution larger than σ_5 which, under our numerical specifications, is equal to 1.67. Figures 7 illustrates the configuration arising when $\sigma = 2$, which corresponds to the regime discussed in

estimates are provided in Basu and Fernald (1995).

…gure 3:

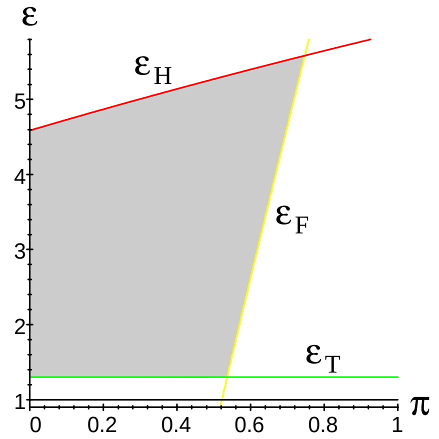


Figure 5 : $¾ = 0$:

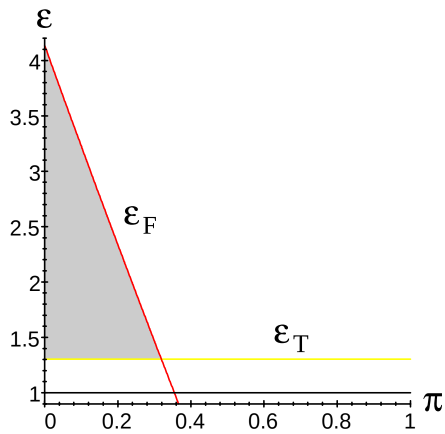


Figure 6 : $¾ = 0{:}1$:

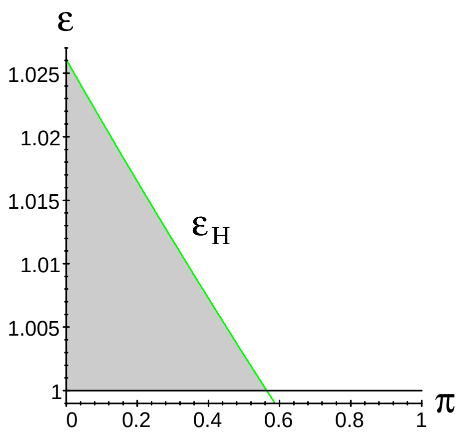


Figure 7 : $¾ = 2$:

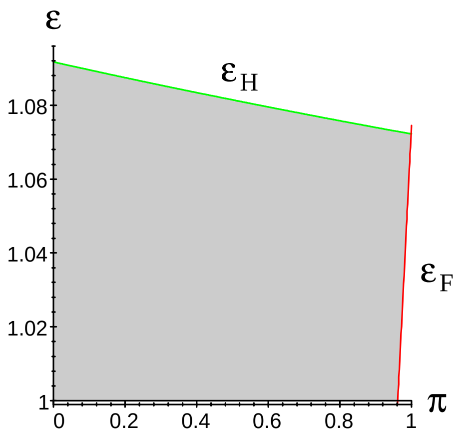


Figure 8 : $¾ = 4$:

The stability region is represented by the area lying below the curve $"_H$ which decreases from $1{:}026$ (when $¼ = 0$) to one (when $¼ = 0{:}56$) so local indeterminacy is ruled out. Since the regime corresponding to $2{:}08 = ¾_6 < ¾ < ¾_7 = 2{:}22$ is qualitatively very similar to the previous one, we do not lose any relevant insight by also referring …gure 7 to it. In …gure 8 we consider, by contrast, the case $¾ = 4$ (which is higher than $¾_6 = 2{:}22$ but lower than $¾_8 = 16{:}67$). The stability region is non-empty for $¼ < 0{:}999$ and is delimited form above by the curve $"_H$; which decreases, approximately, from $1{:}092$ to $1{:}072$ when $¼$ increases from zero to $0{:}999$: When $¼ > 0{:}962$; the stability region becomes in addition bounded from below by the curve $"_F$ which increases very fast and …nally intersects $"_H$:

7. Concluding remarks

The contribution of this paper is to study the evolution of the local dynamics obtained by continuously relaxing the financial constraint in a model with heterogeneous agents and liquidity constrained workers, initially presented in Woodford (1986b) and more recently extended by Grandmont et al. (1998) and Cazzavillan et al. (1998) in order to account for productive factors substitutability and spillover effects in aggregate capital and labor. The general applicability of the geometrical approach adopted makes it possible to not solely base our study on particular specifications for the economic fundamentals but to deal with more general ones, as is the case in the above mentioned contributions. We have shown that the relaxation of the workers' financial constraint, by increasing the possibility of intertemporal arbitrage, reduces the scope for local indeterminacy and sunspot fluctuations, and finally rules them out. Nevertheless, as the sensitivity analysis performed seems to suggest, such phenomena appear to be rather robust and to persist up to very large degrees of liquidity.

There is one final point which is worth mentioning. One may indeed reasonably wonder why, in the model presented in this paper, the scope for indeterminacy vanishes when the degree of liquidity is sufficiently high even when externalities are taken into account. Indeed, this result could appear in contrast with the findings in Benhabib and Farmer (1994) and Farmer and Guo (1994) in which externalities alone are necessary to produce indeterminacy. A rather plausible explanation could be put down to the presence of heterogenous agents which break down the mechanism described in Benhabib and Farmer (1994) and Farmer and Guo (1994). Indeed, as we have already had the opportunity to remark, when $\pi = 1$ the dynamical system reduces to a first order difference equation in a predetermined variable (capital) and therefore one loses one degree of freedom which is crucial for the scope of indeterminacy.

8. Appendix

Proof of Proposition 4 (Liquidity and welfare). Since, as we have already stressed (see footnote 6), we cannot deal with the global inverse of the function U, the proof is valid in a small neighborhood of the (locally) unique steady state. For a given $\pi = \pi^*$, let (a^*, l^*) be an interior stationary solution of system (6) i.e. a solution of the equations in (7). By solving for l the first equation in (7) and setting $\alpha \equiv 1/\beta - (1 - \delta)$, we obtain the function $l = l_1(a) = [\alpha / A\psi(a)\rho(a)]^{1/\gamma}$ defined, positive and differentiable on the open interval $I_1 = (0, +\infty)$. Since U is invertible, the second equation in (7) can be (locally) rewritten as $Al^{\gamma+1}\psi(a)\omega(a) = \lambda(l)$ where $\lambda(l) \equiv U^{-1}(V(l)/(\pi + \gamma(1 - \pi)))$. Dividing it for the first equation, we obtain $\alpha\omega(a)/\rho(a) = \lambda(l)/l$. From the fact that $\omega(a)/\rho(a)$ and $\lambda(l)/l$ are both increasing in their respective arguments, by solving it for l we obtain a second function $l = l_2(a)$ defined, positive, differentiable and increasing in a, i.e. $l_2'(a) > 0$;

on an open interval I$_2$ ½ R$^2_{++}$: By construction, l$_1$ (a) and l$_2$ (a) satisfy l¤ $=$ l$_i$ (a¤) ; i $=$ 1; 2: Let us now suppose that ¼ increases slightly above ¼¤: By continuity, in a neighborhood of ¼¤; system (6) generically has a unique stationary solution (a; l) ; close to (a¤; l¤) and belonging to I$_1$ \ I$_2$: In addition, it is easy to verify that l$_2$ (a) shifts upward in the (a; l) plane when ¼ increases, while l$_2$ (a) is not a¤ected by the amplitude of ¼: This implies, in particular, that the new steady state will lie on the curve l$_1$ (a) : Let us now analyze how the change in ¼ a¤ects the welfare, evaluated at the steady state, of both type of agents. Capitalists at the steady state consume k¤ (1 ¡ ¯) =¯ and therefore are better o¤ if and only if the new solution (a; l) satis…es al > k¤; i.e. if and only if (dl=da)$_{a¤}$ > ¡ l¤=a¤ ´ s$_1$: On the other hand, the induced variation in the workers' utility is u^0 (c) dc ¡ °v^0 (l) dl: Straightforward computations show that workers are then better o¤ if and only if (dl=da)$_{a¤}$ > ¡ f["$_Ã$ (a¤) + s (a¤) =¾ (a¤)] l¤=a¤g = f1 + ° ¡ [¼¤ + (1 ¡ ¼¤) °]g ´ s$_2$: Now, if we di¤erentiate equations l$_1$ (a) and l$_2$ (a) we obtain, respectively, l$_1^0$ (a¤) = f[1 ¡ s (a¤)] =¾ (a¤) ¡ "$_Ã$ (a¤)g l¤= (a¤°) and l$_2^0$ (a¤) = (l¤=a¤) = f¾ (a¤) [" (l¤) ¡ 1]g ´ s$_3$ > 0: The sign of the slope of l$_1$ (a¤) can be either positive or negative, according to the structural parameters. Let us then consider separately the two cases l$_1^0$ (a¤) > 0 and l$_1^0$ (a¤) < 0: When l$_1^0$ (a¤) > 0; one may immediately verify that, when ¼ increases, both agents will be better o¤ when l$_1^0$ (a¤) > l$_2^0$ (a¤) and in the opposite case they will be made worse o¤. The case l$_1^0$ (a¤) < 0 is somewhat richer. Actually, in order to improve the welfare of both agents jl$_1^0$ (a¤)j must be greater than max fjs$_1$j ; js$_2$jg : On the other hand, if js$_2$j < jl$_1^0$ (a¤)j < js$_1$j (js$_1$j < jl$_1^0$ (a¤)j < js$_2$j) workers (capitalists) end up better o¤ and workers (capitalists) worse o¤. Finally, when jl$_1^0$ (a¤)j < min fjs$_1$j ; js$_2$jg both agents end up worse o¤. The above results can be summarized in a more compact way. Indeed, one may claim that, as ¼ increases slightly above ¼¤; both agents are better o¤ if and only if f[1 ¡ s (a¤)] =¾ (a¤) ¡ "$_Ã$ (a¤)g l¤=°a¤ 2 (¡ 1 ; min fs$_1$; s$_2$g) [((l¤=a¤) = f¾ (a¤) [" (l¤) ¡ 1]g ; + 1) ; worse o¤ if and only if it belongs to the interval (max fs$_1$; s$_2$g ; (l¤=a¤) = f¾ (a¤) [" (l¤) ¡ 1]g) and workers (capitalists) are better o¤ and capitalists (workers) are worse o¤ if and only if is included in the interval (s$_1$; s$_2$) ((s$_2$; s$_1$)) : In the light of the above results one may easily draw the conclusion that in the absence of externalities (° = "$_Ã$ = 0) ; by relaxing ¼; one improves the welfare of both types of agents ■

Proof of lemma 5. It is easy to verify that the right inequality in (12) is equivalent to "$_Ã$ (1 + µ) < °: At the same time, the expression (1 + µ"$_Ã$) = (1 + ° ¡ "$_Ã$) is an increasing function of "$_Ã$; therefore the left inequality in (12) is satis…ed for all 0 < "$_Ã$ < ° = (1 + µ) if and only if it is satis…ed with a weak inequality sign for "$_Ã$ = 0 or, equivalently, P (°) ´ µ°^{2} + µ (2 ¡ s) ° + µ (1 ¡ s) ¡ s · 0: Since, under Assumption 3; µ (1 ¡ s) < s; one has P (0) < 0: It follows that the externality parameter ° must satisfy ° < °¤; where °¤ is the unique positive zero of P (°) = 0: Finally, from (12), one has $\theta(1 - s + \nu^*) < s$; i.e. $\nu^* < [s - \theta(1 - s)]/\theta$; by construction ■

Bifurcation values of ε: The bifurcation values of the (local) elasticity of the workers' offer curve are given by:

$$\varepsilon_T = 1 + \frac{\nu}{1 - s - \sigma\varepsilon_\psi};$$

$$\varepsilon_F = 1 - \frac{(1-\pi)\gamma}{\pi + (1-\pi)\gamma} + \frac{[\pi - (1-\pi)\gamma]\{\theta\nu + 2[s - \sigma(\nu - \varepsilon_\psi)]\}}{[\pi + (1-\pi)\gamma][\theta(1 - s) - \sigma(2 + \theta\varepsilon_\psi)]};$$

$$\varepsilon_H = \left[\frac{s - \sigma(1 + \nu - \varepsilon_\psi)}{\theta(1 - s) - \sigma(1 + \theta\varepsilon_\psi)} + \frac{\pi}{\pi + (1-\pi)\gamma}\right]\left[1 + \frac{\theta\nu + \sigma}{\theta(1 - s) - \sigma(1 + \theta\varepsilon_\psi)}\right]$$

9. References

Basu, S., Ferland, J.G., 1995. Are Apparent Productive Spillovers a Figment of Specification Error? Journal of Monetary Economy 36, 165-188.

Baxter, M., King, R.G., 1991. Productive Externalities and Cyclical Volatility. Federal Reserve Bank of Minneapolis, Discussion Paper 53.

Benhabib, J., Farmer, R.E., 1994. Indeterminacy and Increasing Returns. Journal of Economic Theory 63, 19-41.

Benhabib, J., Farmer, R.E., 1996a. Indeterminacy and Sector-Specific Externalities. Journal of Monetary Economics 37, 421-443.

Benhabib, J., Farmer, R.E., 1996b. The Monetary Transmission Mechanism. C.V. Starr Center of Applied Economics, Working Paper 96-13, New-York University.

Benhabib, J., Farmer, R.E., 1997. Indeterminacy and Sunspots in Macroeconomics. Mimeo, New York University.

Bosi, S., Magris, F., 1997. Endogenous Business Cycles: Capital-Labor Substitution and Liquidity Constraint with Increasing Returns to Scale. CORE Discussion Paper 9769, Louvain-la-Neuve, Belgium.

Caballero, R.J., Lyons, R.K., 1992. External Effects in U.S. Procyclical Productivity. Journal of Monetary Economics 29, 209-225.

Cazzavillan, G., 1996. Public Spending, Endogenous Growth, and Endogenous Fluctuations. Journal of Economic Theory 71, 394-415.

Cazzavillan, G., Lloyd-Braga, T., Pintus, P., 1998. Multiple Steady States and Endogenous Fluctuations with Increasing Returns to Scale in Production. Journal of Economic Theory 80, 60-107.

Farmer, R.E., Guo, J.T., 1994. Real Business Cycles and the Animal Spirit Hypothesis. Journal of Economic Theory 63, 42-73.

Grandmont, J.M., 1988. Nonlinear Difference Equations, Bifurcations and Chaos. CEPREMAP Working Paper, Paris.

Grandmont, J.M., Pintus, P., de Vilder, R., 1998. Capital-Labor Substitution and Competitive Nonlinear Endogenous Business Cycles. Journal of Economic Theory 80, 14-59.

Grandmont, J.M., Younès, Y., 1972. On the Role of Money and the Existence of a Monetary Equilibrium. Review of Economic Studies 39, (3), 355-372.

Guckenheimer, J., Holmes, P., 1983. Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields (Springer Verlag, New York).

King, R.G., Plosser, C.I., Rebelo, S., 1988. Production, Growth and Business Cycles. Journal of Monetary Economics 21, 195-232.

Matsuyama, K., 1991. Increasing Returns, Industrialization, and Indeterminacy of Equilibrium. Quarterly Journal of Economics 106, 617-649.

Romer, P.M., 1986. Increasing Returns and Long-Run Growth. Journal of Political Economy 94, (5), 1002-1037.

Woodford, M., 1986a. Stationary Sunspot Equilibria. The Case of Small Fluctuations around a Deterministic Steady State. Mimeo, University of Chicago and New York University.

Woodford, M., 1986b. Stationary Sunspot Equilibria in a Finance Constrained Economy. Journal of Economic Theory 40, 128-137.

Woodford, M., 1994. Monetary Policy and Price Level Indeterminacy in a Cash-in-Advance Economy. Economic Theory 4, 345-80.

Documents de recherche EPEE

2002

02 - 18	**Influence of Parameter Estimation Uncertainty on the European Central Banker Behavior: An Extension** *Jean-Guillaume SAHUC*

2001

01 - 01	**Optimal Privatisation Design and Financial Markets** *Stefano BOSI, Guillaume GIRMENS & Michel GUILLARD*
01 - 02	**Valeurs extrêmes et series temporelles : application à la finance** *Sanvi AVOUYI-DOVI & Dominique GUEGAN*
01 - 03	**La convergence structurelle européenne : rattrapage technologique et commerce intra-branche** *Anne HANAUT & El Mouhoub MOUHOUD*
01 - 04	**Incitations et transitions sur le marché du travail : une analyse des stratégies d'acceptation et des refus d'emploi** *Thierry LAURENT, Yannick L'HORTY, Patrick MAILLE & Jean-François OUVRARD*
01 - 05	**La nouvelle economie et le paradoxe de la productivité : une comparaison France - Etats-Unis** *Fabrice GILLES & Yannick L'HORTY*
01 - 06	**Time Consistency and Dynamic Democracy** *Toke AIDT & Francesco MAGRIS*
01 - 07	**Macroeconomic Dynamics** *Stefano BOSI*
01 - 08	**Règles de politique monétaire en présence d'incertitude : une synthèse** *Hervé LE BIHAN & Jean-Guillaume SAHUC*
01 - 09	**Indeterminacy and Endogenous Fluctuations with Arbitrarily Small Liquidity Constraint** *Stefano BOSI & Francesco MAGRIS*
01 - 10	**Financial Effects of Privatizing the Production of Investment Goods** *Stefano BOSI & Carine NOURRY*
01 - 11	**On the Woodford Reinterpretation of the Reichlin OLG Model : a Reconsideration** *Guido CAZZAVILLAN & Francesco MAGRIS*
01 - 12	**Mathematics for Economics** *Stefano BOSI*
01 - 13	**Real Business Cycles and the Animal Spirits Hypothesis in a Cash-in-Advance Economy** *Jean-Paul BARINCI & Arnaud CHERON*
01 - 14	**Privatization, International Asset Trade and Financial Markets** *Guillaume GIRMENS*
01 - 15	**Externalités liées dans leur réduction et recyclage** *Carole CHEVALLIER & Jean DE BEIR*
01 - 16	**Attitude towards Information and Non-Expected Utility Preferences : a Characterization by Choice Functions** *Marc-Arthur DIAYE & Jean-Max KOSKIEVIC*
01 - 17	**Fiscalité de l'épargne en Europe : une comparaison multi-produits**

2000

Michel BOUTILLIER, Michel GUILLARD & Auguste MPACKO PRISO

00 - 13 **Long-Run Volatility Dependencies in Intraday Data and Mixture of Normal Distributions**
Aurélie BOUBEL & Sébastien LAURENT

1999

99 - 01 **Liquidity Constraint, Increasing Returns and Endogenous Fluctuations**
Stefano BOSI & Francesco MAGRIS

99 - 02 **Le temps partiel dans la perspective des 35 heures**
Yannick L'HORTY & Bénédicte GALTIER

99 - 03 **Les causes du chômage en France : Une ré-estimation du modèle WS - PS**
Yannick L'HORTY & Christophe RAULT

99 - 04 **Transaction Costs and Fluctuations in Endogenous Growth**
Stefano BOSI

99 - 05 **La monnaie dans les modèles de choix intertemporels : quelques résultats d'équivalences fonctionnelles**
Michel GUILLARD

99 - 06 **Cash-in-Advance, Capital, and Indeterminacy**
Gaetano BLOISE, Stefano BOSI & Francesco MAGRIS

99 - 07 **Sunspots, Money and Capital**
Gaetano BLOISE, Stefano BOSI & Francesco MAGRIS

99 - 08 **Inter-Juridictional Tax Competition in a Federal System of Overlapping Revenue Maximizing Governments**
Laurent FLOCHEL & Thierry MADIES

99 - 09 **Economic Integration and Long-Run Persistence of the GNP Distribution**
Jérôme GLACHANT & Charles VELLUTINI

99 - 10 **Macroéconomie approfondie : croissance endogène**
Jérôme GLACHANT

99 - 11 **Growth, Inflation and Indeterminacy in a Monetary « Selling-Cost » Model**
Stefano BOSI & Michel GUILLARD

99 - 12 **Règles monétaires, « ciblage » des prévisions et (in)stabilité de l'équilibre macroéconomique**
Michel GUILLARD

99 - 13 **Educating Children : a Look at Household Behaviour in Côte d'Ivoire**
Philippe DE VREYER, Sylvie LAMBERT & Thierry MAGNAC

99 - 14 **The Permanent Effects of Labour Market Entry in Times of High Aggregate Unemployment**
Philippe DE VREYER, Richard LAYTE, Azhar HUSSAIN & Maarten WOLBERS

99 - 15 **Allocating and Funding Universal Service Obligations in a Competitive Network Market**
Philippe CHONE, Laurent FLOCHEL & Anne PERROT

99 - 16 **Intégration économique et convergence des revenus dans le modèle néo-classique**

1998