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Intraday Exchange Rate Dynamics and Monetary Policy

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Abstract

In this paper, we investigate whether there is a simple relation between an indicator of monetary policy (here, interest rates) and exchange rate volatility. We use an intraday exchange rate model which relies on interactions between four kinds of agents (in domestic and foreign countries) : central banks, speculators, commercial traders and commercial banks. Central banks intervene exogenously in the model, they announce the level of interest rates at every beginning of the day in the Over-The-Counter market. The results suggest an implicit relation between the interest rates and the conditional variance of the exchange rate.

Keywords : exchange rate, intraday, volatility, central bank interest rate, market structure.

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1. Introduction

Market opinion about monetary policy can be very important to take decisions. Indeed, financial market participants and agents in the economy at large attach importance to monetary authorities' information (such as decisions and comments about monetary policy). On the one hand, the central bank has to lead quantitative actions (interventions in the money market or in the foreign exchange market) and on the other hand it has to convey signals to the market (news related to monetary policy's intentions). Market participants closely monitor the central bank's actions to forecast its monetary policy. For instance, official rate changes reveal new information about short-run policy objectives. Official rate changes typically send a signal of the central bank's monetary policy stance and can be easily interpreted by market participants (Hardy (1997, 1998)). Otherwise, the central bank occasionally intervenes in the foreign exchange market. Interventions may signal information regarding monetary policy whenever (Bossaerts and Hillion (1991), Kaminsky and Lewis (1996) and Peiers (1997)). Indeed, central banks have access to information that cannot readily be made available to market (Humpage (1998)). Furthermore, at each time, agents are confronted with an informational shock (i.e. a news) in the foreign exchange market.

In this paper, we try to introduce monetary policy in an intraday exchange rate model. Hence, we use the model of Chauveau and Topol (1996). Chauveau and Topol (hereafter C.T.) attempt to provide a unifying framework for modeling intraday equilibria and the \end of period" equilibrium. Another aim of their study is to derive, from a theorical model, a rationale for time-series properties exhibited by exchange rates (they exhibit conditional heteroscedasticity). Under some assumptions, their model seems to agree with the stylised facts characterising the foreign exchange market. Here, the model is a modified version of C.T. model. Therefore, we present the hypothesis in order to recall quickly and properly the main C.T. hypothesis and to present and to discuss the hypothesis specific to this paper with justifications of the departures. Chauveau and Topol, in particular, suppose that on the very short-run, the central bank keeps zero the difference between foreign and domestic interest rates. We relax this assumption.

Integrations of quantitative interventions and news in that model are left for further research. Also, they will deserve special developments, because most of the time there are only empirical studies on these topics (Chang and Taylor (1998), Dominguez (1998), Goodhart et al. (1993), Ederington and Lee (1993), DeGennaro and Shrieves (1997)). One of the aims of this study is to find a simple relationship between interest rates and exchange rate volatility. To keep the stylised facts characterizing the money market, interest rates are assumed to stay constant all day. Indeed, the intradaily exchange rate volatility is far beyond the money market volatility. We could compare the intradaily volatility according to the stances of monetary policy and examine the foreign exchange market reactions to interest rate changes. The level of interest rates changes at every beginning of the day. So, we could respond to the following question : does the market agitation depend on monetary policy's stance ?

The paper is organised as follows : in order to understand the price formation process

on the foreign exchange market, we need to study on the one hand the market structure (section 2) and on the other hand the speci¯c behaviour of di®erent agents (section 3). The existence of particular relations between agents will be listed to understand the market's functioning (section 4) and underline the price formation (section 5). Section 6 concludes.

2. The market structure

There is an foreign exchange market similar to a bilateral market (with a domestic and foreign country). We note f the foreign currency and d the domestic currency. The relationship between the two currencies is the following : one unit of f = e units of d: On this market, we assume that the demand and the supply of currency come from speculators, commercial traders and commercial banks. There are four kinds of agents in each country : those mentioned above and central banks. The market, that we study, is a \crossed market" because it consists of two di®erent markets (the auction market and the Over-The-Counter (OTC) market), those can be associated to di®erent time scales (daily and intradaily scales). To give henceforth an idea about the relationship between those two markets, we can say that the OTC market ends with an auction market. This determines the equilibrium value of the daily exchange rate.

2.1. The auction market

2.1.1. De¯nition

An auction market takes place at the end of each day. Only banks intervene and submit their buy orders and sell orders of the whole day. Thus, every bank clears its position at the end of each day. Orders conduct this auction market because buy orders and sell orders meet. The equilibrium price is the price which equals demand and supply, it is the price that prevails in the auction market of D-day. At that price, all buy orders and sell orders are realized. There is not market maker to ensure the liquidity of the market, orders passed by banks ensure it.

2.1.2. The daily horizon

We assume that there is a daily horizon for all agents. It is viewed as the moment when the currencies' supply equals the currencies' demand on the whole day[4]. This time scale is de¯ned as long and means the time interval between two auction markets.

     Assumption We assume this horizon common to all agents.

    The equilibrium value of the exchange rate of the day (e_D) sets at the end of every day (at D). The originality of this model is to cross this auction market with an OTC (Over The Counter) market.

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[4]We note the day (D ¡ 1; D) with (D ¡ 1), the time of the last auction market and (D) the time of the next auction market.

2.2. The OTC market

2.2.1. De¯nition

The over-the-counter (OTC) market is opened almost continuously (quotes and transactions are performed in continuous time). The agents which intervene on this market can convey orders at every time. It is an interbank market. When an order ¯nds a counterpart, it is executed every times. The transactions are bilateral unlike auction market's transactions which are multilateral. On this OTC market, there is a new price for each transaction when on the auction market there is only one price for all transactions. This market maker continuously shows a bid price and an ask price[5]. At these prices, the market maker ensures the liquidity of the market in making the counterpart of all orders passed by others banks.

2.2.2. The intradaily horizon

The day $(D_i 1; D)$ is divided into a constant number of sub-periods. The banks, which intervene on the OTC market, quote sequentially in each sub-period. A bank is obliged to quote otherwise it will ¯nally be put o® the market. This time scale is de¯ned as short and represents the time interval between two quotes.

> Assumption The opening period of the OTC market is divided into a constant number of sub-periods.

For the sake of simplicity, the number of sub-periods is 2 (as below). The ¯gure 2.1 represents the two time scales (daily and intradaily) used in the model.
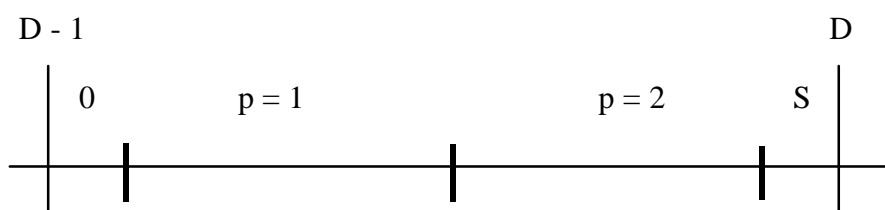
Figure 2.1: The time scales

The period 0 represents the night or the hours between the last auction market at $(D_i 1)$ and the opening period of the OTC market. The period S represents the end of the day (or the auction market).

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[5]We make afterwards a restrictive assumption on these bid and ask prices.

2.3. The agents' intervention dates during the day

In order to understand the agents' role in the foreign exchange market, we present their intervention's dates. The central banks are the ¯rst to intervene, they announce the interest rates value (domestic and foreign interest rates) on the day: They intervene before the opening period of the OTC market and also before the speculators. Speculators take their decisions before the opening period of the OTC market. Consequently, they do not revise their expectations during the day and until the next day. Commercial traders intervene continuously in the OTC market. Commercial banks intervene in the OTC market and in the auction market. At each time t of the day, the banks are either price-maker or price-taker.

> Assumption B banks intervene in the OTC market[6]. The bank quotes just once each sub-period. There are 2B quoting times between two consecutive auction markets (D ¡ 1; D):

We present in the ¯gure 2.2 both the time scales and the agents' intervention dates during the day.
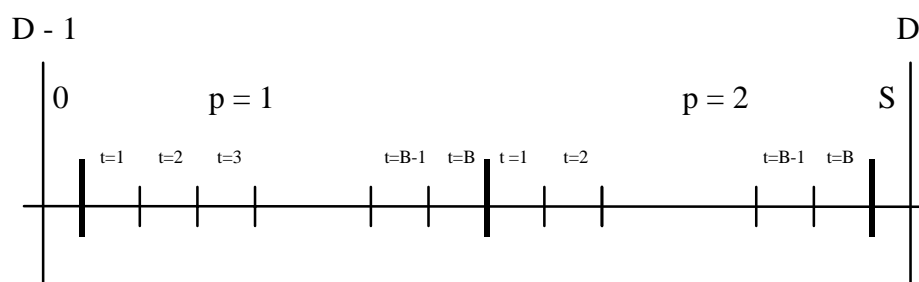
Figure 2.2: The agents' intervention dates

At 0; domestic and foreign central banks announce the daily values of interest rates. When these values are known, speculators take their investment decisions for the day. In $p = 1$, B banks quote. At $t = 1$, the bank 1 quotes whereas the other banks are price takers. At $t = 2$, it is the bank 2 who becomes price-maker, the bank 1 and the $(B ¡ 2)$ others are price-takers. At each time t of the day, the commercial traders pass their orders. At S, only commercial banks intervene in the auction market.

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[6]We could assume that B = 2, for the sake of simplicity, but underlining the exchange rate volatility could be unjusti¯ed.

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3. Agents

3.1. Central banks

The existence of central banks departs from C.T. model. There are two central banks : domestic and foreign central banks. The behaviour of central banks is realistic although elementary. The central banks determine o±cial rate both to de¯ne the range within which they manage short-term interbank rates through open market operations and to signal their short-term policy stance. Here, there is no di®erence between the o±cial rates and the money market rates. Interest rate is viewed as an indicator of monetary policy's stance.

> **Assumption** Central banks (domestic and foreign) intervene exogenously. Before the opening period of the OTC market (at the same time), domestic (foreign) central bank sets the domestic (foreign) interest rate r_D^d (r_D^f) of the D-day. As mentioned in the introduction, central banks may change the level of the interest rate at every beginning of the day. During the day, central banks keep constant their interest rates. Central banks' announcement has not to produce panic reaction in the foreign exchange market (no mimetic behaviour, no contagion...)[7].

> We assume that central banks don't intervene in the foreign exchange market (in terms of volume). Indeed, the purchases and the sales of the central banks on the market are neglected.

3.2. Speculators

3.2.1. The behaviour of speculator

We can characterize the speculator's behaviour by the following assumption :

> **Assumption** Speculators are risk averse[8]. We assume that the speculator k has a CARA[9] utility function U^k :

$$U^k(W^k) = i \ \exp(i \ b^k W^k)$$

> where b^k is the k-speculator's aversion and W^k the speculator's wealth which is denominated in domestic (foreign) currency in the case of a domestic (foreign) speculator.

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[7]This argument will suppose that covariances are zero.
[8]Risk aversion can generate considerable mathematical complexities (Bray (1985)). We can minimise them by using a particular utility function.
[9]CARA = Constant Absolute Risk Aversion

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The speculator k takes his investment decisions once for the whole day, after the fixing of interest rates and before the opening of the OTC market (more precisely, at 0). Let $F^k_{[D-1;0]}$ be the k-speculator's information structure[10] :

$$F^k_{[D-1;0]} = \left\{ \{(e_K; K = 0....D-1)\} \cup \{(r^d_K; r^f_K; K = 0....D)\} \right\}$$

This information structure consists of public information. In particular, there are the exchange rate at the last auction market e_{D-1} and interest rates of D-day, announced by central banks. The speculator does not revise his information structure, so his expectations during the period $(D-1; D)$ are not taken into account[11].

3.2.2. The daily speculative demand of the speculator

In order to set his demand for foreign currency $^fW^{kf}_{D-1}$, the speculator k maximises the expected utility of his wealth at the forthcoming auction market conditional upon its information structure $F^k_{[D-1;0]}$. The domestic[12] speculator's optimising problem reads :

$$\max_{\{^fW^{kf}_{D-1}\}} E\left[U^k(W^{kd}_D) \mid F^k_{[D-1;0]} \right]$$

subject to the budget :

$$W^{kd}_D = W^{kd}_{D-1}R^d_D + {}^fW^{kf}_{D-1}(R^f_D e_D - R^d_D e_{D-1})$$

where :
$$W^{kd}_{D-1} = {}^dW^{kd}_{D-2} + {}^fW^{kf}_{D-2}e_{D-1}$$
$$R^d_D = 1 + r^d_D$$
$$R^f_D = 1 + r^f_D$$

The first order condition of the optimising problem of the domestic speculator gives his demand $^fW^{kf}_{D-1}$ for foreign currency :

$$^fW^{kf}_{D-1} = \frac{g^{kf}_D}{b^k R^f_D (\sigma^k_D)^2}$$

$^fW^{kf}_{D-1}$ is the wealth of speculator k held in foreign currency at the end of D day. g^{kf}_D is the expected gain from holding one unit of foreign currency one day with $g^{kf}_D = E(e_D \mid F^k_{[D-1;0]}) - \frac{R^d_D}{R^f_D}e_{D-1}$; $(\sigma^k_D)^2 = \mathrm{Var}(e_D \mid F^k_{[D-1;0]})$.

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[10] This information structure departs from C.T. model.

[11] We could expect the speculator to continuously revise his expectations and take his decision with respect to his information at time t. Hence, he would behave as a commercial bank (the commercial bank's behaviour is studied later). This assumption can be relaxed by assuming he revises his expectations by using the same method as the banks.

[12] The foreign currency is the risky asset. The foreign speculator's risky asset is the domestic currency. His optimising problem is deduced from the domestic one. We can replace the exchange rate by its inverse and his demand for domestic currency.

Assumption[13] Common knowledge prevails among speculators. Speculators' information structure is given by :

$$F^k_{[D_i \, 1;0]} = F_{[D_i \, 1;0]} = \frac{3}{4} \, \frac{3}{4}(e_D; 0 \cdot \, D^0 \cdot \, D_i \, 1) \, [\, \frac{3}{4}(r^d_K; r^f_K; K = 0 :::: D) \quad 8k$$

If we suppose that the foreign speculator's risk aversion is constant in domestic currency, we obtain the same demand for foreign currency. Hence, the amount of the risky asset (Grossman (1976), (1981)) held by speculator k during one day is proportional to the expected gain g^f_D and inversely proportional to its relative risk aversion and the variance of e_D :

$$^f W^{kf}_{D_i \, 1} = ^f W^{kf}_{D_i \, 1}(g^f_D; b^k; \frac{3}{4}{_D}^2)$$

3.2.3. Speculators' daily aggregated demand for foreign currency

The wealth invested in foreign currency by the whole set of domestic[14] (foreign[15]) speculators is given by the following expression (1) (expression (2)) :

$$(1) \quad ^f W^{df}_{D_i \, 1} = \sum_{k 2 SD_{D_i \, 1}} W^{kf}_{D_i \, 1} = \sum_{k 2 SD_{D_i \, 1}} \frac{g_D}{b^k \frac{3}{4}{_D}^2 R^f_D}$$

$$(2) \quad ^f W^{ff}_{D_i \, 1} = \sum_{k 2 SF_{D_i \, 1}} W^{kf}_{D_i \, 1} = \sum_{k 2 SF_{D_i \, 1}} \frac{g^f_D}{b^k (\frac{3}{4}{^k_D})^2 R^f_D}$$

In order to determine the daily value of the total expected demand for foreign currency ($S\Phi E^{¤f}_D$), we need the expressions of the expected demand for foreign currency by domestic (foreign) speculators, respectively :

$$SP E^{df}_D = ^f W^{df}_{D_i \, 1} \, i \, ^f W^{df}_{D_i \, 2}$$
$$SP E^{ff}_D = ^f W^{ff}_{D_i \, 1} \, i \, ^f W^{ff}_{D_i \, 2}$$

To conclude, the daily value of the total expected demand for foreign currency is :

$$S\Phi E^{¤f}_D = SP E^{df}_D + SP E^{ff}_D = \frac{g^f_D}{b\frac{3}{4}{_D}^2 R^f_D} NS_{D_i \, 1} \, i \, \frac{g^f_{D_i \, 1}}{b\frac{3}{4}{_{D_i \, 1}}^2 R^f_{D_i \, 1}} NS_{D_i \, 2}$$

Assumption The number of speculators is constant, i.e. : $NS_{D_i \, 1} = NS_{D_i \, 2} = NS$

$$S\Phi E^{¤f}_D = ^- \left[\frac{g^f_D}{R^f_D} \, i \, \frac{g^f_{D_i \, 1}}{R^f_{D_i \, 1}} \right]$$

where : $^- = \frac{NS}{b\frac{3}{4}^2}$

The implicit form of $S\Phi E^{¤f}_D$ reads $S\Phi E^{¤f}_D = S\Phi E^{¤f}_D (r)$ where $r \, \acute{} \, (r^d_D; r^f_D; r^d_{D_i \, 1}; r^f_{D_i \, 1})$:

[13]This assumption simpli¯es the expressions of g^{kf}_D and $(\frac{3}{4}{^k_D})^2$.
[14]$SD_{D_i \, 1}$ is the set of domestic speculators during D-day.
[15]$SF_{D_i \, 1}$ is the set of foreign speculators during D-day.

3.3. Commercial traders

3.3.1. The behaviour of the commercial traders

Commercial traders are importers and exporters of goods. The domestic country's importers buy goods from foreign sellers and pay foreign currency for them. The domestic exporters sell domestic goods to foreign clients, who buy with foreign currency. Nevertheless, domestic traders get national currency. Consequently, to settle the domestic goods, domestic importers need foreign currency. It is required for them to trade domestic currency for foreign one. Domestic exporters receive foreign currency, because the foreign clients pay foreign currency for them. So, it is required for us to trade foreign currency for domestic one. Financial counterparts of imports and exports are importers and exporters' orders. These orders reach each bank. They are due to commercial motivations, so we will call them \commercial orders". The exports generate a need of domestic currency for exporters and thus the imports generate a need of foreign currency for importers.

3.3.2. The decomposition of monthly commercial orders in intradaily commercial orders

As the OTC time scale is very small, it is sensible to think of exports and imports at time t being exogenous (they are beyond the forex).

a - Monthly commercial orders :

We assume that commercial orders are noisy, they verify the following relation :

$$\tilde{M}_M^f = M_M^f + \forall_M^m$$
$$\tilde{X}_M^d = X_M^d + \forall_M^x$$

We de⁻ne M_M^f as the deterministic component of monthly imports. It corresponds to the planned transactions of goods by the domestic country, denominated in foreign currency (because it needs an amount of foreign currency M_M^f to perform them). X_M^d is the deterministic component of monthly exports, it corresponds to the planned transactions of goods by the domestic country, denominated in domestic currency. $\forall_M^m$ ($\forall_M^x$) is the monthly noise of imports (exports). These noises cause unforeseen transactions with the bank.

b - Daily commercial orders :

We assume that daily commercial orders are a proportion of monthly orders. The commercial orders on D-day are given by the following equations :

$$M_D^f = {}_5^m_D M_M^f + \forall_D^m$$
$$X_D^d = {}_5^x_D X_M^d + \forall_D^x$$

We de¯ne M_D^f as the daily commercial orders of imports, X_D^d as the daily commercial orders of exports. α_D^m (α_D^x) is the daily proportion of the deterministic component of the monthly imports (exports). v_D^m (v_D^x) is the daily noise of the imports (exports). We suppose that the commercial orders received by the bank i are noisy, they depend on the daily noise (v_D^m and v_D^x): Assuming 30 days per month, the relationship between the noises reads :

$$\sum_{D=1}^{30} v_D^m = v_M^m \qquad \sum_{D=1}^{30} v_D^x = v_M^x$$

Assumption We suppose that exports and imports are uniformly distributed on the whole month, i.e. $\alpha_D = \alpha$:

c - Intradaily commercial orders :

The amount of commercial orders reaching the bank i at time t on $(D_i\ 1; D)$ day reads :

$$M_t^{if} = \alpha_t^{im}\,\alpha_D^m M_M^f + v_t^{im}$$
$$X_t^{id} = \alpha_t^{ix}\,\alpha_D^x X_M^d + v_t^{ix}$$

M_t^{if} (X_t^{id}) is the amount of the import (export) orders reaching the bank i at time t: α_t^{im} (α_t^{ix}) is the intradaily proportion of the deterministic component of the daily imports (exports). v_t^{im} (v_t^{ix}) is the stochastic component of the imports (exports) reaching the bank i time t: These noises are a piece of the bank i's private information. They are assumed to be zero-mean, independent and homoskedastic :

$$E(v_t^{im}) = E(v_t^{ix}) = 0 \qquad E(v_D^m) = E(v_D^x) = 0$$
$$E(v_t^{im}v_u^{jm}) = \delta_{tu}\delta_{ij}(\sigma_v^m)^2 \quad E(v_t^{ix}v_u^{jx}) = \delta_{tu}\delta_{ij}(\sigma_v^x)^2$$
$$E(v_t^{im}v_t^{jx}) = 0$$

where δ_{ij} is the Kronecker delta and $(\sigma_v^m)^2$ $((\sigma_v^x)^2)$ is the variance of the noise of the imports (of the exports). The α, v and the noises verify the following constraints :

$$\sum_{i=1}^{B}\sum_{t=1}^{T} v_t^{im} = v_D^m \qquad \sum_{i=1}^{B}\sum_{t=1}^{T} v_t^{ix} = v_D^x$$
$$\sum_{i=1}^{B}\sum_{t=1}^{T} \alpha_t^{im} = 1 \qquad \sum_{i=1}^{B}\sum_{t=1}^{T} \alpha_t^{ix} = 1$$
$$v_D^m \ll \alpha_D^m v_M^m \qquad v_D^d \ll \alpha_D^f v_M^x$$
$$v_t^{im} \ll \alpha_t^{im} v_D^m \qquad v_t^{id} \ll \alpha_t^{if} v_D^x$$

3.4. Commercial banks

3.4.1. The bank's features

Banks are labelled from i = 1 to B, where B is the total number of banks. BD (BF) is the total number of domestic (foreign) banks, that is to say B = BD + BF

Assumption Each bank is risk neutral, acts myopically and not strategically. None of the banks manipulates the market.

3.4.2. Relations between the bank i and its clients

All orders (commercial and speculative) are centralized by banks. The amount of commercial orders, received by the bank i during the opening of the OTC market, fluctuates (they are stochastic because they depend on noises v_t^{im} and v_t^{ix}): Speculative orders sent to the bank i are given by the following equation :

$$SPE_t^{if} = \beta_t^{if} SPE_D^{if}(r)$$

where $\sum_{i=1}^{P} \sum_{t=1}^{P} \beta_t^{if} = 1, \ 0 \cdot \beta_t^{if} \cdot 1$ and $r \ \acute{} \ (r_D^d; r_D^f; r_{D_i 1}^d; r_{D_i 1}^f)$: SPE_D^{if} is uniformly divided among banks.

> **Assumption** Transactions costs are negligible.

The total of orders to be executed at time t by bank i is defined as the number of units of foreign currency against the domestic one :

$$TO_t^{if} = \left(\frac{X_t^{id}}{e_t} \ _i \ M_t^{if} \ + \ SPE_t^{if}(r) \right)$$

where e_t, is the exchange rate[16] at time t: Then, the bank i intervenes in the OTC market, to trade foreign for domestic currency (and vice-versa) for its clients.

3.4.3. The bank i's program

According to the time t, the bank i is either price taker or price maker. When the bank i is price maker, on the one hand it has to determine the value of exchange rate prevailing at time t and the other hand it has to make counterpart of all orders passed by others banks. When the bank i is price taker, it has to choose the amount of currency to trade with the price maker. Within the intraday period, bank i maximises the expected value of its wealth (s_D^{id})[17] at the forthcoming auction market D, conditional upon its information structure[18] F_t^i: The bank i's optimizing problem[19] reads :

$$\underset{\substack{d_{t+k2_i}^{if} \\ fe_{t+k2_i}g}}{n \ Max \ o} \ E\left[\text{s}_D^{id} j F_t^i \right]$$

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[16] There is only one price (bid price = ask price)

[17] The wealth s_D^{id} is valued in domestic currency.

[18] We will give more details about this information structure below.

[19] The optimising problem of a foreign bank is similar to domestic one although conditional expectations are relative to the inverse of exchange rate $\frac{1}{e_t} = E(\frac{1}{e_D} j F_t^i)$. The optimal behaviour of a foreign bank is identical to the one of domestic bank with a linearisation of its optimal program.

$$\text{subject to :}$$

$$\hat{ps}_D^{id} = ps_{t-1}^{id} + ps_{t-1}^{if}e_D + s_t^{if}(e_D - e_t) + \sum_{\substack{k=1 \\ (t+k\geq\iota^i)}}^{2\mathbb{P}-t} s_{t+k}^{if}(e_D - e_{t+k})$$

$$+ \sum_{\substack{k=1 \\ (t+k\geq\iota^i)}}^{2\mathbb{P}-t} s_{t+k}^{if}(e_D - e_{t+k})$$

with :

$ps_t^{if} = ps_{t-1}^{if} + s_t^{if}$ and $ps_0^{if} = 0$

$s_t^{if} = Q_t^{if} + TOI_t^{if}$ if the bank is quoting

$s_t^{if} = d_t^{if} + TOI_t^{if}$ if the bank is not quoting

$ps_t^{id} = ps_{t-1}^{id} - (Q_t^{if} + TOI_t^{if})e_t$ if the bank is quoting

$ps_t^{id} = ps_{t-1}^{id} - (d_t^{if} + TOI_t^{if})e_t$ if the bank is not quoting

$ps_0^{id} = W_0^{id}$

Bank i determines its quote e_{t+k} ($k \geq 0$) when it is quoting (ι^i is the bank i's quoting time). Bank i determines its demand for foreign currency d_{t+k}^{if} ($k \geq 0$) when it is not quoting. $\hat{ps}_D^{id}$ is the expected position of bank i at time D; denominated in domestic currency: ps_{t-1}^{id} (ps_{t-1}^{if}) is the position of bank i at time $t-1$, denominated in domestic (foreign) currency. s_t^{id} (s_t^{if}) is the change in the bank i's position at time t, denominated in domestic (foreign) currency. e_t is the OTC exchange rate at time t: e_{t+k} is the exchange rate at the forthcoming intraday times. TOI_t^{if} is the total of orders of bank i to be executed at time t. d_t^{if} is the bank i's demand for foreign currency at the time t: Q_t^{if} is the transactions the bank i has to accept when it is quoting, i.e. the sum of the other banks' demands. When the bank i is price maker under the assumption of an infinite elasticity of demand Q_t^{if}; its optimal quote is :

$$e_t = E\left[e_D \mid F_t^i\right]$$

When the bank i is price taker, its optimal demand for the foreign currency is :

$$d_t^{if} = sign\left[E(e_D \mid F_t^i) - e_t\right] q^{f\alpha}$$

where $q^{f\alpha}$ is a transaction bound in foreign currency between a price taker bank and the price maker bank.

4. The functioning of the market

4.1. Interbank relations in the OTC market

a- The determination of the current exchange rate e_t :

The price maker i uses its expectations $E(e_D \mid F_t^i)$ to determine the quote e_t. The value of this quote is the OTC market exchange rate prevailing at the same time :

$$e_t = E(e_D \mid F_t^i)$$

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b- Information conveyed by the quote e_t :

The price maker's quote is published. This quote is totally informative, it conveys some information (public and private) from one bank to another.

Assumption The price maker's quote does not contain any information relative to volumes[20] of its correspondents' orders (more precisely, purchase or sale).

c- Foreign currency demand of the price taker :

Each price taker uses its conditional expectation $E(e_D \mid F_t^j)$ to set its foreign currency demand. It depends on the di®erence between the expectation and the current exchange rate :

$$d_t^j = sign(E(e_D \mid F_t^j) - e_t)q^{\pi f}$$

If $E(e_D \mid F_t^j) = e_t$, the solution is indetermined, the demand for currency is set to zero, $d_t^{jf} = 0$:

If $E(e_D \mid F_t^j) > e_t$, the bank j anticipates an appreciation of the foreign currency, its demand reads $d_t^{jf} = q^{\pi f}$ (foreign currency purchase).

If $E(e_D \mid F_t^j) < e_t$, the bank j anticipates a depreciation of the foreign currency, its demand reads : $d_t^j = -q^{\pi f}$ (foreign currency sale).

d - Orders' execution received by the price maker :

The price maker i has to accept the whole transactions of price takers. The sum of these transactions is Q_t^{if}.

Assumption Each bank has exactly $(B - 1)$ correspondents.

The ¯gure

4.2. The interbank equilibrium at the auction market

At the auction market, commercial traders and speculators can no more pass any order. The auction market is an interbank market. The equilibrium price at the auction market is the price at which equals demand and supply :

Assumption[21] The banks' speculation is intradaily. Indeed, the bank i's position sets zero both at the beginning and at the end of the day, i.e. :

$$ps_0^{if} = 0$$
$$ps_D^{if} = 0$$

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[20] It is realistic, because volumes are not generally public information in the foreign exchange market.
[21] This assumption departs from C.T. model.

If the bank gets speculative gains at the end of the day, it has to convert foreign currency into domestic currency.

Every bank clears its position at the end of the day, its position at $t = 2B$ is ps_{2B}^{if}. We note FIX_D^{if} the foreign currency demand which clears the bank i's position, at D :

$$ps_{2B}^{if} + FIX_D^{if} = 0$$

If we take into account the whole of banks, we note :

$$\sum_{i=1}^{B} FIX_D^{if} + \sum_{i=1}^{B} ps_{2B}^{if} = 0$$

After some algebra calculations, we obtain :

$$\sum_{i=1}^{B} FIX_D^{if} = \sum_{i=1}^{B} TOI_D^{if}$$

The interbank equilibrium at the auction market reads :

$$\sum_{i=1}^{B} FIX_D^{if} = (X_D^f - M_D^f) + SPE_D^{\pi f}$$

As central banks do not intervene in the foreign exchange market, the equilibrium at the auction market reads :

$$\sum_{i=1}^{B} FIX_D^{if} = (\hat{X}_D^f - \hat{M}_D^f) + S\hat{P}E_D^{\pi f} = 0$$

By using the expression of the daily trade balance[22] $(\hat{X}_D^f - \hat{M}_D^f)$, we obtain a new equilibrium's expression :

$$S\hat{P}E_D^{\pi f} = u_D - \pm_D e_D$$

where :
$$u_D^{\pi} = \jmath_D \left(\frac{2X_M^d}{e_{M-1}} - M_M^f + \frac{\forall_D^x}{e_{M-1}} - \forall_D^m \right)$$
$$\pm_D = \jmath_D \frac{X_M^d}{e_{M-1}^2}$$

4.3. Determination of the bank's expectation

At each time t, the bank i gets public and private information. Only the private information changes during the period of the OTC market, because it depends on noises. Within the intraday period, each bank i, either as a price taker or as a price maker, has to determine its expectation of the exchange rate e_D at the next auction market conditionally to its information structure. We note this conditional expectation $E(e_D \mid F_t^i)$.

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[22]The daily trade balance is determined in appendix 1.

4.3.1. Public information

We split the public information into three parts. This public information decomposition departs from C.T. model. To determine the bank's expectation, only the first two parts are taken into account.

a - Public information due to past days :

It consists of the past values of the exchange rate at auction markets $(e_K; K = 1::::D_i 1)$ and the past values of domestic and foreign interest rates $(r_K^d; r_K^f; K = 1:::D_i 1)$: We note this information structure :

$$F_{D_i 1}^{Pub} = \frac{3}{4} \frac{3}{4}(e_K; K = 1::::D_i 1) [\frac{3}{4}(r_K^d; r_K^f; K = 1:::D_i 1)´$$

b - Public information due to current day :

(1) Public information before the opening period of the OTC market : this information consists of domestic and foreign interest rates announced by the central banks $(r_D^d; r_D^f)$:

$$F_{0;D}^{Pub} = \frac{3}{4}(r_D^d; r_D^f)$$

(2) Public information due to OTC market : this information consists of the quotes within the current OTC period $(e_t; t = 1:::2B)$: This set starts with the opening period of the OTC market. At $t = 1$, this information is not available. The information structure reads :

$$F_{t;D}^{Pub} = \frac{3}{4}(e_t; t = 1:::2B)$$

c- Public information due to news :

The economic news[23] of foreign and domestic countries are not taken into account.

The general public information structure is given by the following expression :

$$F_{D_i 1;D}^{Pub} = \frac{3}{4} F_{D_i 1}^{Pub} [F_{0;D}^{Pub} [F_{t;D}^{Pub}´$$

4.3.2. Private information

Banks get private information from their clients' orders. Private information is viewed as generated by a sequence of implicit exchange rates (s_t^i), i.e. the ones that would clear the cumulated orders reaching bank i :

$$s_t^i = \frac{\sum_{i=1}^{P} X_i^{id}}{\sum_{i=1}^{P} M_i^{if} + \sum_{i=1}^{P} S P E_i^{if}}$$

We suppose here, that each bank observes its orders $(X_i^{id}; M_i^{if}, \text{ and } E_i^{if})$ but not the corresponding noises $(v_t^{im}; v_t^{ix})$: The case when each bank observes the noises relative to its clients' orders is studied in Chauveau and Topol's paper (1996). The private information, which has been conveyed to the market by the quote, belongs to the public information. So, we define the strict private information as the private information which has been received since the last quoting time of the bank. Consequently, the private information splits into two components : the private information conveyed to the market and the private information received since the last quote of the bank.

The private information s_t^i may be split into two components[24]. The first component is the private information which has been conveyed to the market by the last quote of the bank i. This private information conveyed to the market belongs to the public information. The second component of the private information is the private information which has been received since the last quote of the bank. The set of private information generates the following information structure :

$$F_t^{priv;i} = \sigma(s_t^i; t = 1...2B) \qquad \forall i = 1 \text{ to } B$$

The private information received since the last quote of the bank can consist of a component which is correlated with variables that generate the public information ($E(s_t^i \mid F_{t-1})$) and a non correlated component (ν_t^i, the strict private information) :

$$s_t^i = E(s_t^i \mid F_{t-1}) + \nu_t^i$$

4.3.3. The banks' estimations

Undertaking the exact calculation of the conditional expectation $E(e_D \mid F_t^i)$ is a somewhat complicated task. To overcome this difficulty, banks are supposed to look for a good predictor of the exchange rate at the next auction market. If expectations are rational, this predictor can be thought as a linear combinaison of the observed variables[25].

a- Expectation based on public information :

This expectation is not clarified in the C.T. model. At t = 1; all banks determine the expectation of the exchange rate at the next auction market conditionally to their information structure. This information structure reads :

$$F_{D-1;0}^{Pub} = \sigma\left(F_{D-1}^{Pub} \cup F_{0;D}^{Pub}\right)$$

$$F_{D-1;0}^{Pub} = \sigma\left(\sigma(e_K; K = 1....D-1) \cup \sigma(r_K^d; r_K^f; K = 1...D-1) \cup \sigma(r_D^d; r_D^f)\right)$$

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[24] This private information decomposition departs from C.T. model.

[25] These observed variables are related to private and public information.

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This information structure consists of public information available before the opening period of the OTC market: Every bank has to choose an estimator of $E(e_D \mid F_{D-1;0}^{Pub})$. We will assume that banks use Covered Interest Parity (CIP) to determine a good predictor of exchange rate e_D. It reflects the expectation of the next auction market's exchange rate, i.e.:

$$\hat{e}_D^{Pub} = E(e_D \mid F_{D-1;0}^{Pub}) = \left(e_{D-1} \frac{R_D^d}{R_D^f} \right)$$

We will assume that this expectation does not change during the day. There is no risk premium in this model, because banks are risk neutral ; we leave the assumption of risk-averse banks and risk premia for further work.

b- Strict private information's estimation :

In order to determine $E(e_D \mid F_t^i)$, the bank i uses its strict private information (definition mentioned above). The strict private information reads :

$$\xi_t^i = s_t^i - E(s_t^i \mid F_{t-1})$$

where F_{t-1} is the public information at $t-1$: ξ_t^i and s_t^i depend on domestic and foreign interest rates. Every bank has to choose an estimator of $E(s_t^i \mid F_{t-1})$: For the sake of simplicity, the expectation $E(s_t^i \mid F_{t-1})$ is determined by the orthogonal projection of s_t^i onto $L^2(F_{t-1})$:

$$E(s_t^i \mid F_{t-1}) = \sum_{k=0}^{t-1} \hat{a}_{t;k}^i (e_k - e_{k-1})$$

with $e_0 = \hat{e}_D^{Pub}$ and $e_{-1} = 0$ and where estimators of $\hat{a}_{t;k}^i$ are $\hat{a}_{t;k}^i = \frac{E\left[s_t^i (e_k - e_{k-1}) \right]}{Var(e_k - e_{k-1})}$

Bank i can value the strict private information from :

$$\xi_t^i(r) = s_t^i(r) - \sum_{k=0}^{t-1} \hat{a}_{t;k}^i(r)(e_k - e_{k-1})$$

where $r \equiv (r_D^d; r_D^f; r_{D-1}^d; r_{D-1}^f)$

By using a geometrical approach, the bank's expectation reads :

$$E(e_D \mid F_t^i) = e_{t-1} + \circledR_t^i \xi_t^i$$

where :
$$\circledR_t^i = \frac{E\left[e_D \xi_t^i \right]}{Var(\xi_t^i)}$$
$$\xi_t^i = s_t^i - \sum_{k=0}^{t-1} \hat{a}_{t;k}^i (e_k - e_{k-1})$$

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5. The intradaily exchange rate dynamics and its statistical properties

The intradaily exchange rate dynamics is given by the following expressions :

$$e_t = e_{t_{i-1}} + \circledR_t^i(r) \hbar_t^i(r)$$

$$e_t = \hat{e}_D^{Pub} + \sum_{t=1}^{\mathsf{X}} \circledR_t^i(r) \hbar_t^i(r)$$

$$e_t = \left(e_{D_{i-1}} \frac{R_D^d}{R_D^f} \right)^{\tilde{A}} + \sum_{t=1}^{\mathsf{X}} \circledR_t^i(r) \hbar_t^i(r)$$

where $r \; \acute{} \; (r_D^d; r_D^f; r_{D_{i-1}}^d; r_{D_{i-1}}^f)$

The level of exchange rate depends on interest rates, strict private information of quoting banks and proportionality coe±cient. The change in the OTC exchange rate reads :

$$\mathbb{C}e_t = \circledR_t^i(r) \hbar_t^i(r)$$

where $r \; \acute{} \; (r_D^d; r_D^f; r_{D_{i-1}}^d; r_{D_{i-1}}^f)$

This change is proportional to the strict private information of the quoting bank. The proportionality coe±cient and the strict private information's estimation depend on r_D^d; $r_{D_{i-1}}^d$; r_D^f and $r_{D_{i-1}}^f$: The change in the OTC exchange rate is an implicit function of domestic and foreign interest rates. Conditional expectation and conditional variance of the change in the OTC exchange rate read :

$$E(\mathbb{C}e_t \; j \; F_{t_{i-1}}) = 0$$
$$Var(\mathbb{C}e_t \; j \; F_{t_{i-1}}) = h_t = (\circledR_t^i(r))^2 \, Var(\hbar_t^i(r) \; j \; F_{t_{i-1}})$$

We can see that the conditional variance depends on interest rates. Nevertheless, it is di±cult to estimate it because it is an implicit form, unless to express conditional variance with a linear form. Simulations could be undertaken to show that volatility models (ARCH, stochastic model.....) can ¯t the simulated data quite well.

6. Conclusion

It is the task of future research to study the foreign exchange market responses both to central bank interventions and to monetary policies' news in terms of volatility. Indeed, the central bank interventions can in°uence exchange rates over the horizons of intraday.

Central bank interventions can take several forms ranging from direct market intervention, such as open market operations to outright depreciation and appreciation. We will suppose that the central bank intervenes during the day only if the rate of depreciation of the exchange rate would exceed a value if no intervention takes place. Whatever its form, central bank intervention will introduce skewness in the distribution of exchange rate as perceived by traders. Another approach is to assess the impact of a subset of news related to the monetary policy instruments of central banks on intraday exchange rate dynamics.

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Appendix 1 : The daily commercial trade

Note : $f(e_D) = X_D^f \, \textrm{i} \, M_D^f$

where
$$\hat{X}_D^d = {}_{,D}X_M^d + \forall_D^x; \quad \hat{X}_D^f = \frac{\cancel{X}_D^d}{e_D} + {}_{,D}\frac{X_M^d}{e_D} + \forall_D^x$$
$$\hat{M}_D^f = {}_{,D}M_M^f + \forall_D^m$$

$$f(e_D) = X_D^f \, \textrm{i} \, M_D^f = {}_{,D}\frac{X_M^d}{e_D} + \frac{\forall_D^x}{e_D} \, \textrm{i} \, {}_{,D}M_M^f \, \textrm{i} \, \forall_D^m$$

$$f(e_{M\,\textrm{i}\,1}) = {}_{,D}\frac{X_M^d}{e_{M\,\textrm{i}\,1}} + \frac{\forall_D^x}{e_{M\,\textrm{i}\,1}} \, \textrm{i} \, {}_{,D}M_M^f \, \textrm{i} \, \forall_D^m$$

$$f(e_D) = f(e_{M\,\textrm{i}\,1}) + f^0(e_{M\,\textrm{i}\,1})(e_D \, \textrm{i} \, e_{M\,\textrm{i}\,1})$$

We obtain :

$$\hat{X}_D^f \, \textrm{i} \, \hat{M}_D^f = {}_{,D}\frac{2X_M^d}{e_{M\,\textrm{i}\,1}} + \frac{\forall_D^x}{e_{M\,\textrm{i}\,1}} \, \textrm{i} \, {}_{,D}M_M^f \, \textrm{i} \, \forall_D^m \, \textrm{i} \, {}_{,D}\frac{X_M^d}{e_{M\,\textrm{i}\,1}^2}e_D + \forall_D^x \left[\frac{e_{M\,\textrm{i}\,1} \, \textrm{i} \, e_D}{e_{M\,\textrm{i}\,1}^2} \right]^{\#}$$

The daily trade balance $(X_D^f \, \textrm{i} \, M_D^f)$ is a linear function of the exchange rate :

$$\hat{X}_D^f \, \textrm{i} \, \hat{M}_D^f \, \frac{1}{4} \, \mathtt{u}_D \, \textrm{i} \, \pm_D e_D$$

where $\mathtt{u}_D$ and $\pm_D$ are respectively :
$$\mathtt{u}_D = u_D^\mathtt{¤} + \forall_D$$
$$u_D^\mathtt{¤} = {}_{,D}\frac{2X_M^d}{e_{M\,\textrm{i}\,1}} \, \textrm{i} \, {}_{,D}M_M^f$$
$$\forall_D = \frac{\forall_D^x}{e_{M\,\textrm{i}\,1}} \, \textrm{i} \, \forall_D^m$$
$$\pm_D = {}_{,D}\frac{X_M^d}{e_{M\,\textrm{i}\,1}^2}$$

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